University of Wisconsin-Madison
Engineering Physics Department
Spring 2014 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

Some potentially useful identities

$$\hat{f}(s) = L[f(t)] = \int_0^\infty dt\, e^{-st} f(t)$$

$$L[\frac{t^{n-1}}{(n-1)!}] = \frac{1}{s^n}, \qquad L[\frac{2^n t^{n-1/2}}{1*3*5*...*(2n-1)\sqrt{\pi}}] = s^{-(n+1/2)}$$

$$L[e^{-at}] = \frac{1}{s+a}, \qquad L[te^{-at}] = \frac{1}{(s+a)^2}$$

$$L[\sin(at)] = \frac{a}{a^2+s^2}, \qquad L[\cos(at)] = \frac{s}{a^2+s^2}$$

$$L[\frac{e^{at}-e^{bt}}{a-b}] = \frac{1}{(s-a)(s-b)}, \qquad L[\frac{ae^{at}-be^{bt}}{a-b}] = \frac{s}{(s-a)(s-b)}$$

$$L[e^{at}\cos bt] = \frac{s-a}{(s-a)^2+b^2}, \qquad L[e^{at}\sin bt] = \frac{b}{(s-a)^2+b^2}$$

$$L[\sinh at] = \frac{a}{s^2-a^2}, \qquad L[\cosh at] = \frac{s}{s^2-a^2}$$

Student No.________________

Engineering Physics Department
Spring 2014 Qualifying Exams

Mathematics

Problem 1.

Consider the curve in the complex plane defined by:

$$\Gamma(t) = \sin(\frac{\pi t}{2}) + i\cos(\frac{3\pi t}{2}) \qquad -1 \leq t \leq 1$$

a) (25%) Sketch this curve in the complex plane.

b) (75%) Calculate the value of the following integral where the integration path is along the parameterized curve *Γ(t)* from *t = -1* to *t = 1*.

$$I = \int_{\Gamma} \frac{dz}{z}$$

Student No._________________

Engineering Physics Department
Spring 2014 Qualifying Exams

Mathematics

Problem 2.

Calculate *p(t)* for all $t \geq 0$ that satisfies the coupled set of differential equations

$$\frac{d^2 p}{dt^2} = 7p + 6q - 3t^2$$

$$\frac{dq}{dt} = -p + t$$

subject to the initial conditions

$$p(0) = 20, \quad \frac{dp}{dt}(0) = 0, \quad q(0) = 0$$

Student No._________________

Engineering Physics Department
Spring 2014 Qualifying Exams

Mathematics

Problem 3.

A matrix equation $Ax = y$ is given by:

$$\begin{bmatrix} 1 & 0 \\ 2 & 2 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ -1 \end{Bmatrix}$$

a) (50%) Using a vector space argument, show that the system has no exact solution.

b) (50%) Derive a unique approximate solution which minimizes the magnitude of the resulting error vector $Ax - y$.

Student No.________________

Engineering Physics Department
Spring 2014 Qualifying Exams

Mathematics

Problem 4.

Consider the differential equation

$$\frac{dy}{dx} - e^{y} \cos x = 0$$

a) (80%) What is the general solution?

b) (20%) What is the particular solution that satisfies the boundary condition $y(0) = 0$?

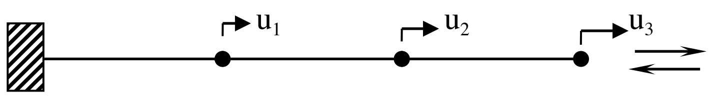
Student No.________________

Engineering Physics Department
Spring 2014 Qualifying Exams

Mathematics

Problem 5.

A simple model of the longitudinal vibrations of a continuous bar consists of the three-bar finite element model shown below:



When fixed boundary conditions are implemented, the modal analysis of this problem reduces to:

$$KD = \omega^2 MD$$

Here the eigenvalue ω is the natural frequency of vibration and the eigenvector **D** is a three element vector containing the three nodal displacements u_1, u_2 and u_3 shown in the figure above.

$$\mathbf{K} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathrm{M} = \begin{bmatrix} 2 & 1/2 & 0 \\ 1/2 & 2 & 1/2 \\ 0 & 1/2 & 1 \end{bmatrix} \quad ; \quad \mathrm{D} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

(a) (75%) Find the eigenvalues for this system of equations.
(b) (25%) One of the values for ω^2 is an integer. For ***only*** this eigenvalue, find the corresponding eigenvector.

Student No.__________________

Engineering Physics Department
Spring 2014 Qualifying Exams

Mathematics

Problem 6.

Consider the complex inverse function,

$$w(z) = \sin^{-1} z$$

(a) (80%) Using elementary function definitions, find an alternative functional expression for $w(z)$ that eliminates the inverse.
(b) (20%) Using the expression derived in part a), calculate w when $z = 1/2$.