University of Wisconsin-Madison
Engineering Physics Department
Spring 2015 Qualifying Exams

Classical Physics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

$\mu_0 = 4\pi \times 10^{-7}$ H/m　　$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

$G = 6.67 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2}$　　$M_e = 5.97 \times 10^{24}$ kg　　$R_e = 6{,}371$ km

Center of mass moments of inertia:

$I_{disk} = \frac{1}{2}MR^2$　　$I_{sphere} = \frac{2}{5}MR^2$　　$I_{rod} = \frac{1}{12}ML^2$

$I = Mk^2$ (where k is the radius of gyration)

Electron: $m_e = 9.1 \times 10^{-31}$ kg　　$q_e = -e = -1.6 \times 10^{-19}$ C

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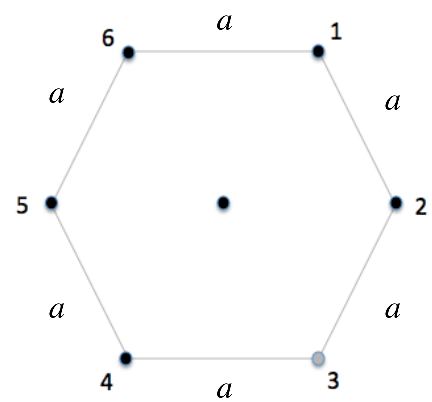
Problem 1. Five electrons are located on the corners of a symmetric hexagon with sides of length $a = 5\times10^{-5}$ m. A proton is located in the center of the hexagon. All charges lie in the same plane.



a) (4 points) What is the electrical potential of this charge distribution in the corner of the hexagon that does not have a charge?

b) (1 point) Assume that the original five electrons and the central proton remain fixed. How much work is needed to place a sixth electron at the corner that does not have one?

c) (1 point) Assume that the original five electrons and the central proton remain fixed. What is the final velocity of the sixth electron if it is released from the last corner of the hexagon?

d) (4 points) Again starting from the hexagon configuration with all six electrons and the proton, what is the final velocity of each of the six electrons if they are released simultaneously?

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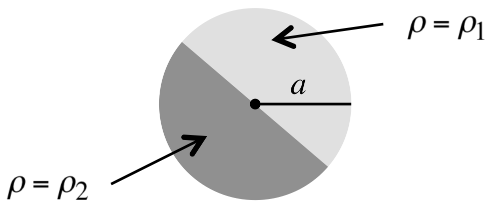
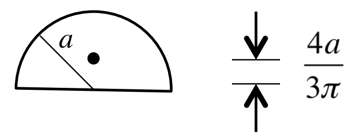
Problem 2. A solid cylinder of radius a and length L is composed of two different materials, one with mass density ρ_1 and the other with mass density ρ_2, as illustrated in the sketch below.



End view of cylinder.

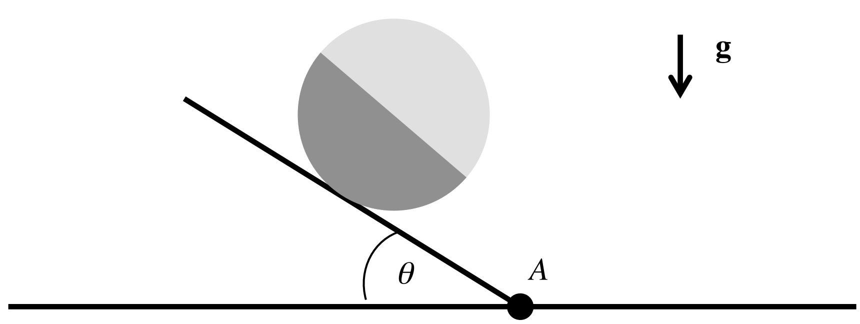


Note: the centroid of a semi-circle of radius a is $4a/3\pi$ above its straight side.

a) (2 points) What is the mass moment of inertia of the solid cylinder about its geometric axis in terms of L, a, ρ_1, and ρ_2?

b) (3 points) In terms of a, ρ_1, and ρ_2, how far is the center of mass of the cylinder from its axis?

c) (5 points) The cylinder is placed on a surface hinged at point A such that the cylinder's axis remains in the horizontal plane. In terms of the given parameters, what is the largest angle θ to which the surface can be inclined keeping the cylinder in static equilibrium? Assume that the cylinder does not slip over the surface and that the orientation of the cylinder adjusts to maintain static equilibrium as long as possible while the surface is lifted from horizontal. [Hint: consider potential energy.]



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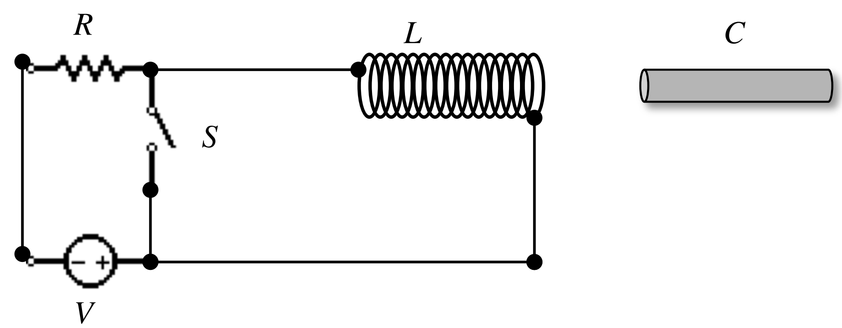
Problem 3. The inductor of inductance L in the circuit shown below is a tightly wound solenoid of N' turns per unit length. Its length is l_s, and its cross section is circular with radius $a_s << l_s$. The circuit also has a resistor of resistance R, a source of voltage V, and a switch S.

The electrically conducting cylinder C has length l_s and radius $a_c < a_s$, and it may be inserted into the inductor L. Assume that the cylinder is initially located outside the inductor for both parts of this problem, and consider all motion to be slow so as to avoid transient effects.



a) (5 points) The source applies a constant voltage V. Initially, the switch is open. The switch is then closed. If the conducting cylinder is then inserted into the inductor, how much current flows through the inductor in the final state? Express your answer in terms of the source voltage V, the resistance R, and the geometrical parameters of the inductor and the conducting cylinder.

b) (5 points) The source applies a constant voltage V with the switch left open. How much mechanical work is required to insert the conducting cylinder into the inductor? Use positive values to indicate mechanical work done on the electrical circuit and negative values to indicate work done by the electrical circuit.

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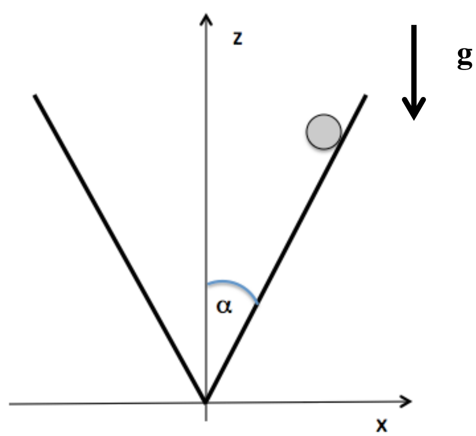
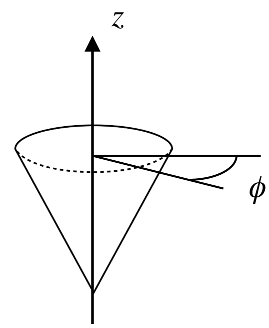
Problem 4. A small ball (negligible radius) of mass m slides without friction along the inside of a cone with opening angle 2α. In general it can move azimuthally and along the cone's surface in the $x-z$ plane.





a) (4 points) Find an expression for the Lagrangian of the motion in terms of the azimuthal angle (ϕ) and the height (z) of the ball above the point of the cone.

b) (4 points) Find an equation of motion for the height of the ball in terms of any constants of the motion.

c) (2 points) Show that circular orbits can be obtained from this equation of motion.

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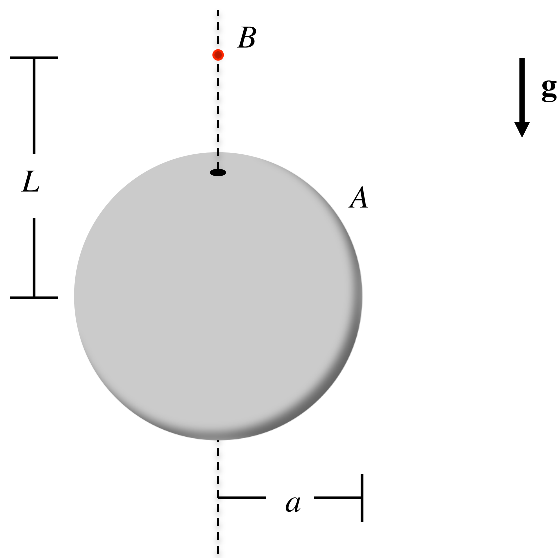
Problem 5. Spherical shell (A) has an electrical charge density of σ units of charge per unit area, where $\sigma > 0$, and the radius of the shell is a. The shell has two small holes, and the line passing through the two holes also passes vertically through the geometric center of the shell. Shell A is fixed in place.

A small object (B) of mass m_B has electrical charge $q_B < 0$. It is released from rest at a distance L above the geometric center of sphere A, i.e. along the line that passes through the two holes in A.



a) (6 points) Assuming that all forces except the Coulomb force and the force due to gravity are negligible and that object B freely passes through the holes in shell A, what charge density σ is needed to make object B stop and reverse its motion at a distance $2L$ below the geometric center of shell A? [Express your answer in terms of m_B, g, a, L, q_B and the permittivity of free space ε_0.]

b) (4 points) For the conditions of part a), what is the speed of object B as it passes through the hole at the bottom of shell A?

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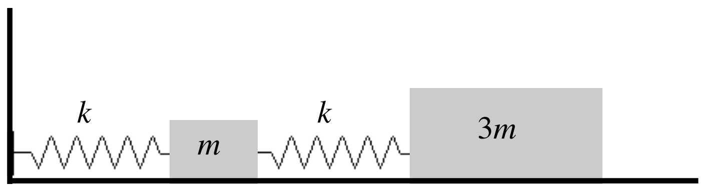
Problem 6. Two blocks of mass m and $3m$ lie on a frictionless surface and are coupled to each other by a spring of spring constant k. Another spring with the same spring constant k is attached to the smaller mass and a fixed wall.



a) (6 points) What are the frequencies of oscillation for this coupled system?

b) (4 points) For each mode, find the relative orientation and relative amplitude of the displacement of the two blocks (at least approximately) and briefly describe the motion.