University of Wisconsin-Madison
Engineering Physics Department
Spring 2013 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1.

Determine the most general solution to the differential equation

$$x\frac{dy}{dx} + y = xy^3$$

Hint: Consider a change of variables $v = v(y)$.

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Problem 2.

For the matrix shown:

$$A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

(a) (50%) Compute the eigenvalues and eigenvectors. Show all of your work.

Hint: The eigenvalues are integers.

(b) (50%) Use eigenvector decomposition to invert the matrix.

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Problem 3.

Consider upper or lower triangular matrices.

(a) (30%) Prove that the eigenvalues of a triangular matrix are the entries on the main diagonal.

(b) (60%) Show that if λ is an eigenvalue of $\mathbf{A}$, then λ^2 is an eigenvalue of $\mathbf{A}^2$; more generally, show that λ^n is an eigenvalueof $\mathbf{A}^n$ if n is a positive integer.

(c) (10%) Use the previous results to find the eigenvalues of $\mathbf{A}^9$, where $\mathbf{A}$ is:

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 7 & 11 \\ 0 & -1 & 3 & 8 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

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Problem 4. The form of the Cauchy-Riemann equations in polar coordinates is

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \quad ; \quad \frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$$

If $f(z) = u + iv$ is analytic, determine the following:

(a) (50%) v when $u = \frac{1}{r^2}\sin 2\theta$

(b) (50%) u when $v = r^3(1-4\cos^2\theta)\sin\theta$

Make sure you determine the *most general* form of u and v.

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Problem 5.

a) (50%) Find all $z = x + iy$ in the complex domain such that

$$\sin z = i.$$

b) (50%) What is the value of the following integral?

$$I = \frac{1}{2\pi i} \oint_{|z|=1} \frac{dz}{\sin z - i},$$

The integration contour is the unit circle centered at the origin.

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Problem 6.

Calculate the general solution to the differential equation

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} = -1.$$

Hint: Consider using reduction of order.