University of Wisconsin-Madison
Engineering Physics Department
Fall 2005 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1.

1. Consider the following differential equation

$$\frac{d}{dx}(e^{\frac{x^2}{2}}\frac{dy}{dx}) + e^{\frac{x^2}{2}}(1-2x^2)y = 0,$$

with boundary conditions $y(0) = 0$, $dy/dx(x=0) = 1$.

a) Use a series solutions to calculate y(x) near x = 0. Find the first three non-zero contributions to the series.

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Problem 2.

2. We are interested in calculating solutions to the matrix equation

$$\begin{pmatrix} 1 & 2 & n \\ 0 & 2 & -1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = b \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

where b and n are parameters.

a) Under what conditions for b and n is the solution $x = y = z = 0$ the only solution?
b) Under what conditions for b and n are there no values of x, y, and z that satisfy the above equation?
c) From the general solution, under what conditions for b and n is $x > y$ when $z > 0$?

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Problem 3.

Evaluate the following integral given that the parameter a has magnitude $|a| < 1$.

$$\int_0^{2\pi} \frac{\cos^3\theta}{1 - 2a\cos\theta + a^2}\, d\theta$$

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Problem 4.

Given the matrix **A**

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

a) Use the Gram-Schmidt procedure to construct an orthonormal basis set from the column vectors of **A**. Take the first basis vector to be the first column of **A**; that is $\mathbf{u}_1 = \mathbf{v}_1 = (1,1,1)$.

b) Find the **QR**-decomposition of the matrix **A**.

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Problem 5.

Find the complete solution $y(x)$ to the following differential equation by employing the method of variation of parameters.

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 2e^{-x}$$

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Problem 6.

Evaluate the following integral of the complex variable $z = x + iy$, for all nonnegative integer values of n. Here, z_o is a constant value of z, and C is a closed contour that encloses both z_o and the origin.

$$\oint_C \frac{dz}{z^n(z - z_0)}.$$