University of Wisconsin-Madison
Engineering Physics Department
Spring 2011 Qualifying Exams

Classical Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, <u>NOT</u> ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

$\mu_0 = 4\pi \times 10^{-7}$ H/m $\qquad \varepsilon_0 = 8.854 \times 10^{-12}$ F/m

Center of mass moments of inertia:

$I_{disk} = \frac{1}{2} MR^2 \qquad I_{sphere} = \frac{2}{5} MR^2 \qquad I_{rod} = \frac{1}{12} ML^2$

$I = Mk^2$ (where k is the radius of gyration)

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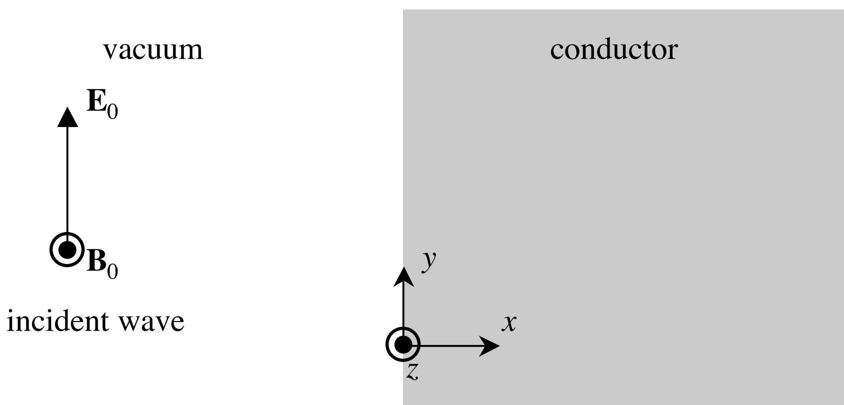
Problem 1.

A plane electromagnetic wave propagates in vacuum at frequency $f = 2 \times 10^9$ Hz and is normally incident on the planar surface of a semi-infinite conductor (infinite in y and z and semi-infinite in x). There is a reflected wave, but the conductivity of the material is finite, $\sigma = 2 \times 10^4$ Ohm^{-1}m^{-1}, and charge-current density in the conductor is related to electric field as $\mathbf{J} = \sigma \mathbf{E}$.



Sketch of an electromagnetic wave that propagates in vacuum as $\mathbf{E} = \mathrm{Re}\{E_0 \exp(ikx - i\omega t)\}\hat{\mathbf{y}}$ and $\mathbf{B} = \mathrm{Re}\{B_0 \exp(ikx - i\omega t)\}\hat{\mathbf{z}}$, and is incident upon the conductor.

Determine the distance within the conductor ($x > 0$) wherein the amplitude of the transmitted signal decreases by a factor of $1/e$. [Provide a derivation and/or physical reasoning for your numerical answer, and indicate any approximations.]

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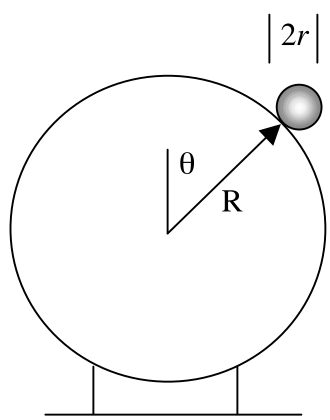
Problem 2.

A small, solid sphere of radius r, mass m, and moment of inertia I (I=0.4 m r^2) rests at the top of a fixed cylinder. It is then allowed to roll, without slipping, down the cylinder surface.

a) (8 points) What is the angle (θ) at which the sphere separates from the surface of the cylinder?
b) (2 points) Is the no-slip condition physically realistic up to the separation angle? Provide a brief justification of your answer.



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Problem 3.

A non-conducting sphere of radius a carries a uniform charge density ρ.

a) (2 points) What is the electric field within the sphere?
b) (1 point) What is the electric field outside the sphere?
c) (1 point) If a spherical cavity of radius d is created at the center of the non-conducting sphere, what is the electric field within the cavity?
d) (6 points) If a spherical cavity of radius d is created such that the center of the cavity is a distance e from the center of the sphere, where $d + e \leq a$, what is the electric field within the cavity?

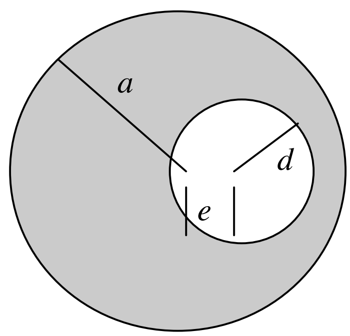

Figure for part d.

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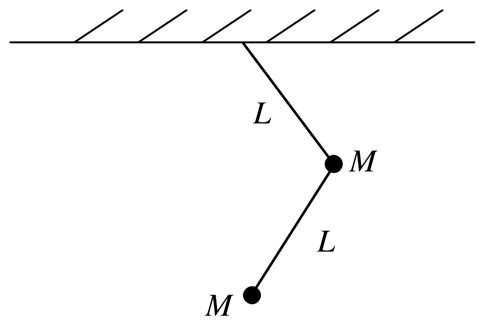
Problem 4.

A double pendulum is made from two small objects, each of mass M, and two rods of negligible mass, as shown in the figure below. Each rod has length L, and there are pivots connecting the first rod to the support and at the joint between the two rods.



a. (5 points) Find the equations of motion for the two masses for small displacements from equilibrium. Assume that motion is co-planar with the vertical equilibrium position, and express your equations in terms of M, L, and the gravitational constant g.

b. (5 points) Find the frequencies of oscillation for this system.

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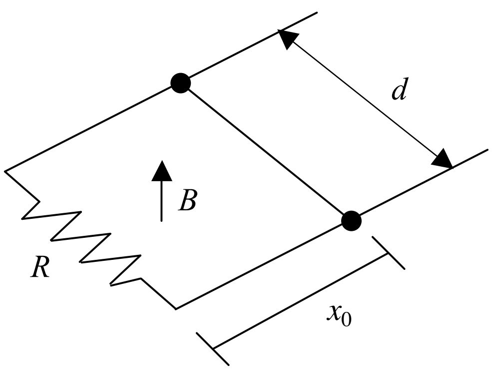
Problem 5.

A simple circuit consists of a resistor of resistance R, two parallel wires separated by a distance d, and a third wire that is free to move while providing an electrical connection between the parallel wires. The circuit is immersed in an external orthogonal magnetic field of uniform magnitude B.

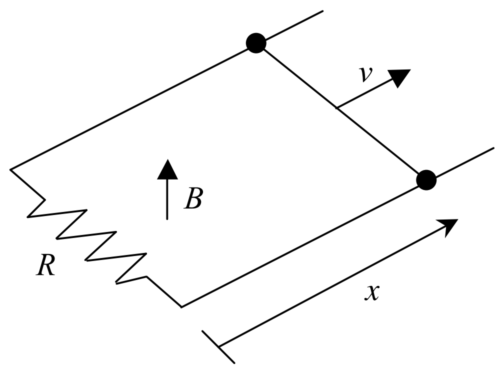
For $t < 0$, the third wire is stationary and located at a distance $x = x_0$ from the resistor. For $t \geq 0$, the third wire moves at a constant velocity v away from the resistor. The self-inductance of the circuit is proportional to the location of the third wire, $L = L_1 x$.

a. (5 points) Find a relation for the current in the circuit as a function of time for $t \geq 0$.

b. (5 points) Solve the relation from part a. for $vt << x_0$.



Circuit configuration for $t < 0$.



Circuit configuration for $t \geq 0$.

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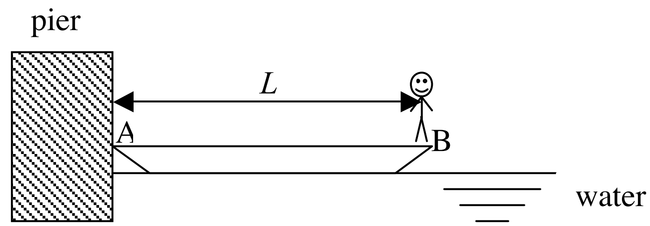
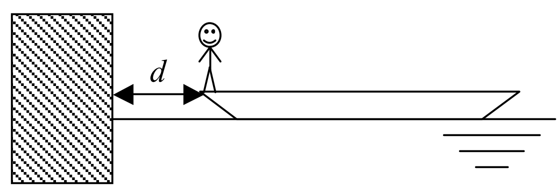
Problem 6.

End A of a boat of mass M and length L is in contact with a pier. A man of mass m, initially standing on End B of the boat, walks toward End A. What will be his distance d from the pier when he reaches End A? Ignore friction between the boat and the liquid.



Initial position.



Final position.