University of Wisconsin-Madison
Engineering Physics Department
Fall 2005 Qualifying Exams

Modern Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

Physical Constants

$m_e = 9.1 \times 10^{-31}$ kg $= 511$ keV/c^2

$e = 1.6 \times 10^{-19}$ C $\quad k = 1.6 \times 10^{-19}$ Joules / eV

$\hbar = 1.06 \times 10^{-34}$ J-sec $= 6.582 \times 10^{-16}$ eV-sec

$h = 4.136 \times 10^{-15}$ eV-sec

$c = 3 \times 10^{8}$ m / s

$c^2 = 931.5$ MeV / amu

$G = 6.67 \times 10^{-11}$ $m^3\, kg^{-1}\, s^{-2}$

Solar mass and radius: $M_s = 1.99 \times 10^{30}$ kg $\quad R_s = 6.96 \times 10^{8}$ m

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1. **Fission reaction**

Consider the uranium fission reaction with a thermal neutron in a fission reactor

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe^{*} + ^{96}_{38}Sr^{*} \xrightarrow[1\,s]{after} ^{139}_{54}Xe + ^{95}_{38}Sr + 2\left(^{1}_{0}n\right) + 7(\gamma)$$

Rest masses in amu.

^{235}U	^{1}n	^{140}Xe	^{96}Sr	^{139}Xe	^{95}Sr
235.043923	1.008665	139.921640	95.921680	138.918787	94.919358

The 2 fission neutrons have a total kinetic energy of 5.2 MeV.

The prompt gamma rays have a total energy of 6.7 MeV.

a) What is the prompt energy locally absorbed from the Xe and the Sr from this event?
b) What is the kinetic energy of the ^{140}Xe* product?
c) Describe qualitatively the expected energy distributions for the product neutrons and gammas.

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2. **Compton scattering**

In 1922, Compton observed that X-rays, with incident wavelength λ and frequency ν, scattered from electrons (assumed to be at rest) with an increase in their wavelength. He found that the scattered wavelength, λ', was described solely by the scattering angle of the photon, θ using the formula:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

a) Derive the relationship above using the assumption that the X-ray should be treated as a particle with linear momentum and energy.
b) If the detection limit for the change in wavelength is 1 part in 10,000 for X-rays backscattered at 180 deg, at what photon energy would one no longer be able to detect a change in wavelength?
c) Based on b), why does the wave model of light work reasonably well for describing scattering of a *visible* wavelength laser from electrons?

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3. **Gravitational redshift**

A prediction of general relativity is that time will be slower in a gravitational well. Specifically, if Δt_1 is a time interval measured by a clock where the gravitational potential is ϕ_1 , the same interval will be measured by another clock as Δt_2 where the potential is ϕ_2. This law is given by the relationship,

$$\frac{\Delta t_2 - \Delta t_1}{\Delta t_1} \cong \frac{1}{c^2}\left(\phi_2 - \phi_1\right)$$

(Gravitational potential is the potential energy per unit mass and so its relationship to local gravitational force is, $F = - m\, d\phi/dr$.)

a) Derive the equation above using simple time contraction from special relativity and the equivalence principle ("A homogenous gravitational field is equivalent to uniformly accelerated reference frame"). To do this, consider two clocks placed on a *slowly* rotating circular platform of radius r with constant rotation velocity ω. One clock is in the center of rotation and the other on the edge of the rotating platform. *Hint: Compare time differences of the clocks and assume local uniform centripetal acceleration*.
b) Estimate the redshift expected for terrestrial observation of H-α radiation (λ=656 nm) emitted from the surface of the sun. *Solar mass and radius are given in the table of constants*.

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4. **Gamma-ray recoil**

A free nucleus, with mass m, undergoes gamma decay from an excited state and emits a photon with energy E_γ as a result of a transition from E_i to E_f. We will examine the nuclear recoil energy.

a. Write the conservation laws for the gamma decay.

b. Obtain the first order approximation of the recoil energy by expressing E_γ in terms of $\Delta E = E_i - E_f$.

c. Taking $E_\gamma = 0.1$ MeV gamma ray and $m = 100$ amu, determine the recoil energy.

d. Compare the recoil energy to the energy width of the excited state given that the excited state lifetime is 1 ns.

e. Can the photon be absorbed by an identical free nucleus at rest? and why?

f. What happens if the nucleus is in an atom that is tightly bound to a solid?

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5. Radioactive dating

Determine the age of a mineral using the following information: the minerals contains N_P radioactive parents with decay rate λ, and N_D stable daughter nuclei. Suppose that no daughter nuclei were initially present in the mineral.

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6. Wave scattering

A plane wave representing a non-relativistic particle with energy E interacts with a potential step of ΔV. See diagram below

Case (1): $\Delta V = -3\,E$

Case (2): $\Delta V = +3/4\,E$

a) Find the reflected amplitude of the wave for both cases.

b) Compare the results for cases 1) and 2). Is this a general result or specific to the sizes of the potential steps?