University of Wisconsin-Madison
Engineering Physics Department
Fall 2006 Qualifying Exams

Modern Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

Contains energy levels table and log paper.

USEFUL CONSTANTS

$m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $k = 1.6\times10^{-19}$ Joules/eV

$\hbar = 1.06 \times 10^{-34}$ J-s = 6.582×10^{-16} eV-s, $c = 3 \times 10^8$ m/s

1 amu = 931.48 MeV, m_e=0.511 MeV, 1 barn = 10^{-24} cm^2

1 Ci (Curie) = 3.7×10^{10} disintegrations/s

USEFUL RELATIONS and EQUATIONS

$$P(\vec{r}) = \int \left|\psi\psi^*\right| dV \ , \ j = \frac{\hbar}{2mi}\left[\psi^* \frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right]$$

Trigonometric identities: $\cos(\theta \pm \phi) = \cos(\theta)\cos(\phi) \mp \sin(\theta)\sin(\phi)$,
$\sin(\theta \pm \phi) = \sin(\theta)\cos(\phi) \pm \sin(\phi)\cos(\theta)$ and $\tan(\theta \pm \phi) = \dfrac{\tan(\theta) \pm \tan(\phi)}{1 \mp \tan(\theta)\tan(\phi)}$

$\cosh^2(x) - \sinh^2(x) = 1$, $\cos(x) = \dfrac{(e^{ix} + e^{-ix})}{2}$, $\sin(x) = \dfrac{(e^{ix} - e^{-ix})}{2i}$

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Problem 1.

For the following nuclear transformations, give a brief description of the nuclear processes taking place and which, if any, nuclear particle (neutron, proton or electron) is being converted, captured, created or emitted. Also provide any mass or energy criterion for the process.

i) ${}^{40}_{19}K_{21} \rightarrow {}^{40}_{18}Ar_{22} + \_\_ \ldots$
ii) ${}^{54}_{25}Mn_{29} + \_\_ \rightarrow {}^{54}_{24}Cr_{30} + \_\_$
iii) ${}^{203}_{80}Hg^{*}_{123} \rightarrow e^{-} + {}^{203}_{80}Hg_{123}$

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Problem 2.

6.5 MeV neutrons are scattered by ^{9}Be initially at rest. The ^{9}Be nucleus has several excited states, the first at 3.36 MeV, a second at 5.96 MeV and a third at 7.37 MeV. Assuming that these are the only states of ^{9}Be that can be excited, what energy neutrons are detected at an angle of 60 degrees from the incoming neutron direction for the case of isotropic scattering in the laboratory reference frame (no angular preference for scattering in any direction)? Use integer masses. (Hint: this problem is more easily solved in the laboratory system than in the center-of-mass system).

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Problem 3.

A radioactive sample was produced by neutron bombardment in a nuclear reactor. After irradiation the radioactive decay of the sample was measured as a function of time with a high purity Germanium detector. The following table contains the data collected:

Time, [min]	[counts/s]
0	366
1	289
2	241
3	210
4	189
6	161
7	151
10	128
15	99
20	78
25	63
35	42
45	30

After several hours a constant background of 15 counts/s was observed. Compute the half-lives and initial activities of the radioactive isotopes in the sample. (Use the graph paper provided).

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Problem 4.

a. A free nucleus, with excited state mass m, gamma decays and emits a photon with energy E_g as a result of a transition from E_i to E_f. Determine the nuclear recoil energy. Express E_g in terms of $\Delta E = E_i$ to E_f.

b. Taking $E_g = 0.1$ MeV gamma ray, atomic mass $A = 100$, and assuming an excited state lifetime = 1 nsec, determine the recoil energy.

c. Show that the photon cannot be absorbed by a similar free nucleus.

d. How fast does a similar nucleus have to be spun in a centrifuge to make it possible to absorb the photon?

e. Show that the emitted photon can be absorbed if both the emitting and absorbing nuclei are very tightly bound to solids. (This is known as the Mössbauer effect.)

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Problem 5.

Consider the shell model of a nucleus (Fig. 5.6, given at the beginning of the exam).

a. Give the reason why the $1d_{5/2}$ level is lower in energy than the $1d_{3/2}$ level.

b. Give an estimate of the spin and parity of the ground state of ${}^{57}_{26}Fe$ and ${}^{26}_{14}S$.

c. In the presence of a magnetic field, indicate the splitting of the ground state as a function of magnetic field.

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Problem 6.

a. Show $\frac{dE}{dt} = Fv$ is valid relativistically with $F = \frac{dp}{dt}$.

b. Show $T = E - mc^2$ reduces to the classical expression for kinetic energy in the limit of $v \ll c$.