University of Wisconsin-Madison
Engineering Physics Department
Spring 2006 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1.

Calculate the following integral in the complex plane. The integration path is the unit circle centered on $z = 0$.

$$I = \oint_{|z|=1} z^m e^{-\frac{1}{z}} dz$$

Here, m ≥ 0 is an integer.

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Problem 2.

The Laplace transform of *y(t)* is given by

$$Y(p) = \int_0^\infty dt\, e^{-pt} y(t).$$

Suppose Y(p) satisfies

$$Y(p) = K(p)L(p)M(p)$$

where *K(p), L(p)* and *M(p)* are the Laplace transforms of *k(t), l(t)* and *m(t)*, respectively,

a) Use the convolution theorem to construct an integral expression for *y(t)* in terms of *k(t)*, *l(t)* and *m(t)*.

b) Explicitly calculate *y(t)* for the special case

$$k(t) = t,$$
$$l(t) = \frac{1}{t + c},$$
$$m(t) = H(t - c),$$

where $c > 0$ is a constant and $H(t - c)$ is the Heaviside step function.

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Problem 3.

Consider the following matrix equation

$$Ax = b$$

where x is column vector of unknowns and b is a column vector.

For each of the following cases, first consider if a solution x exists. If it does, what is the dimension of the solution space for x

a) if $b = 0$ and A is a 23×19 matrix whose rank is 18?

b) if $b \neq 0$ and A is a 17×17 matrix with $\det A \neq 0$?

c) $b \neq 0$ and A is a 17×17 matrix with $\det A = 0$?

d) if A is a 17 X 17 matrix with $\det A \neq 0$ and b is a solution to the matrix equation

$$Bb = 0,$$

where $\boldsymbol{B}$ is a matrix whose rank is 4.

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Problem 4.

Solve the following equation subject to the boundary conditions: $y(1) = 2$, $y'(1) = 1$:

$$y'(x)y''(x) - x = 0$$

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Problem 5.

Evaluate the following integral, assuming *a* is real and positive:

$$I = \int_0^\infty \frac{dx}{x^4 + 4a^4}$$

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Problem 6.

Consider the vectors $\mathbf{P} = P_x\mathbf{i} - 2\mathbf{j} + P_z\mathbf{k}$, $\mathbf{Q} = \mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$, and $\mathbf{S} = -6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

(a) For $P_x = 4$, find the value of P_z that results in **P** being coplanar with **Q** and **S**.
(b) Find the values of P_x and P_z that result in **P** perpendicular to the plane defined by **Q** and **S**.
(c) For $P_x = 4$ and $P_z = 5$, find the minimum distance between the point (4,-2,5) defined by the head of **P** and the plane defined by **Q** and **S**.