University of Wisconsin-Madison
Engineering Physics Department
Fall 2012 Qualifying Exams

Modern Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1. A 30-year old astronaut makes a trip from the Earth to a star that is 6 light years away at an average velocity of $0.9c$.
(a) (5 pts) Neglecting the time it takes to accelerate to its travel velocity and the time to turn the spaceship around for the return, how old is the astronaut upon return to Earth?
(b) (5 pts) Upon the astronaut's return, how old is the astronaut's boss, who was 40 years old when the astronaut left for the trip?

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Problem 2. A purely radioactive isotope N_1 with a decay constant λ_1, decays to radioisotope N_2 which has a decay constant λ_2. The concentrations of these radioisotopes at time $t = 0$ are $N_1(0) = N_0$ and $N_2(0) = 0$.

(a) (4 pts) Determine the radionuclide concentrations for N_1 and N_2 as a function of time.

(b) (2 pts) Determine the time at which the activity of N_2 reaches it maximum value.

(c) (2 pts) Determine the time at which the total activity $(A_{tot} = A_1 + A_2)$ reaches its maximum value.

(d) (2 pts) Determine the relationship between λ_2 and λ_1 for which the total activity will peak before the activity of nuclide 2 peaks.

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Problem 3. A photon of energy E_γ Compton scatters and the scattered photon induces pair production.

a) (7 pts) For the case of $E_\gamma = 10\, m_e c^2$ and using the basic momentum and energy conservation equations, determine the scattering angle θ when the scattered photon's energy is equal to the threshold energy to induce pair production.

b) (3 pts) For the case of $E_\gamma >> m_e c^2$, show that the angle θ approaches 60°.

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Problem 4. Two alpha particles are observed coming from the α-decay of a nucleus of mass 200 amu, one with energy 4.808 MeV and the second with energy 4.650 MeV. Both populate excited states of the daughter nucleus followed by gamma emission. Gamma photons of energies 150, 161 and 305 keV are observed. If needed, use integer masses for all particles.

(a) (7 pts) From this information construct a decay scheme and show how it explains the observed alpha and gamma energies.

(b) (3 pts) The decaying parent state has spin 1 and negative parity, and the ground state of the daughter has spin zero and also negative parity. Explain why there is no direct alpha decay to the ground state.

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Problem 5.
a) (7 pts) Find the normalized wavefunctions and energy levels for a particle confined in a two-dimensional rectangular box with the potential given by:

$$V(x,y) = \begin{cases} +\infty & x \le 0 \text{ and } x \ge a; \text{ and } y \le 0 \text{ and } y \ge b, \\ 0 & 0 < x < a \text{ and } 0 < y < b. \end{cases}$$

b) (3 pts) Determine the lowest energy state which has two sets of quantum number pairs $(n_{x,} m_y)$ that give the same energy (i.e. is a degenerate state) when $b = 2a$. It may be helpful here to make a simple energy diagram for the lower quantum states.

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Problem 6. The Bohr theory of atomic hydrogen postulates a simple planetary model for the atom, where an electron of mass m_e revolves in circular orbits around a heavy nucleus of mass M, and where the orbital circumference is quantized to be an integral number of deBroglie wavelengths $(\lambda = h/mv)$.

(a) (6pts) Derive a formula for the energy of quantum state n (= 1, 2, ...) when the nucleus is taken to be infinitely massive $(M = \infty)$. With this, give an expression for the wavelength of light emitted by a transition between two of these quantized states, n_1 and n_2.

(b) (2pts) How would that wavelength change if the nuclear mass M is finite?

(c) (2pts) The H_α spectral line of hydrogen arises from a transition between the $n_{initial}$ = 3 and n_{final}= 2 states. Estimate the mass ratio of deuterium to hydrogen if, respectively, their H_α lines have wavelengths of 656.101 nm and 656.280 nm.

Data: $m_e / M_H = 1/1836$