University of Wisconsin-Madison
Department of Engineering Physics
Spring 2006 Qualifying Exams

Modern Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

USEFUL CONSTANTS

$m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $k = 1.6 \times 10^{-19}$ Joules/eV

$\hbar = 1.06 \times 10^{-34}$ J-s $= 6.582 \times 10^{-16}$ eV-s, $c = 3 \times 10^{8}$ m/s

1 amu = 931.48 MeV, m_e=0.511 MeV, 1 barn = 10^{-24} cm^2

1 Ci (Curie) = 3.7×10^{10} disintegrations/s

USEFUL RELATIONS and EQUATIONS

$$P(\vec{r}) = \int \left|\psi\psi^*\right| dV \ , \ j = \frac{\hbar}{2mi}\left[\psi^* \frac{\partial\psi}{\partial x} - \psi \frac{\partial\psi^*}{\partial x}\right]$$

Trigonometric identities: $\cos(\theta \pm \phi) = \cos(\theta)\cos(\phi) \mp \sin(\theta)\sin(\phi)$,
$\sin(\theta \pm \phi) = \sin(\theta)\cos(\phi) \pm \sin(\phi)\cos(\theta)$ and $\tan(\theta \pm \phi) = \dfrac{\tan(\theta) \pm \tan(\phi)}{1 \mp \tan(\theta)\tan(\phi)}$
$\cosh^2(x) - \sinh^2(x) = 1$, $\cos(x) = \dfrac{(e^{ix} + e^{-ix})}{2}$, $\sin(x) = \dfrac{(e^{ix} - e^{-ix})}{2i}$

For the following reaction $X(a,b)Y$ where X is the stationary target in the laboratory reference frame, the following expression is obtained for the kinetic energy of the reaction product b in terms of θ its scattering angle relative to the incoming direction of particle a and the kinetic energy T_a of particle a:

$$T_b^{1/2} = \frac{(m_a m_b T_a)^{1/2}\cos(\theta) \pm \left(m_a m_b T_a \cos^2(\theta) + (m_Y + m_b)(m_Y Q + (m_Y - m_a)T_a)\right)^{1/2}}{m_Y + m_b}.$$

The threshold energy for a reaction is given by

$$T_{Th} = (-Q)\left(\frac{m_Y + m_b}{m_Y + m_b - m_a}\right).$$

Student No.__________________

Engineering Physics Department
Spring 2006 Qualifying Exams

Modern Phsyics

Problem 1.

It is desired to study the low-lying states of ${}^{35}_{17}Cl\left(\frac{+3}{2}\right)$ through the reaction ${}^{32}_{16}S(\alpha,p){}^{35}_{17}Cl$. The ground state (GS) and low-lying excited states are

Level	Energy [MeV]	Spin
5	3.003	$\left(\frac{+5}{2}\right)$
4	2.694	$\left(\frac{+3}{2}\right)$
3	2.646	$\left(\frac{+7}{2}\right)$
2	1.763	$\left(\frac{+5}{2}\right)$
1	1.219	$\left(\frac{+1}{2}\right)$
GS	0	$\left(\frac{+3}{2}\right)$

(a) Compute the Q-value of the reaction. (b) If the incident α particle has 4.8 MeV of energy, which of the excited states can be reached? (c) These excited states decay by gamma emission. Indicate the primary type and multipole of the emitted gamma radiation associated with all possible transitions as the nucleus de-excites to the ground state. The atomic masses of the nucleons in amu are:

M(${}^{1}_{1}H$) = 1.007825	M(${}^{4}_{2}He$) = 4.002603
M(${}^{35}_{17}Cl$) = 34.968853	M(${}^{32}_{16}S$) = 31.972071

Student No.___________________

Engineering Physics Department
Spring 2006 Qualifying Exams

Modern Phsyics

Problem 2.

Suppose you wish to produce a radioactive tracer isotope by irradiating a sample in a proton accelerator. The nuclear reaction and decay scheme is

$$ {}^{A}_{Z}X + p \rightarrow {}^{A+1}_{Z+1}X^{*} \rightarrow {}^{A+1}_{Z+1}X + \gamma . $$

You have no place to store the radioactive material once it has been produced and it requires one hour to deliver the material from the proton beam accelerator to your laboratory. If you require 2 mCi of material delivered at 2:00 PM, when should you start irradiating the sample?

Sample mass	= 1 gram
Atomic weight	= 5 grams/mole
Reaction cross section	= 10 barns
Product half-life	= 2.5 hours
Beam intensity	= 1.5 x 10^8 protons/cm^2-s

Student No.________________

Engineering Physics Department
Spring 2006 Qualifying Exams

Modern Phsyics

Problem 3.

Muons are produced by cosmic ray interactions in the Earth's upper atmosphere. The muons then approach the Earth with a speed near that of light (0.998c). They are unstable, with a rest mass of 107 MeV/c^2 and have a half-life of 2.2×10^{-6} s at rest.

a) If 1 million muons are created at an altitude of 18 km, how many muons are detected at sea level (0 m elevation)?
b) If the muons were created at twice the altitude of part a), would we still be able to detect muons at seal level? If you answer is yes, how many?
c) What is the total energy of the created muons? What is their kinetic energy?

Student No.________________

Engineering Physics Department
Spring 2006 Qualifying Exams

Modern Phsyics

Problem 4.

a) List and define the three major methods of interaction for gamma-ray photons with matter.

b) Scintillators are used for gamma-ray detection. Scintillators operate by detecting visible light produced by charge-particle interaction in the scintillation crystal. A scintillator is used to detect gamma-rays from a source that produces monenergetic 10 MeV gamma-rays. The measured energy spectrum has the expected peak at 10 MeV, but also exhibits two other peaks at energies of 0.511 and 1.022 MeV. Explain the origin of these two “extra” peaks.

c) Based on b) would one expect similar extra peaks with a 1 MeV gamma source?

d) A 1 MeV monoenergetic photon beam is available to an experimentalist for an experiment. However, the experimentalist only requires 800 keV photons. At what angle relative to the incoming direction of the photon beam should the experiment be setup so that scattered photons of 800 keV are intercepted? (Assume that only scattering events produce these photons and that the scattering medium is an iron cube of 27 cc. The distance from the block to the experiment is 75 cm).

Student No._________________

Engineering Physics Department
Spring 2006 Qualifying Exams

Modern Phsyics

Problem 5.

a) Write down an expression for the relativistic kinetic energy T.

b) Show that in the limit of velocities $v \ll c$ this reduces to the usual classical, low energy, definition of kinetic energy.

c) Find an expression for the kinetic energy in the limit $T \gg mc^2$.

Student No.__________________

Engineering Physics Department
Spring 2006 Qualifying Exams

Modern Phsyics

Problem 6.

A plane wave function of an atom of energy $E < V_0$ encounters a barrier potential V_0 with thickness a. Calculate the transmission and reflection coefficients.