University of Wisconsin-Madison
Engineering Physics Department
Spring 2005 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ON THE FRONT PAGES ONLY.

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, on the first six solutions you provided will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

Calculators with symbolic manipulation capability are NOT allowed

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Problem 1.

1. Evaluate the following integral analytically. You may find calculus of residues useful in its evaluation:

$$\int_0^{2\pi} \frac{\sin^2\theta}{5+4\cos\theta}\, d\theta$$

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Problem 2.

2. Consider the general eigenvalue problem of the form:

$$\overline{\overline{A}}\,\overline{x} = \lambda \overline{\overline{M}}\,\overline{x}$$

where both $\overline{\overline{A}}$ and $\overline{\overline{M}}$ are matrices and λ is the eigenvalue. Given:

$$\overline{\overline{A}} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \qquad \overline{\overline{M}} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

find the eigenvalues and eigenvectors for $\overline{x}$ in this problem.

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Problem 3.

Consider the following differential equation for $y(x)$:

$$x^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - x \frac{\mathrm{d}y}{\mathrm{d}x} + y = 5 .$$

(a) Is this equation linear or nonlinear? Explicitly demonstrate that your answer is correct.

(b) Find the general solution to the equation.

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Problem 4.

(a) Write the standard form of the eigenvalue-eigenvector equation for the n x n matrix **A**.
(b) *Show* what the *weakest* restriction on **A** is so that all its eigenvalues will be real.
(c) Assuming **A** satisfies the restriction in (b), will it always have n independent eigenvectors? Show whether its eigenvectors must be orthogonal – make sure to consider all possibilities.
(d) What is a necessary and sufficient condition on **A** so that at least one eigenvalue is zero?
(e) Under what conditions will **A** have a zero eigenvector?

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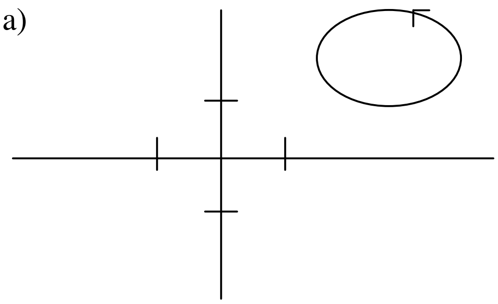
Problem 5.

Consider the following integral

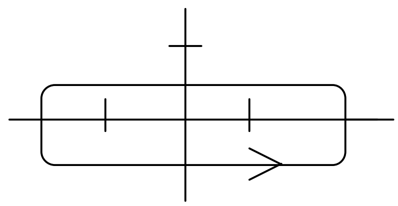
$$I = \oint_C dz \frac{e^{\pi z}}{z^4 - 1}$$

where C is a closed oriented curve in the complex plane. Calculate I for each of the following possible curves for C. The four hash marks refer to the points z = 1, z = i, z = -1, and z = -i.

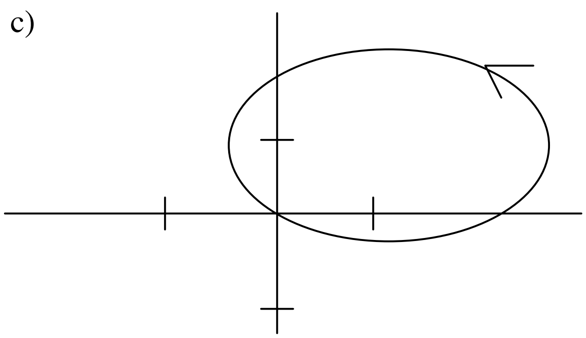
a)



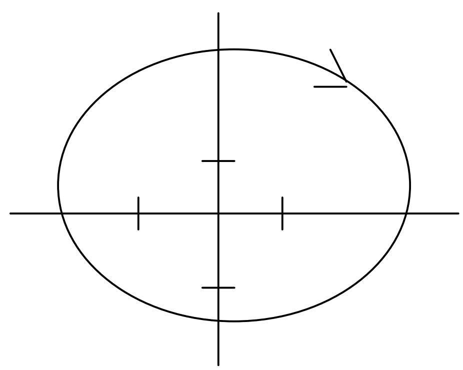
b)



c)



d)



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Problem 6.

Laplace transform the following differential equation

$$\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} = F(t)$$

and solve for y(t) for the special cases

a) $F(t) = 0, y(t=0) = y_o, \frac{dy}{dt}(t=0) = y_1,$

b) $F(t) = F_0\delta(t) - F_1\delta(t-\tau), y(t=0) = 0, \frac{dy}{dt}(t=0) = 0.$

Here $\delta(t)$ is the Dirac-delta function, F_0, F_1, y_o, y_1, γ and τ are positive constants.