University of Wisconsin-Madison
Engineering Physics Department
Fall 2008 Qualifying Exams

Modern Physics

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1.

Consider a particle in a one-dimensional infinite square well of width L that is centered at the origin x=0. [You need not derive the normalized wavefunctions if you know them.]

a. (3 pts) What are the values of the normalized eigenfunctions of the two lowest energy states ($\psi_1(0)$ and $\psi_2(0)$) evaluated at x=0?

b. (3 pts) What are the values of $\langle x\rangle$ for states n=1 and n=2?

c. (4 pts) What are the values of $\langle x^2\rangle$ for the states n=1 and n=2?

Some potentially useful integrals.

$$\int x\sin(x)dx = \sin(x) - x\cos(x)$$

$$\int x\sin^2(x)dx = \frac{x^2}{4} - \frac{x}{4}\sin(2x) - \frac{1}{8}\cos(2x)$$

$$\int x^2\sin^2(x)dx = \frac{x^3}{6} - \frac{x}{4}\cos(2x) - \frac{1}{4}\left(x^2 - \frac{1}{2}\right)\sin(2x)$$

$$\int x\cos(x)dx = \cos(x) + x\sin(x)$$

$$\int x\cos^2(x)dx = \frac{x^2}{4} + \frac{x}{4}\sin(2x) + \frac{1}{8}\cos(2x)$$

$$\int x^2\cos^2(x)dx = \frac{x^3}{6} + \frac{x}{4}\cos(2x) + \frac{1}{4}\left(x^2 - \frac{1}{2}\right)\sin(2x)$$

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Problem 2.

(10 points) The period of a pendulum is measured to be 3.00 s in the reference frame of the pendulum. What is the period when measured by an observer moving at a speed of 0.950*c* relative to the pendulum? [*c* is the speed of light.]

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Problem 3.

The energy spectrum for the decay particles in beta decay can be approximated by

$$N(T_e) = \frac{K}{c^5}\sqrt{T_e^2 + 2T_e m_e c^2}\,(Q - T_e)^2\,(T_e + m_e c^2)$$

where T_e is the beta energy, m_e is the beta rest mass, and Q is the peak beta energy.

a. (2 points) What is an approximation to this distribution in the limit $T_e \gg m_e c^2$?

b. (2 points) Sketch the energy distribution in part (a) including relevant features.

c. (6 points) What is the average beta energy for the approximate distribution in part (b)? How does it compare to the commonly accepted average energy of $Q/3$?

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Problem 4.

Consider the following alpha decay.

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha$$

a. (5 points) Given the Q-value of this decay process, show that if $Q \ll M_\alpha c^2$, then the momentum of the alpha particle is

$$p = \sqrt{\frac{2M_\alpha Q}{1 + \frac{M_\alpha}{M_Y}}}$$

b. (5 points) After the decay, the alpha particle moves into a mass spectrometer, which uses a uniform magnetic field to measure the mass of the particle. Assuming the magnetic field is perpendicular to the particle's momentum vector, we can write the force on the particle as

$$\vec{F} = q\vec{v} \times \vec{B}$$

where q is the charge of the particle, B is the magnetic field, and v is the particle velocity. Using this relation, find the mass of the alpha particle in terms of the radius of the resulting motion, Q, and B.

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Problem 5.

The Potassium isotope ^{40}K is naturally present at very low levels in lava flowing out of volcanoes. ^{40}K beta decays to ^{40}Ar with a $1.28\times10^{9}\ y$ half-life. Argon gas escapes from the flowing lava. Once the lava solidifies, any argon produced in the decay of ^{40}K is trapped inside and cannot escape.

a. (3 points) Write down the decay equation of beta decay of ^{40}K including all terms.

b. (7 points) A geologist brings you a piece of solidified lava in which you find the ($^{40}Ar/\,^{40}K$) ratio to be 0.12. What is the age of the rock?

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Problem 6.

Explain the process of alpha particle decay of heavy nuclei by answering the following questions.

a. (2 pts) Why do heavy nuclei decay with alpha particles ${}_{2}^{4}He$ and not, for instance, deuterons ${}_{1}^{2}H$ or tritons ${}_{1}^{3}H$?

b. (2 pts) Draw a graph of the potential energy inside and around a heavy nucleus that describes the α-decay process. Describe the functional form of the graph where appropriate.

c. (2 pts) Draw a line on the graph in part (b) representing the Q-value of the α decay energetics.

d. (2 pts) Using the graph from parts (b) and (c) identify the relevant parameters that determine the half-life of the α-decay. Give a short description referring to the graph as necessary. You may wish to label some additional points on the graph to use in your description.

e. (2 pts) Assume that two isotopes of the element ^{98}Cf (A~250), differing in neutron number by 4, have identical alpha decay Q-values. Which isotope should have the longer half life and why? You can refer to the previous results in stating your answer.