University of Wisconsin-Madison
Engineering Physics Department
Fall 2013 Qualifying Exams

Modern Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Constants available for the Exam

$\hbar = 6.58 \times 10^{-16}$ eV-s
$\hbar c = 197.3$ eV-nm
$c = 2.998 \times 10^{8}$ m/s
$\dfrac{e^2}{4\pi\varepsilon_0} = 1.44$ MeV-fm
$k = 1.38 \times 10^{-23}$ J/K
$m(H_2) = 3.34 \times 10^{-27}$ kg

Possible useful integrals:

$$\int_0^{2\pi} (\sin^2 x)\,dx = \pi$$
$$\int_0^{2\pi} (\cos^2 x)\,dx = \pi$$
$$\int_0^{2\pi} (\sin x \cos x)\,dx = 0$$

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Problem 1. Consider the following β^- decay chain with the half-lives indicated:

$$^{210}\mathrm{Pb} \xrightarrow[22\,y]{} {}^{210}\mathrm{Bi} \xrightarrow[5.0\,d]{} {}^{210}\mathrm{Po}.$$

A sample contains 30 MBq of ^{210}Pb and 20 MBq of ^{210}Bi at time t_1 and it was originally pure ^{210}Pb at time $t = 0$

(a) (7pts) What is the value of time t_1?

(b) (3pts) When is the activity of ^{210}Pb equal to the activity of ^{210}Bi?

Solution

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Problem 2.

(a) (8 pts) Compute the threshold energy for the $^{12}\text{C} + \gamma \rightarrow 3\ ^{4}\text{He}$ reaction, where the ^{12}C target is at rest.

(b) (2 pts) Why would this reaction require more energy than the threshold energy if two of the alphas come off in the same direction but the third does not?

(note: $m_C = 12$ u, $m_{He} = 4.002603$ u, and 1 u = 931.5 MeV).

Solution

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Problem 3. A neutron is confined inside a cubic box. The potential is given by

$$V(x,y) = \begin{cases} +\infty \text{ for } x \le 0 \text{ and } x \ge a;\ y \le 0 \text{ and } y \ge a;\ z \le 0 \text{ and } z \ge a \\ 0 \text{ for } 0 < x < a,\ 0 < y < a \text{ , and } 0 < z < a\ . \end{cases}$$

a) (7pts) Determine the 3-D normalized wave functions.
b) (3pts) What are the energies, levels of degeneracy and quantum numbers for the first three energy states?

<u>Solution</u>

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Problem 4. The Bohr theory of atomic hydrogen postulates a simple planetary model for the atom, where an electron of mass m_e revolves in circular orbits around a heavy nucleus of mass M, and where the orbital circumference is quantized to be an integer number of deBroglie wavelengths $(\lambda = h/mv)$.
(a) (6pts) Derive a formula for the energy of the quantum states n (= 1, 2, ...) when the nucleus is taken to be infinitely massive $(M = \infty)$.
(b) (2pts) Derive an expression for the wavelength of light emitted by a transition between two of these quantized states, n_1 and n_2.
(c) (2pts) Before the quantum mechanical description of the atom was developed, it was found empirically that the visible and ultraviolet spectral emission lines of hydrogen can be represented by the formula:

$$\lambda_n = \text{constant } \times \frac{n^2}{n^2 - 4} \qquad \text{where } n = 3, 4, 5, \ldots$$

Derive this equation from your answer in part (b) above.

<u>Solution</u>

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Problem 5. A gas comprised of particles with mass *m* at thermodynamic equilibrium with temperature *T* has a velocity distribution given by the Maxwell velocity distribution:

$$f(\vec{v})d^3\vec{v} = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT} d^3\vec{v}$$

where *k* is the Boltzmann constant.

(a) (6 pts) Derive an expression for the energy distribution of the particles.
(b) (4 pts) Calculate the average energy of the particles.

Useful integrals:

$$I(n) = \int_0^\infty e^{-\alpha x} x^n \, dx = \frac{\Gamma(n+1)}{a^{n+1}} \quad \text{where } n \geq 0.$$

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi} \qquad \Gamma(n+1) = n\Gamma(n)$$

Solution

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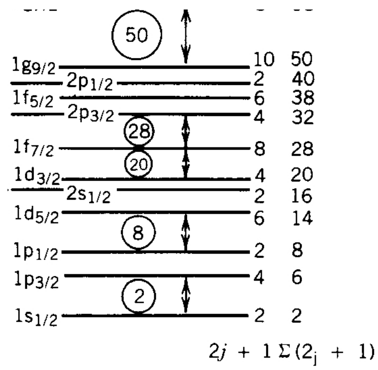
Problem 6. The low-lying levels of the ^{13}C nucleus are: ground state ($\frac{1}{2}^-$), 3.09 MeV ($\frac{1}{2}^+$), 3.68 MeV ($\frac{3}{2}^-$), and 3.85 MeV ($\frac{5}{2}^+$). The next states are about 7 MeV and above. Explain these lowest four states (level populations and rationale for the assigned populations, spin and approximate energies) using the shell model, given below.



Solution