University of Wisconsin-Madison
Engineering Physics Department
Spring 2008 Qualifying Exams

Mathematics

You must solve 4 out of the 6 problems.
Start each problem on a new page.

SHOW ALL YOUR WORK.
WRITE ONLY ON THE FRONT PAGES OF THE WORKSHEETS, NOT ON THE EXAM PAGES

Grading is based on both the final answer and work done in reaching your answer. All problems receive an equal number of points.

Clearly indicate which problems you want graded. If you do not indicate which problems are to be graded, the first four solutions you provide will be graded.

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______

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Problem 1.

Consider the following function in the complex plane:

$$f(z) = \frac{1}{\sqrt{2} - z}$$

a) (50%) Writing $f(z)$ as the following power series about the point $z = z_0 = e^{i\pi/4} = (1+i)/2^{1/2}$,

$$f(z) = \sum_{n=0}^{\infty} c_n (z - e^{(i\pi/4)})^n \,,$$

determine the coefficients c_n and the radius of convergence of the series.

b) (50%) Evaluate the following integral:

$$I_N = \oint \frac{dz}{(z - e^{(i\pi/4)})^N} f(z)$$

where the integration path is the circle of radius 10^{-3} centered at $z = z_0$ and N is a positive integer.

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Problem 2.

Calculate the solution to the following differential equation (80%)

$$\frac{d^2y}{dt^2} + v\frac{dy}{dt} = F_0[\delta(t-10) - \delta(t-20)]$$

where v and F_0 are constants, $\delta(t)$ is the Delta function and the initial conditions are $y(0) = dy/dt(0) = 0$. Sketch the solution (20%).

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Problem 3.

Use the method of variation of parameters to determine the general solution of the second-order differential equation:

$$\frac{d^2 y}{dx^2} - 4y = e^x$$

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Problem 4.

Given the linear system of equations $Ax = y$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \end{bmatrix} \qquad y = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

a) (40%) Determine the null space of the matrix A.

b) (30%) Use the result of part (a) to construct the general solution for the vector x.

c) (30%) Find the solution with the smallest Euclidean length for the vector x.

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Problem 5. Given the matrix **A** below, determine if **A** is diagonalizable. If so, find a matrix **P** that diagonalizes **A** and determine $\mathbf{P}^{-1}\mathbf{AP}$. If not, find the eigenvalues and eigenvectors of **A**.

$$\mathbf{A} = \begin{bmatrix} 19 & -9 & -6 \\ 25 & -11 & -9 \\ 17 & -9 & -4 \end{bmatrix}$$

Hint: You should find the eigenvalues are integers.

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Problem 6. Use residue calculus to evaluate the following integral:

$$I = \int_0^\infty \frac{x^3 \sin mx}{x^4 + 4a^4}\, dx$$

where a and m are both real and positive ($a > 0, m > 0$).