University of Wisconsin-Madison
Engineering Physics Department
Spring 2012 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

Some potentially useful identities

$$\hat{f}(s) = L[f(t)] = \int_0^\infty dt\, e^{-st} f(t)$$

$$L[\frac{t^{n-1}}{(n-1)!}] = \frac{1}{s^n}, \qquad L[\frac{2^n t^{n-1/2}}{1*3*5*...*(2n-1)\sqrt{\pi}}] = s^{-(n+1/2)}$$

$$L[e^{-at}] = \frac{1}{s+a}, \qquad L[te^{-at}] = \frac{1}{(s+a)^2}$$

$$L[\sin(at)] = \frac{a}{a^2+s^2}, \qquad L[\cos(at)] = \frac{s}{a^2+s^2}$$

$$L[\frac{e^{at}-e^{bt}}{a-b}] = \frac{1}{(s-a)(s-b)}, \qquad L[\frac{ae^{at}-be^{bt}}{a-b}] = \frac{s}{(s-a)(s-b)}$$

$$L[e^{at}\cos bt] = \frac{s-a}{(s-a)^2+b^2}, \qquad L[e^{at}\sin bt] = \frac{b}{(s-a)^2+b^2}$$

$$L[\sinh at] = \frac{a}{s^2-a^2}, \qquad L[\cosh at] = \frac{s}{s^2-a^2}$$

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Problem 1. Solve the following differential equation for *x(t)*

$$\frac{d^2x}{dt^2} - \gamma^2 x = 23\delta(t-10)$$

subject to the initial conditions $x(0) = dx/dt(0) = 0$. Here, γ is a positive real constant and $\delta(t)$ is the delta-function

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Problem 2. Consider the following integral

$$I = \frac{1}{2\pi i}\oint_C dz \frac{\alpha + 4iz - \alpha z^2}{(z-i)(z^2-1)}$$

where α is a constant. For each of the following cases, draw a closed oriented curve C in the complex plane such that

a) (25%) I = 0
b) (25%) I = -2
c) (25%) I = 1+i
d) (25%) I = α

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Problem 3. Let A be a diagonalizable matrix, and assume that S is a matrix, which diagonalizes A. Prove that a matrix T diagonalizes A if and only if it is of the form $T=CS$ where C is an invertible matrix such that $AC=CA$.

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Problem 4. Find the general solution $y(x)$ for the ordinary differential equation

$$y'' + 2x(y')^2 = 0$$

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Problem 5. Let matrix **A** be defined by:

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Find a matrix **P** that diagonalizes **A** and determine $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$.

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Problem 6. With respect to analytic functions in the complex plane,

(a) (50%) if $3x^2y - y^3$ is the real part of an analytic function of z, determine the imaginary part.

(b) (50%) prove that xy^2 cannot be the real part of an analytic function of z.