University of Wisconsin-Madison
Engineering Physics Department
Spring 2014 Qualifying Exams

Modern Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

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Problem 1. A photon of energy $E_\gamma = 20\ m_e c^2$ Compton scatters from a nearly stationary electron. The scattered photon emerges at an angle θ and the scattered electron at an angle ϕ, both with respect to the incident photon direction.

a) (6pts) Using the basic momentum and energy conservation equations, determine the electron scattering angle ϕ such that ϕ minimizes the kinetic energy of the electron.

b) (3pts) For the same incoming photon energy, determine the electron scattering angle ϕ such that ϕ maximizes the kinetic energy of the electron.

c) (1pts) What is the probability of the electron being emitted in the angular range $\pi/2 < \phi \le \pi$?

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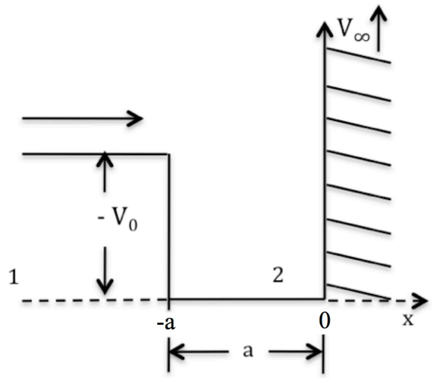
Problem 2. A source at $x = -\infty$ emits particles of mass m with an energy $E > 0$ and a positive waveform encounters a step potential drop at $x = -a$ and an infinite potential barrier at $x = 0$ as shown in the figure below.



$$V(x) = \begin{cases} 0 & x < -a \\ -V_0 \quad \text{for:} & -a < x < 0 \\ +\infty & x > 0 \end{cases}$$

(a) (3pts) Solve for the wavefunctions in region 1 and 2 in the figure.
(b) (4pts) Calculate the total reflection coefficient, and comment on whether it makes intuitive sense.
(c) (3pts) Derive a condition for a maximum in amplitude in region 2 and give a physical interpret of this.

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Problem 3. A clock in a satellite in a circular orbit appears to an observer on earth to be running at a rate governed by two factors:

i) Special relativistic time dilation
ii) The effect of acceleration in the gravitational field of the earth

The second effect increases the clock rate by a fraction $\Delta\phi/2$, where $\Delta\phi$ is the difference in gravitational potential between the positions of the clock of mass *m* and the observer.

a) (6pts) Derive an expression for the combined effect on the difference between the satellite clock rate and the earth clock rate.
b) (3pts) Estimate the difference in time between the two observers per year for a low-earth orbit.
c) (1 pt) The best accuracy attainable with a cesium clock is of order 10^{-12} seconds. Given this, would this total effect be measureable?

(Note the radius of the earth is 6400 km).

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Problem 4. Alpha decay is the spontaneous emission of an alpha particle from a nucleus.
a) (8pts) Use the Semi-Empirical Mass Formula given below to derive an expression that can be used to predict the stable nuclide mass number A beyond which spontaneous alpha decay (emission) becomes possible. Assume that the binding energy of the alpha particle (i.e B($^{4}_{2}$He)) is 23.4 MeV.

b) (2pts) Use the expression from part (a) to determine the stability of $^{226}_{90}$Th.

$$M(Z,A)c^2 = A\,M_n c^2 + Z(M_H - M_n)c^2 - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_{sym}\frac{(A-2Z)^2}{A} - \delta$$

where $\delta = \begin{cases} +a_p A^{-3/4} & \text{for even/even} \\ 0 & \text{for even/odd} \\ -a_p A^{-3/4} & \text{for odd/odd} \end{cases}$

and
A (mass number), Z (charge),
M_n (neutron mass = 1.008665 amu), M_H (hydrogen mass = 1.007825 amu),
a_v= 15.5 MeV, a_s= 16.8 MeV, a_c= 0.72 MeV, a_{sym}= 23 MeV, and a_p= 34 MeV.

(Power series expansion: $(1+x)^q = 1 + q\,x + \frac{q(q-1)}{2!}x^2 + \ldots$ for $|x|<1$).

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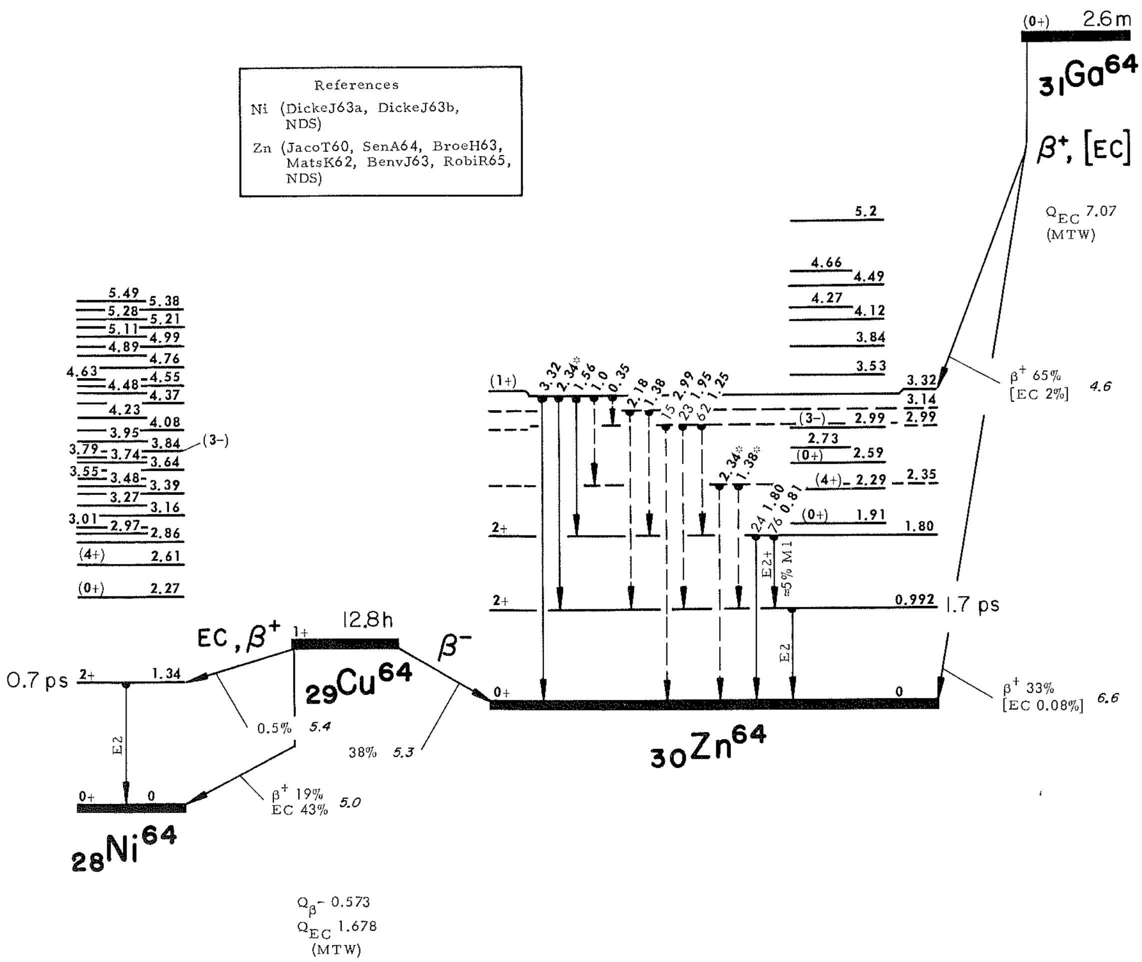
Problem 5. A sample of pure $^{63}_{29}$Cu is irradiated for 24 hours by a low energy neutron beam. Neutrons are captured and produce the radioisotope $^{64}_{29}$Cu. The decay scheme for $^{64}_{29}$Cu is shown in the figure provided.

a) (5 pts) What is the activity of $^{64}_{29}$Cu at the end of the 24 hour irradiation period?

b) (5 pts) How many atoms of $^{64}_{28}$Ni are produced by electron capture to the ground state of $^{64}_{28}$Ni during irradiation?

Sample mass	= 10 gram
Atomic weight	= 63.5 grams/mole
$^{63}_{29}$Cu reaction cross section	= 5 barns
Copper half-life	= 12.8 hours
Beam intensity	= 2×10^{12} neutrons/cm^2 –s

1 Ci = 3.7×10^{10} dis/sec, 1 barn = 10^{-24} cm^2, N_{Av}= 6.023×10^{23} atoms/mole



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Problem 6. A proton undergoes the reaction p + ${}^{4}_{2}\mathrm{He} \rightarrow {}^{2}_{1}\mathrm{H} + {}^{3}_{2}\mathrm{He}$.
a) (2pts) Calculate the Q-value of the reaction.

b) (8pts) What is the threshold energy for this reaction?

Mass [amu] (where1 amu = 931.502 MeV/c^2)	Mass [amu]
$M({}^{1}_{1}\mathrm{H})$ = 1.007825	$M({}^{2}_{1}\mathrm{H})$ = 2.014102
$M({}^{3}_{2}\mathrm{He})$ = 3.016029	$M({}^{4}_{2}\mathrm{He})$ = 4.002603