

BenPrescott_MiniAssignment2

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1 Mini Programming Assignment: Implement a Selection Sort Algorithm

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Executive Summary Description:

This notebook provides a comparison of execution speeds for a selection sort algorithm. This algorithm has many practical use cases, such as sorting products by price or customer review score.

Algorithm Explanation

Selection sort is an algorithm that sorts some values using a defined condition. In the case of our example code, the algorithm is sorting various arrays from smallest number to largest. It does this by adding the first number to a 'sorted group', then reviewing the unsorted data again to identify if the next number is larger or smaller than the one added to the 'sorted group'. If it is smaller, it places it before the previous smallest number. It then repeats this process until all values have been sorted. This algorithm takes $O(n^2)$ time, as the algorithm has to continually search through the initial dataset and the 'sorted group' to identify new values against existing sorted.

Approach

I created five different arrays consisting of random, uniformly distributed values between 0 and 25,000. The total count of numbers in each array is 5,000, 10,000, 15,000, 20,000, and 25,000 respectively. A selection sort algorithm was then ran against each of these arrays, with the sorting algorithm time recorded. The array size and execution time of the algorithm were then added to a simple Pandas DataFrame for quick review, followed by a visual plot to easily identify the trend in execution times.

Results

The results of this experiment confirm that the sorting algorithm is not exactly fast, especially as the size of an array grows. The first array of 5,000 numbers took ~88 milliseconds, with the second array (10,000 values - double the size of the first) taking ~3.56 seconds. This is approximately four times slower when doubling the array size. The same is true when doubling from 10,000 values to 20,000.

However, selection sort brings a benefit of being an ‘in-place’ sorting algorithm, as it does not need additional space in memory to sort the array. It is also fast on smaller array sizes, but struggles when dealing with scale.

```
[189]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from timeit import default_timer as timer
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
```

```
[3]: # Finds the smallest value in an array
def findSmallest(arr):
    # Stores the smallest value
    smallest = arr[0]
    # Stores the index of the smallest value
    smallest_index = 0
    for i in range(1, len(arr)):
        if arr[i] < smallest:
            smallest_index = i
            smallest = arr[i]
    return smallest_index

# Sort array
def selectionSort(arr):
    newArr = []
    for i in range(len(arr)):
        # Finds the smallest element in the array and adds it to the new array
        smallest = findSmallest(arr)
        newArr.append(arr.pop(smallest))
    return newArr

print(selectionSort([5, 3, 6, 2, 10]))
```

[2, 3, 5, 6, 10]

```
[231]: np.random.seed(10) # Setting random seed for repeatable array creation
array5000 = list(np.random.randint(0, 25000, 5000))
array10000 = list(np.random.randint(0, 25000, 10000))
array15000 = list(np.random.randint(0, 25000, 15000))
array20000 = list(np.random.randint(0, 25000, 20000))
array25000 = list(np.random.randint(0, 25000, 25000))
```

```
[228]: arrays = [array5000, array10000, array15000, array20000, array25000] # Defining
→ a list of the arrays for a loop
results = pd.DataFrame() # Creating an empty DataFrame to store future
→ execution times
```

```

idx = 0 # Initial index to start storing data in the DataFrame

# For loop that loops through the 'arrays' list, storing their size and
↳ execution times in the DataFrame
for array in arrays:
    results.at[idx, 'array_size'] = len(array)
    start = timer()
    selectionSort(array)
    end = timer()
    results.at[idx, 'execution_time'] = end-start
    idx += 1

results['array_size'] = results['array_size'].astype(int) # Converting the
↳ array_size values to integer to remove the floating decimal .0

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[229]: # Reviewing the execution time results  
results
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```
[229]:      array_size  execution_time
0         5000         0.879144
1        10000         3.557370
2        15000         7.828711
3        20000        13.852441
4        25000        22.043596
```

```
[230]: # Plotting the results using array_size on the X axis and execution_time on the Y axis
# Overlaying a scatter plot to easily see the points on the line
plt.figure(figsize=(16,8))
plt.plot(results['array_size'], results['execution_time'])
plt.scatter(results['array_size'], results['execution_time'])
plt.xticks(results['array_size'])
plt.xlabel('Array Size')
plt.ylabel('Execution Time')
plt.title('Selection Sort Execution Times by Array Size')
plt.plot()
```

```
[230]: <Figure size 1152x576 with 0 Axes>
```

```
[230]: [<matplotlib.lines.Line2D at 0x1f27b48b248>]
```

```
[230]: <matplotlib.collections.PathCollection at 0x1f27b74d3c8>
```

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[230]: ([<matplotlib.axis.XTick at 0x1f27ba3ad08>,
      <matplotlib.axis.XTick at 0x1f27ba3a508>,
      <matplotlib.axis.XTick at 0x1f27b485548>,
      <matplotlib.axis.XTick at 0x1f27bd2f808>,
      <matplotlib.axis.XTick at 0x1f27bd553c8>],
      <a list of 5 Text major ticklabel objects>)
```

```
[230]: Text(0.5, 0, 'Array Size')
```

```
[230]: Text(0, 0.5, 'Execution Time')
```

```
[230]: Text(0.5, 1.0, 'Selection Sort Execution Times by Array Size')
```

```
[230]: []
```

