University of Wisconsin-Madison
Engineering Physics Department
Spring 2005 Qualifying Exams

# Modern Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

**SHOW ALL YOUR WORK.**
**WRITE ON THE FRONT PAGES ONLY.**

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, on the first six solutions you provided will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

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**Problem 1.** A 5.0 MeV proton beam is available to produce neutrons from the reaction ${}^{7}_{3}Li(p,n){}^{7}_{4}Be$. A neutron beam of 1.75 MeV is needed for a particular experiment.

(a) What is the Q-value of the reaction?

(b) In what direction relative to the incident beam should we look to find 1.75 MeV neutrons? Assume that the target nucleus is at rest.

The atomic masses of the nucleons in amu are:

| | |
|---|---|
| M(${}^{7}_{3}Li$) = 7.016003 | M(${}^{7}_{4}Be$) = 7.016923 |
| M($n$) = 1.008665 | M($p$) = 1.007276 |

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**Problem 2.** Consider the simple radioactive decay chain depicted below. Iodine decays to Xenon, which in turn decays to Cesium. Determine the time at which the Xenon concentration has reached its maximum value. Assume the initial concentrations of Xenon and Iodine are $X_0$ and $I_0$ and the corresponding decay constants are $\lambda_X$ and $\lambda_I$. Cesium is a stable nuclide and its initial concentration is zero.

$$I \xrightarrow{\lambda_I} Xe \xrightarrow{\lambda_{Xe}} Cs$$

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**Problem 3.** The wave function in a one-dimensional, infinite square well is

$$\psi_n(x) = \sqrt{\frac{2}{L}}\ \sin\frac{n\pi x}{L} \quad ,n = 1, 2, 3, ...$$

where the well (zero potential) extends from $x = 0$ to $x = L$ and the potential is infinite elsewhere. The quantity $n$ is the quantum number. For the ground state wave function,

(a) Find the expectation values for the particle position, $x$, namely $\langle x \rangle$.

Hint: $\int x \sin^2(ax)dx = \frac{x^2}{4} - \frac{x\sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2}$.

(b) Find the expectation values for the particle momentum, $p$, namely $\langle p \rangle$ and its variance, namely $\langle p^2 \rangle$.

(c) What is the physical significance of the variance of the momentum?

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**Problem 4.** The Zeeman effect is an experimentally measured splitting of spectral emission lines when an external magnetic field is applied.

(a) For a particle of charge $q$ and mass $m$ in circular motion, derive the expected change in angular frequency caused by the application of a magnetic field perpendicular to the plane of the particle's motion. Assume that the relative change in angular frequency is small.

(b) Applying quantum mechanics, what is the expected change in the energy level of the particle?

(c) Show that the expected energy differences caused by the application of a 1 T field to a hydrogen atom are a slight perturbation to the unperturbed energies.

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**Problem 5.** Let's consider the Debroglie wavelength of an electron.

(a) If an electron is accelerated through a potential of 10 V, what is the electrons wavelength?

(b) Electron diffraction (Bragg scattering) measurements are carried out with single crystal nickel, which has inter-atomic spacing of $2.15 \times 10^{-10}$ m. Estimate the voltage, $V$, necessary to maximize the scattered electron intensity near 50 degrees scattering angle if the electron beam is incident normal to the surface of the crystal.

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**Problem 6.** ${}^{17}_{10}Ne(-\frac{1}{2})$ decays to an excited state of ${}^{17}_{9}F$ through the emission of positrons. ${}^{17}_{9}F$ decays to the ground state of ${}^{16}_{8}O$ emitting a 10.6 MeV proton.

(a) For positron emission to take place, one type of nucleon in the nucleus is converted to another type. Write down the nucleon conversion process for the positron emission.

(b) What is the maximum energy of the positrons emitted in the decay to the ${}^{17}_{9}F$ excited state?

(c) What is the degree of forbiddenness if the positrons decay to the $(-\frac{3}{2})$ excited state of ${}^{17}_{9}F$?

The atomic masses of the nucleons in amu are:

| | |
|---|---|
| $M({}^{17}_{9}F) = 17.002095$ | $M({}^{17}_{10}Ne) = 17.017690$ |
| $M({}^{16}_{8}O) = 15.994915$ | $M(p) = 1.007276$ |

USEFUL CONSTANTS

$m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $k = 1.6\times10^{-19}$ Joules / eV

$\hbar = 1.06 \times 10^{-34}$ J-sec, $c = 3 \times 10^{8}$ m / s

1 amu = 931.48 MeV, $m_e$=0.511 MeV