University of Wisconsin-Madison
Engineering Physics Department
Fall 2008 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____

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Problem 1.

Given two $n \times n$ matrices A and B, where it is known that A is nonsingular, prove that the eigenvalues of the matrix products AB and BA are the same, even if $AB \neq BA$.

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Problem 2.

Solve the following boundary value problem for $y(x)$:

$$x^2 \frac{d^2 y}{dx^2} - 3x\frac{dy}{dx} + 4y = \ln(x) \quad , \quad x > 1$$

where

$$y(1) = 0, \qquad \frac{dy}{dx}(1) = 0$$

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Problem 3.

Consider the linear transformation $\mathbf{T}: \mathbf{R}^4 \rightarrow \mathbf{R}^4$, which maps a vector $\mathbf{V}$ according to:

$$\mathbf{T}\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \begin{bmatrix} 2c \\ a + c \\ b - 2c \\ d \end{bmatrix}$$

Find the eigenvalues and corresponding eigenvectors of $\boldsymbol{T}$.

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Problem 4.

Evaluate the integral:

$$I = \int_0^{2\pi} \frac{d\theta}{A + B\cos\theta}$$

Here A and B are both real and positive with $A > B$.

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Problem 5.

Consider the following Sturm-Liouville problem in the domain $0 \le x \le a$.

$$\frac{d^2y}{dx^2} + \lambda y = 0,$$

where the eigenvalues λ are determined by the boundary conditions

$$a\frac{dy}{dx}(0) = y(0),$$

$$a\frac{dy}{dx}(a) = -\beta^2 y(a).$$

Here, a and β^2 are positive constants ($a > 0$, $\beta^2 > 0$).

a) Show that the eigenvalues are solutions to the transcendental equation (5 points)

$$\tan(\sqrt{\lambda}a) = \frac{\sqrt{\lambda}a(1+\beta^2)}{\lambda a^2 - \beta^2}$$

b) For $\beta < \pi/2$, show that the smallest eigenvalue satisfies $\beta^2 < \lambda_1 a^2 < \pi^2/4$. (2.5 points)
c) For large λ ($\lambda a^2 >> 1$, $\lambda a^2 >> \beta^2$), provide an approximate expression for the discrete eigenvalues. (2.5 points)

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Problem 6.

a) Find an expression for all values in the complex plane $z = x + iy$ that satisfy the equation (8 points).

$$\sin(z) = 2i$$

b) Sketch the location of these points in the complex plane (2 points).