University of Wisconsin-Madison
Engineering Physics Department
Fall 2006 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, <u>NOT</u> ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1.

One of the eigenvectors of the eigenvalue problem $\boldsymbol{AX} = \lambda\boldsymbol{X}$ is

$$X = \frac{1}{3}\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

where the matrix A is

$$A = \begin{pmatrix} 2 & 0 & 1 \\ -4 & -1 & 7 \\ 4 & 3 & -1 \end{pmatrix}$$

a) Determine all of the eigenvalues.
b) Let $\boldsymbol{B} = (2 + 3i)\boldsymbol{A}$ (every element of the matrix $\boldsymbol{B}$ is 2 + 3i times the corresponding element in $\boldsymbol{A}$). How are the eigenvalues and eigenvectors of the problem $\boldsymbol{BX} = \lambda\boldsymbol{X}$ related to the corresponding eigenvalues and eigenvectors of the problem $\boldsymbol{AX} = \lambda\boldsymbol{X}$?

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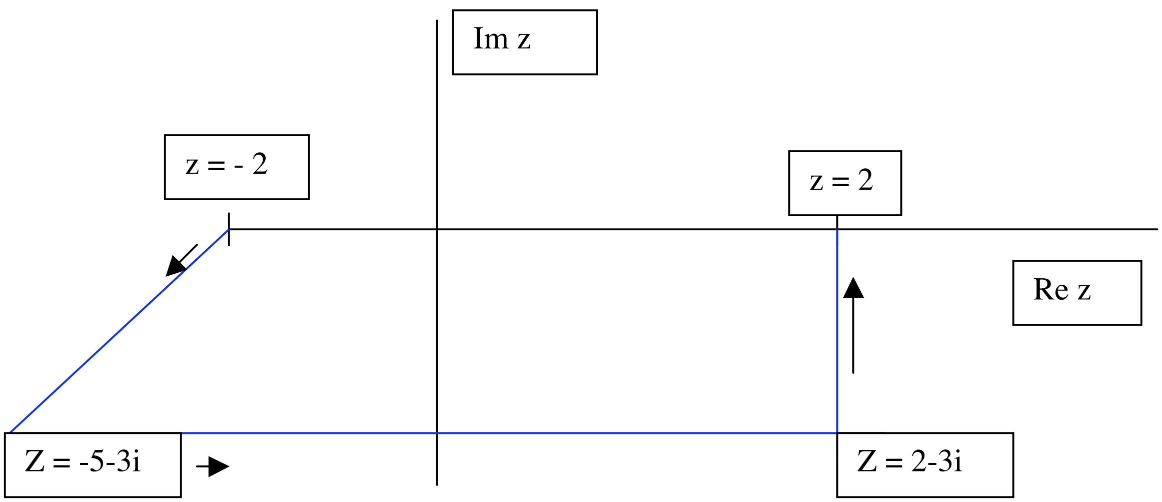
Problem 2.

Consider the integral in complex space

$$I = \int_{-2}^{2} \frac{dz}{1+z^2}$$

Determine the integral if the integration path is:

a) Along the real axis $z = x; -2 \le x \le 2$
b) On the semi-circle $z = 2e^{i\theta}; 0 \le \theta \le \pi$
c) Along the path $\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3$. Γ_1 : z = -2 –(3 + 3i)t; $0 \le t \le 1$, Γ_2: z = -5- 3i +7t; $0 \le t \le 1$, Γ_3: z = 2 – 3i +3it; $0 \le t \le 1$.



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Problem 3.

Construct a solution for y(t) for all $t \geq 0$ where y(t) satisfies

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} = 3\delta(t-2) - 3\delta(t-3)$$

with the initial conditions $y(0) = 0$, $dy/dt(0) = 0$. Here $\delta(t - \tau)$ is the Dirac Delta function.

Sketch the solution.

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Problem 4. Consider the differential equation,

$$x^2 \frac{d^2 y}{dx^2} + (1 + 2x)\frac{dy}{dx} = 0$$

Solve for the general solution of y(x).

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Problem 5. By making use of suitably indented contours in the complex plane, evaluate the following integral:

$$\int_{-\infty}^{+\infty} \frac{\sin x}{x(x^2 + a^2)} dx, \quad (a \text{ is real and positive})$$

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Problem 6. Consider the matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$. Find the matrix $\boldsymbol{P}$ that renders $\boldsymbol{A}$ orthogonally diagonalizable. Find the corresponding diagonal matrix $\boldsymbol{D}$.