University of Wisconsin-Madison
Engineering Physics Department
Spring 2009 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1.

Use residue calculus to evaluate I.

$$I = \int_0^{2\pi} \frac{d\vartheta}{a^2 + \sin^2 \vartheta}$$

for $a > 0$ and real.

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Problem 2.

Give the general solution to the following differential equation

$$\frac{dy}{dx} = \frac{y^2 + 2xy}{x^2}$$

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Problem 3.

Find the general solution to the matrix equation

$$\frac{dX}{dt} = AX$$

where

$$X = \begin{pmatrix} x_1(t) & x_2(t) \\ x_3(t) & x_4(t) \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$$

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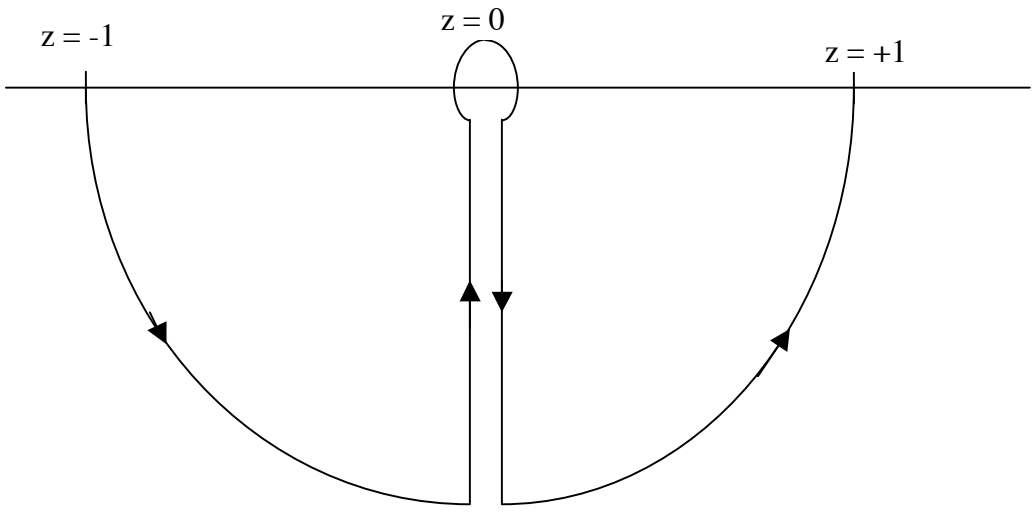
Problem 4.

Calculate the value of the following integral in the complex plane for all integer values of n with $n \geq 1$ for the two integration paths specified.

$$I = \int_{-1}^{1} dz \frac{1}{z^n}$$

a) (50%) The integration path is along the semi-circular orbit $z = e^{i\theta}$ with $0 \leq \theta \leq \pi$.
b) (50%) Along the integration path shown in the figure below



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Problem 5.

Determine the complete solution $y(x)$ to the ordinary differential equation

$$\left(1+x^2\right)\frac{dy}{dx} = x^3 y$$

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Problem 6.

Given the matrix equation $Ax = y$, where A is of dimension $m \times n$ for each of the cases listed below, determine under what condition, there is:

a. an exact and unique solution

b. no exact solution

c. an infinite number of exact solutions

Be Specific! Each Case is worth 1/3.

Case 1: m = n

Case 2: m < n

Case 3: m > n