Student No.__________________

University of Wisconsin-Madison
Engineering Physics Department
Spring 2008 Qualifying Exams

Classical Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, <u>NOT</u> ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

Useful constants and formulae:

$\mu_0 = 4\pi \times 10^{-7}$ H/m $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

c $= 3 \times 10^8$ m/s $g = 9.81$ m/s^2

$I_{cyl} = \frac{1}{2} MR^2$ $I_{sphere} = \frac{2}{5} MR^2$

Table of Integrals:

$$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2x}$$

$$\int \frac{dx}{x^3\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{2a^2x^2} + \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int x\sqrt{x^2+a^2}\,dx = \frac{(x^2+a^2)^{3/2}}{3}$$

$$\int x^2\sqrt{x^2+a^2}\,dx = \frac{x(x^2+a^2)^{3/2}}{4} - \frac{a^2x\sqrt{x^2+a^2}}{8} - \frac{a^4}{8}\ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int x^3\sqrt{x^2+a^2}\,dx = \frac{(x^2+a^2)^{5/2}}{5} - \frac{a^2(x^2+a^2)^{3/2}}{3}$$

$$\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} - a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \frac{\sqrt{x^2+a^2}}{x^2}\,dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int \frac{\sqrt{x^2+a^2}}{x^3}\,dx = -\frac{\sqrt{x^2+a^2}}{2x^2} - \frac{1}{2a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

$$\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$$

$$\int \frac{x^2\,dx}{(x^2+a^2)^{3/2}} = \frac{-x}{\sqrt{x^2+a^2}} + \ln\left(x+\sqrt{x^2+a^2}\right)$$

$$\int \frac{x^3\,dx}{(x^2+a^2)^{3/2}} = \sqrt{x^2+a^2} + \frac{a^2}{\sqrt{x^2+a^2}}$$

$$\int \frac{dx}{x(x^2+a^2)^{3/2}} = \frac{1}{a^2\sqrt{x^2+a^2}} - \frac{1}{a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \frac{dx}{x^2(x^2+a^2)^{3/2}} = -\frac{\sqrt{x^2+a^2}}{a^4x} - \frac{x}{a^4\sqrt{x^2+a^2}}$$

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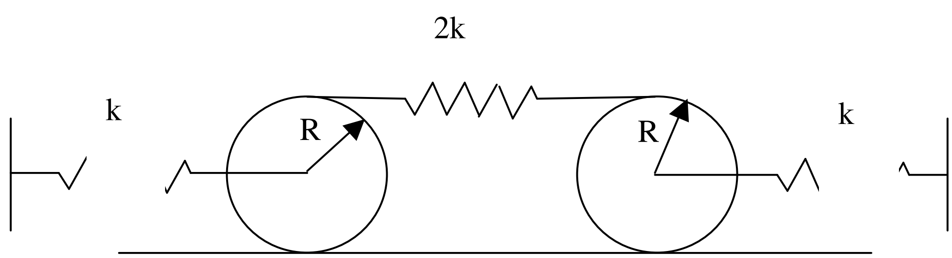
Problem 1. Consider the mechanism shown below. The springs are massless and the two wheels are each of mass *m*.

a) (4 points) Derive the equations of motion, assuming that the wheels roll without slipping.
b) (4 points) What are the natural frequencies for this mechanism?
c) (2 points) Qualitatively describe the motion associated with each natural frequency.



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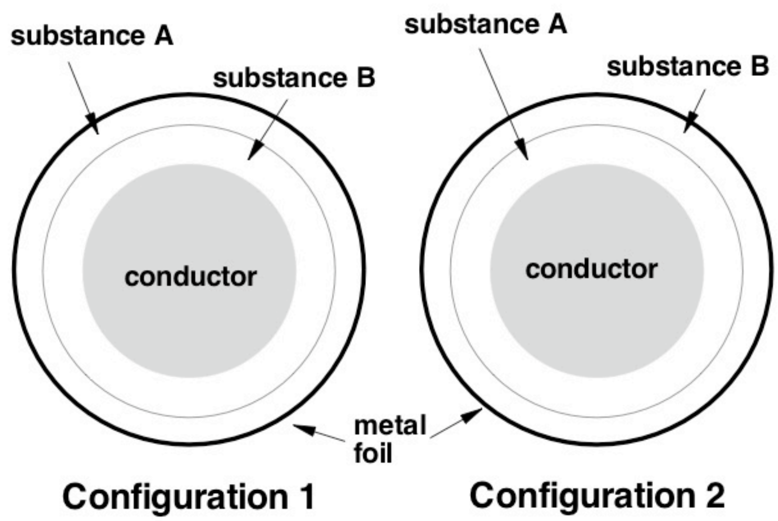
Problem 2. A capacitor is constructed from a spherical conductor of radius 0.04 m, two different dielectric substances that are layered over the spherical conductor, a metal foil that is wrapped over the outer surface of the dielectric materials, and a wire lead (that we shall ignore). One of the two dielectric substances (substance A) has a permittivity of 2×10^{-11} F/m, the other (substance B) has a permittivity of 4×10^{-11} F/m, and 1×10^{-4} m^3 of each substance are used.

a. (5 points) Which of the two configurations shown in the sketch has the SMALLER capacitance? **Explain your reasoning**.



b. (5 points) Compute the value of capacitance for the solution from part a.

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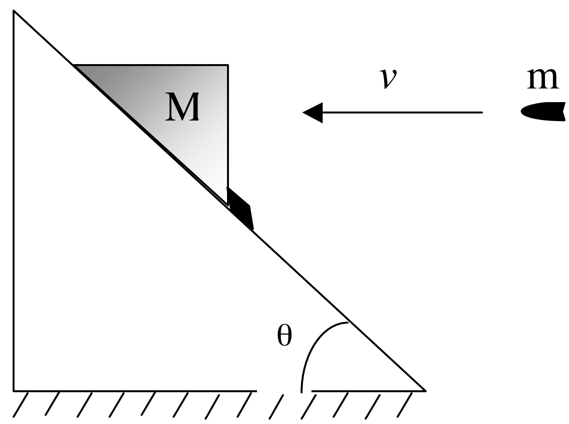
Problem 3. The wedge shown below has mass *M* and is held on an immobile ramp in the position shown by a small stop. The surface on which it lies is frictionless. A bullet of mass *m* is shot at the wedge and embeds within it, entering with velocity *v*. Determine the distance the wedge travels up along the ramp. Assume that the wedge remains in contact with the ramp at all times.



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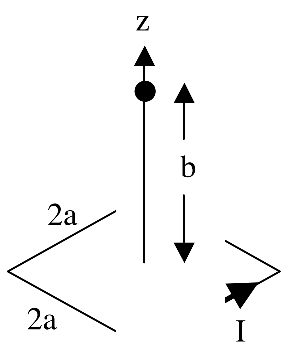
Problem 4. Consider a square current loop, which is of length *2a* on each side, lying in the *x-y* plane. It is centered on the origin and carries current *I*.

1. (5 points) Calculate the magnetic field at a height *b* from the origin (at $z=b$).
2. (5 points) If the square loop is replaced by a circular loop, what radius would give the same field at $z=b$. [It is sufficient here to set up the equation without solving it.] What is the value of this radius if $b>>a$? [Evaluate this case.]



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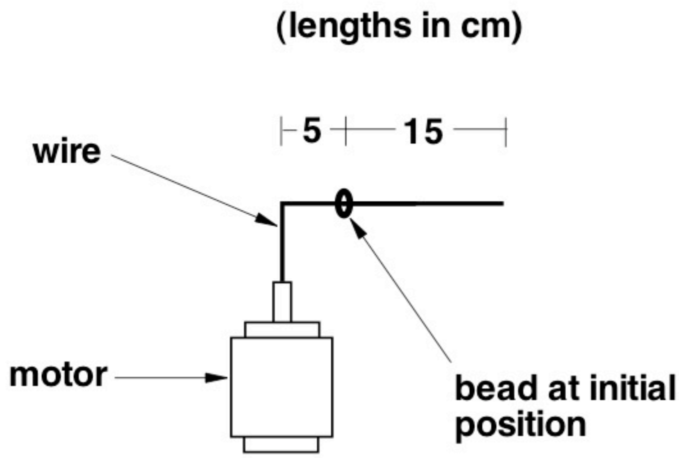
Problem 5. A rigid "L"-shaped wire is mounted on a motor such that the segment from the motor attachment to the right-angle bend is aligned with the motor's shaft and points vertically. The segment from the bend to the free end lies in a horizontal plane. The horizontal segment is 20 cm in length. While the motor is off, a bead is placed on the wire 5 cm from the bend (15 cm from the free end of the wire) and is temporarily held in place by string. After the motor is activated, it rotates the wire at a constant 10 revolutions per second.



a. (8 points) With the motor turning at the constant rate of rotation of 10 revolutions per second, the bead is released. If the friction between the bead and the wire is negligible, what is the speed of the bead when it leaves the wire?

b. (2 points) If there is friction between the bead and the wire, is the force due to friction constant from the time that the bead moves until it leaves the wire? Why or why not?

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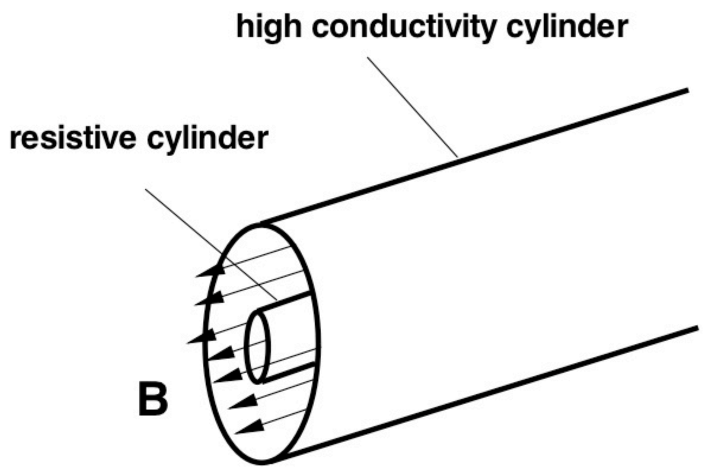
Problem 6. Two thin concentric cylindrical shells have length L, which is much greater than the radius of either shell. The outer cylinder is of radius b and is a very good electrical conductor, while the inner cylinder of radius a, acts as a resistor of strength R to any current that loops azimuthally around that shell. An initially uniform axial magnetic field passes through both cylinders.



The inner cylinder is suddenly filled with pressure, causing it to uniformly expand with constant radial velocity v outward toward the outer cylinder. Find a differential equation that describes the time-dependent current flowing azimuthally around the inner surface of the outer cylinder as a function of the parameters a, b, L, R, v, the magnitude of the initial magnetic field B_0, and time t.