University of Wisconsin-Madison
Engineering Physics Department
Fall 2013 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

Some potentially useful identities

$$\hat{f}(s) = L[f(t)] = \int_0^\infty dt\, e^{-st} f(t)$$

$$L[\frac{t^{n-1}}{(n-1)!}] = \frac{1}{s^n}, \qquad L[\frac{2^n t^{n-1/2}}{1*3*5*...*(2n-1)\sqrt{\pi}}] = s^{-(n+1/2)}$$

$$L[e^{-at}] = \frac{1}{s+a}, \qquad L[te^{-at}] = \frac{1}{(s+a)^2}$$

$$L[\sin(at)] = \frac{a}{a^2+s^2}, \qquad L[\cos(at)] = \frac{s}{a^2+s^2}$$

$$L[\frac{e^{at}-e^{bt}}{a-b}] = \frac{1}{(s-a)(s-b)}, \qquad L[\frac{ae^{at}-be^{bt}}{a-b}] = \frac{s}{(s-a)(s-b)}$$

$$L[e^{at}\cos bt] = \frac{s-a}{(s-a)^2+b^2}, \qquad L[e^{at}\sin bt] = \frac{b}{(s-a)^2+b^2}$$

$$L[\sinh at] = \frac{a}{s^2-a^2}, \qquad L[\cosh at] = \frac{s}{s^2-a^2}$$

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Problem 1.

By making use of integration around suitably indented contours in the complex plane, evaluate the following real integral:

$$I = \int_{-\infty}^{+\infty} \frac{\sin x}{x(x^2 + a^2)}\, dx, \qquad a \text{ real and positive}$$

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Problem 2. Find the general solution to:

$$\frac{d^2 y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

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Problem 3.

Consider the linear equation $Ax = b$ where

$$A = \begin{pmatrix} 3 & -1 & 3 & 1 \\ 1 & 1 & 1 & -1 \\ 3 & -2 & 3 & 2 \end{pmatrix}$$

$$b = \begin{pmatrix} \alpha \\ 10 \\ 0 \end{pmatrix}$$

Use linear algebra techniques to determine for which values of the constant α this equation has:

a) No solutions (30%)
b) A unique solution --- find the solution (30%)
c) An infinite number of solutions --- write the solution as a sum of column vectors (40%)

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Problem 4.

Consider the following function in the complex plane

$$f(z) = \frac{z-1}{z-i}$$

a) (50%) Writing *f(z)* as the following power series about the point $z = z_0 = 2 + i$

$$f(z) = \sum_{n=0}^{\infty} c_n (z-2-i)^n = \sum_{n=0}^{\infty} c_n (z-z_0)^n$$

determine the coefficients c_n and the radius of convergence of the series

b) (50%) Evaluate the following integral:

$$I_N = \oint \frac{dz}{(z-2-i)^N} f(z)$$

where the integration path is a circle of radius *0.001* centered at $z = 2 + i$ and N is a positive integer greater than unity.

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Problem 5.

Assume A is a real symmetric positive definite matrix.

a) (70%) Show that $\left(A^{1/2}\right)^{-1} = \left(A^{-1}\right)^{1/2}$, where $A^{1/2}A^{1/2} = A$.

b) (30%) If $A = \begin{bmatrix} 7 & 3 \\ 3 & 7 \end{bmatrix}$, compute $A^{1/2}$.

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Problem 6.

Find the general solution for $y(x)$ to the following ordinary differential equation.

$$\frac{d^2y}{dx^2} - \frac{3}{2x}\frac{dy}{dx} = x$$