# Safe Reinforcement Learning by Imagining the Near Future

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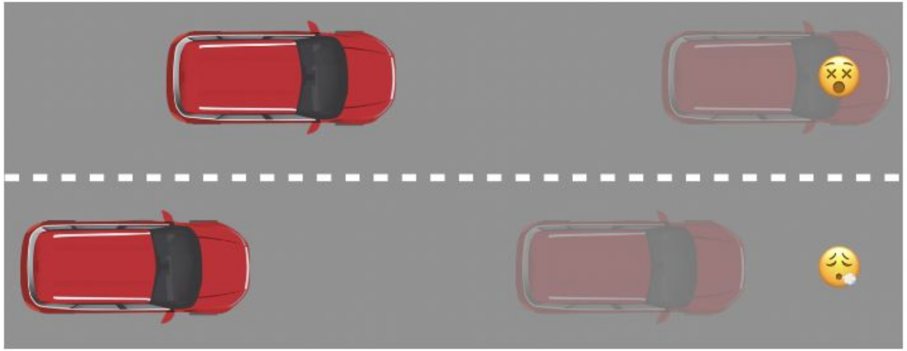


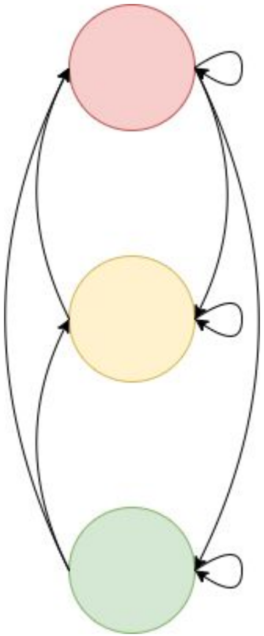
## In a nutshell

If:

- irrecoverable and unsafe states are known
- there exists a safe policy

we can guarantee choosing a safe policy

# Irrecoverable state

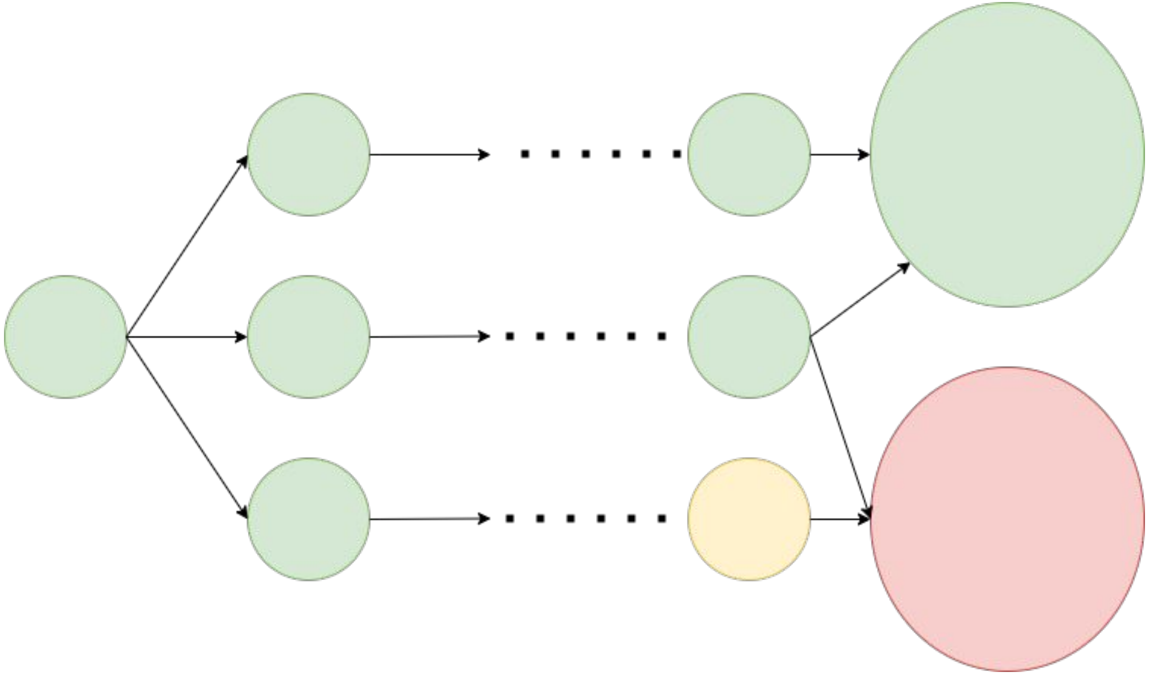


$$s \in S_{unsafe}$$

$$s \notin S_{unsafe}, \text{ given } s_{t+1} = \mathcal{T}(s_t, a_t) \text{ with } s_0 = s,$$
$$\forall t \in \mathbb{N}, \ \mathcal{T} \text{ satisfies } s_{\bar{t}} \in S_{unsafe} \text{ for some } \bar{t} \in \mathbb{N}$$

**Assumption 3.1.** *There exists a horizon $H^* \in \mathbb{N}$ such that, for any irrecoverable states $s$, any sequence of actions $a_0, \ldots, a_{H^*-1}$ will lead to an unsafe state. That is, if $s_0 = s$ and $s_{t+1} = T(s_t, a_t)$ for all $t \in \{0, \ldots, H^* - 1\}$, then $s_{\bar{t}} \in \mathcal{S}_{\text{unsafe}}$ for some $\bar{t} \in \{1, \ldots, H^*\}$.*

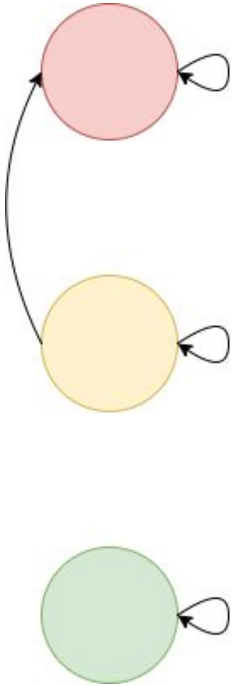
# Idea



$$\{\mathcal{T} \mid \forall t \in \{1, .., H^*\}, \mathcal{T}(s_t, a_t) \in S_{safe}\}$$

$$\{\mathcal{T} \mid \exists t \in \{1, .., H^*\}, \mathcal{T}(s_t, a_t) \notin S_{safe}\}$$

# Reward Penalty Framework



$$\left(\tilde{r}(s,a), \widetilde{T}(s,a)\right) = \begin{cases} (r(s,a), T(s,a)) & s \notin \mathcal{S}_{\text{unsafe}} \\ (-C, s) & s \in \mathcal{S}_{\text{unsafe}} \end{cases}$$

With big enough C

$$\sum_{t=0}^{H^*-1} \gamma^t r_{max} - \sum_{t=H^*}^{\infty} \gamma^t C = \frac{r_{max}(1-\gamma^{H*}) - C\gamma^{H*}}{1-\gamma}$$

$$\sum_{t=0}^{\infty} \gamma^t r_{min} = \frac{r_{min}}{1-\gamma}$$

$$\frac{r_{max}(1-\gamma^{H*}) - C\gamma^{H*}}{1-\gamma} < \frac{r_{min}}{1-\gamma}$$

# Known model assumptions

$$C > \frac{r_{max} - r_{min}}{\gamma^{H*}} - r_{max}$$

# Unknown model assumptions

$$C > \frac{r_{max} - r_{min}}{\gamma^{H*}} - r_{max}$$

$\hat{T} : S \times A \rightarrow \mathcal{P}(S)$ is **calibrated** if:
$T(s,a) \in \hat{T}(s,a) \quad \forall (s,a) \in (S \times A)$

# 1 - Bellmin Operator

➢ $$\underline{\mathcal{B}}^{*}Q(s,a) = \tilde{r}(s,a) + \gamma \min_{s' \in \hat{T}(s,a)} max_{a'} Q(s^{'}, a^{'})$$

- $\underline{\mathcal{B}}^{*}$ is a $\gamma - contraction$ in the $\infty - norm$

- Banach's fixed-point theorem

  ★ $\underline{\mathcal{B}}^{*}$ has a unique fixed point $\underline{Q}^{*}$

# 2 - Optimal Q

- for a **calibrated** $\hat{T}$, $\underline{Q}^*(s,a) \leq \tilde{Q}^*(s,a)$ for all $(s,a)$

  - $\underline{\mathcal{B}}^* Q(s,a) = \tilde{r}(s,a) + \gamma \min_{s' \in \hat{T}(s,a)} max_{a'} Q(s',a')$

  - $\mathcal{B}^* Q = r(s,a) + \gamma \max_{a'} Q(s',a')$

# 3 - Safety

- If there is a safe action $a$ at state $s$, then $argmax_a \underline{Q}^*(s,a)$ is a safe action

    - $$a \in A_{Unsafe} \Rightarrow \underline{Q}^* \leq \underline{\tilde{Q}}^* \leq \frac{r_{max}(1-\gamma^{H*}) - C\gamma^{H*}}{1-\gamma}$$

    - $$a \in A_{Safe} \Rightarrow \frac{r_{min}}{1-\gamma} \leq \underline{Q}^* \leq \underline{\tilde{Q}}^*$$

# Algorithm

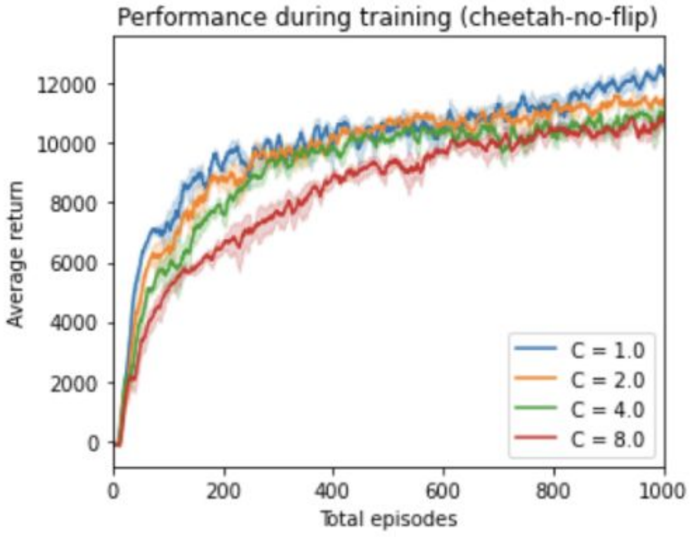
**Algorithm 1** Safe Model-Based Policy Optimization (SMBPO)

**Require:** Horizon $H$

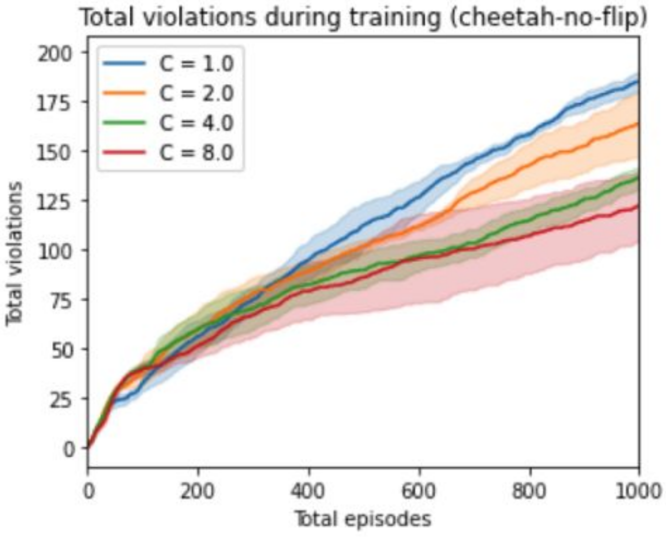
Initialize empty buffers $\mathcal{D}$ and $\widehat{\mathcal{D}}$, an ensemble of probabilistic dynamics $\{\widehat{T}_{\theta_i}\}_{i=1}^N$, policy $\pi_\phi$, critic $Q_\psi$.
Collect initial data using random policy, add to $\mathcal{D}$.
**for** episode $1, 2, \ldots$ **do**
    Collect episode using $\pi_\phi$; add the samples to $\mathcal{D}$. Let $\ell$ be the length of the episode.
    Re-fit models $\{\widehat{T}_{\theta_i}\}_{i=1}^N$ by several epochs of SGD on $L_{\widehat{T}}(\theta_i)$ defined in (9)
    Compute empirical $r_{\min}$ and $r_{\max}$, and update $C$ according to (3).
    **for** $\ell$ times **do**
        **for** $n_{\text{rollout}}$ times (in parallel) **do**
            Sample $s \sim \mathcal{D}$.
            Startin from $s$, roll out $H$ steps using $\pi_\phi$ and $\{\widehat{T}_{\theta_i}\}$; add the samples to $\widehat{\mathcal{D}}$.
        **for** $n_{\text{actor}}$ times **do**
            Draw samples from $\mathcal{D} \cup \widehat{\mathcal{D}}$.
            Update $Q_\psi$ by SGD on $L_Q(\psi)$ defined in (10) and target parameters $\bar{\psi}$ according to (12).
            Update $\pi_\phi$ by SGD on $L_\pi(\phi)$ defined in (13).

Based on MBPO (Janner et al., 2019) and Soft Actor Critic (Haarnoja et al., 2018)

# Parameter C




(a) Performance with varying $C$




(b) Cumulative safety violations with varying $C$

# Experiments - exploration

$$L_Q(\psi) = \mathbb{E}_{(s,a,r,s')\sim\mathcal{D}\cup\widehat{\mathcal{D}}}[(Q_\psi(s,a) - (r + \gamma V_{\bar{\psi}}(s'))^2]$$

$$V_{\bar{\psi}}(s') = \begin{cases} -C/(1-\gamma) & s' \in \mathcal{S}_{\text{unsafe}} \\ \mathbb{E}_{a'\sim\pi(s')}[Q_{\bar{\psi}}(s',a') - \alpha \log \pi_\phi(a' \mid s')] & s' \notin \mathcal{S}_{\text{unsafe}} \end{cases}$$

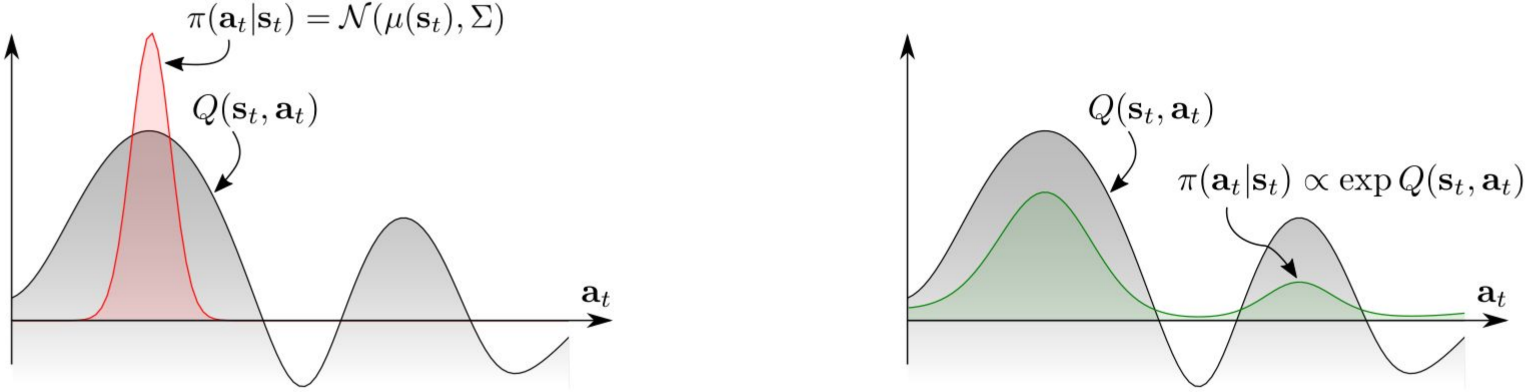
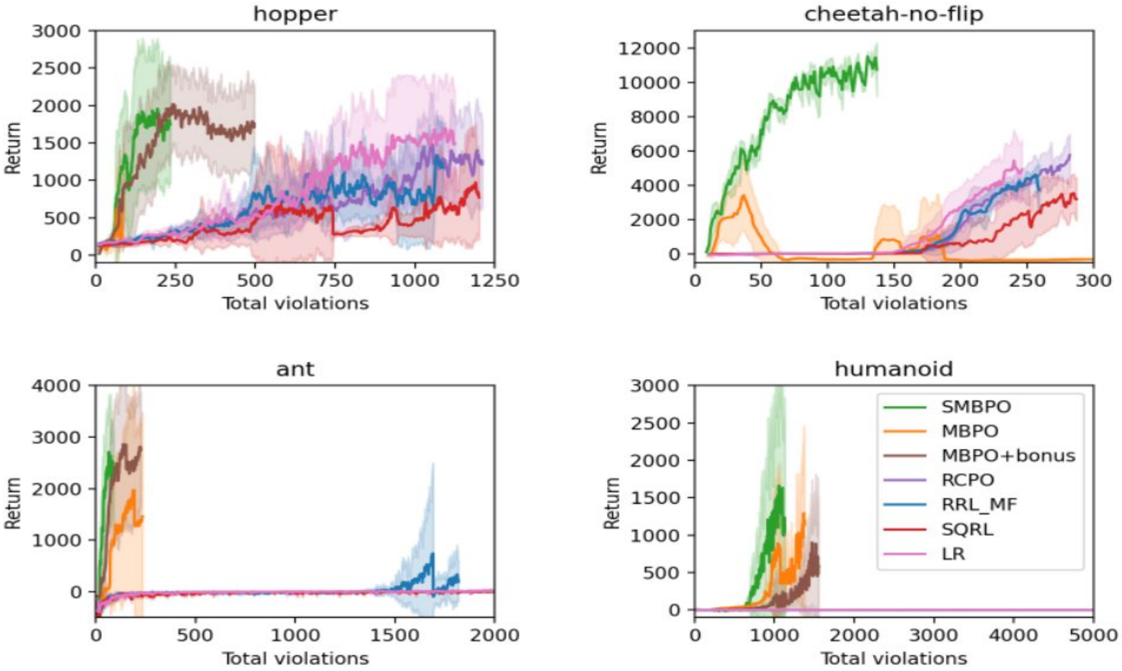



Figure from (Tang & Haarnoja, 2017)

# Experiments



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