Multi-path Summation for Decoding 2D Topological Codes

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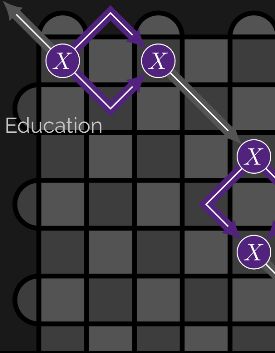
November 30, 2018



Outline

- Quantum Computing ∩ Social & Economic Justice

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- The Surface Code & Decoding

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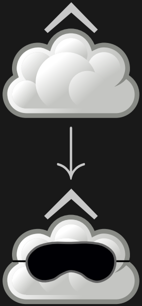
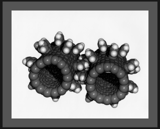
- Results

Quantum Computing: The Good …



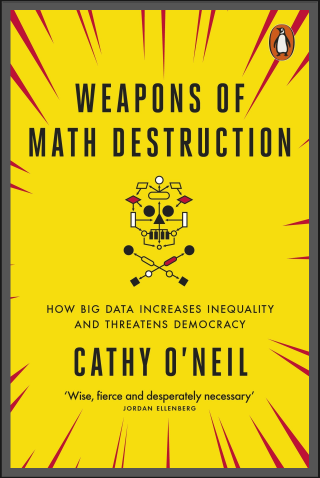
- quantum networks allow private communication
- 'blind' quantum computation allows private processing



- simulating quantum physics allows 'quantum CAD'

The Bad ...

- people are impressed by algorithms
- they trust algorithms to be objective and correct
- important decisions are being made with unverified machine learning
- quantum computers: more impressive, but less verifiable?



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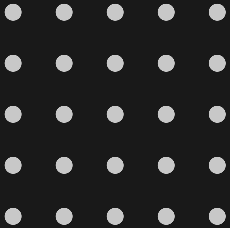
- low-depth circuits for producing random numbers
  - single niche application = “supremacy”?

The Surface Code

For now, large-scale quantum computation needs surface codes:



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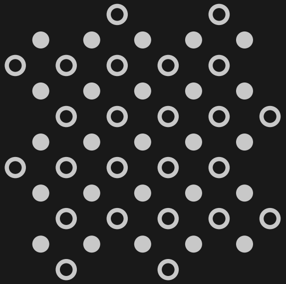

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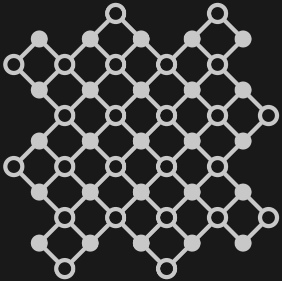

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- connect each ancilla to a small number of data qubits using constant-length planar links

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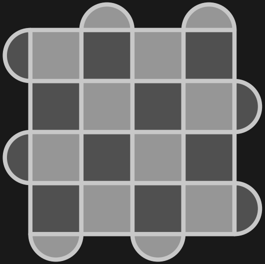

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- add local ancilla qubits
- connect each ancilla to a small number of data qubits using constant-length planar links
- measure *stabilisers* indirectly using H/CNOT gates

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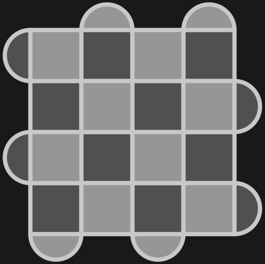

- : weight-4 X check
- : weight-4 Z check
- Weight-2 boundary checks: ,
- logical Z operator runs from left to right, logical X from top to bottom
- $[\![d^2, 1, d]\!]$ code

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Justification: We'll still be able to observe the effects of different decoding algorithms in this simpler scenario.

Decoding Algorithms

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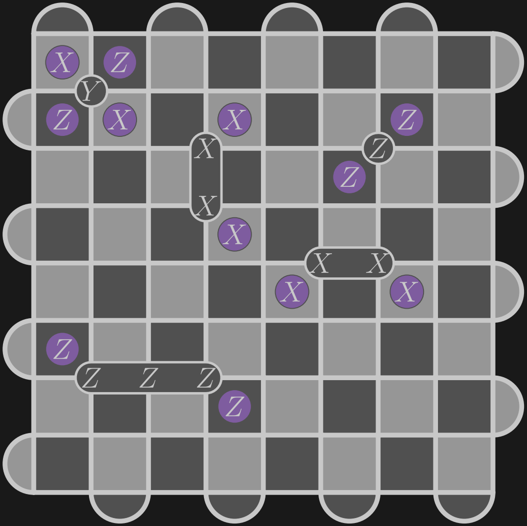

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"we" (Dennis/Kitaev/Landahl/Preskill '01) can do a derivation …

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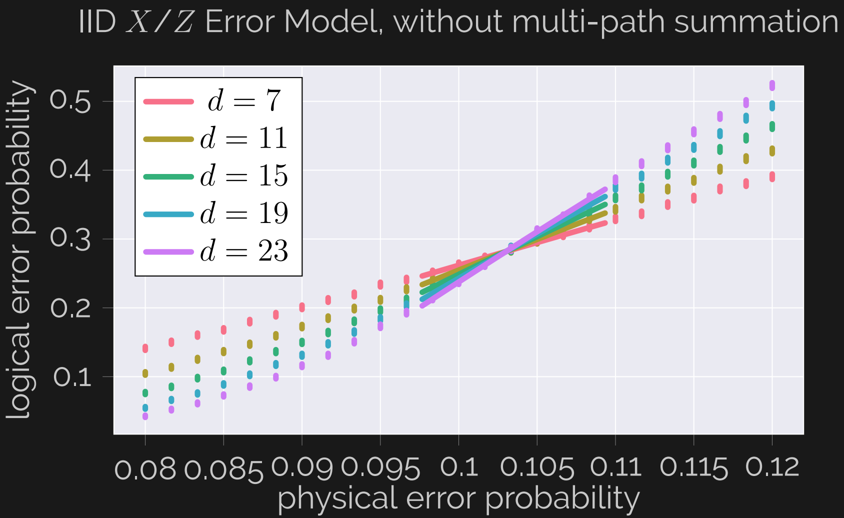
$$\propto -\sum_{e \in \mathrm{edges}(E)} |e|$$

Decoding Algorithms

this results in a threshold error rate of 10.3%:



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optimal threshold against this error model near 10.9%

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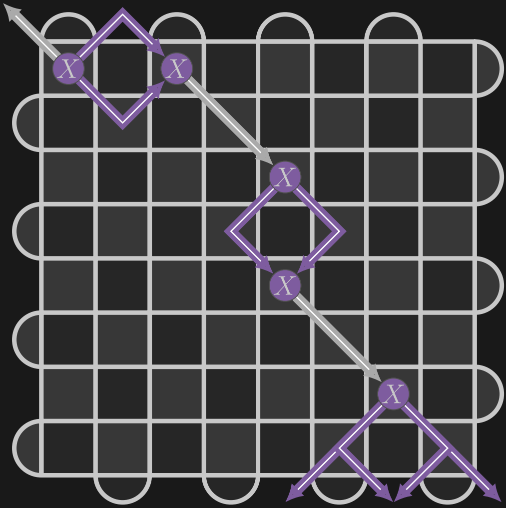

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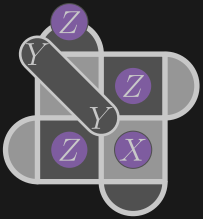
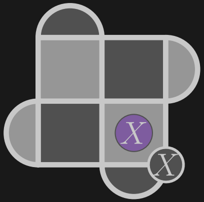
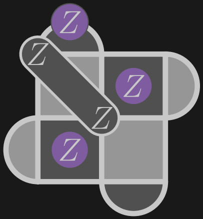
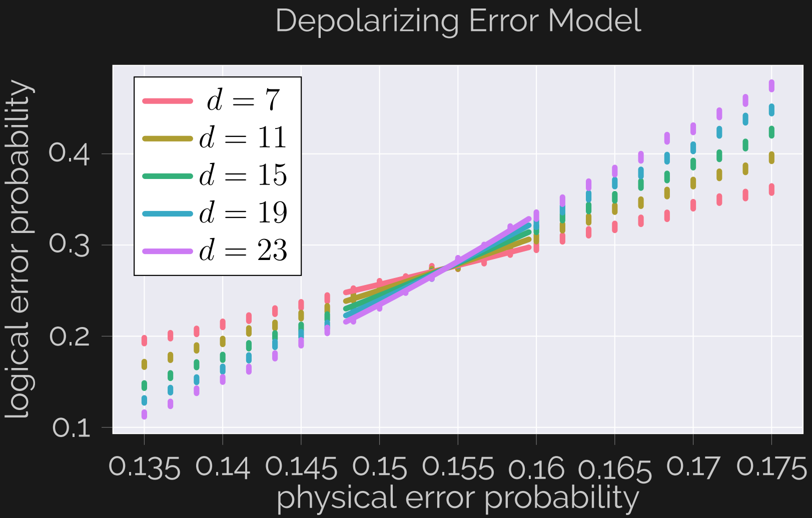






Decoding Algorithms

threshold vs. depolarising noise: 15.4% (optimal: 18.9%)



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- we only consider minimum-length paths
- we consider two possibilities, either the vertices being considered are joined by a single path, or none of the relevant qubits are in error

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$$p(E) = \prod_q (1 - p_q) \times \prod_{e \in \text{edges}(E)} \sum_{p \in \text{paths}(e)} \prod_{q \in p} o_q$$
$$\sim \sum_{e \in \text{edges}(E)} \log \left(\sum_{p \in \text{paths}(e)} \prod_{q \in p} o_q \right)$$

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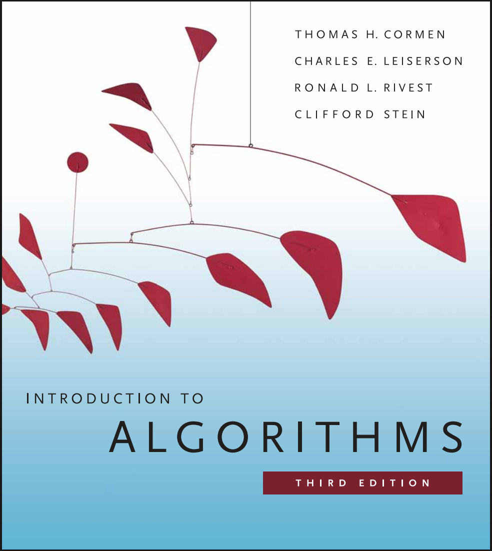

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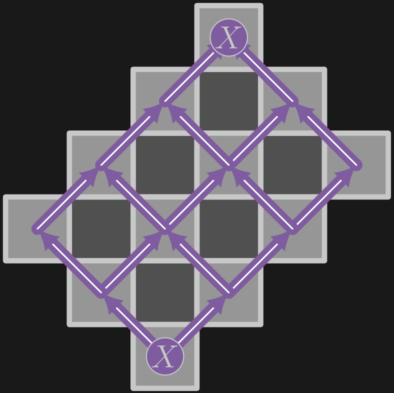

Belief Propagation

approximate marginal probabilities (Poulin/Chung '08)

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processors associated with ancillas calculate messages to be passed to neighbouring processors associated with data qubits

$$m_{c\to q}(E_q) \propto \sum_{E_{q'},\, q' \in \mathrm{supp}(c) \setminus q} \left(\delta_{\mathrm{synd}_c, S_c \cdot E_c} \prod_{q' \in \mathrm{supp}(c) \setminus q} m_{q' \to c}(E_{q'}) \right)$$

qubit processors calculate new messages to be passed back:

$$m_{q\to c}(E_q) \propto p(E_q) \prod_{c' \in \mathrm{supp}(q) \setminus c} m_{c' \to q}(E_q),$$

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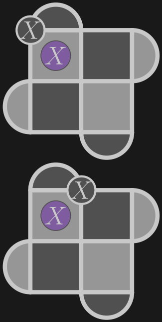
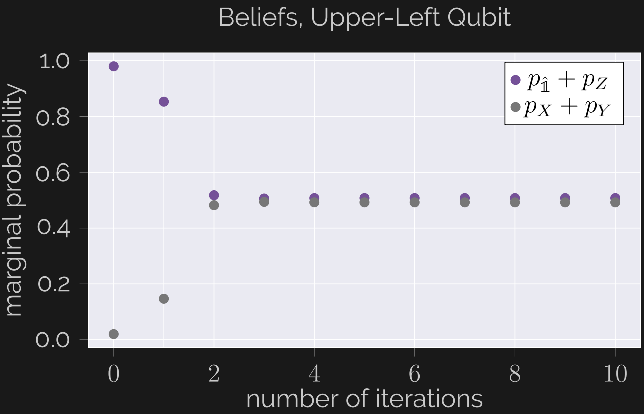
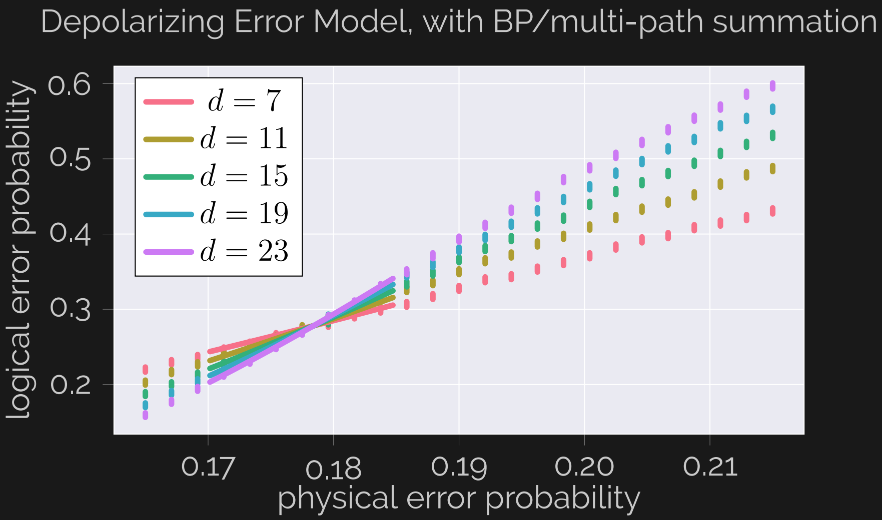




Results

threshold goes about $^2/_3$ of the way toward optimal



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- uses parallel algorithms with time cost proportional to d
- **question:** can probability of logical error be quickly calculated as a path sum? (See Bravyi/Suchara/Vargo '14)
- **question:** does the threshold increase appreciably when we extend this to the realistic case?

To Learn More

- QuTech: `qutech.nl` & `qutechacademy.nl`
- IGDORE: `igdore.org`
- Quantum Error Correction & Fault Tolerance: `pirsa.org/C17045`
- My other slides: `https://github.com/bcriger/slide_decks`
- My research: `scirate.com/ben-criger/papers`
- My Twitter: `@BenCriger`