University of Wisconsin-Madison
Engineering Physics Department
Spring 2007 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1.

Use Laplace transform techniques to solve the following differential equation for $y(t)$

$$t\frac{d^2y}{dt^2} + (t-1)\frac{dy}{dt} - y = 0$$

subject to the conditions $y(0) = 0$, $y(1) = 1$.

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Problem 2.

Evaluate the following integral

$$I = \int_{-\infty}^{+\infty} dx \frac{e^{iax} + e^{-ibx}}{x^2 + 2x + 1 + c^2},$$

where $a > 0$, $b > 0$ and $c > 0$.

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Problem 3.

Consider the linear equation $\boldsymbol{Ax} = \boldsymbol{b}$, where $\boldsymbol{A}$ is the matrix given below, (containing a real constant c) and $\boldsymbol{x}$ and $\boldsymbol{b}$ are three-component vectors.

$$A = \begin{pmatrix} 2 & c & 0 \\ c & 3 & 2 \\ 0 & 2 & 2 \end{pmatrix}$$

For each of the two cases of the vector $\boldsymbol{b}$ below, explain whether and why any restrictions must be placed on c (and if so, determine these restrictions) so that the equation $\boldsymbol{Ax} = \boldsymbol{b}$ will have a nontrivial (that is, nonzero) solution for the vector $\boldsymbol{x}$**.** Explain whether and why this solution for $\boldsymbol{x}$ will be unique. Find *all* solution(s) for $\boldsymbol{x}$ in each case.

(i) $\boldsymbol{b} = \{2, -1, 2\}$

(ii) $\boldsymbol{b} = \{0,\ 1,\ 1\}$

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Problem 4.

For z a complex variable and z_o a complex constant, evaluate the following integral around an *arbitrary* closed curve C, when $z = 0$ and $z = z_o$ both lie inside C, for:

a) (2 points) $n = 0$
b) (8 points) for all integer values of $n > 0$:

$$\oint_C \frac{dz}{z^n (z - z_o)}.$$

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Problem 5.

An important class of problems in mathematical physics involves Bessel functions, defined by a differential equation of the form,

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + (x^2 - p^2)y = 0$$

where p is real and non-negative.

(a) (7 points) Beginning from an assumed form:

$$y(x) = \sum_{k=0}^{\infty} B_k x^{2k+s}$$

find the required values of s and the recursion relation for the coefficients B_k.

(b) (3 points) For what values of p do your results from part a) provide two linearly independent solutions?

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Problem 6.

Let

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ 1 & 3 & 8 & 9 \\ 1 & 3 & 2 & 2 \end{bmatrix}$$

(a) (6 points) Find $\mathbf{A}^{-1}$ by directly employing row operations on **A.**

(b) (4 points) $\mathbf{A}^{-1}$ can alternatively be computed from det(**A**) and adj(**A**). Verify that the first column of $\mathbf{A}^{-1}$ from part (a) is consistent with this alternative method of calculation.