University of Wisconsin-Madison
Engineering Physics Department
Spring 2008 Qualifying Exams

Modern Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1.

Consider a particle in a one-dimensional infinite square well of width L that is centered at the origin x=0. [You need not derive the normalized wavefunctions if you know them.]

a) (3 pts) What are the values of the normalized eigenfunctions of the two lowest energy states ($\psi_1(0)$ and $\psi_2(0)$) evaluated at x=0?
b) (3 pts) What are the values of <x> for states n=1 and n=2?
c) (4 pts) What are the values of $\langle x^2 \rangle$ for the states n=1 and n=2?

Some potentially useful integrals.

$$\int x\sin(x)dx = \sin(x) - x\cos(x)$$

$$\int x\sin^2(x)dx = \frac{x^2}{4} - \frac{x}{4}\sin(2x) - \frac{1}{8}\cos(2x)$$

$$\int x^2\sin^2(x)dx = \frac{x^3}{6} - \frac{x}{4}\cos(2x) - \frac{1}{4}\left(x^2 - \frac{1}{2}\right)\sin(2x)$$

$$\int x\cos(x)dx = \cos(x) + x\sin(x)$$

$$\int x\cos^2(x)dx = \frac{x^2}{4} + \frac{x}{4}\sin(2x) + \frac{1}{8}\cos(2x)$$

$$\int x^2\cos^2(x)dx = \frac{x^3}{6} + \frac{x}{4}\cos(2x) + \frac{1}{4}\left(x^2 - \frac{1}{2}\right)\sin(2x)$$

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Problem 2.

(10 pts) In a Compton scattering event, what is the maximum possible kinetic energy K_e that can be transferred to the scattered electron for a given initial photon energy $E=h\nu$ expressed as a function of E and other constants of the problem?

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Problem 3.

A particle of mass m_0 has a velocity in the laboratory frame of $v_0=3/5c$. Suddenly it decays into two particles. Particle 1 with mass $m_1=2/3m_0$ has a velocity of $v_1=4/5c$ in the same direction as the original particle.

a) (8 pts) What are the velocity v_2 (magnitude & direction) and the mass m_2 of Particle 2?

b) (2 pts) What physical interpretation do you give to the result in part (a)?

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Problem 4.

Explain the process of alpha particle decay of heavy nuclei by answering the following questions.

a) (2 pts) Why do heavy nuclei decay with alpha particles ${}_{2}^{4}He$ and not, for instance, deuterons ${}_{1}^{2}H$ or tritons ${}_{1}^{3}H$?

b) (2 pts) Draw a graph of the potential energy inside and around a heavy nucleus that describes the α-decay process. Describe the functional form of the graph where appropriate.

c) (2 pts) Draw a line on the graph in part (b) representing the Q-value of the α decay energetics.

d) (2 pts) Using the graph from parts (b) and (c) identify the relevant parameters that determine the half-life of the α-decay. Give a short description referring to the graph as necessary. You may wish to label some additional points on the graph to use in your description.

e) (2 pts) Assume that two isotopes of the element ^{98}Cf (A~250), differing in neutron number by 4, have identical alpha decay Q-values. Which isotope should have the longer half life and why? You can refer to the previous results in stating your answer.

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Problem 5.

Consider the nuclear reaction ${}^{1}_{1}p + {}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{1}_{0}n$ or in shorthand ${}^{14}_{6}C\left(p,n\right){}^{14}_{7}N$. This endothermic reaction has a Q-value of $Q = -1.598454\text{ MeV}$.

a) (6 pts) What is the proton threshold kinetic energy necessary to induce this reaction if the carbon nucleus is initially at rest? [Data: $m_p = 1.007276\text{ amu}$, $m_n = 1.008665\text{ amu}$, $M_{{}^{14}N} = 14.003241\text{ amu}$.]
b) (4 pts) Why is the threshold kinetic energy different than the Q-value?

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Problem 6.

(10 pts) Radioactive Isotope 1 has an initial number $N_1(0) = N_{10}$ and radioactive Isotope 2 has an initial number $N_2(0) = 0$. Isotope 1 decays into Isotope 2 with a decay constant λ_1. Isotope 2 decays with a decay constant λ_2, where $\lambda_2 > \lambda_1$. At what time are the activities of the two isotopes the same?