University of Wisconsin-Madison
Engineering Physics Department
Spring 2011 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1.

If A, B, C, and D are $n \times n$, $n \times m$, $m \times n$, and $m \times m$ matrices, respectively, it can be shown that

$$\begin{vmatrix} A & -B \\ C & D \end{vmatrix} = |A||D + CA^{-1}B| \qquad \text{if } |A| \neq 0,$$

and

$$\begin{vmatrix} A & -B \\ C & D \end{vmatrix} = |D||A + BD^{-1}C| \qquad \text{if } |D| \neq 0$$

a) (5 points)

Using the properties of the identity matrix and the two equations given above, show that

$$|I_n + BC| = |I_m + CB|$$

where I_n and I_m are appropriately sized identity matrices.

b) (5 points)

Use the result in part (a) to show that

$$|A + BDC| = |A||D + DCA^{-1}BD| / |D|$$

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Problem 2.

Consider the second-order ODE initial value problem

$$m\ddot{y} + c\dot{y} + ky = 0$$
$$y(0) = a$$
$$\dot{y}(0) = b$$

where a and b are constants. Here, m, c and k are all positive real constants.

a) (2.5 points)

Rewrite the ODE as a system of first-order ODEs.

b) (2.5 points)

Use an eigenvalue analysis of the system matrix to identify a relation between the coefficients that leads to an oscillatory solution. What is the frequency of oscillation, and what is the decay rate?

c) (5 points)

What is the solution when the system is not oscillatory? Determine all possible solutions for all the cases when the contants m,k and c do not satisfy the condition determined in part b).

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Problem 3.

Given a real, positive parameter a, find the general solution $y(x)$ to the following differential equation:

$$(a+x)^2 \frac{d^2 y}{dx^2} - 2y = 1$$

[Hint: A change of variables may be useful.]

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Problem 4.

For parameters a, b and m real and positive, evaluate:

$$I = \int_{-\infty}^{+\infty} \frac{\cos mx}{(x+b)^2 + a^2} dx$$

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Problem 5.

Construct a Laurent series for the function f about $z = 0$ defined on the annulus $1 < |z| < 2$.

$$f(z) = \frac{3+i}{(z-i)(z+3)}$$

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Problem 6.

Consider the eigenvalue problem $AX=\lambda X$ where $A \neq 0$ is a real symmetric matrix. Are the following statements true? If so, provide a proof. If not, provide a counterexample where A is a *2x2* matrix.

a) (4 points) All eigenvalues are real.
b) (3 points) No two eigenvalues can be the same.
c) (3 points) 0 cannot be an eigenvalue