

University of Wisconsin-Madison
Engineering Physics Department
Fall 2011 Qualifying Exams

Modern Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE
WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1. A beam of 0.68 pm photons undergoes Compton scattering.

- (a) (5pts) Derive the formula for Compton scattering by conserving total energy and momentum.
 - (b) (2pts) What is the energy of the photons that emerge at a 45° angle with respect to the incident beam?
 - (c) (3pts) At which angle relative to the incoming direction will the outgoing electron be scattered from the Compton scattering event in part b?
- (Note: $hc = 1239.85 \text{ MeV fm}$).

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Problem 2. (a) (5pts) From $F = \frac{dp}{dt}$, show that $\frac{dE}{dt} = v F$ for a relativistic particle where E is the total energy, p is the relativistic momentum, F is the force, and v is the particles velocity.
(b) (5pts) Show that the relativistic expression for kinetic energy reduces to the Newtonian expression for kinetic energy when a particle's velocity is small.

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Problem 3. $^{139}_{55}\text{Cs}$ is a beta emitter produced from fission and is far from the “valley” of stability. Use the Semi-Empirical Mass Formula below to derive an expression that can be used to predict the stable nuclide in a sequence of beta decays. Which of the following nuclides is predicted to be the stable nuclide: $^{139}_{58}\text{Ce}$, $^{139}_{57}\text{La}$, $^{139}_{56}\text{Ba}$, $^{139}_{54}\text{Xe}$, or $^{139}_{53}\text{I}$?

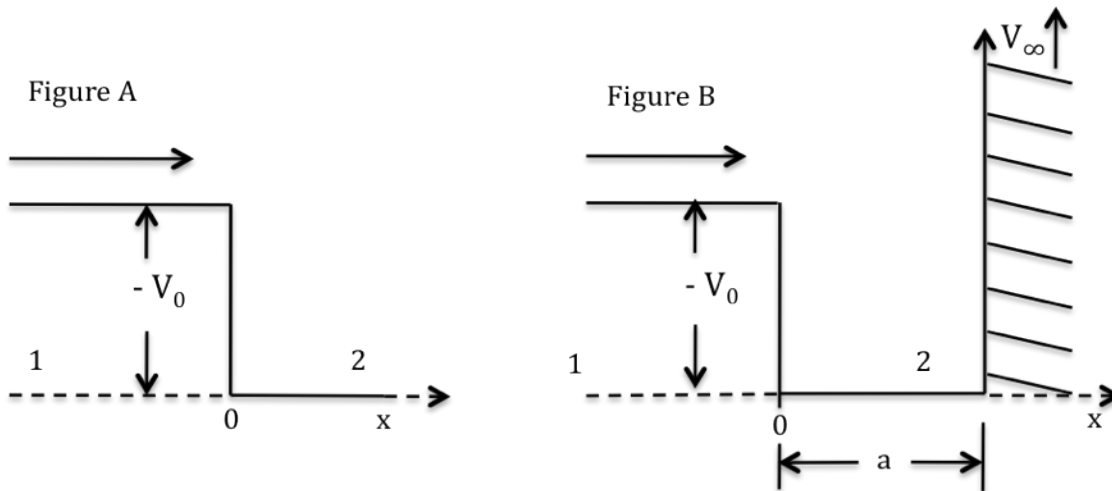
$$M(Z,A)c^2 = A M_n c^2 + Z(M_H - M_n)c^2 - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_{\text{sym}} \frac{(A-2Z)^2}{A} - \delta$$

where $\delta = \begin{cases} +a_p A^{-3/4} & \text{for even/even} \\ 0 & \text{for even/odd} \\ -a_p A^{-3/4} & \text{for odd/odd} \end{cases}$

and A (mass number), Z (charge), M_n (neutron mass = 1.008665 amu), M_H (hydrogen mass = 1.007825 amu), $a_v = 15.5$ MeV, $a_s = 16.8$ MeV, $a_c = 0.72$ MeV, $a_{\text{sym}} = 23$ MeV, and $a_p = 34$ MeV.

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Problem 4. A beam of particles of mass m emitted at negative infinity encounters a step potential drop as shown in figure A below.



- (a) (2pts) What are the wavefunctions in regions 1 and 2 in figure A?
- (b) (4pts) What are the reflection and transmission coefficients at the origin in figure A?
- (c) (2pts) An infinite potential wall is placed a distance a from the origin in region 2 of figure A with the resulting configuration depicted in figure B. What are the wavefunctions in regions 1 and 2?
- (d) (2pts) What is the reflection coefficient at the origin in figure B?

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Problem 5. (a) (3pts) Assuming the electron in a hydrogen atom has an intrinsic spin \vec{S} , use a simple Bohr model argument to show that the shift in the energy of a bound state of the atom due to the Spin-Orbit effect can be represented by $\Delta E \propto \vec{L} \cdot \vec{S}$ where \vec{L} is the orbital angular momentum of the electron in the atom.

(b) (5pts) Draw a simple energy diagram of the first 3 bound states of hydrogen ($n=1, 2, 3$), indicating the relative shifts in states due to the spin-orbital perturbation.

(c) (2pts) The fine structure splitting of $^2P_{3/2}$ and $^2P_{1/2}$ states in hydrogen is 4.5×10^{-5} eV. Taking this shift to be due to the interaction of the spin magnetic moment with the field arising from the orbital motion, estimate the magnetic field \vec{B} that the 2p electron experiences.

(Note: $\mu_B = 5.8 \times 10^{-5}$ eV/T).

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Problem 6. The reaction $^{13}\text{C}(\text{d},\text{p})^{14}\text{C}$ ($Q = 5.95$ MeV) has a resonance at a deuteron energy (lab frame) of 2.45 MeV. From this, determine at which alpha energy (lab frame) the reaction $^{11}\text{B}(\alpha,\text{n})^{14}\text{N}$ ($Q = 0.15$ MeV) may be expected to have a resonance. (Hint: consider the compound nucleus formed in each case.)

Mass [amu] (where 1 amu = 931.502 MeV/c ²)	Mass [amu]
$M(^1_1\text{H}) = 1.007825$	$M(^1_0\text{n}) = 1.008665$
$M(^2_1\text{H}) = 2.014102$	$M(^4_2\text{He}) = 4.002603$
$M(^{13}_6\text{C}) = 13.003355$	$M(^{14}_6\text{C}) = 14.003242$
$M(^{11}_5\text{B}) = 11.009305$	$M(^{14}_7\text{N}) = 14.003074$
$M(^{15}_7\text{N}) = 15.000109$	---