University of Wisconsin-Madison
Engineering Physics Department
Spring 2005 Qualifying Exams

Classical Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ON THE FRONT PAGES ONLY.

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, on the first six solutions you provided will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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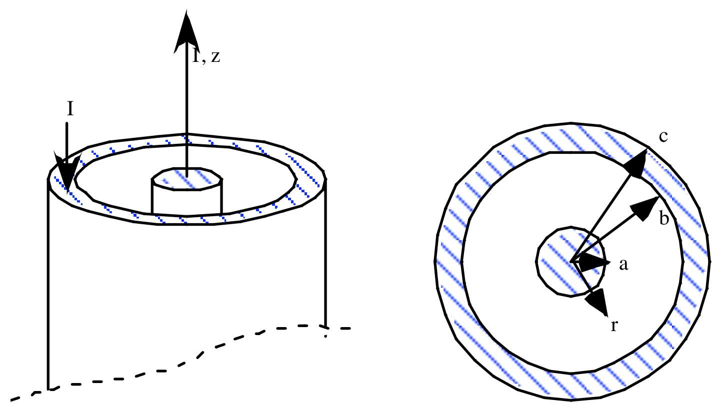
Problem 1.

Equal and opposite current flows in the infinite coaxial cylindrical conductors shown below. Assume the current is uniformly distributed in the conductors.

(a) Find expressions for the magnetic field as a function of radius in all regions of space.

(b) Assuming a very thin outer conductor, so that you can ignore the contributions from that region, find the self-inductance per unit length of the system.



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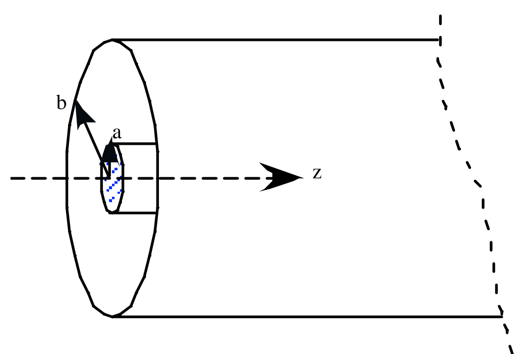
Problem 2.

Consider a transverse electromagnetic mode (TEM) wave traveling along a coaxial transmission line that consists of a long straight wire of radius a, surrounded by *a* conducting cylinder of radius *b*. Assume perfect conductors. (For a TEM, $E_z = 0$ and $B_z = 0$, while the wave vector $\boldsymbol{k}$ is directed along the *z*-axis.)

(a) Use Maxwell's equations and the resulting boundary conditions on the *E,B* fields to determine the $E(r,\theta,z,t)$ and $B(r,\theta,z,t)$ distributions in the cavity region, within an arbitrary scale factor.

(b) Derive an expression for the total time-averaged power flow traveling down the waveguide.



Note: useful formulas on the next pages

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

$$\textit{Gradient}: \qquad \nabla t = \frac{\partial t}{\partial s}\,\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\,\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\,\hat{\mathbf{z}}$$

$$\textit{Divergence}: \qquad \nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\textit{Curl}: \qquad \nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

$$\textit{Laplacian}: \qquad \nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

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Problem 3.

A spherically symmetric arrangement of charges has the following charge density distribution within a sphere of 1 m radius:

$$\rho_q(r) = \rho_0\left(1 - r^2\right)$$

for r in m, and ρ_0=3×10^{-6} C/m^3. Answer the following for conditions where the permittivity is the permittivity of free space, ε_0=8.85×10^{-12} F/m.

a) Find the electric field distribution (magnitude and direction) for all r (within the sphere and outside the sphere).

b) Find the electrostatic potential distribution for all r.

c) For an *asymmetric* distribution of positive charges within a sphere of 1m with the same net charge as in the above problem, qualitatively describe how you would estimate the maximum deviation of electrostatic potential 10 m from the center of the sphere from the result computed with the answer to part b).

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Problem 4.

A particle of mass *m* moving in a central force field executes the trajectory

$$r = r_0\, e^{\theta}$$

Find the potential energy function and force field as functions of *r* and the constants of the motion.

(Hint: consider conservation laws, and note that $\dot{r} = r\dot{\theta}$).

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Engineering Science

Problem 1. A small block has a mass of 0.1 kg and lies at rest on a frictionless surface. A narrow strip of metal of length l_s=30 cm and mass m_s=0.2 kg is also free to slide across the surface and has an initial speed of 1 m/s (translation without rotation). If the block lies in the path of the end of the strip of metal (see figure), what are the velocities of the block and the metal strip and the rate of rotation of the metal strip following an elastic collision? [The angular moment of inertia of the strip of metal is $m_s l_s^2 / 12$.]

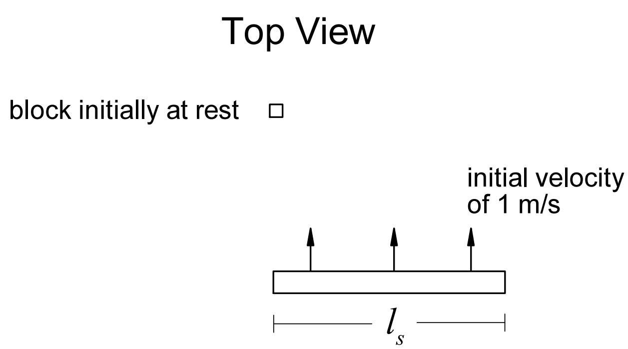

Diagram showing initial conditions.

Assume that the left end of the strip of metal makes contact with the block when the collision occurs.

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Problem 6.

A cylinder with radius a=20 cm and mass 0.2 kg (I=0.5ma^2) is placed on a 45° wedge that has a mass of 0.4 kg, as shown in the figure. The wedge is free to slide without friction on the horizontal surface, but the cylinder rolls without slip on the top surface of the wedge. With the acceleration due to gravity of 9.81 m/s^2 acting downward, find the acceleration of the wedge with respect to the horizontal surface and the acceleration of the center of mass of the cylinder with respect to the top surface of the wedge, being specific on the direction of each.