University of Wisconsin-Madison
Engineering Physics Department
Spring 2012 Qualifying Exams

Modern Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

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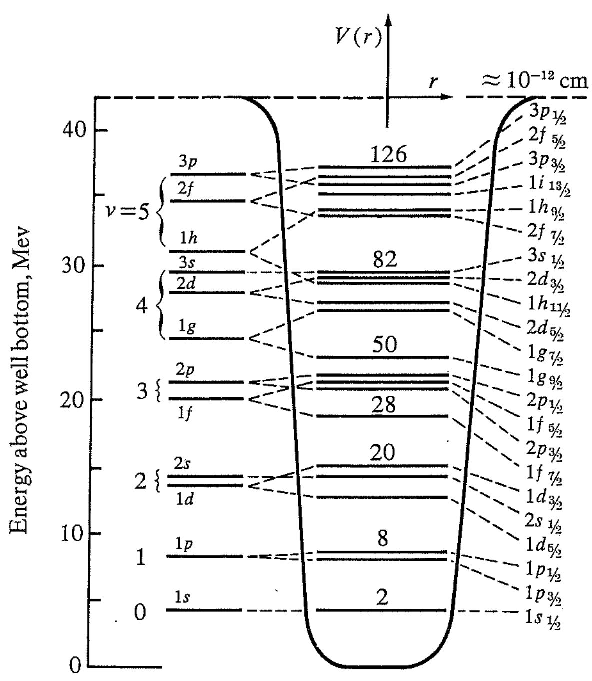
Constants available for the Exam

$\hbar = 6.58 \times 10^{-16}$ eV-s
$\hbar c = 197.3$ eV-nm
$c = 2.998 \times 10^{8}$ m/s
$\dfrac{e^2}{4\pi\varepsilon_0} = 1.44$ MeV-fm

Figure of the Shell Model of the Nucleus for Problem 2.



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Problem 1.

(a) (4 pts) A free nucleus, with excited state mass m and atomic number A, gamma decays and emits a photon with energy E as a result of a transition from E_i to E_f. Determine an expression for the energy E of the photon in terms of $\Delta E = E_i - E_f$.

(b) (2 pts) Taking the energy of the gamma as $E = 0.1$ MeV and $A = 100$, determine the recoil energy of the nucleus.

(c) (2 pts) Taking the excited state lifetime τ as 1 nsec, show that the 0.1 MeV photon cannot be absorbed by a similar free nucleus.

(d) (2 pts) In contrast, explain why the emitted photon can be absorbed if both the emitting and absorbing nuclei are very tightly bound to solids.

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Problem 2. Consider the shell model of a nucleus (see level energy diagram on page 2).
(a) (3 pts) Give the reason that the $1d_{5/2}$ level is lower in energy than the $1d_{3/2}$.
(b) (4 pts) Determine the spin and parity of the ground state of ${}^{57}_{26}\mathrm{Fe}$ and ${}^{26}_{14}\mathrm{S}$.
(c) (3 pts) In the presence of a magnetic field B, into how many levels does the ground state of ${}^{57}_{26}\mathrm{Fe}$ split and how does the splitting relate to the magnetic field?

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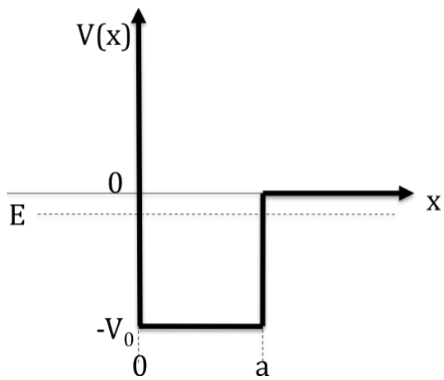
Problem 3. A particle with total energy E is bound in the one-dimensional potential well:



$$V(x) = \begin{cases} +\infty & x < 0 \\ -V_0 \quad \text{for:} & 0 < x \le a \\ 0 & x > a \end{cases}$$

(a) (7 pts) Derive a relationship for the allowed values of the energy E.
(b) (3 pts) What is the minimum potential well depth V_0 required to have a bound state?

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Problem 4. Consider the beta-decay of ^{6}He: $^{6}\text{He} \rightarrow {}^{6}\text{Li} + \beta^{-} + \overline{\upsilon}$ where Q = 3.57 MeV.
(a) (8 pts) Derive an equation for the kinetic energy of the beta and evaluate it for the case where the beta is observed to move away from the ^{6}Li nucleus while the ^{6}Li remains at rest in the laboratory reference frame after the decay. (Note: the ^{6}He is at rest before the decay occurs).
(b) (2 pts) If the neutrino remains at rest after the decay rather than the ^{6}Li, explain qualitatively how your answer in (a) would change. Would the beta's kinetic energy be larger or small than for case (a)?

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Problem 5. Long-lived $^{232}_{90}\mathrm{Th}$ ($t_{1/2} = 14.05 \times 10^{9}$ yr) decays through a series of much shorter-lived daughters to the stable isotope $^{208}_{82}\mathrm{Pb}$. A rock is determined to contain 1.37 g of $^{232}_{90}\mathrm{Th}$ and 0.31 g of $^{208}_{82}\mathrm{Pb}$.

(a) (8 pts) Determine the age of the rock. (Assume there are no initial nuclide of $^{208}_{82}\mathrm{Pb}$ present at time zero and the concentration levels of the intermediate isotopes between $^{232}_{90}\mathrm{Th}$ and $^{208}_{82}\mathrm{Pb}$ are negligibly small.)

(b) (2 pts) Seven alpha particles are emitted during the series decay of $^{232}_{90}\mathrm{Th}$ to $^{208}_{82}\mathrm{Pb}$. The rock is pulverized and the alphas are collected as helium gas. Assuming the rock was large enough that all the alphas emitted during the ages were collected, what volume of gas could be collected from this rock at standard temperature and pressure (760 mm and 0°C)? (Note: Standard Pressure = 1.013×10^{5} N/m^2, R = 8.31 J/K-mole, and $k_B = 1.381 \times 10^{-23}$ J/K)

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Problem 6. At time $t = 0$, 10^8 muons with at velocity of $0.998c$ directed towards the earth are created at an altitude of 9000 m. The muon has a mean lifetime of 2 μsec and a rest mass energy of about 106 MeV.

(a) (3 pts) What is the kinetic energy of the muons when they are created?

(b) (2 pts) What is the lifetime of the muon in earth's reference frame?

(c) (2 pts) What is the distance the muons travel in their reference frame before colliding with the earth?

(d) (3 pts) How many muons reach the earth's surface?