University of Wisconsin-Madison
Engineering Physics Department
Fall 2009 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

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Problem 1.

Evaluate the following integral using complex analysis and the residue theorem

$$I = \int_0^\infty dx \frac{1}{(i + x^2)^2}$$

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Problem 2.

Consider the following set of experimental data for parameters *a* and *b*

a	1	2	3	4	5
b	1.8	3.2	3.9	4.9	6.2

We want to fit this data to a linear polynomial model given by

$$b = x_1 a + x_2$$

where x_1 and x_2 are constants to be determined.

a. (2 points) Does an exact solution exist for x_1 and x_2? Why or why not?

b. (7 points) Derive an approximate solution that will minimize the Euclidean norm of the error in the fit of the data.

c. (1 point) For the data given, what matrix equation does your solution for x_1 and x_2 actually satisfy?

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Problem 3.

Find the general solutions for $y(x)$ and $z(x)$ that satisfy

$$\frac{d^2y}{dx^2} = z + x$$

$$\frac{d^2z}{dx^2} = y + 2x$$

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Problem 4.

Determine the general solution to the differential equation

$$\frac{dy}{dx} = \frac{x^2 - y^2}{x^2 + 5yx}$$

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Problem 5.

In the following matrix equations, A and B are nxn matrices of constants

$$\frac{dX}{dt} = AX$$

$$\frac{dY}{dt} = BX$$

The matrix A is diagonalizable, i. e., there exist a matrix P, such that

$$P^{-1}AP = D = \begin{pmatrix} \lambda_1 & 0 & ... & 0 \\ 0 & \lambda_2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \lambda_n \end{pmatrix}$$

Use this information to solve for Y given the initial conditions

$$X(t = 0) = X_0$$

$$Y(t = 0) = Y_0$$

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Problem 6.

a) (8 points) Calculate all $z = x + iy$ in the complex domain that satisfy the following equation.

$$\sinh z^2 = i$$

b) (2 points) Sketch the location of these points in the complex plane.