University of Wisconsin-Madison
Engineering Physics Department
Fall 2014 Qualifying Exams

Modern Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Constants available for the Exam

$c = 2.998 \times 10^{8}$ m/s

$\dfrac{e^2}{4\pi\varepsilon_0} = 1.44$ MeV-fm

$hc = 1.24 \times 10^{4}$ eV-Å

$\mu_B = \dfrac{eh}{4\pi m} = 5.79 \times 10^{-5}$ eV/T

Conversion Factors

1 MW-s = 1.602×10^{-19} MeV

1 Å = 10^{-8} cm

Possible useful integrals:

$$\int_0^{2\pi} (\sin^2 x)\,dx = \pi$$
$$\int_0^{2\pi} (\cos^2 x)\,dx = \pi$$
$$\int_0^{2\pi} (\sin x \cos x)\,dx = 0$$

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Problem 1. The synchrotron radiation process limits the beam energy because of the energy loss per circulation, given by

$U_{per\ turn} = \frac{e^2}{3\varepsilon_0} \frac{\beta^3 \gamma^4}{\rho}$ where ρ is the bending radius and β, γ are the usual relativistic factors.

For a desired 3 TeV beam energy, the losses are impractical, and it has been suggested that muons, with a rest mass energy (mc^2) of 105.6 MeV (compared to the electron rest mass of 0.511 MeV), be used instead.

a) (3pts) For the same bending radius and at 3 TeV beam energy, what is the ratio of the energy loss of muons to the energy loss of the electrons?

b) (3pts) A muon at rest has a lifetime of 2.2 microseconds. What is the lifetime in the laboratory of the 3 TeV muon beam?

c) (1pts) For a 3.8 km circumference storage ring, how many circulations would the muon beam make in one laboratory lifetime?

d) (3pts) If the pulse of muons in the storage ring is 10 microseconds long in the laboratory frame, how long would the pulse appear in the muon rest frame?

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Problem 2. Consider two identical spin ½ particles, called A and B. The spin wavefunction of the two particle system can be symmetric or anti-symmetric under the interchange of the particle labels.

a) (1.5pts) What is the total spin of the symmetric spin state(s)?

b) (1.5pts) What is the total spin of the anti-symmetric spin states?

c) (2pts) If these two particles together have either total orbital angular momentum L=0 or L=1, what spin state is allowed for L=0 and for L=1? Explain your reasoning in words.

d) (3pts) What total angular momenta J, the sum of the spin and orbital angular momentum, are allowed for L=1 and L=0? Explain your reasoning.

e) (2pts) If a spin 1 particle decays into particles A and B, what is its parity if we assume that parity is conserved in the decay?

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Problem 3. Consider an atom with zero spin ($S = 0$) but with orbital angular momentum L. Transitions occur in the atom between a $L = 2$ and a $L = 1$ state in a magnetic field of 0.6T.

a) (5pts) Sketch an energy level diagram for the perturbed states and indicate the allowed transitions between states.

b) (5pts) If the emission wavelength observed before the field is turned on is 5000Å, determine the wavelength observed when the magnetic field is on.

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Problem 4. A gas comprised of particles with mass m at thermodynamic equilibrium with temperature T has a velocity distribution given by the Maxwell velocity distribution function:

$$f(\vec{v})d^3\vec{v} = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}\, d^3\vec{v}$$

where k is the Boltzmann constant.

(a) (6 pts) Derive an expression for the energy distribution *f(E)dE* of the particles.
(b) (4 pts) Calculate the average energy of the particles.

Useful integrals:

$$I(n) = \int_0^\infty e^{-\alpha x}\, x^n\, dx = \frac{\Gamma(n+1)}{a^{n+1}} \quad \text{where } n \geq 0.$$

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi} \qquad \Gamma(n+1) = n\,\Gamma(n)$$

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Problem 5. A molten salt fueled nuclear reactor utilizes the Thorium/Uranium fuel cycle to both breed and burn the fissile isotope ^{233}U. The complete transmutation and decay chain scheme for the production of ^{233}U is

$$^{232}_{90}Th + n \longrightarrow {}^{233}_{90}Th \xrightarrow{\beta^-} {}^{233}_{91}Pa \xrightarrow{\beta^-} {}^{233}_{92}U + n \longrightarrow fission\,.$$

Thorium-232 is added to the reactor through the molten salt feed stream. In order to produce 3000 MWth of power from ^{233}U, ^{232}Th must be replenished at a rate of F atoms/s. The neutron flux of the reactor is $\phi = 2.0\text{x}10^{15}$ n/cm^2-s and each fission releases 200 MeV of energy.

a) (2pts) Write the rate equation and an expression for the equilibrium concentration of each nuclide in the chain.

b) (2pts) Compute the thorium feed rate (replenish rate) required to produce the 3000 MWth of power.

c) (6pts) What are the ^{232}Th and ^{233}Th concentrations 110 days after startup of the reactor assuming that the ^{232}Th feed rate is $2.0\text{x}10^{18}$ atoms/s during the startup period? Assume that the initial concentrations of these two isotopes are zero.

Required data:

^{233}Th: $t_{1/2}$= 23 min.	^{233}Pa: $t_{1/2}$= 27 days	$\sigma_a^{233Th} \cong 0$ barns	$\sigma_a^{233Pa} \cong 0$ barns
$\sigma_a^{232Th} = 4.0$ barns	$\sigma_f^{233U} = 260$ barns	$\sigma_\gamma^{233U} = 23$ barns	1 barn = 10^{-24} cm^2

(Note: 1 MW-s = 1.602×10^{-19} MeV)

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Problem 6. Bromine-87 a radioactive nuclide produced from fission, β^--decays with a half-life of 55 seconds. The daughter, ${}^{87}_{36}\mathrm{Kr}$ is left in an excited state and there is a 10% chance of decaying by neutron emission and a 90% chance by photon emission.

a) (1pts) What is the β^--decay reaction for a single neutron?

b) (2pts) Calculate the separation energy of the last neutron of the daughter.

c) (5pts) If the beta decay leaves the daughter nucleus in the excited state, *En*, what is the energy of this state if a 250 keV neutron is emitted?

d) (2pts) Consider the case where, instead of neutron emission, a photon is emitted from the excited level. The excited level has a spin 2^+ and the photon is emitted to the ground state of the daughter nuclides, which has a spin 1^-. What is the multipole of the radiation field?

Mass [amu] (where1 amu = 931.502 MeV/c^2)	Mass [amu]
$M({}^{1}_{0}\mathrm{n})$ = 1.00866501	$M({}^{87}_{36}\mathrm{Kr})$ = 86.913354
$M({}^{87}_{35}\mathrm{Br})$ = 86.920711	$M({}^{86}_{36}\mathrm{Kr})$ = 85.910610
$M({}^{86}_{35}\mathrm{Br})$ = 85.918797