University of Wisconsin-Madison
Engineering Physics Department
Fall 2011 Qualifying Exams

Classical Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

$\mu_0 = 4\pi \times 10^{-7}$ H/m $\qquad \varepsilon_0 = 8.854 \times 10^{-12}$ F/m

Center of mass moments of inertia:

$I_{disk} = \frac{1}{2}MR^2 \qquad I_{sphere} = \frac{2}{5}MR^2 \qquad I_{rod} = \frac{1}{12}ML^2$

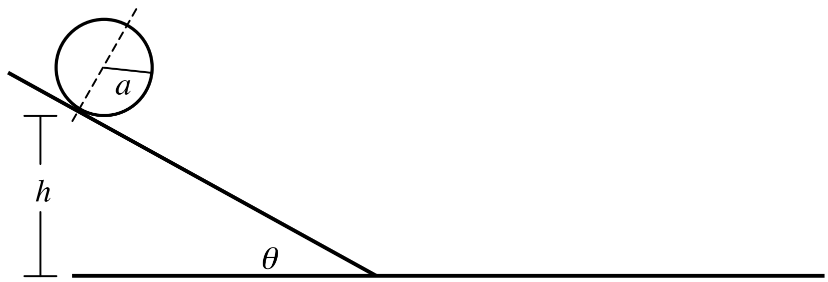
$I = Mk^2$ (where k is the radius of gyration)

Student No.________________

Engineering Physics Department
Fall 2011 Qualifying Exams

Classical Physics

Problem 1.
A thin cylindrical shell of radius a and mass M rolls without slip down an inclined plane, starting from rest from height h, as shown in the figure. The shell contacts a rough horizontal surface at the bottom of the plane and continues to roll without slip.



a) (5 points) What is the angular velocity (rate of rotation in rad/time) of the shell just prior to contact with the horizontal plane?

b) (5 points) What is the angular velocity of the shell after contact with the horizontal surface?

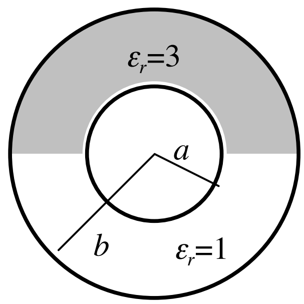
Express your answers in terms of a, h, the gravitational constant g, and the angle of inclination of the plane θ.

Student No.________________

Engineering Physics Department
Fall 2011 Qualifying Exams

Classical Physics

Problem 2.
A capacitor is formed from a spherical conductor with outer radius a, surrounded by a spherical conducting shell of inner radius $b>a$. As shown in the figure below, one hemisphere of the gap between the conductors is filled with a linear isotropic dielectric material of relative permittivity $\varepsilon_r=3$.



Cross-section of spherical capacitor.

a) (6 points) What is the capacitance of this configuration in terms of a, b, and the permittivity of free space, ε_0?

b) (1 points) How much energy is stored by the capacitor when it is charged to V volts?

c) (3 points) What is the distribution of surface charge density (charge per unit area) on the inner conductor when the capacitor is charged to V volts?

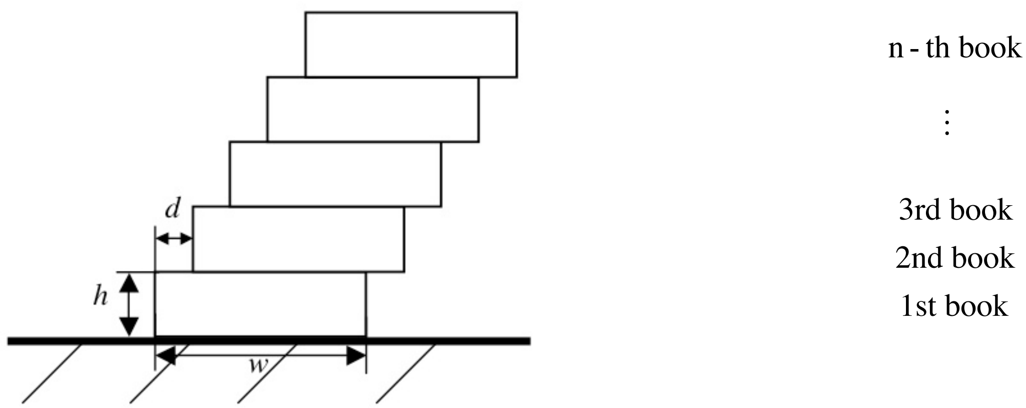
Student No.________________

Engineering Physics Department
Fall 2011 Qualifying Exams

Classical Physics

Problem 3.

A total of n books of uniform mass density ρ, width w, height h and length L are stacked on top of each other. Each book is misaligned, in the direction of w, by the same distance d with respect to the book on its top and the book on its bottom. What is the maximum d beyond which this leaning tower of book would fall? [The figure below shows the example of n = 5.]



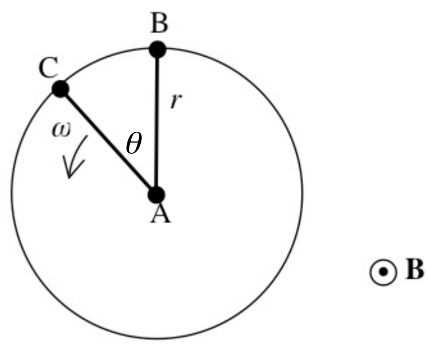
Student No.________________

Engineering Physics Department
Fall 2011 Qualifying Exams

Classical Physics

Problem 4.

Rods AB and AC are in electrical contact with each other through a pin at point A. The opposite ends of the rods (points B and C) are in electrical contact with a circular loop of wire that is centered at point A. Rod AB is fixed, rod AC rotates about point A with angular velocity ω, and the rods and circular loop remain coplanar. This configuration is immersed in an orthogonal magnetic field **B**. The wire and the rods have a resistance per unit length ρ, sufficiently large that self-inductance effects from the current in the wire can be ignored. Calculate the currents flowing in the wire and the rods as a function of time t in the interval $0 < t < 2\pi/\omega$, where the angle made by the rods satisfies $\theta = \omega t$, and make a sketch of their directions.



Student No.________________

Engineering Physics Department
Fall 2011 Qualifying Exams

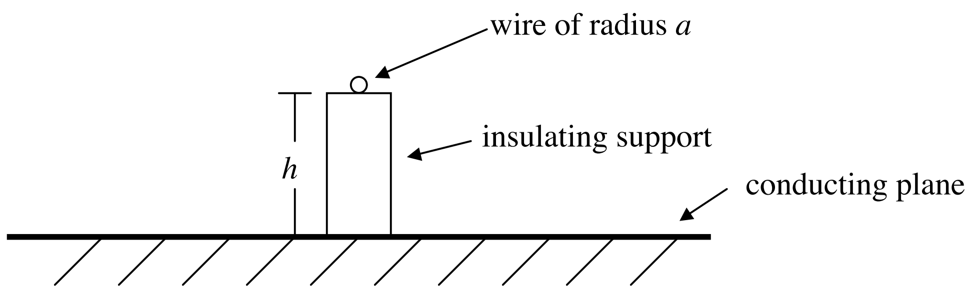
Classical Physics

Problem 5.

Insulating mechanical supports hold the bottom of a long straight wire a distance h above an electrically conducting horizontal plane. The wire has linear mass density of σ (mass per unit length) and radius a, where $a \ll h$. An external circuit induces current (I) along the length of the wire, and the weight of the leads is negligible relative to the weight of the wire. (The length of the wire L is much larger than h; $L \gg h$.)

a) (6 points) Find an expression for the current required to just lift the wire from its supports in terms of h, σ, g (gravitational acceleration), and μ_0 (permeability of free space). Assume that the wire remains oriented in a horizontal plane.

b) (4 points) Determine the total magnetic flux per unit length along the wire for the conditions of part a).



Cross-section view of the configuration.

Student No._________________

Engineering Physics Department
Fall 2011 Qualifying Exams

Classical Physics

Problem 6.
A pendulum consists of a mass (m) at the end of a massless inextensible cable of length l modulated in time by a rotating mechanism (pictured) of radius r, angular frequency ω, and initial phase ϕ, according to $l(t)=l_0+ r \sin(\omega t+\phi)$. Consider the limit of small oscillations ($\theta \ll \pi$) and small modulation ($r \ll l$). Formulate the equation of motion without solving it.