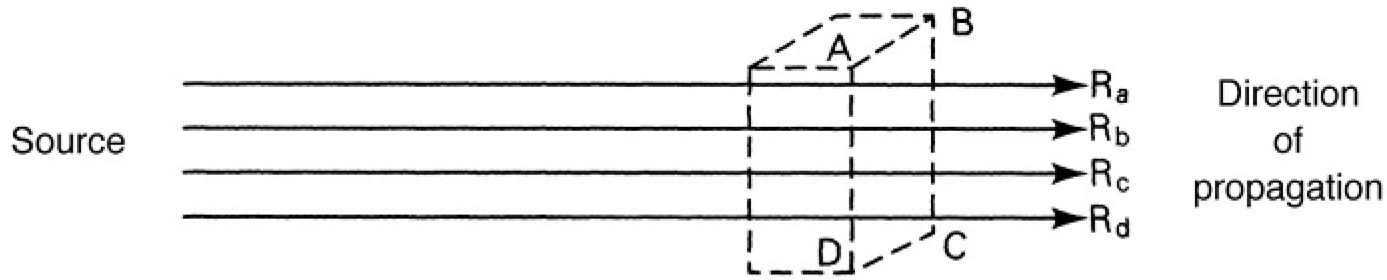
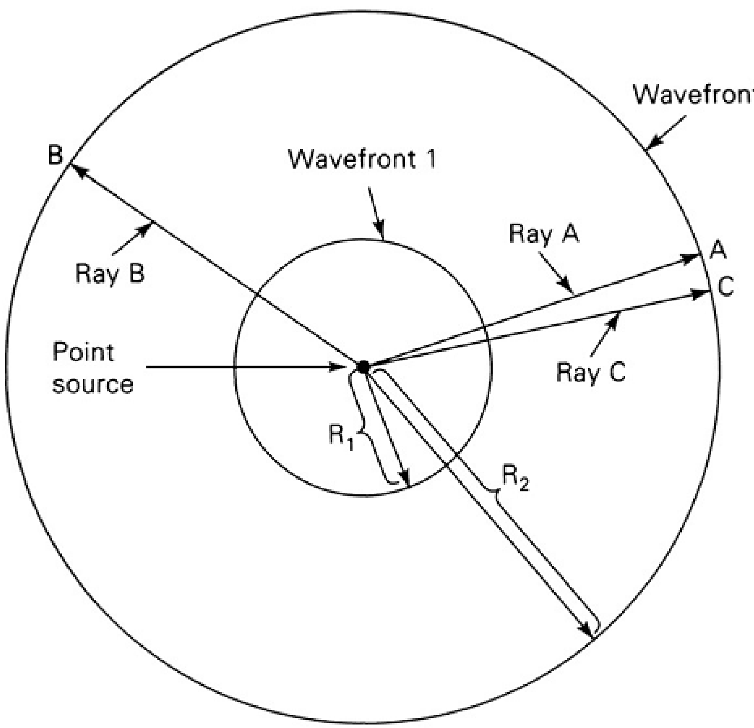
Wireless Communications Channels: Characterization & Modeling

LOS wireless communications link: a review

Basic radio propagation
Line-of-sight (LOS) link design
path engineering
LOS point-to-point digital communications
design considerations

PLANE WAVE AND WAVE FRONT





WAVE FRONT FROM AN ISOTROPIC SOURCE

IMPEDANCE OF FREE SPACE:

$Z_{FS}=(\mu_o/\varepsilon_o)^{1/2}=(1.26E\text{-}6/8.85E\text{-}12)^{1/2}=377\Omega$

μ_o: MAGNETIC PERMEABILITY (in H/m)

ε_o: ELECTRIC PERMITIVITY (in F/m)

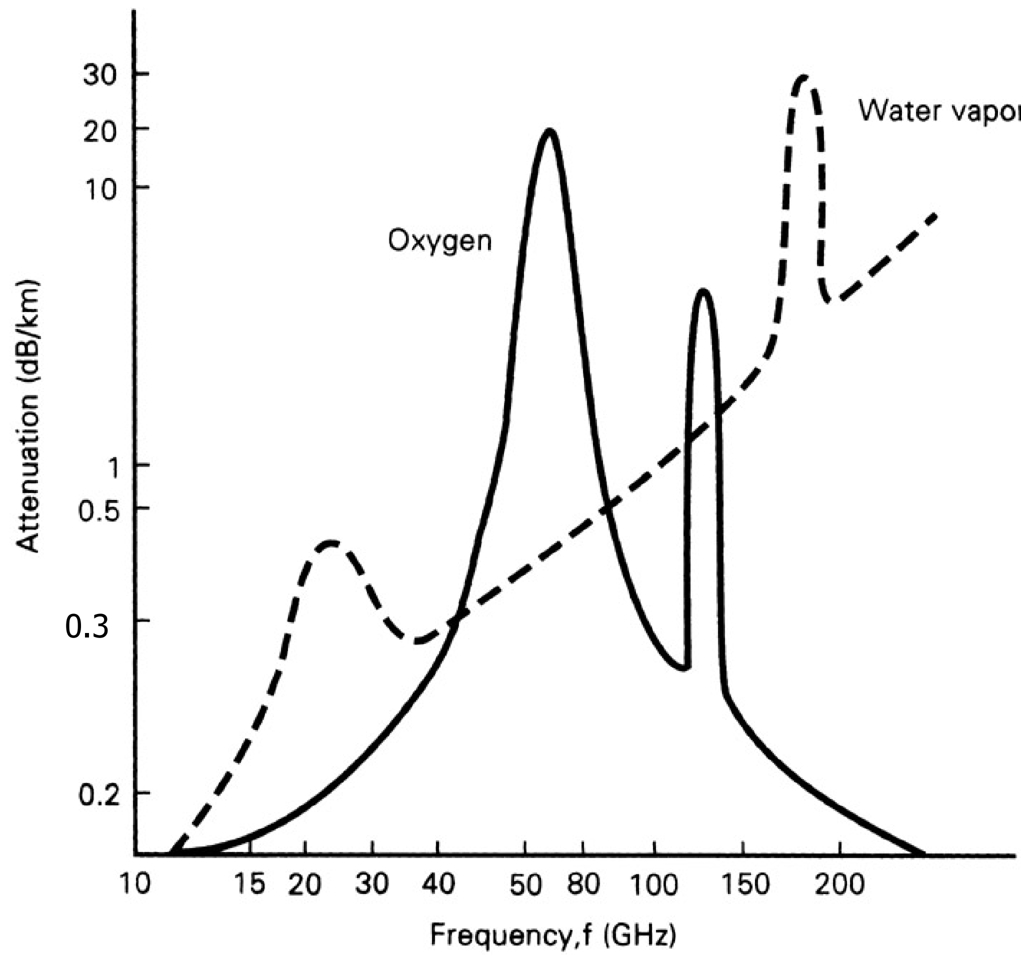
FREE-SPACE LOSS: $L_{FREE\text{-}SPACE}=(4\pi Rf/c)^2$

POWER DENSITY PER UNIT AREA AT R

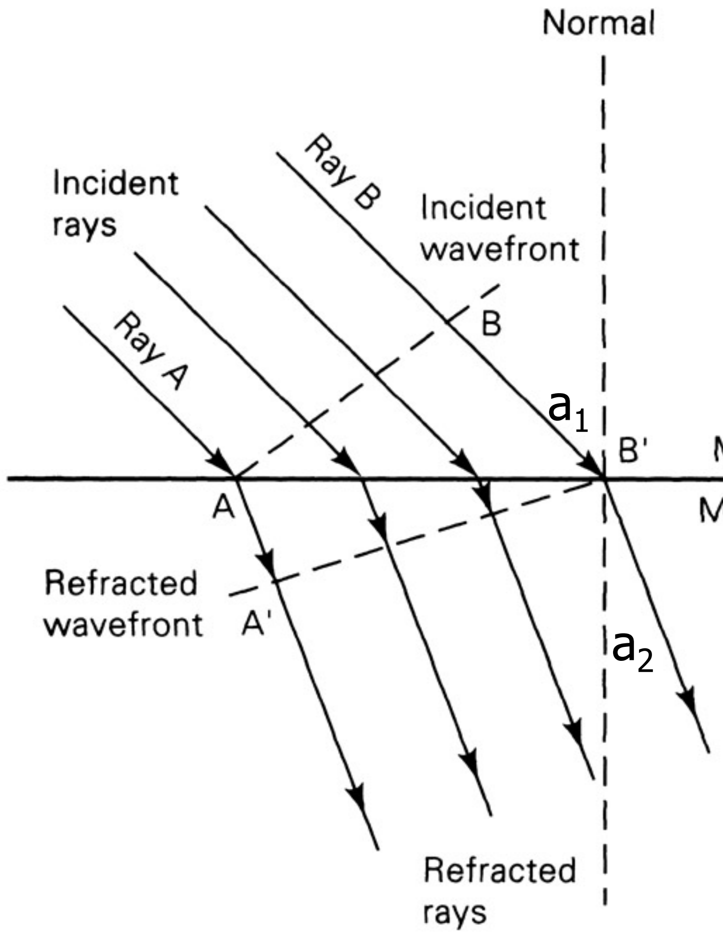
$p_R=P/\mathbf{4\pi R^2}$

$p_{R2}=p_{R1}(R1/R2)^2$: square-law

Atmospheric absorption of electromagnetic waves



Refraction at a plane boundary between two media



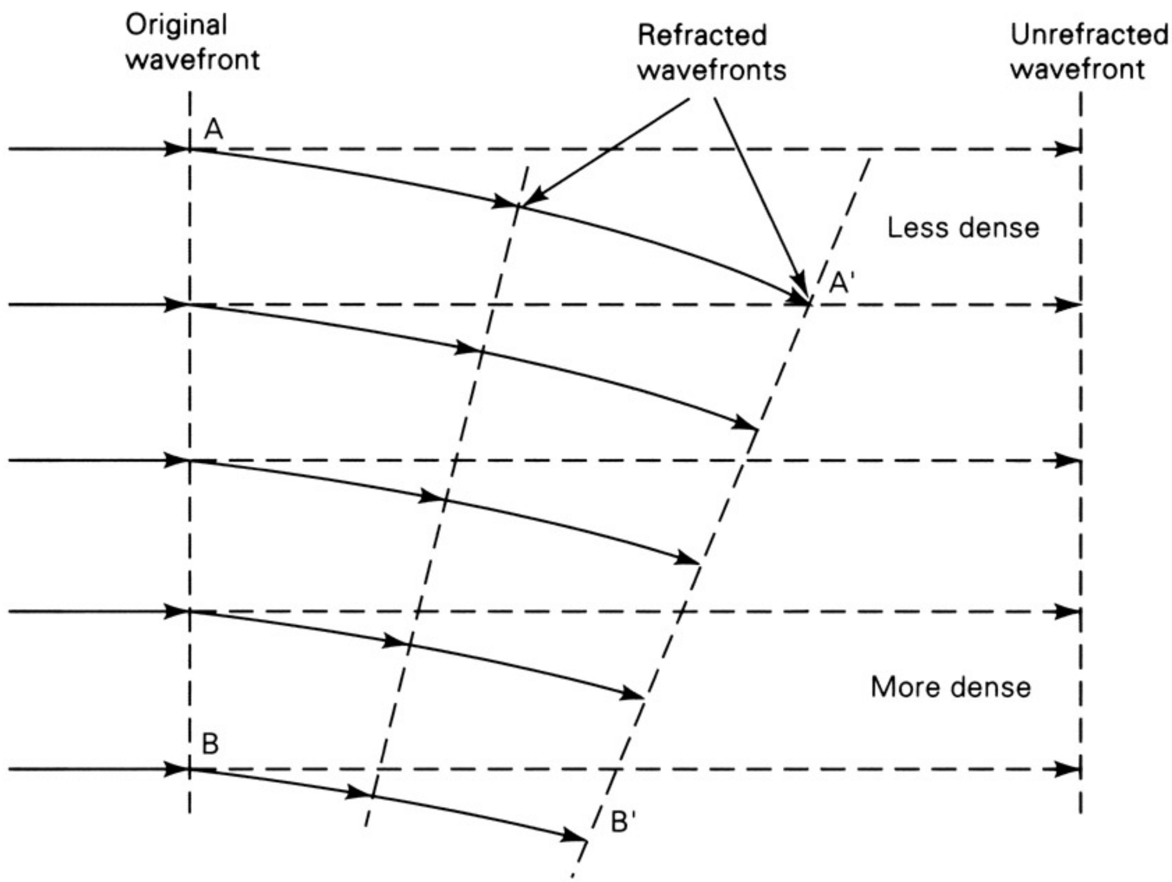
Media with different propagation velocities $v_1=c/n_1$, $v_2=c/n_2$
n_1, n_2: refractive index
Snell's law: $n_1 \sin(a_1)=n_2 \sin(a_2)$
a_1=angle of incidence
a_2=angle of refraction

(v_1, n_1)

$(n_2>n_1, v_2<v_1)$

The refractive index n(h) is a function of h, the height above the earth. Since n(h) is too close to 1, it is more convenient to define the **refractivity** N=(n-1)1E6 in N units,
Vertical gradient of refractive index: dn/dh

Wavefront refraction in a gradient medium



The rays are bent toward the region of higher refractive index

N proportional to $(\varepsilon_r)^{1/2}$. Hence, $dn/dh=0.5(d\varepsilon_r/dh)$

The rate of change of the dielectric constant $(d\varepsilon_r/dh)$ **is nearly constant** for the first few hundred meters above the earth's surface

K: EFFECTIVE EARTH RADIUS FACTOR

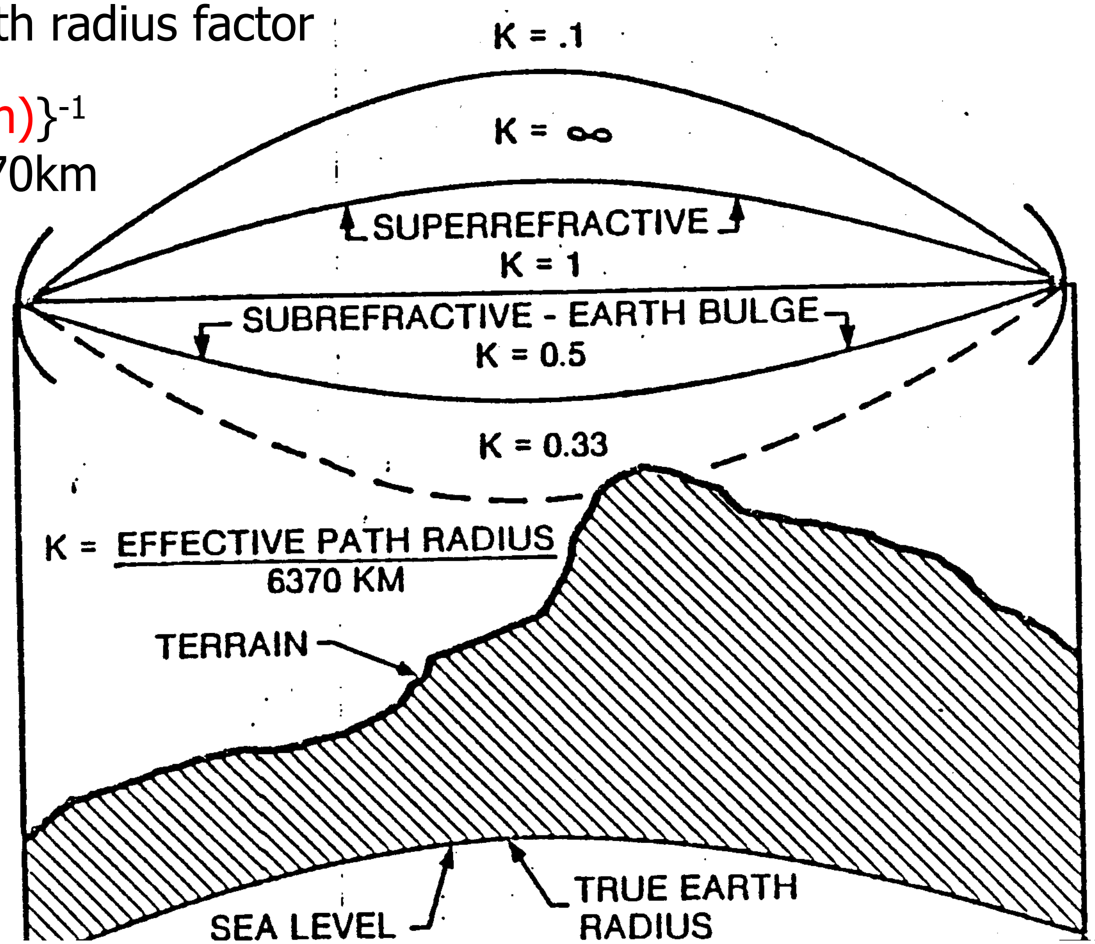
If dn/dh is constant, the net effect of refraction is the same as if the radio waves continued in a straight line but over an earth whose EFFECTIVE radius is $\mathbf{r_e = K.r}$ where K is called the effective earth radius factor

$K=\{1+r(dn/dh)\}^{-1} =\{1+0.5r\ (d\varepsilon_r/dh)\}^{-1}$

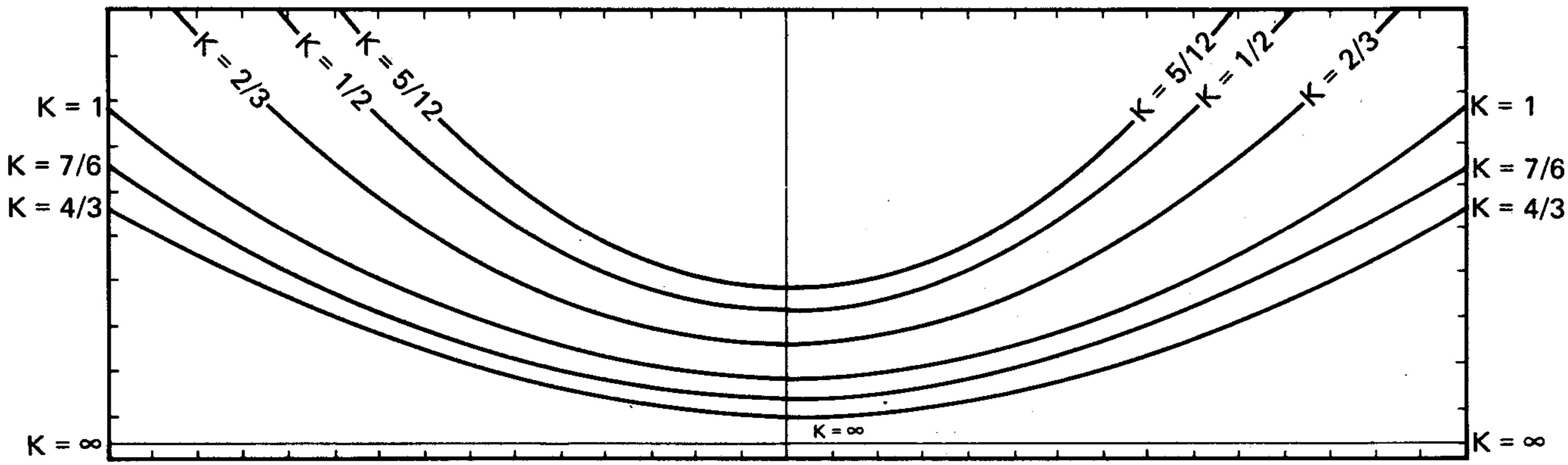
r is true radius of the earth, r= 6370km

$K=\{1+ (dN/dh)/157\}^{-1}$ where (dN/dh) in N units per km.

(dN/dh)	**K**
314	0.33
157	0.5
0	1
-157	∞
-314	-1



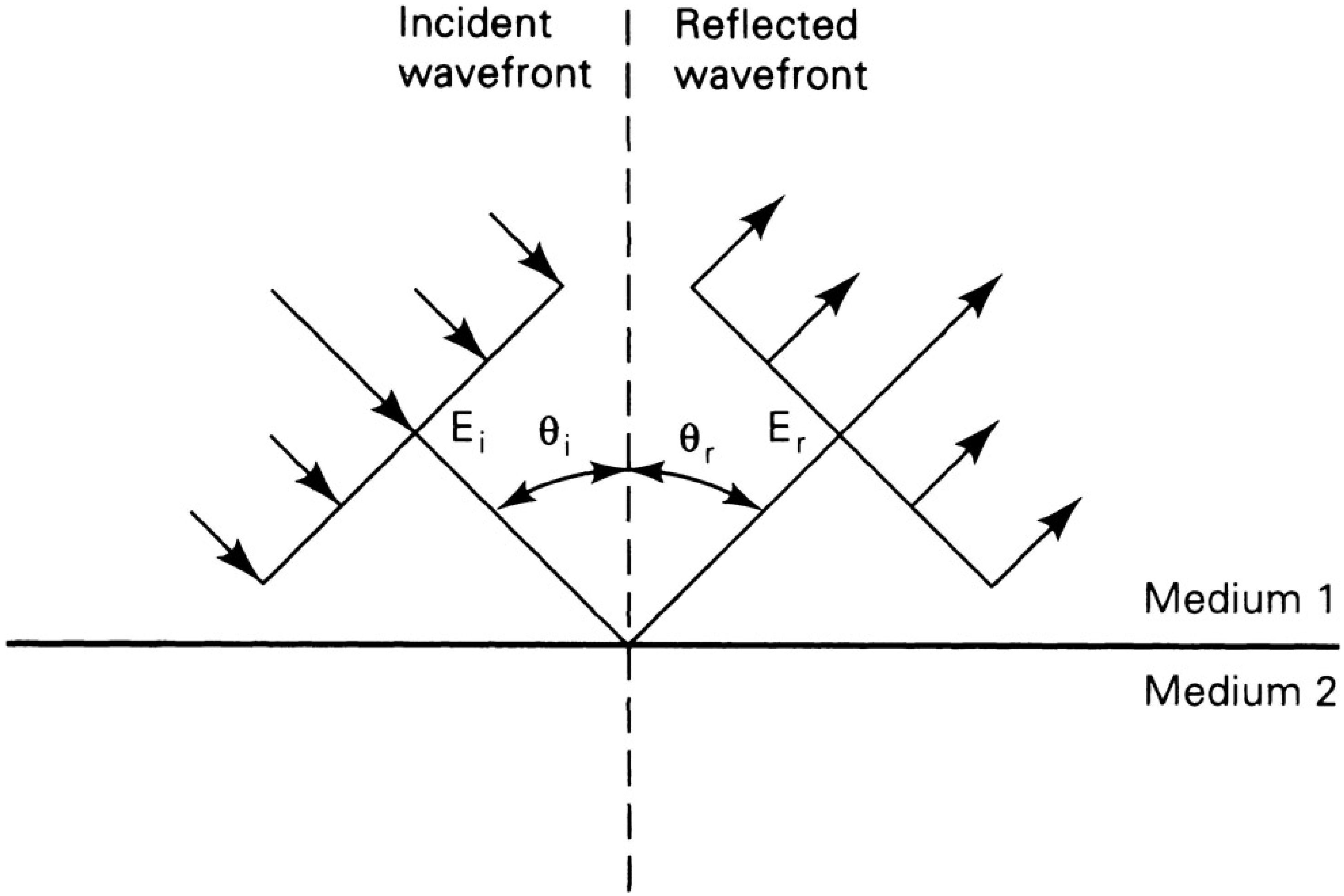
EQUIVALENT EARTH PROFILE CURVES



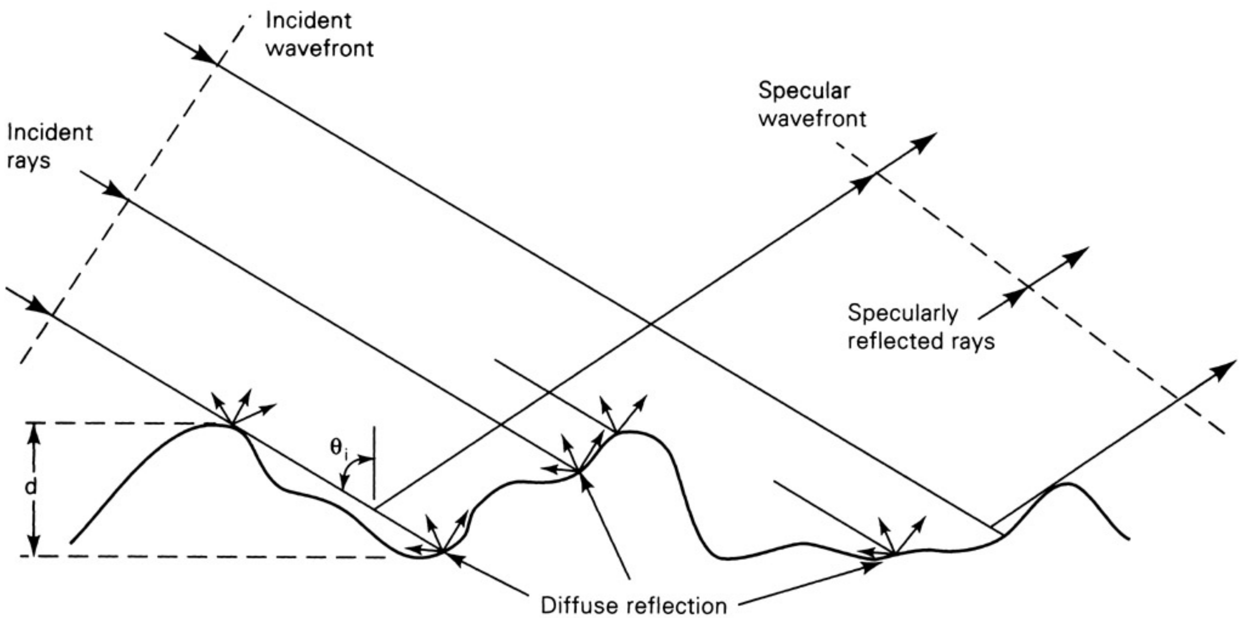
K-FACTOR GUIDE:

K	**Propagation**	**weather**	**terrain**
4/3	perfect	standard atmosphere	temperate zone, no fog
1-4/3	ideal	no surface layers, fog	dry, mountainous, no fog
2/3-1	average	substandard, light fog	flat, temperate, some fog
0.5-2/3	difficult	surface layers, ground fog	coastal
0.4-0.5	bad	fog moisture, over water	coastal, water, tropical

reflection at a plane boundary of two media



Reflection from a semi-rough surface

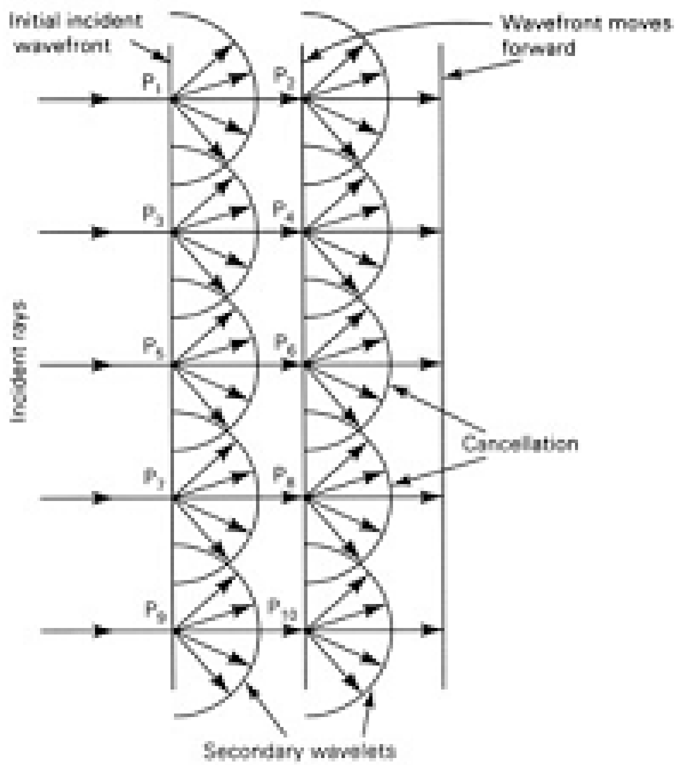


RAYLEIGH CRITERION: SEMIROUGH SURFACE WILL REFLECT AS A SMOOTH SURFACE WHENEVER $\cos(\theta_i) > \lambda/8d$

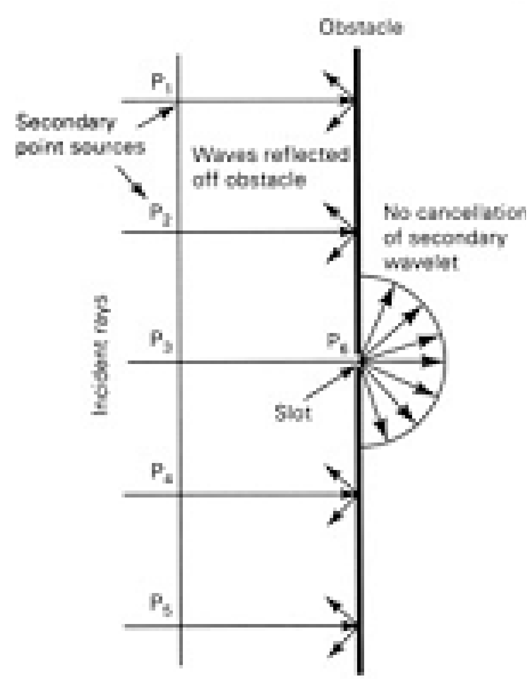
WHERE d: DEPTH OF THE SURFACE IRREGULARITY.

WAVE DIFFRACTION

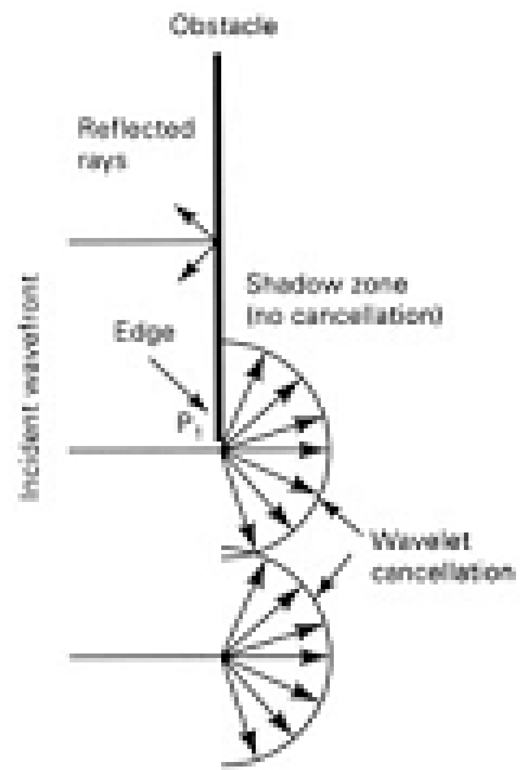
(a) Huygens's principle for a plane wavefront



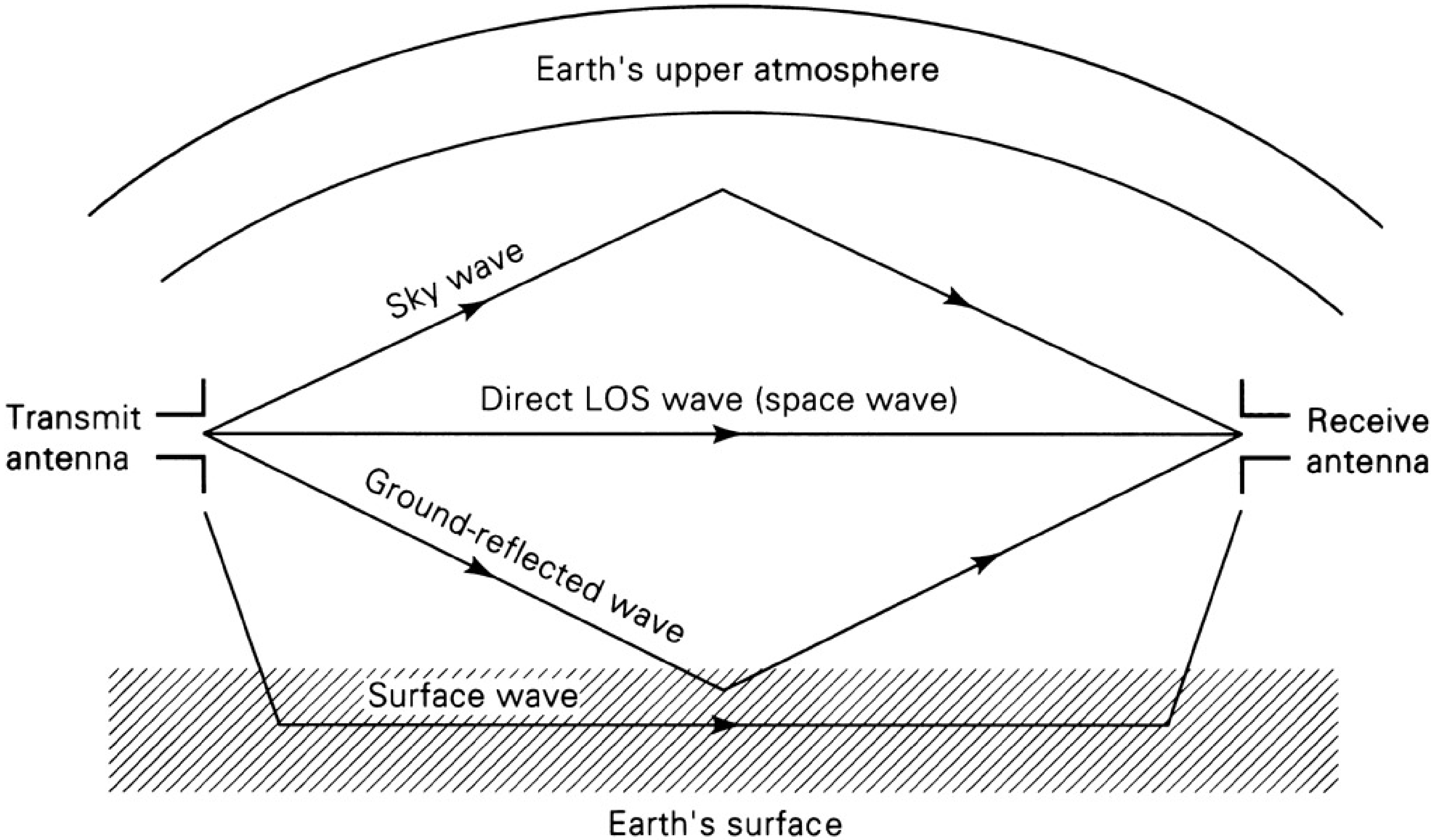
(b) finite wavefront through a slot



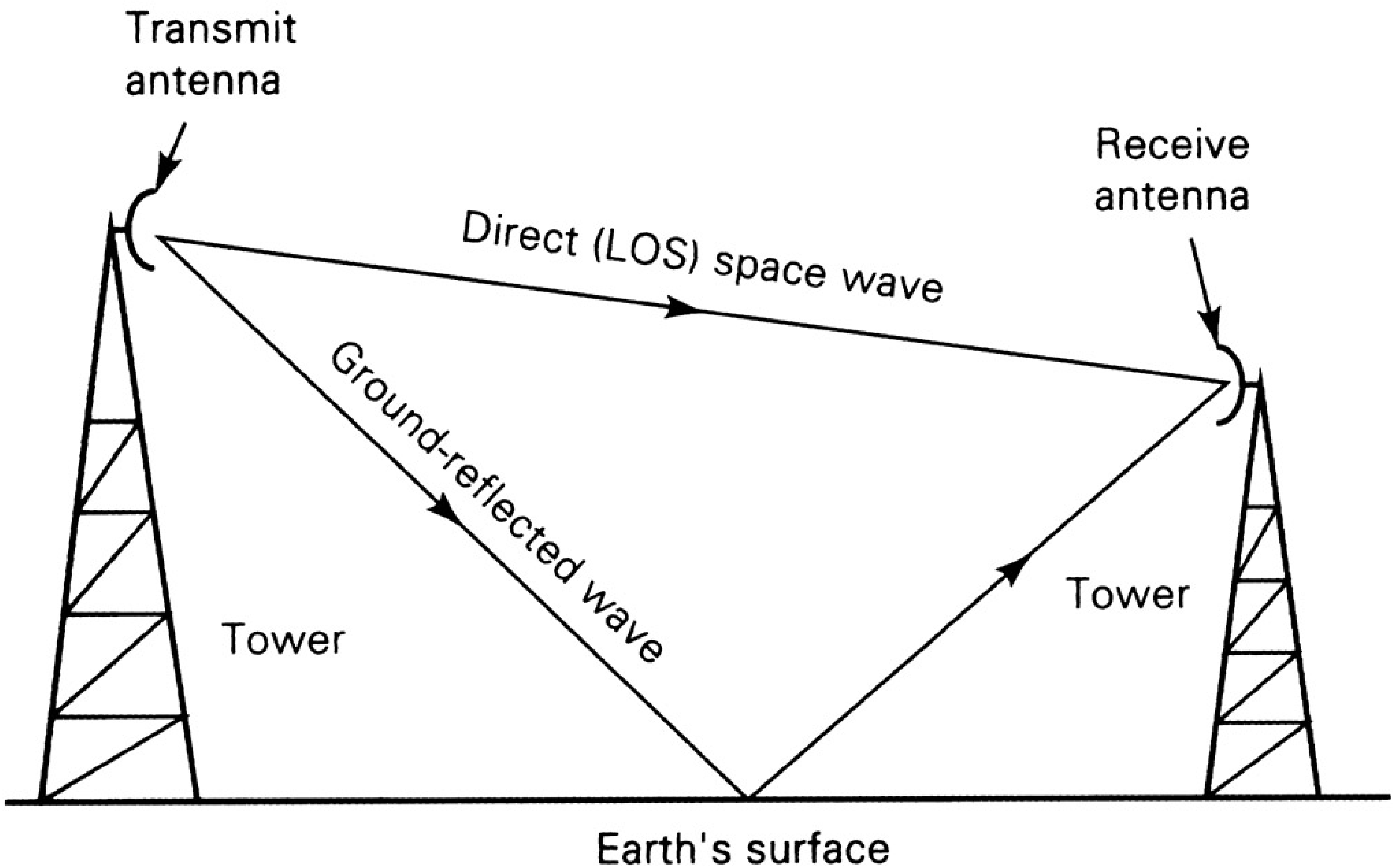
(c) around an edge



Normal modes of wave propagation

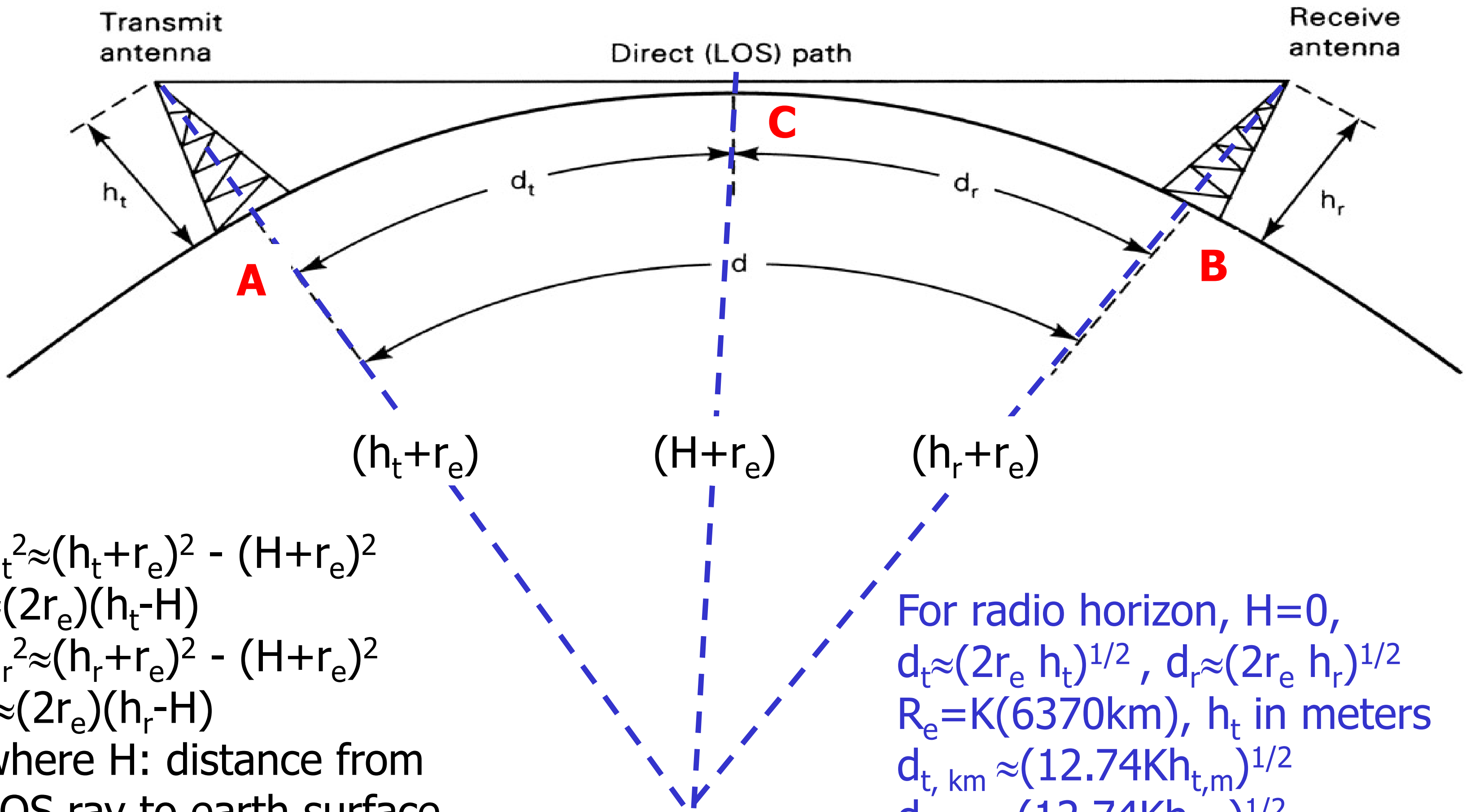


Space-wave propagation: line-of-sight (LOS)



Space waves and radio horizon

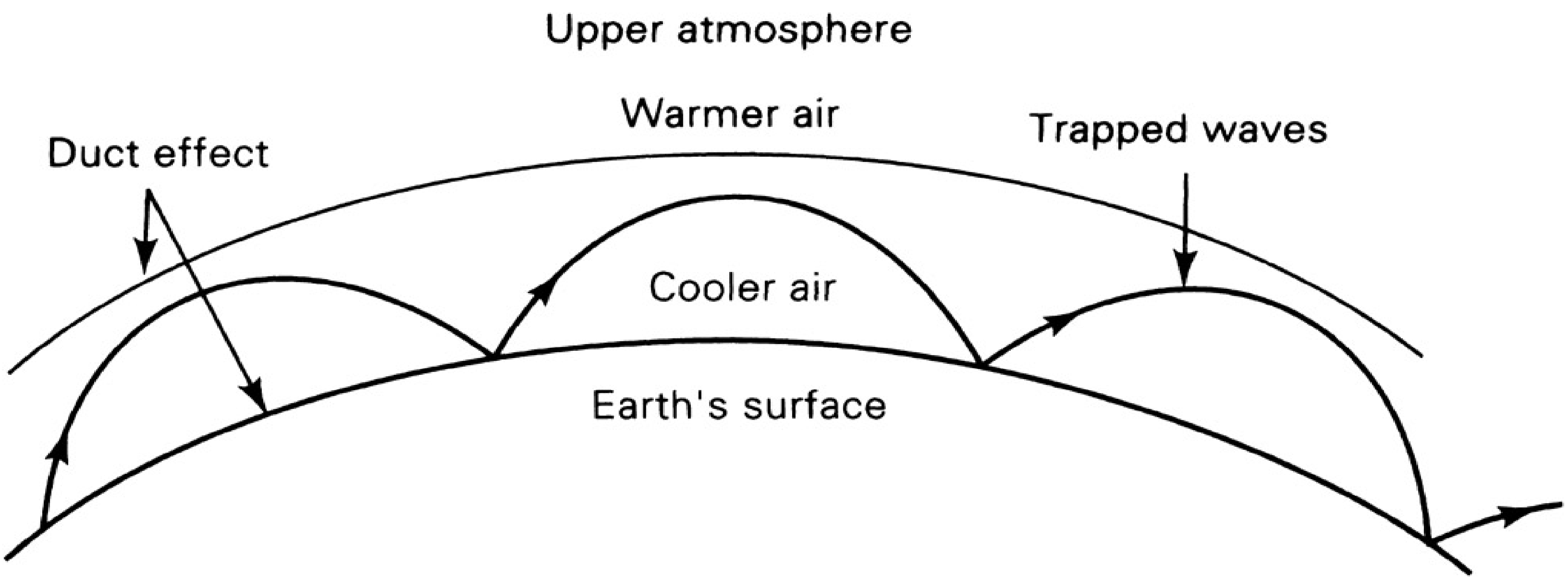
RADIO HORIZON= OPTICAL HORIZON for K=1



$d_t^2 \approx (h_t+r_e)^2 - (H+r_e)^2$
$\approx (2r_e)(h_t-H)$
$d_r^2 \approx (h_r+r_e)^2 - (H+r_e)^2$
$\approx (2r_e)(h_r-H)$
where H: distance from LOS ray to earth surface

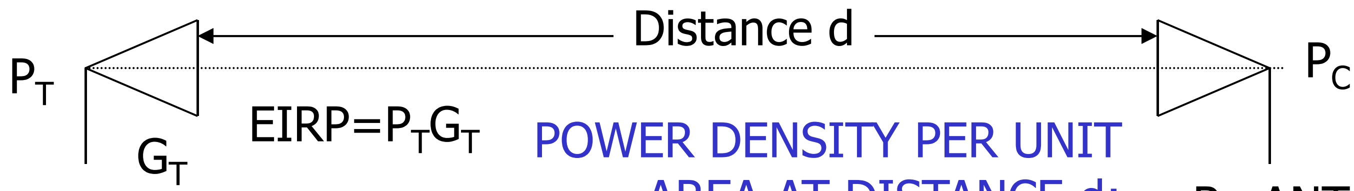
For radio horizon, H=0,
$d_t \approx (2r_e\, h_t)^{1/2}$, $d_r \approx (2r_e\, h_r)^{1/2}$
R_e=K(6370km), h_t in meters
$d_{t,\ km} \approx (12.74Kh_{t,m})^{1/2}$
$d_{r,\ km} \approx (12.74Kh_{r,m})^{1/2}$
Longest $d_{km} = d_{t,\ km} + d_{r,\ km}$
$\approx (12.74Kh_{t,m})^{1/2} + (12.74Kh_{r,m})^{1/2}$

Duct propagation



ATMOSPHERIC DUCTS: DIELECTRIC WAVE-GUIDE-LIKE REGION
CAN EXTEND HUNDREDS OF KM BEYOND NORMAL RADIO HORIZON

LOS: FREE-SPACE LOSS



$EIRP=P_TG_T$

POWER DENSITY PER UNIT AREA AT DISTANCE d: $p_d=(P_TG_T)/(4\pi d^2)$

Tx POWER P_T
Tx ANT POWER GAIN G_T

Rx ANT POWER GAIN G_R
Rx (CAPTURED) POWER P_C

p_d : amount of power incident on each unit area of an imaginary surface (perpendicular to the direction of propagation of the electromagnetic wave).
effective capture area of the rx antenna: $A_C=(G_R\lambda^2)/(4\pi)$
where $\lambda=c/f$: wavelength

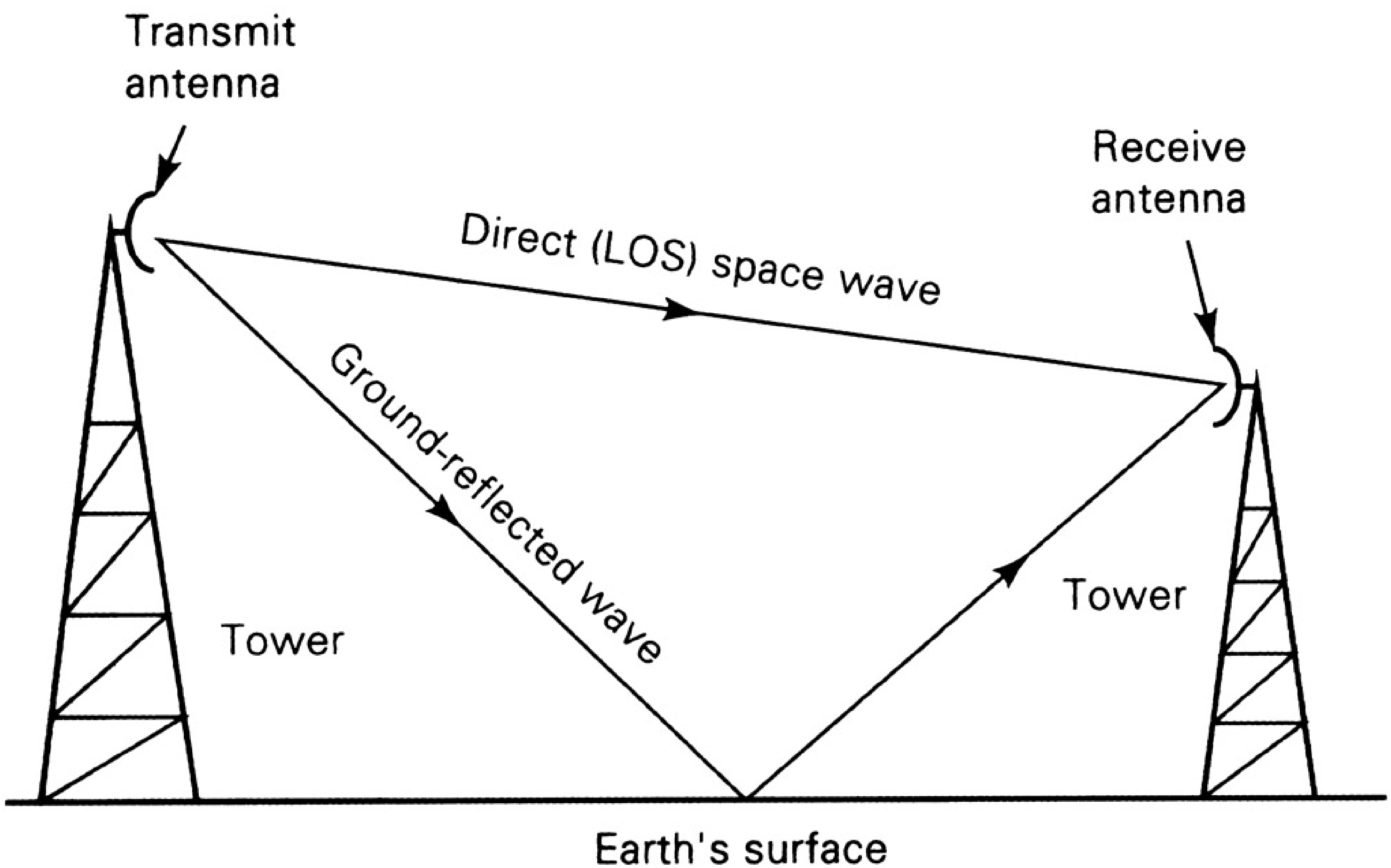
Rx CAPTURED POWER: $P_C=A_C\, p_C =(G_RP_TG_T\, \lambda^2)/(4\pi d)^2=P_T(\, G_TG_R)/(4\pi d/\lambda)^2$
FREE-SPACE LOSS: $L_{FREE\text{-}SPACE}=(4\pi df/c)^2$, i.,e., proportional to d^2 and f^2
$P_{C,dBm} =P_{T,dBm}+ (G_{T,dB}+G_{R,dB}\ )- L_{FS,\ dB}$
$L_{FS,\ dB}= 10\log_{10}(L_{FREE\text{-}SPACE}) =92.44+20\log_{10}(f_{\ GHz}\ )+20\log_{10}(d_{km}\ )$

Decibels: dB, dBm, dBW, dBi

- **dB** (Decibel) = $10 \log_{10} (P_r/P_t)$
  Log-ratio of two signal levels. Named after Alexander Graham Bell. System gains and losses can be added/subtracted, especially when changes are in several orders of magnitude.

- **dBm (dB of mW),** power relative to 1mW, i.e., 0 dBm is 1 mW. X mW is $10\log_{10}(X)$ dBm
- **dBW (dB of W),** power relative to 1W, i.e., 0 dBW is 1W.
  Y W is $10\log_{10}(Y)$ dBW , 0dBW=30dBm
- **dBi (dB isotropic)** The gain a given directional antenna has over a theoretical isotropic (point source) antenna.

Space-wave propagation: line-of-sight (LOS)



LOS TRANSMISSION CONSIDERATION

The presence of the ground modifies the generation and propagation of radio waves so that the received power is ordinarily less than would be expected in free space (P_R)

$$V_r = \sqrt{\frac{P_r}{P_R}} = \left| 1 + \underbrace{R e^{j\Delta}}_{\text{reflected wave}} + \underbrace{(1-R)A e^{j\Delta}}_{\text{surface wave}} + \underbrace{\ldots\ldots\ldots\ldots}_{\text{induction field and secondary effects of the ground}} \right|$$

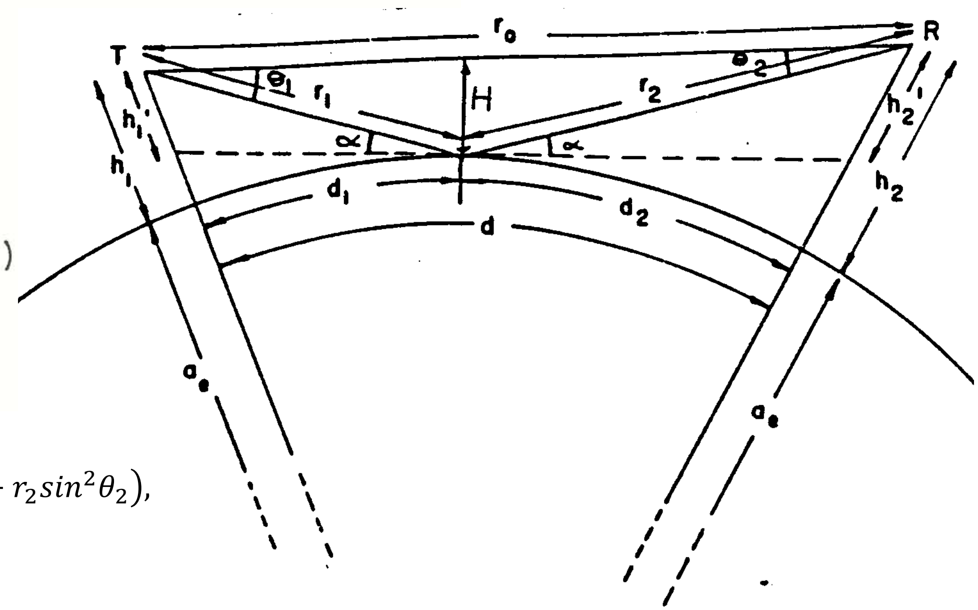
direct wave

where R :reflection coefficient of the ground

A :" surface wave " attenuation factor

$$\Delta = \frac{2\pi}{\lambda}(r_1 + r_2 - r_0) \simeq \left(\frac{\pi}{\lambda} \cdot \frac{dH^2}{d_1 . d_2}\right)$$

$$r_1 + r_2 - r_0 \approx \frac{dH^2}{2d_1 . d_2}$$;H :path clearance



$$r_0 = r_1\sqrt{1 - sin^2\theta_1} + r_2\sqrt{1 - sin^2\theta_2} \approx r_1 + r_2 - \frac{1}{2}\left(r_1 sin^2\theta_1 + r_2 sin^2\theta_2\right),$$

$$r_i \sin\theta_i = H \quad , \sin\theta_i \approx tan\theta_i = \frac{H}{d_i}, i = 1,2$$

FRESNEL ZONES

For near grazing paths and $h_1,h_2 > \lambda$,

R~ -1(worst-case) and A~ 0, and

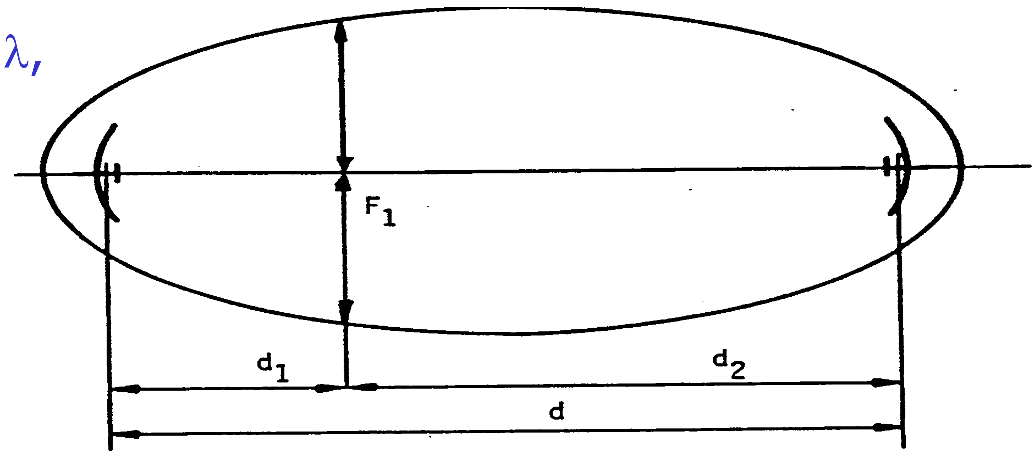
$V_r = 2 \sin \Delta/2$

$= 2 \sin((\pi/2\lambda) .dH^2/(d_1.d_2))$

For $dH^2/(d_1.d_2) = n\lambda$,

$V_r = 2 \sin(n\pi/2)$

the received signal is enhanced for odd n and reduced (cancelled) for even n



FIRST FRESNEL ZONE

The regions in space where these reflections take place are called FRESNEL ZONES, i.e., n^{th} Fresnel zone clearance

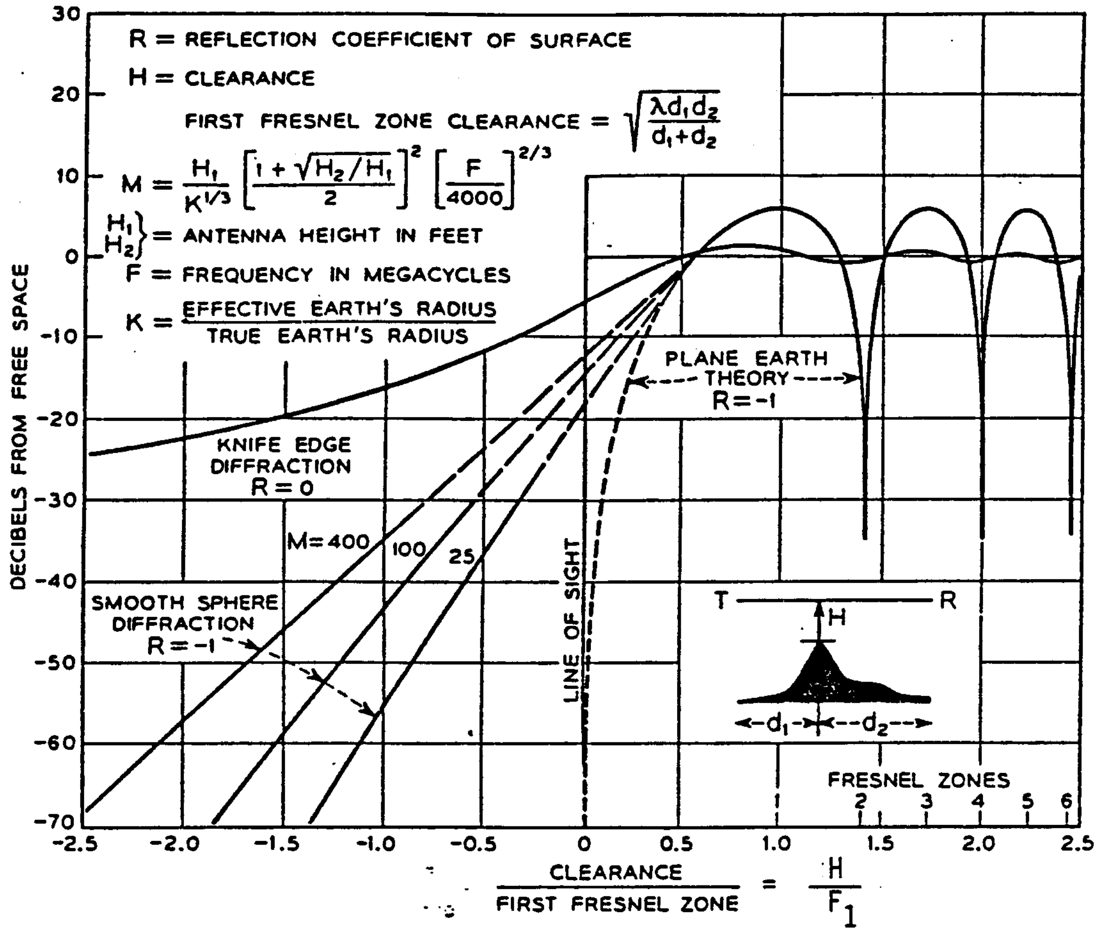
$F_n = \{n\lambda d_1.d_2/d\}^{1/2}$, $F_n = F_1\, n^{1/2}$

It is found in practice that only signals reflected within the first Fresnel zone have a large enough signal amplitude to produce significant interference. As much as possible, precautions are taken to keep this zone free of any obstacles.

TRANSMISSION LOSS VERSUS CLEARANCE

REQUIRED CLEARANCE

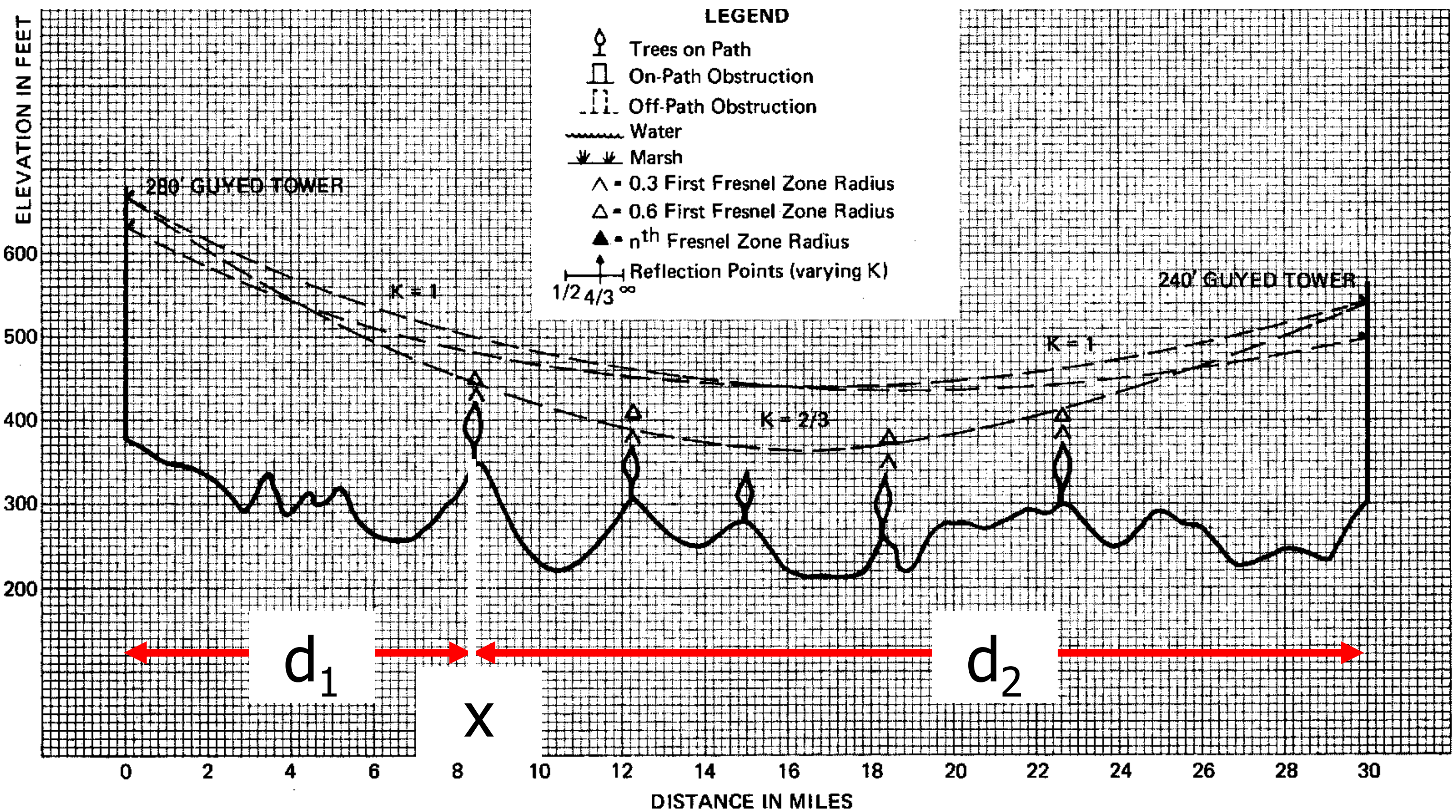
Heavy-route, or highest reliability systems:
- At least 0.3 F_1 @ K=2/3 or
- At least 1.0 F_1 @ K=4/3
- whichever requires the greater heights.
- In areas of very difficult propagation, it may be necessary also to ensure a clearance of at least grazing at K=1/2.
- All criteria should be evaluated along entire path.



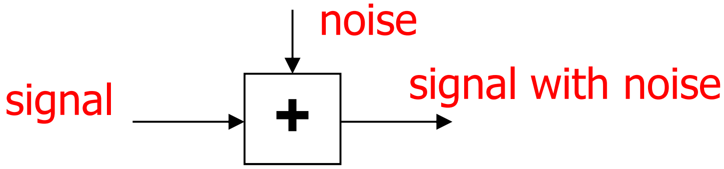
Light-route/ medium reliability systems: At least 0.6 F_1 + 10 feet @ K=1

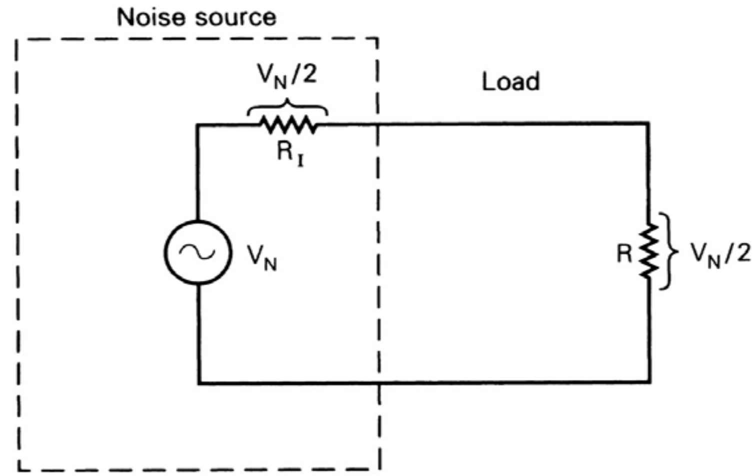
PATH ENGINEERING

for a given link, using up-to-date map plot the terrain profile at each point $x=(d_1,d_2)$ along the link, identify required clearance plot the corresponding los ray and determine the antenna heights



THERMAL NOISE IN RECEIVER





Thermal noise produced by random motion of charged particles (e.g., electrons) has a Gaussian distribution and a power spectral density (PSD): $S_n(f)=h|f|/\{\exp(h|f|/kT)-1\}$ W/Hz

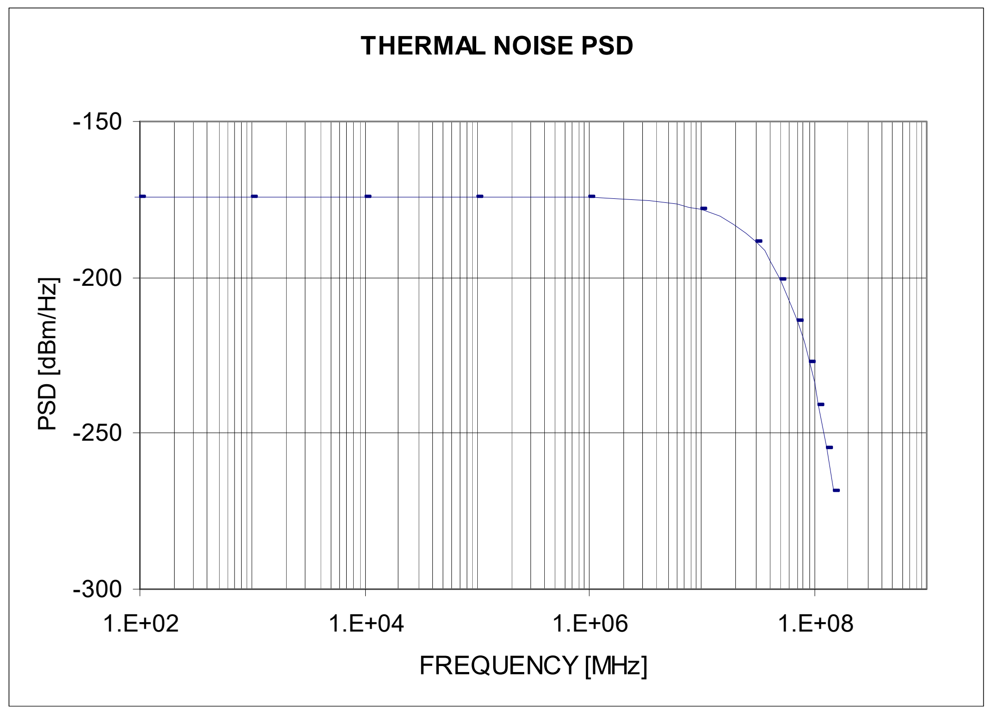
where k=1.38E-23 Joules/°K (Boltzman's constant)

h=6.62E-34 Joules.sec (Plank's constant), °K= 273+ °C

For $|f|<0.1kT/h$ (about 1E12 Hz),

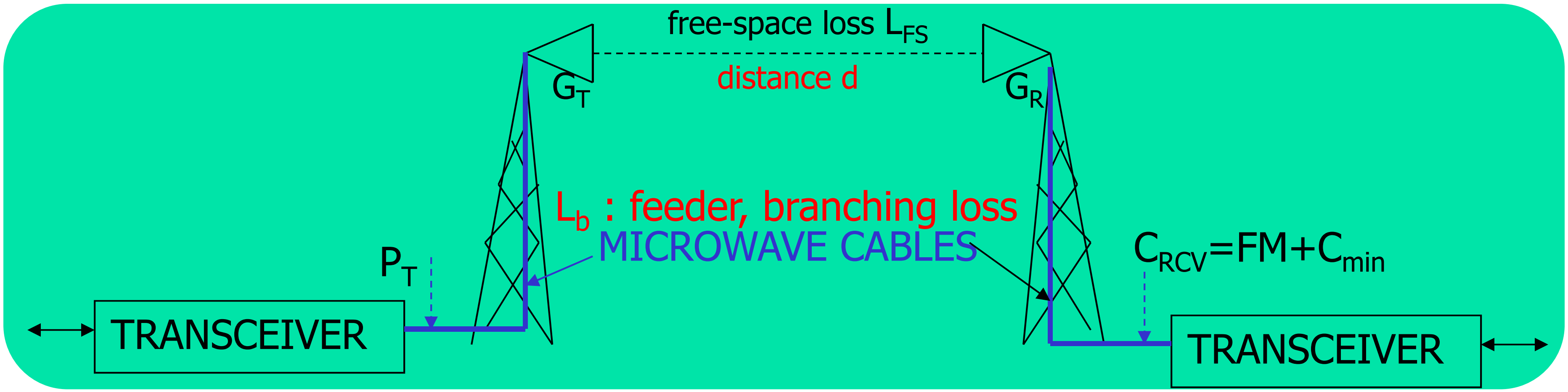
@ room temperature (290°K)

$$S_n(f) \approx kT=-174\text{dBm/Hz}$$



LOS TRANSMISSION EQUATIONS FOR DIGITAL COMMUNICATIONS

- P_T : Transmitter output power excluding antenna gains. (in dBm)
  G_T , G_R (in dBi) :Tx and Rx antenna gains, L_b : feeder, branching loss (Tx/Rx).
  effective isotropically radiated power (in dBm) $EIRP = P_T + G_T - L_b$
  L_{FS} : free-space loss $\mathbf{L_{FS,\,dB} = 92.44+20\log_{10}(f_{GHz})+20\log_{10}(d_{km})}$
  $C_{RCV,\,dBm} = EIRP - \mathbf{L_{FS,\,dB}} + G_R - L_b = P_T + G_T - L_{FS,\,dB} + G_R - 2L_b$



TRANSCEIVER System gain: $G_s = P_T - C_{min}$ in dB
P_T : Transmitter output power excluding antenna gains. (in dBm)
C_{min} : min received power (in dBm) for required quality objective (in BER)
Minimum received power: $C_{min}=10\log_{10}(kT)+NF+10\log_{10}(f_b)+E_b/N_o$
$10\log_{10}(kT)= -174$ dBm/Hz; NF: noise figure of the receiver (dB)
f_b: transmission bit rate E_b/N_o: required for certain threshold BER.

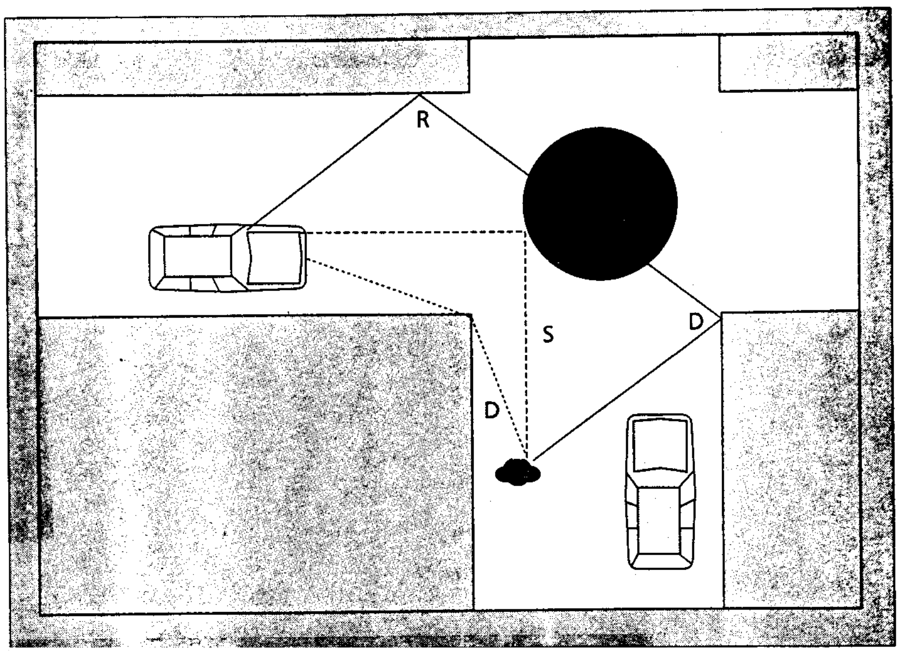
Fade margin: $FM = G_s + G_T + G_R - L_{FS} - 2L_b$

mobile communications channels:
characterization

MULTIPATH PROPAGATION

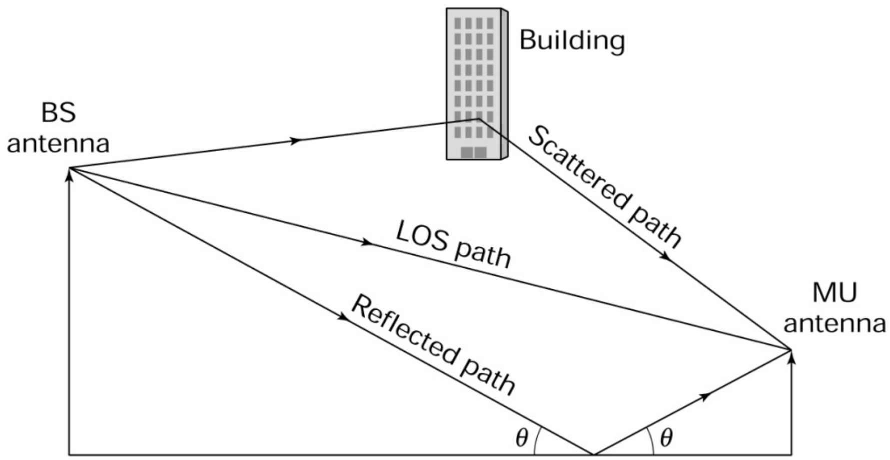
Three basic propagation mechanisms:

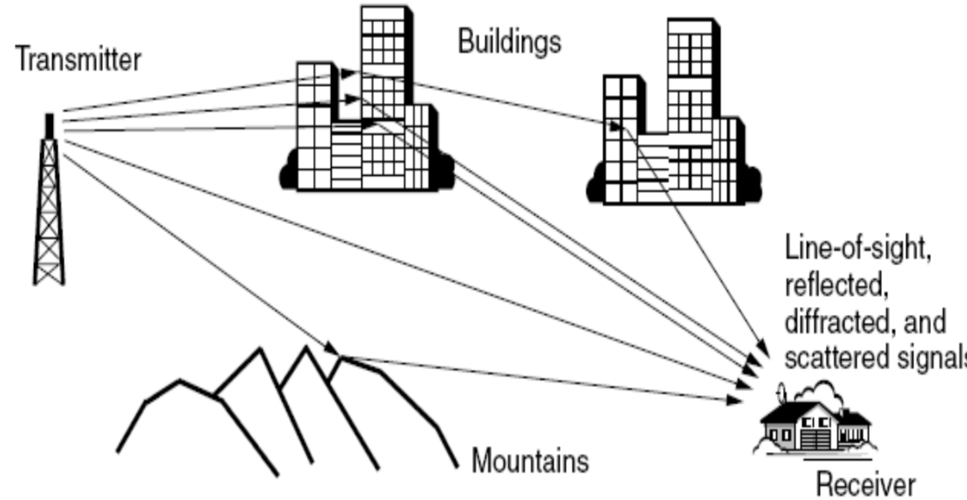
- **Reflection** (R) occurs when a propagation radio wave impinges upon an obstruction with dimensions very large compared to the wavelength of the radio wave.
- **Diffraction** (D) occurs when the radio path between the transmitter and receiver is obstructed by an impenetrable body. Based on Huygens' principle, secondary waves are formed behind the obstructing body even though there is no LOS between the transmitter and receiver.
- **Scattering** (S) occurs when the radio channel contains small objects, rough
- surfaces with dimensions that are on the order of the wavelength or less of the propagating wave, e.g., foliage, lampposts, street signs

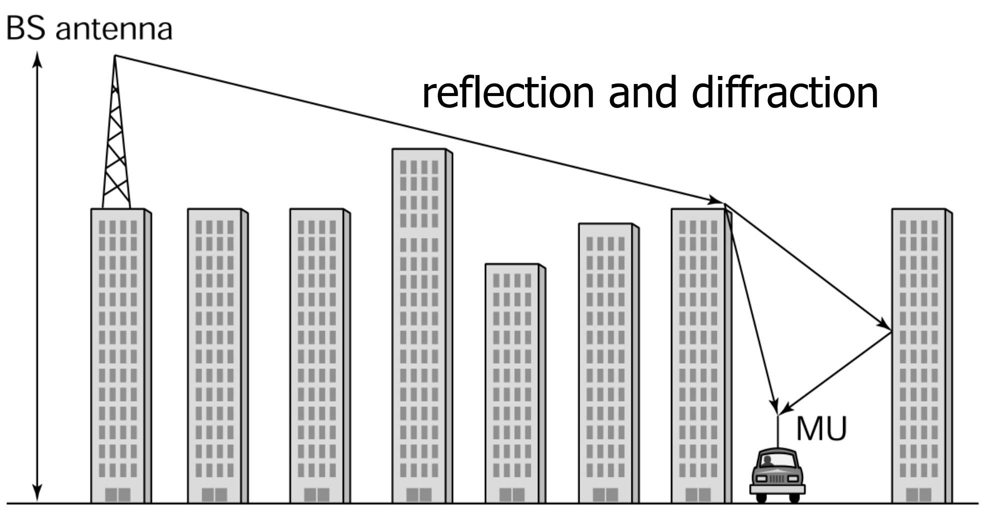


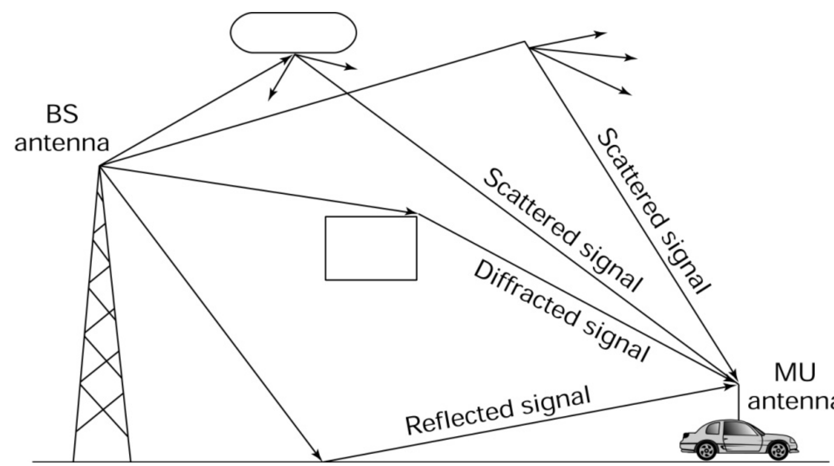
Scattering, which follows the same physical principles as diffraction, causes energy from a transmitter to be reradiated in many different directions. It is the most difficult mechanism to predict.

Multipath Propagation

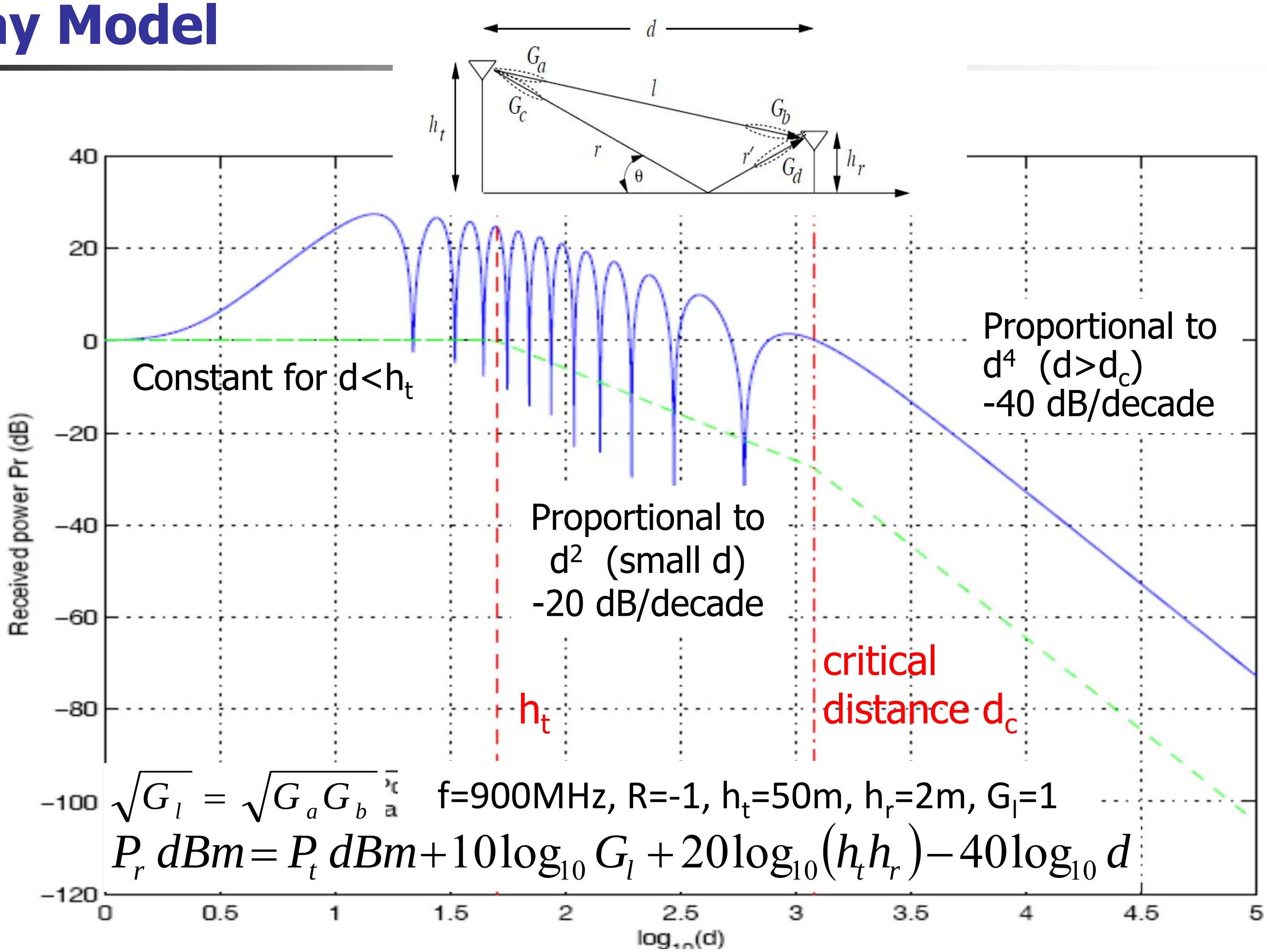








2-Ray Model

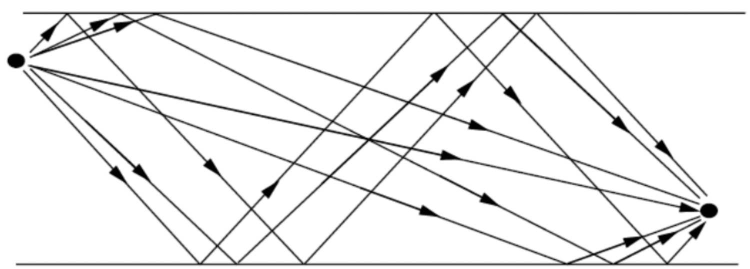


$\sqrt{G_l} = \sqrt{G_a G_b}$ f=900MHz, R=-1, h_t=50m, h_r=2m, G_l=1

$$P_r\, dBm = P_t\, dBm + 10\log_{10} G_l + 20\log_{10}(h_t h_r) - 40\log_{10} d$$

- Received signal power falls off independent of λ(f) since the cancellation of the two rays changes the effective area of the receive antenna

10-Ray Model: Urban Microcells



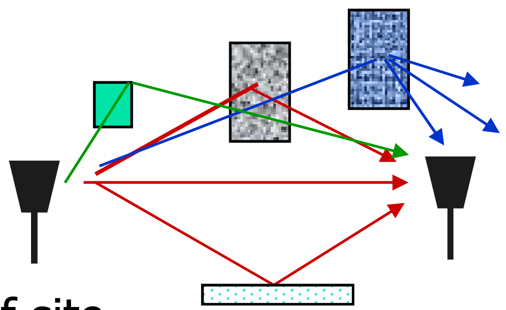
- Ground and 1-3 wall reflections
- Falloff with distance squared (d^{-2})!
  - Dominance of the multipath rays decaying as d^{-2}, …
  - over the combination of the LOS and ground-reflected rays (the two-ray model), which decays as d^{-4}.
- Empirical studies: $d^{-\gamma}$, where $2 \leq \gamma \leq 6$

Ray Tracing:

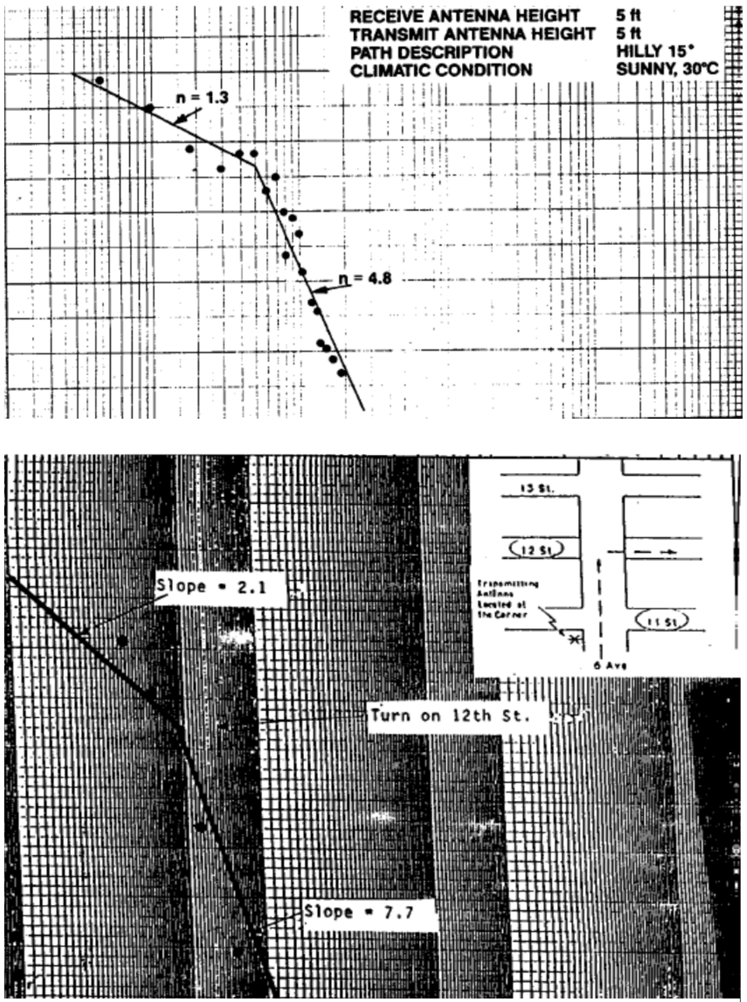
- Represent wavefronts as simple particles
- Effects of reflection, diffraction and scattering on the wavefront can be approximated using simple geometric equations.
- The error is smallest if the receiver is far from the nearest scatterer and the number of scatterers is large
- Typically includes reflected rays, can also include scattered and diffracted rays.
- Requires site parameters
  - Geometry
  - Dielectric properties

General Ray Tracing (GRT):

- Models all signal components
  - Reflections
  - Scattering
  - Diffraction
- Requires detailed geometry and dielectric properties of site
  - Similar to Maxwell, but easier math.
- Computer packages often used



Path-Loss Modeling Techniques



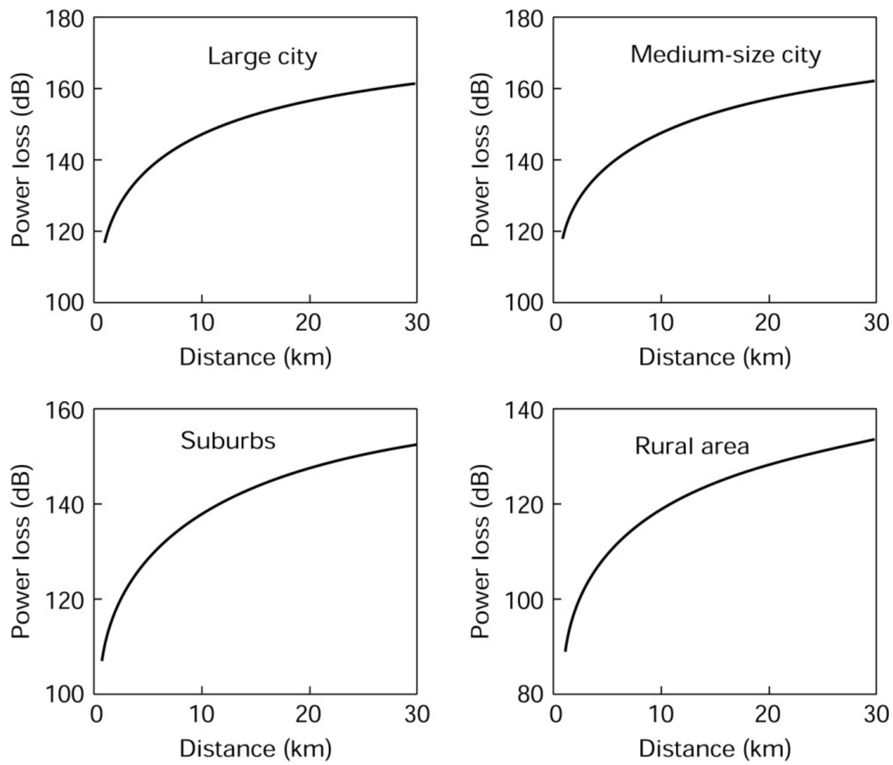
Examples of measured Path-Loss

- **Free-space 2-path loss model:** (too simple) Ground reflection approximately cancels LOS path above a critical distance. Hence, loss Proportional to d^2 (small d) or d^4 ($d>d_c$)
- **Maxwell's equations** (impractically complex)
- **Ray-tracing models:** (simpler to Maxwell)
  - requires site-specific information (e.g., detailed geometry and dielectric properties) to model all signal components (Reflections, Scattering, Diffraction)
- **Empirical Models:** (good for high-level analysis) environment-specific, with simplified power falloff models. $P_r=P_tK[d_0/d]^\gamma$, $2\leq\gamma\leq 8$.
  - Captures main characteristics of path loss
  - Used when path loss dominated by reflections.
  - Most important parameter is the path loss exponent γ, determined empirically.

Examples of Empirical Models:

- Okumura model: based on empirical data in Tokyo
- Hata model: Analytical approximation to Okumura model
- COST 231: Extensions to 2 GHz

Examples of Path Loss based on the Hata model with f = 900 MHz, antenna height: BS: 150 m, MS:1.5m.



Indoor Models

- 900 MHz: 10-20dB attenuation for 1-floor, 6-10dB/floor for next few floors (and frequency dependent)
- Partition loss each time depending upon material (see table)
- Outdoor-to-indoor: building penetration loss (8-20 dB), decreases by 1.4dB/floor for higher floors. (reduced clutter)
- Windows: 6dB less loss than walls (if not lead lined)

Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26

Shadowing

- attenuation from obstructions: random due to the number and types of obstructions are random, e.g., mobile/terminal travels into a propagation shadow behind a building or a hill or other obstacle much larger than the wavelength of the transmitted signal, and the associated received signal level is attenuated significantly.
- typically follows a log-normal distribution, i.e., power in dB value is Gaussian

$$\varepsilon_{dB} \sim N(0, \sigma_\varepsilon^2) \qquad f_{\epsilon(\mathrm{dB})}(x) = \frac{1}{\sqrt{2\pi}\sigma_\epsilon} \exp(-\frac{x^2}{2\sigma_\epsilon^2}).$$

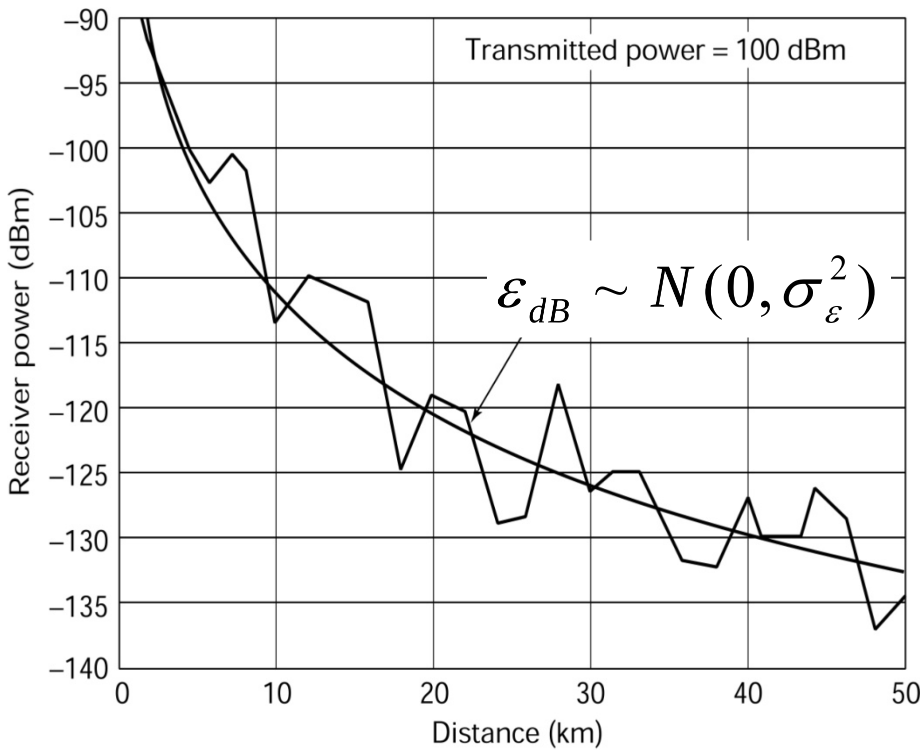
  - zero mean since actual mean was included in the path loss),
  - $4<\sigma_\varepsilon<12$ (empirical)
- decorrelated over some distance called *decorrelation distance*

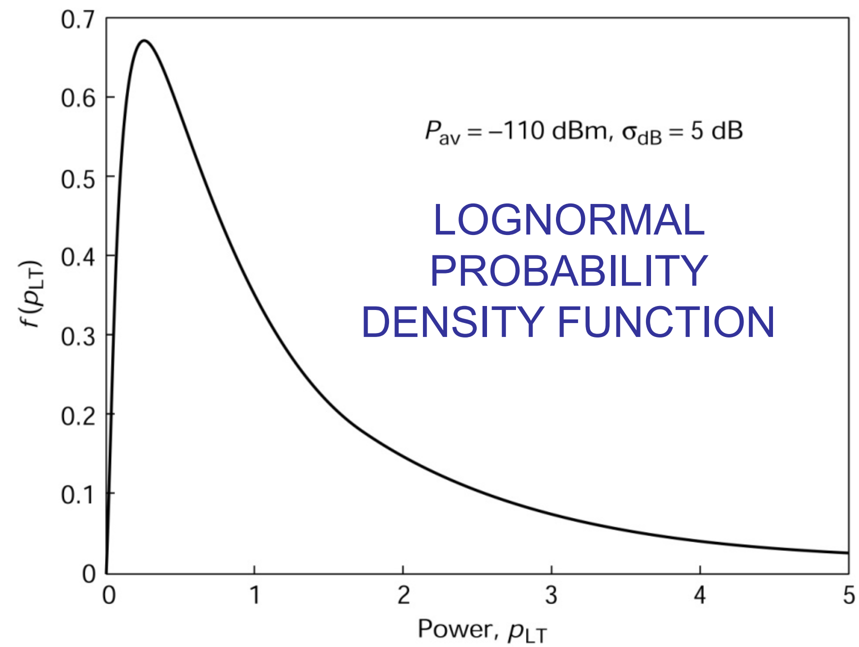
- $\epsilon_{(\mathrm{dB})}$ follows the Gaussian (normal) distribution $\Longrightarrow$ ϵ in linear scale is said to follow a log-normal distribution with pdf given by

$$f_\epsilon(y) = \frac{20/\ln 10}{\sqrt{2\pi}y\sigma_\epsilon} \exp[-\frac{(20\log_{10} y)^2}{2\sigma_\epsilon^2}].$$

- σ_ϵ: 8 dB for an outdoor cellular system and 5 dB for an indoor environment.

Received power under path-loss & shadowing



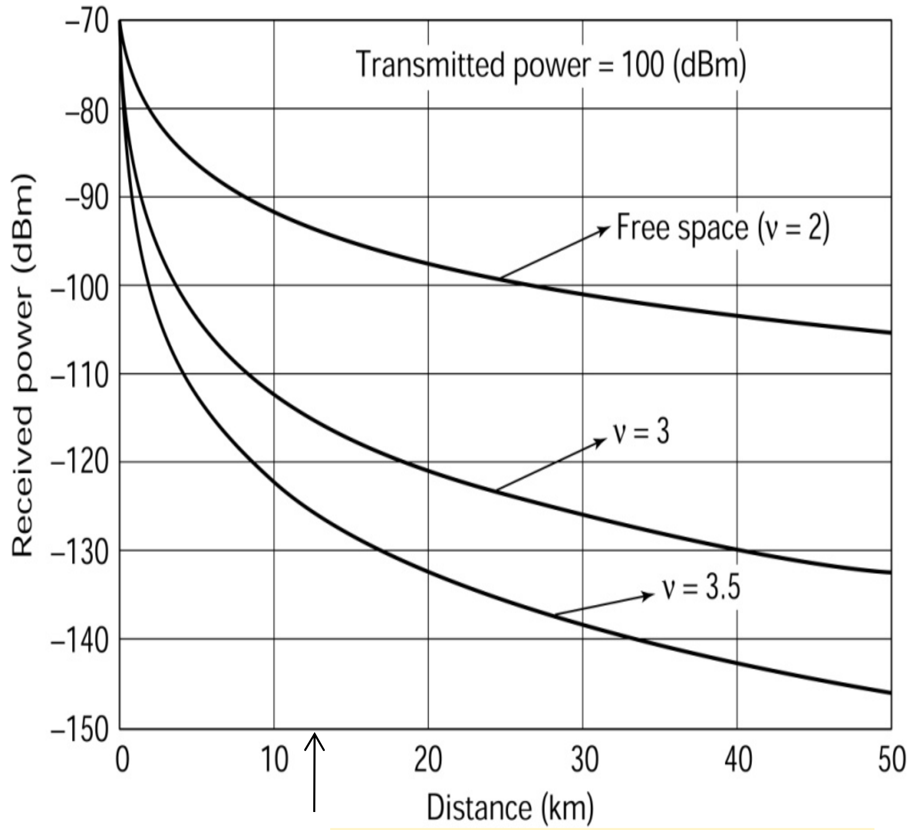


Large-scale Path Loss and Shadowing Models

- Recall: fixed LOS free-space loss: $L_{FS,\,dB} = 92.44 + 20\log_{10}(f_{GHz}) + 20\log_{10}(d_{km})$
- general Path Loss and Shadowing: $\varepsilon_{dB} \sim N(0, \sigma_{\varepsilon}^2)$
  $L_p(d,f) = L_o + 10\kappa\log_{10}(d/d_o) + 10n\log_{10}(f/f_o) + \varepsilon_{dB}$
- L_o obtained from measurements at d_0 (=1km, macrocell, 100m, microcell outdoor, 10mpicocell indoor)
- κ: path-loss exponent usually=$2 \leq \kappa \leq 8$, n: frequency-loss exponent, are MMSE estimates based on data
- Shadowing variance is estimated variance of measured data relative to straight-line path-loss

environment	**Path-loss exponent**
Free-space	2
Urban microcells	2.7-3.5
Urban macrocells	3.7-6.5
Office (same floor)	1.6-3.5
Office (multi-floor)	2-6
store	1.8-2.2
factory	1.6-3.3
home	3

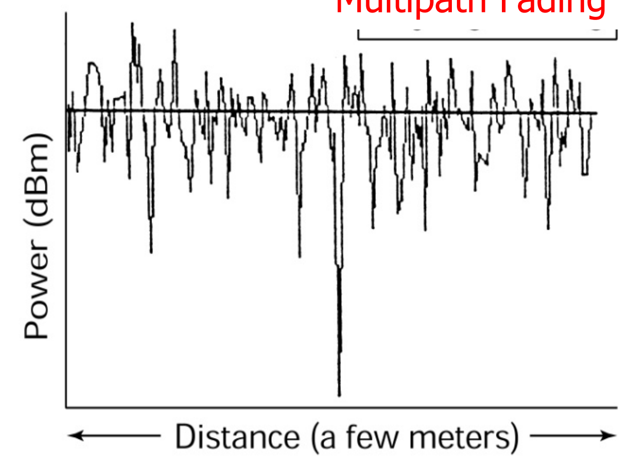
POWER LOSSES



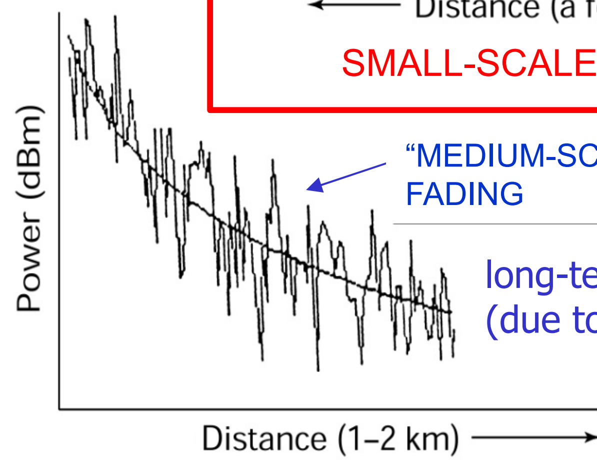
LARGE-SCALE FADING

VERY LONG-TERM PATH LOSS
(includes *average* shadowing)
in dB= $A+10v\log_{10}(\text{distance})$

short-term
Multipath Fading



SMALL-SCALE FADING

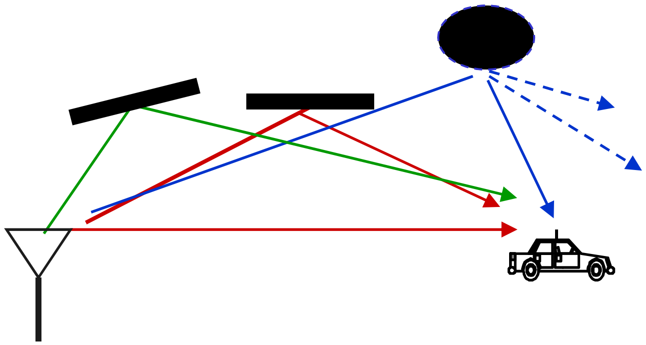


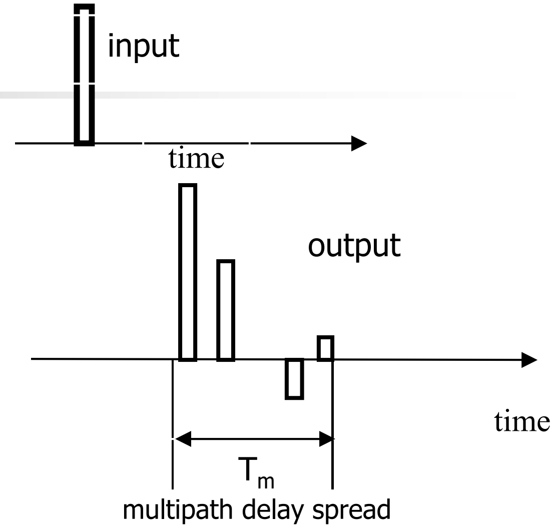
“MEDIUM-SCALE” FADING

long-term shadowing
(due to obstructions)

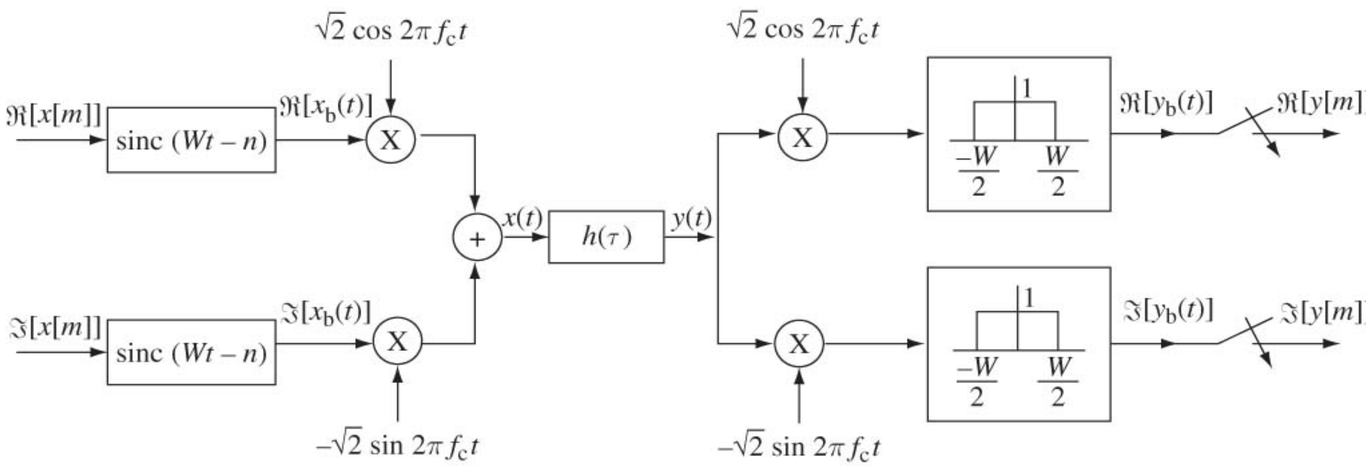
multipath fading channels: modeling

Multipath Modeling





- Channel consists of a random number of path components, each with random amplitude, phase, Doppler shift, delay, changing with time. Multipath fading due to constructive and destructive interference of the transmitted waves.
- W: signal bandwidth, sampling rate: 1/W
- Transmission at passband [f_c-W/2, f_c+W/2] and processing at baseband [-W/2,+W/2].



LINEAR TIME-VARIANT CHANNEL MODEL OF MULTIPATH PROPAGATION

- The (baseband) impulse response of an LTV channel, h(τ; t), is the channel output at t in response to an impulse applied to the channel at (t-τ), i.e., τ is how long ago impulse was put into the channel for the current observation.
- Each path of h(τ; t) is associated with a delay and a complex gain

Multipath channel due to N scatters characterized

by amplitude $\alpha_n(t)$ and delay $\tau_n(t)$ for n=1,2,...,N:

Tx signal: $\mathrm{Re}\left[x(t)e^{-j\omega_c t}\right], x(t): complex - baseband$

$\Rightarrow$ Rx signal (without noise): $\mathrm{Re}\left[r(t)e^{-j\omega_c t}\right]$

$$\text{where } r(t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j\omega_c \tau_n(t)} x(t - \tau_n(t))$$

$$h(\tau;t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j\omega_c \tau_n(t)} \delta(t - \tau_n(t))$$

Linear time-variant (LTV) channel:

Received signal consists of many components with slow amplitude changes, but fast phase changes, introducing constructive and destructive addition of signal components.

$$r(t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j\omega_c\tau_n(t)}x(t-\tau_n(t))$$

$$h(\tau;t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j\omega_c\tau_n(t)}\delta(t-\tau_n(t))$$

$$r(t)=\lim_{\partial\tau\to 0}\sum_{m=-\infty}^{+\infty}h(m\partial\tau;t)x(t-m\partial\tau)\partial\tau=\int_{-\infty}^{+\infty}h(\tau;t)x(t-\tau)d\tau$$

$$\text{input: } x_1(t)\rightarrow \text{ output: } r_1(t)=\int_{-\infty}^{+\infty}h(\tau;t)x_1(t-\tau)d\tau$$

$$\text{input: } x_2(t)=x_1(t-t_1)\rightarrow \text{ output: } r_2(t)=\int_{-\infty}^{+\infty}h(\tau;t)x_1(t-t_1-\tau)d\tau\neq r_1(t-t_1)$$

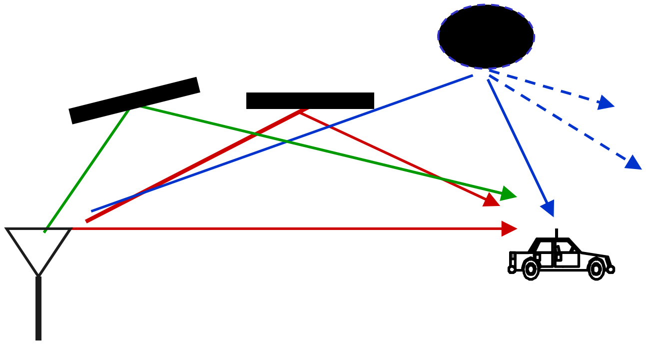
$$\text{since } h(\tau;t)\neq h(\tau;t-t_1) \text{ in general}$$

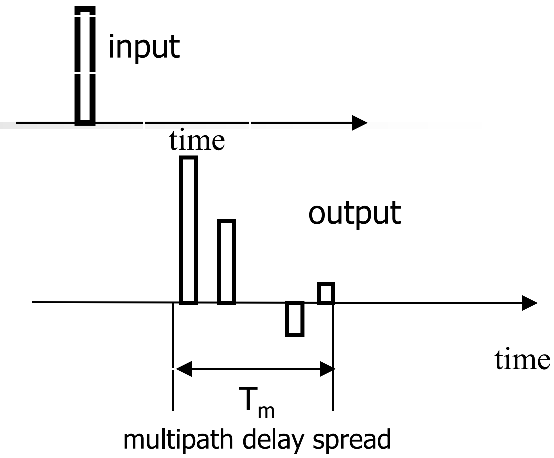
$$\text{Transfer function: } H(f;t)=\mathrm{F}_\tau\left\{h(\tau;t)\right\}=\int_{-\infty}^{+\infty}h(\tau;t)e^{-j2\pi f\tau}d\tau$$

$$H(f;t)\leftrightarrow h(\tau;t)=\mathrm{F}_f^{-1}\left\{H(f;t)\right\}=\int_{-\infty}^{+\infty}H(f;t)e^{j2\pi f\tau}df$$

$$R(f;t)=H(f;t)X(f) \text{ where } X(f)=\mathrm{F}\left\{x(t)\right\}$$

Narrowband frequency-flat fading:





- The channel consists of random number of paths, each with random amplitude, phase, delay and Doppler shift.
- Delay spread: $T_m = \max_{m,n} |\tau_n(t) - \tau_m(t)|$
- If $T_m \ll 1/W$, W:signal BW, then $x(t) \approx x(t-\tau_n)$.
- Received signal given by

$$r(t) \approx x(t) \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\varphi_n(t)} = a(t)x(t)$$

- Multipath effects: represented by a single complex random fading tap $a(t)$.
- No signal distortion (no spreading in time, frequency-flat fading)
- For large N(t), the Im and Re parts of a are jointly Gaussian (they are i.i.d., stationary if $\varphi_n(t) \sim U[0,2\pi]$)
- Received signal characterized by its mean, autocorrelation, and cross correlation.
- $T_m \ll 1/W$: single tap (resolvable path), frequency-flat fading

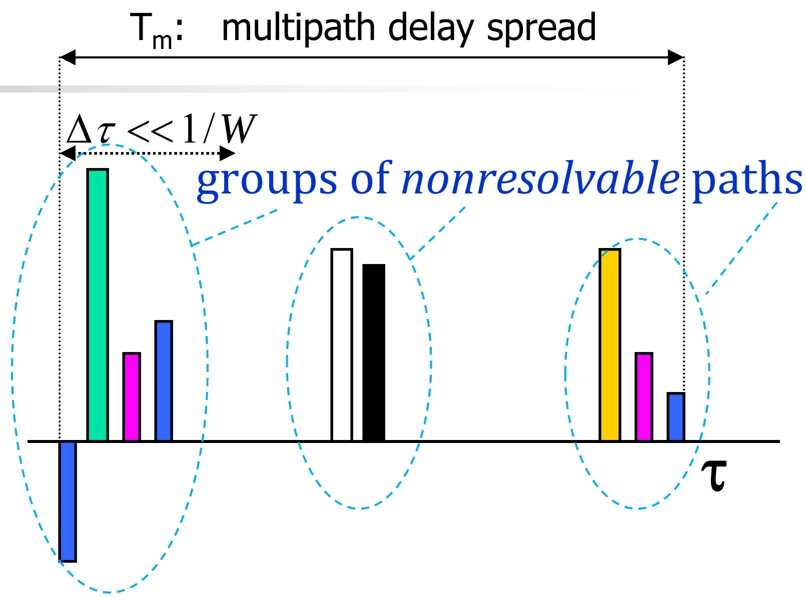
Frequency-Selective Fading

- If $T_m > 1/W$, W:signal BW, then significant time spreading → substantial signal distortion (wideband fading)
- Two paths with delays τ_1, τ_2 are *resolvable* if $|\tau_1 - \tau_2| >> W^{-1}$ where W^{-1} is the *Tx symbol interval*. Otherwise, the paths are *nonresolvable*.
- A group of *nonresolvable* paths are represented (modeled) by one *single* fading component with an amplitude undergoing fast variation.
- A physical multipath channel can be represented by a number of *resolvable* fading components. Sampled baseband-equivalent received signal:

$$r[m] = r(m/W) = \sum_{k=K_1}^{K_2} h_m[k]\, x[m-k] + w[m],$$

$w[m]$: zero-mean complex Gaussian noise

$$h_m[k] \approx \sum_{n^*} \alpha_n(t) e^{-j\omega_c \tau_n(t)}: \text{complex, random tap}$$



- n^* denotes *all* n's corresponding to delays $\tau_n(t)$'s within the time interval $[(k-1)/2W, (k+1)/2W]$, and all possible paths τ_n's are in the time interval

  $[(K_1-1)/2W, (K_2+1)/2W]$.
- Delay spread $T_m \le (K_2 - K_1 + 1)/W$. Coherence BW$=1/T_m$
- $T_m > 1/W$: multiple taps (resolvable paths), frequency-selective fading

Time-variant & Doppler effects

$$\tau_n(t) = \int \tau'_n(t)\,dt \rightarrow \omega_c \tau_n(t) = 2\pi \int f_c \tau'_n(t)\,dt,$$

$$f_{dn}(t) = f_c \tau'_n(t) : \text{Doppler frequency shift in path } n$$

$$\text{Doppler frequency spread: } B_d = f_c \max_{l,n} \left| \tau'_n(t) - \tau'_l(t) \right|$$

- Coherence time: $(\Delta t)_c = 1/B_d$,
- For $1/B_d$ >>1/W, slow (TIME-FLAT) fading
- For $1/B_d$ < 1/W, fast (TIME-SELECTIVE) fading
- $T_m < 1/B_d$: *under-spread* channels (typical):
  - Delay spread T_m depends on distance to scatterers, on the order of nanoseconds (indoor) to microseconds (outdoor).
  - Coherent time $1/B_d$ depends on carrier frequency and vehicular speed, on the order of milliseconds or more.
  - over a long-time scale, channel can be considered as time-invariant.
- $T_m << T_s << 1/B_d$: slow and frequency-flat fading transmission

STATIONARY RANDOM PROCESS: REVIEW

- **Definition: Strictly** or strict-sense stationary (SSS) random process X(t):
  - Its 1st-order distribution function **independent of t**:
    $F_{X(t)}(x) = F_{X(t+t')}(x)$, for $\forall t,t'$
  - Its 2nd-order distribution function depends only on the **time difference** (between 2 observation times t, t'):
    $F_{X(t),X(t')}(x,x') = F_{X(0),(t'-t)}(x,x')$, $\forall t,t'$
- Results: **Strictly** stationary random process X(t) has:
  - **Constant mean**: $m_X(t) \equiv E\{X(t)\} = m_X$ for $\forall t$
  - **Autocorrelation function** depends only on **time difference**: d= t-t'
    $R_X(t,t') \equiv 0.5E\{X^*(t)X(t')\} = R_X(d) = R_X(-d)$; $R_X(0) = E\{|X(t)|^2\} \geq |R_X(d)|$,
    power spectral density (psd): $$S_X(f) \equiv \int_{-\infty}^{+\infty} R_X(\tau) e^{-j2\pi f\tau} d\tau$$

$R_X(t-t')$ describes the interdependence of 2 RV's obtained by observing X(t) at times t and t': $R_X(t-t')$ with a wide pulse-width indicates a slowly fluctuating X(t)

The above results are not sufficient to guarantee that X(t) is **strictly stationary.** If X(t) ONLY has the above characteristics, it is called **wide-sense stationary (WSS)**, or **2nd-order** or **weakly stationary**

WSSUS MODEL OF MULTIPATH CHANNEL

- A channel is **wide-sense stationary uncorrelated scattering** (WSSUS) when:
  (a) its impulse response h(τ; t) is a wide-sense stationary (WSS) process;
  (b) US: its impulse responses at τ_1 and τ_2 , h(τ_1; t) and h(τ_2; t), are uncorrelated if $\tau_1 \neq \tau_2$ for any t.
- **autocorrelation function of h(τ; t) in WSSUS case:**

$$\text{WSS: } \phi_h(\tau,\tau+\Delta\tau,\Delta t) = 0.5E\left\{h^*(\tau;t)h(\tau+\Delta\tau;t+\Delta t)\right\} \text{ depends on } \Delta t \text{ and } \tau,\tau+\Delta\tau$$

$$\text{US: } E\left\{h^*(\tau;t)h(\tau+\Delta\tau;t+\Delta t)\right\} = E\left\{h^*(\tau;t)h(\tau;t+\Delta t)\right\}\delta(\Delta\tau)$$

$$\text{WSSUS: } \phi_h(\tau,\tau+\Delta\tau,\Delta t) = \phi_h(\tau,\tau,\Delta t)\delta(\Delta\tau)\text{: delay cross-power density}$$

$$\text{For } \Delta t = 0, \phi_h(\tau,\tau,\Delta t) \equiv \phi_h(\tau)\text{: multipath intensity profile (or delay power density)}$$

multipath intensity profile (WSSUS case)

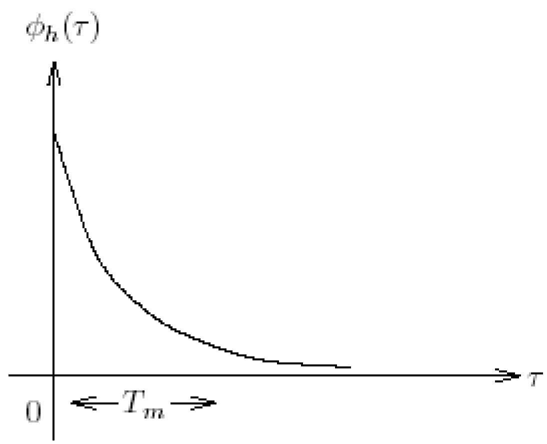
- **multipath intensity profile** (or *delay power density*) provides the average power at the channel output as a function of the propagation delay, τ.

$$\Delta t = 0, \phi_h(\tau, \tau, \Delta t) \equiv \phi_h(\tau)$$

Approximate max delay of significant multipath:

- **multipath delay spread**, T_m: nominal width of the multipath intensity profile $\phi_h(\tau)$: range of τ over which $\phi_h(\tau)$ is essentially non-zero
- Usually, it is assumed that $T_m \approx \sigma_\tau$

$$\sigma_\tau = \left[\frac{\int (\tau - \bar{\tau})^2 \phi_h(\tau) d\tau}{\int \phi_h(\tau) d\tau} \right]^{1/2} \qquad \bar{\tau} = \frac{\int \tau \phi_h(\tau) d\tau}{\int \phi_h(\tau) d\tau}$$



autocorrelation function of transfer function H(f; t)

$H(f;t) = \mathrm{F}_\tau\{h(\tau;t)\}$: time-varying channel transfer function

time-frequency correlation function: $\phi_H(\Delta f;\Delta t) = 0.5E\{H^*(f;t)H(f+\Delta f;t+\Delta t)\}$

$$\phi_H(\Delta f;\Delta t) = \iint_{-\infty}^{+\infty} 0.5E\{h^*(\tau;t)h(\tau';t+\Delta t)\}\, e^{j2\pi f\tau - j2\pi((f+\Delta f)(\tau+\Delta\tau))} d\tau d\tau', \quad \tau' = \tau + \Delta\tau$$

$$\phi_H(\Delta f;\Delta t) = \iint_{-\infty}^{+\infty} \phi_h(\tau,\tau',t,t+\Delta t)\, e^{j2\pi f\tau - j2\pi((f+\Delta f)(\tau+\Delta\tau))} d\tau d\tau'$$

WSSUS: $\phi_h(\tau,\tau+\Delta\tau,\Delta t) = \phi_h(\tau,\tau,\Delta t)\delta(\Delta\tau)$: depends only on $\tau, \Delta t$

$\rightarrow$WSSUS: $\phi_H(\Delta f;\Delta t) = \int_{-\infty}^{+\infty} \phi_h(\tau,\tau,\Delta t) e^{-j2\pi\Delta f\tau} d\tau = \mathrm{F}_\tau\{\phi_h(\tau,\tau,\Delta t)\}$

frequency-correlation function: for $\Delta t = 0$, $\quad \phi_H(\Delta f) \leftrightarrow \phi_h(\tau) = \mathrm{F}_{\Delta f}^{-1}\{\phi_H(\Delta f)\} = \int_{-\infty}^{+\infty} \phi_H(\Delta f) e^{j2\pi\Delta f\tau} d\Delta f$

US$\Rightarrow \phi_H(\Delta f) = 0.5E\{H^*(f;t)H(f+\Delta f;t)\}$ depends only on Δf : WSS in frequency

Fourier transform relations for WSSUS h(τ; t)

auto-correlation function of $h(\tau;t)$: $\phi_h(\Delta\tau;\Delta t) = 0.5E\{h^*(\tau;t)h(\tau+\Delta\tau;t+\Delta t)\} = \phi_h(\tau,0,\Delta t)\delta(\Delta\tau)$;

for Δt=0, $\phi_h(\tau)$: multipath intensity profile

time-frequency correlation function of $H(f;t)$: $\phi_H(\Delta f;\Delta t) = 0.5E\{H^*(f;t)H(f+\Delta f;t+\Delta t)\}$

Fourier transform relations:

$H(f;t) = \mathrm{F}_\tau\{h(\tau;t)\}$: time-varying channel transfer function

$H(f;v) = \mathrm{F}_t\{H(f;t)\}$: Doppler-spread function

$\phi_H(\Delta f;\Delta t) = \mathrm{F}_\tau\{\phi_h(\tau;\Delta t)\}, \phi_h(\tau;\Delta t) = \phi_h(\tau,\tau,\Delta t)\delta(\Delta\tau)$: US

for Δt=0, $\phi_H(\Delta f) = \mathrm{F}_\tau\{\phi_h(\tau)\}$: delay power spectrum,

correlation of channel gains at f and Δf for any t.

$S_H(\Delta f;v) = \mathrm{F}_{\Delta t}\{\phi_H(\Delta f;\Delta t)\} = \mathrm{F}_\tau\{S_h(\tau;v)\}$;

$S_H(\Delta f;v) = 0.5E\{H^*(f;v)H(f+\Delta f;v+\Delta v)\}$: Auto-correlation function of $H(f;v)$

for Δf=0, Doppler power spectrum $S_H(v) = \int_{-\infty}^{+\infty} S_h(\tau;v)d\tau$

$S_h(\tau;v) = \mathrm{F}_{\Delta t}\{\phi_h(\tau;\Delta t)\}$: scattering function

scattering function in case of WSSUS h(τ; t)

scattering function:

- measures power vs delay and Doppler
- used to characterize channel rms delay and Doppler spread.

$S_h(\tau;\nu) = \mathrm{F}_{\Delta t}\left\{\phi_h(\tau;\Delta t)\right\}$: scattering function

time-frequency correlation function of $H(f;t)$:

$$\phi_H(\Delta f;\Delta t) = \mathrm{F}_{\tau}\left\{\phi_h(\tau;\Delta t)\right\} = \mathrm{F}_{\tau}\left\{\mathrm{F}_{\nu}^{-1}\left\{S_h(\tau;\nu)\right\}\right\} = \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} S_h(\tau;\nu)e^{j2\pi\Delta t\nu}d\nu\right]e^{-j2\pi\Delta f\tau}d\tau$$

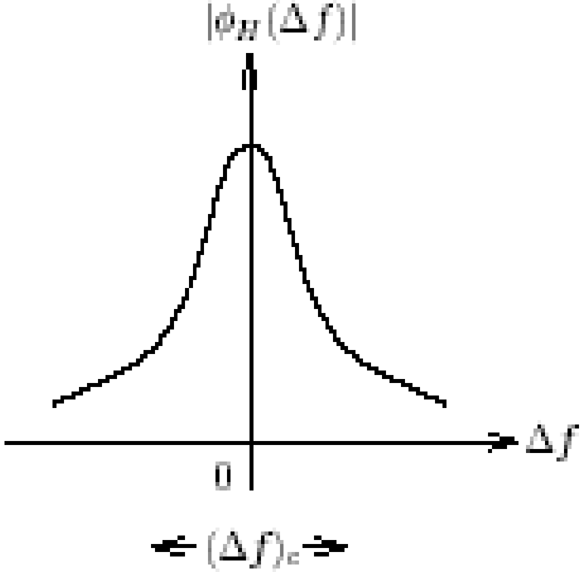
for Δt=0, delay power spectrum: $\phi_H(\Delta f) = \int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty} S_h(\tau;\nu)d\nu\right]e^{-j2\pi\Delta f\tau}d\tau$

Delay spread & frequency-selective fading

$\phi_h(\tau)$: multipath intensity profile $\rightarrow \phi_H(\Delta f) = \mathrm{F}_\tau\{\phi_h(\tau)\}$: delay power spectrum, correlation of channel gains at f and Δf for any t

$\phi_H(\Delta f)=0$ implies signals separated in frequency by Δf will be uncorrelated after passing through channel

coherence bandwidth of the channel, $(\Delta f)_c \approx 1/T_m$: The maximum frequency difference for which the signals are still strongly correlated. Two sinusoids with frequency separation larger than $(\Delta f)_c$ are affected differently by the channel at any t.



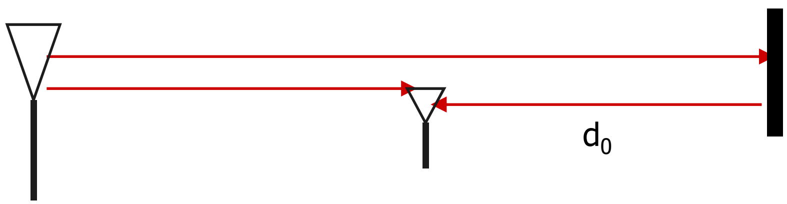
W: bandwidth of the transmitted signal.

If $(\Delta f)_c$ < W, frequency-selective fading channel: severe ISI;

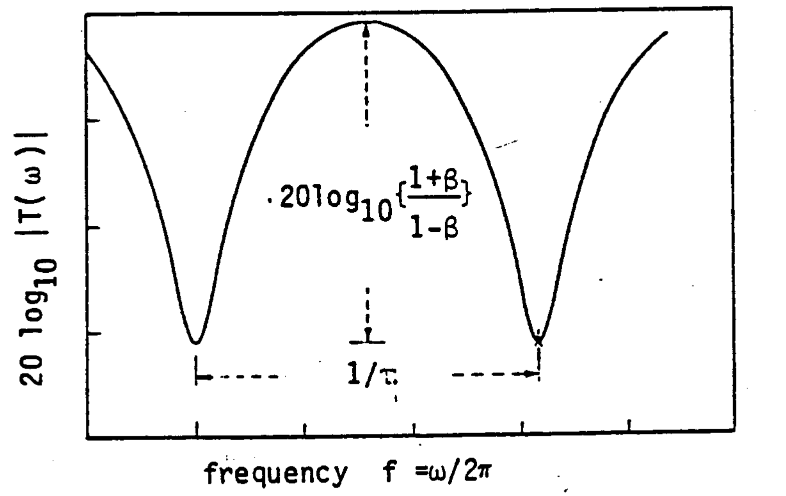
If $(\Delta f)_c$ >> W, flat fading channel: negligible ISI.

$$\text{ISI-free channel: } \phi_H(\Delta f) \approx \phi_o\text{:constant} \leftrightarrow \phi_h(\tau) = \phi_o \delta(\tau)$$

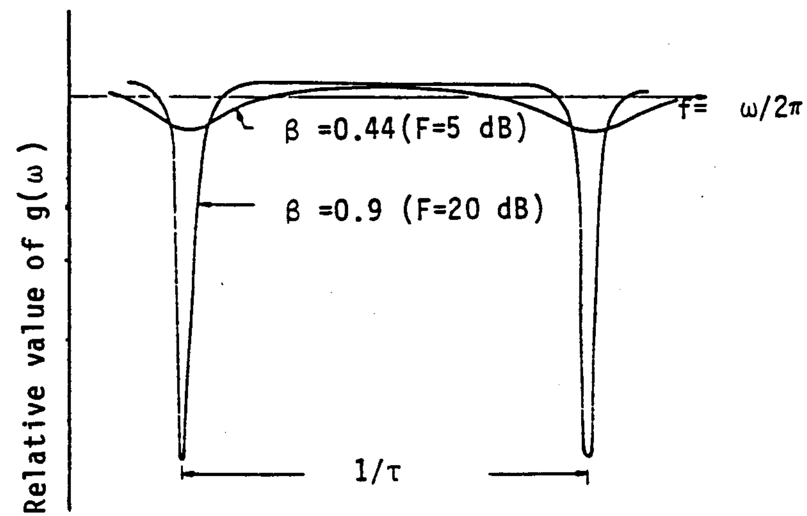
EXAMPLE OF 2-PATH MODEL



At receiver, the received signal is
$r(t)= x(t) + \beta\ x(t-\tau)$
where x(t) : the main path
β : relative level between the main and reflected paths
$\tau=2d_0/c$: relative time delay between the main and reflected path,
Channel transfer function $T(\omega) = 1 + \beta e^{-j\omega\tau}$
Amplitude distortion:
$|T(\omega)|^2= 1 + \beta^2 + 2\beta\cos\omega\tau = 1 + \beta^2 + 2\beta\cos\omega\tau$
phase distortion:
$\Phi(\omega) = \tan^{-1}[\beta\sin\omega\tau/(1 + \beta\cos\omega\tau)]$
group delay distortion $g(\omega) = d\Phi/d\omega$
$g(\omega)=\beta\tau(\beta+\cos\omega\tau)/(1 + \beta^2 + 2\beta\cos\omega\tau)$



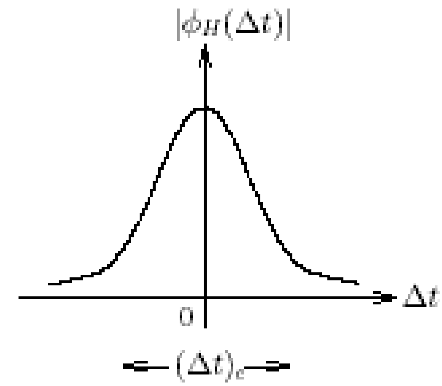
(a) AMPLITUDE DISTORTION (| T(ω)| in dB)

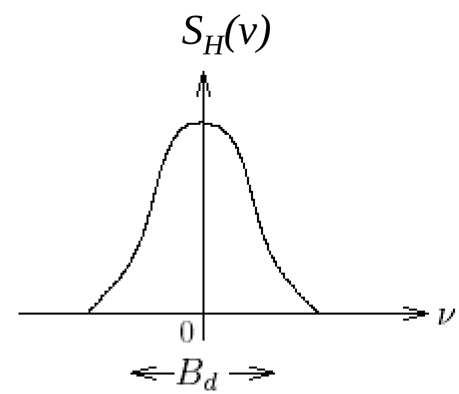


(b) GROUP DELAY DISTORTION (g(ω))

Time correlation function $\phi_H(\Delta t)$ & Doppler power spectrum $S_H(\nu)$

- $\phi_H(\Delta t)$ is independent of f due to the US assumption: US in the time domain is equivalent to WSS in the frequency domain (depends only on Δf) .
- $\phi_H(\Delta t)$ characterizes, on average, how fast the channel transfer function changes with time.
- $\phi_H(\Delta t)=0$ implies signals separated in time by Δt will be uncorrelated after passing through channel.
- **coherence time** of the fading channel, $(\Delta t)_c$: Maximum time over which $\phi_H(\Delta t)>0$: nominal width of $\phi_H(\Delta t)$.
- $(\Delta t)_c >> T$(symbol interval of the Tx signal): slow fading.
- Doppler power spectrum $\phi_H(\nu)$: Fourier transform of the time correlation function $\phi_H(\Delta t)$
- Doppler spread B_d is maximum Doppler for which $\phi_H(\nu)\to 0$: nominal width of $\phi_H(\nu)$, $B_d \approx 1/(\Delta t)_c$





Time correlation function $\phi_H(\Delta t)$ & Doppler power spectrum $S_H(v)$

$H(f;t) = \mathrm{F}_\tau\{h(\tau;t)\}$: time-varying channel transfer function

time-frequency correlation function of $H(f;t)$:

$\phi_H(\Delta f;\Delta t) = 0.5E\{H^*(f;t)H(f+\Delta f;t+\Delta t)\}$

WSSUS $\rightarrow \phi_H(\Delta f;\Delta t) = \mathrm{F}_\tau\{\phi_h(\tau;\Delta t)\}$

for $\Delta t=0$, $\phi_H(\Delta f) = \mathrm{F}_\tau\{\phi_h(\tau)\}$: delay power spectrum,

correlation of channel gains at f and Δf for any t.

$S_H(\Delta f;v) = \mathrm{F}_{\Delta t}\{\phi_H(\Delta f;\Delta t)\}$;

for $\Delta f=0$, $\phi_H(\Delta t)$: time-correlation function of $H(f;t)$, Doppler power spectrum $S_H(v)$

$\left(S_H(\Delta f;v) = \mathrm{F}_\tau\{S_h(\tau;v)\},\ S_h(\tau;v) = \mathrm{F}_{\Delta t}\{\phi_h(\tau;\Delta t)\}:\ \text{scattering function}\right)$

$$\bar{v} = \left[\int v S_H(v)dv\right]\left[\int S_H(v)dv\right]^{-1},\quad \sigma_v^2 = \left[\int [v-\bar{v}]^2 S_H(v)dv\right]\left[\int S_H(v)dv\right]^{-1}$$

Doppler spread: $B_d \approx \sigma_v$

Doppler-spread function H(f,ν):

$$H(f;t)=\mathrm{F}_{\tau}\left\{h(\tau;t)\right\}:\ \text{channel transfer function} \rightarrow H(f;\nu)=\mathrm{F}_{t,\nu f}\left\{H(f;t)\right\}:\ \text{Doppler-spread function}$$

$$\phi_H(\Delta f;\Delta t)=\mathrm{F}_{\tau}\left\{\phi_h(\tau;\Delta t)\right\}:\text{time-frequency correlation function of } H(f;t)$$

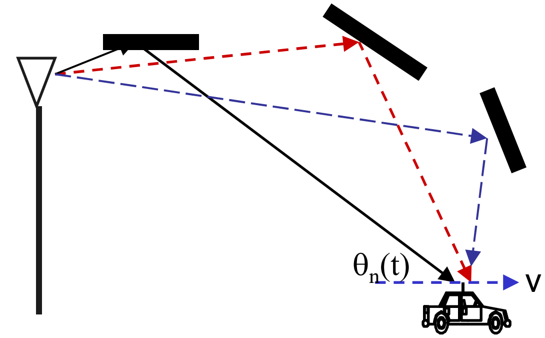
$$\text{for } \Delta t=0,\ \phi_H(\Delta f)=\mathrm{F}_{\tau}\left\{\phi_h(\tau)\right\}:\text{delay power spectrum, correlation of channel gains at } f \text{ and } \Delta f \text{ for any } t.$$

$$S_H(\Delta f;\nu)=\mathrm{F}_{\Delta t}\left\{\phi_H(\Delta f;\Delta t)\right\};\ \text{for } \Delta f=0, \text{Doppler power spectrum } S_H(\nu)$$

$$S_H(\Delta f;\nu)=0.5E\left\{H^*(f;\nu)H(f+\Delta f;\nu+\Delta\nu)\right\}:\text{Auto-correlation function of } H(f;\nu)$$

- being time-variant in the time domain can be equivalently described by having Doppler shifts in the frequency domain.
  - **DOPPLER-SPREAD ⇒ TIME-SELECTIVE FADING**
- Doppler power spectrum: function of the Doppler shift, ν, Fourier transform of the time-correlation function $\phi_H(\Delta t)$

Doppler Spread in land-mobile channel



$$h(\tau;t)=\sum_{n=1}^{N(t)}\alpha_n(t)e^{-j\omega_c\tau_n(t)}\delta(t-\tau_n(t))$$

$$\omega_c\tau_n(t)=\varphi_{n0}+2\pi\int f_{dn}(t)dt,\quad \varphi_{n0}=2\pi d_n/c,$$

$f_{dn}(t)=f_c\left[v(t)/c\right]\cos\theta_n(t)$: Doppler frequency shift in path n

$\rightarrow$ Doppler frequency spread:

$$B_d=\max_{l,n}\left|f_{dn}(t)-f_{dl}(t)\right|=2f_c\left[v(t)/c\right]$$

$$H(f;t)=\mathrm{F}_{\tau,f}\left\{h(\tau;t)\right\}=\sum_{n=1}^{N(t)}\alpha_n(t)e^{-j2\pi\left[(f/f_c)-1\right]\left[\varphi_{n0}+2\pi\int f_{dn}(t)dt\right]}$$

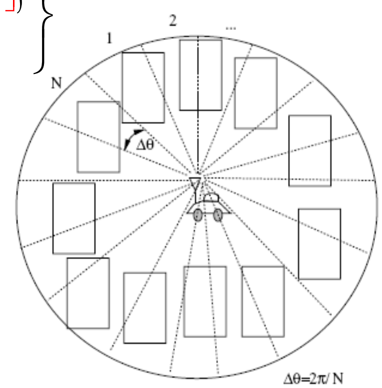
$$\phi_H(\Delta t)=0.5\sum_{l=1}^{N(t)}\sum_{n=1}^{N(t)}E\left\{\alpha_n{}^*(t)\alpha_l(t+\Delta t)e^{j\left[(f/f_c)-1\right]\left\{\left[\varphi_{n0}+2\pi\int f_{dn}(t)dt\right]-\left[\varphi_{l0}+2\pi\int f_{dl}(t+\Delta t)dt'\right]\right\}}\right\}$$

For independent n,l terms and uniformly distributed θ_n:

$$\phi_H(\Delta t)=0.5\sum_{n=1}^{N(t)}E\left\{\left|\alpha_n\right|^2\right\}E_{\theta_n}\left\{e^{-j\pi B_d\Delta t\cos\theta_n}\right\}=PJ_0(\pi B_d\Delta t),$$

$$J_0(x)=\frac{1}{\pi}\int_0^{\pi}e^{-jx\cos\theta}d\theta,\quad 2P=\sum_{n=1}^{N(t)}E\left\{\left|\alpha_n\right|^2\right\}$$

uniform scattering environment [Clarke, Jakes]



AUTOCORRELATION & DOPPLER POWER SPECTRUM OF A LAND-MOBILE RADIO CHANNEL

- Correlation over time can be specified by autocorrelation function and power spectral density of fading process.

•For an omnidirectional mobile antenna and received plan waves uniformly distributed in arrival angle,

- **time correlation function**:

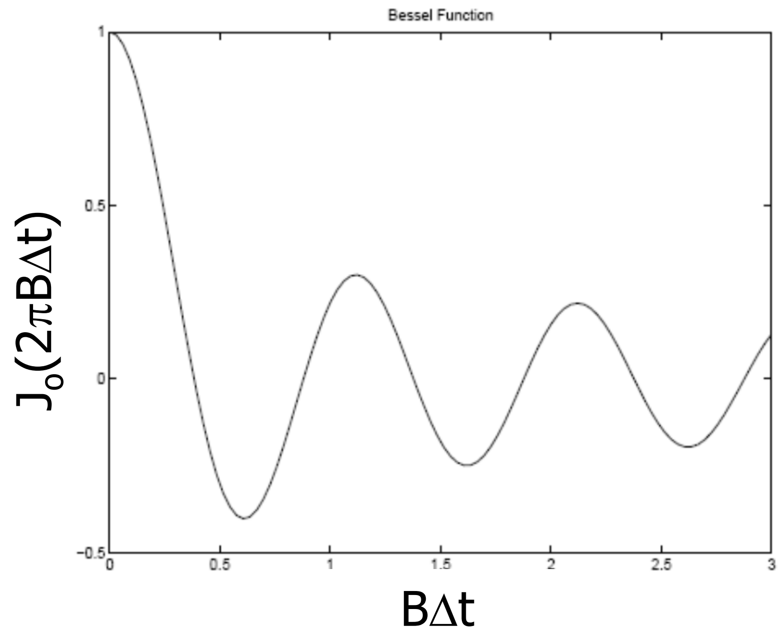
  $\phi_H(\Delta t) = PJ_o(2\pi B\Delta t)$,

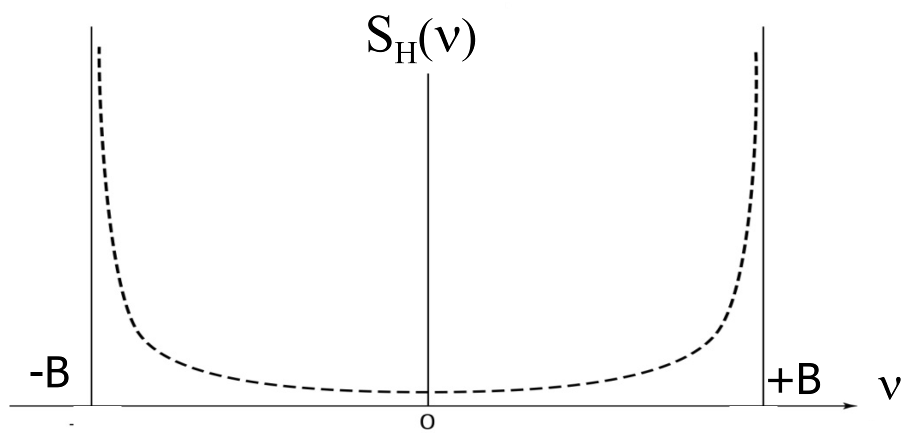
  $J_o(x)$ is the 0^{th}-order Bessel function of the 1^{st} kind.

- **Doppler power spectrum** $S_H(\nu)$: Fourier transform of time-correlation function $\phi_H(\Delta t)$,

  $S_H(\nu) = P/\{\pi[B^2 - \nu^2]^{1/2}\}$,

  $|\nu| < B = B_d/2 = vf_c/c$





STATISTICAL MULTI-TAP MODELS

- multi-tap model for design and performance analysis based on statistical ensemble of channels rather than specific physical channel

$$r[m] = r(m/W) = \sum_{k=K_1}^{K_2} h_m[k]\, x[m-k] + w[m],$$

$$w[m]: \text{zero-mean complex Gaussian noise}$$

$$h_m[k] \approx \sum_{n^*} \alpha_n(t) e^{-j\omega_c \tau_n(t)}: \text{complex, random tap}$$

- Non-LOS: many small scattered paths, complex circular symmetric Gaussian tap. $\rightarrow$ signal envelope follows Rayleigh distribution (power is exponential)
- Near-LOS (with LOS component): 1 line-of-sight plus scattered paths. $\rightarrow$ signal envelope follows Ricean distribution.

- In some environments, measured results support Nakagami distribution (Similar to Ricean, but models "worse than Rayleigh", better to obtain closed-form BER expressions)

Small-Scale Multipath Fading: Rayleigh fading (NLOS propagation) case

$$r(t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} x(t - \tau_n(t))$$

$$\approx [\sum_{n=1}^{N} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)}] x(t - \bar{\tau}).$$

(approximation for case of 1 resolvable tap)

$$Z(t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j2\pi f_c \tau_n(t)}$$

$$= Z_c(t) - jZ_s(t)$$

$$Z_c(t) = \sum_{n=1}^{N} \alpha_n(t) \cos\theta_n(t)$$

$$Z_s(t) = \sum_{n=1}^{N} \alpha_n(t) \sin\theta_n(t)$$

$$Z(t) = \alpha(t) \exp[j\theta(t)]$$

$$\alpha(t) = \sqrt{Z_c^2(t) + Z_s^2(t)}, \quad \theta(t) = \tan^{-1}[Z_s(t)/Z_c(t)]$$

central limit theorem: when N is sufficiently large, $Z_c(t)$ and $Z_s(t)$ are approximately independent Gaussian random variables with zero mean and equal variance

$$\sigma_z^2 = \frac{1}{2} \sum_{n=1}^{N} E[\alpha_n^2]$$

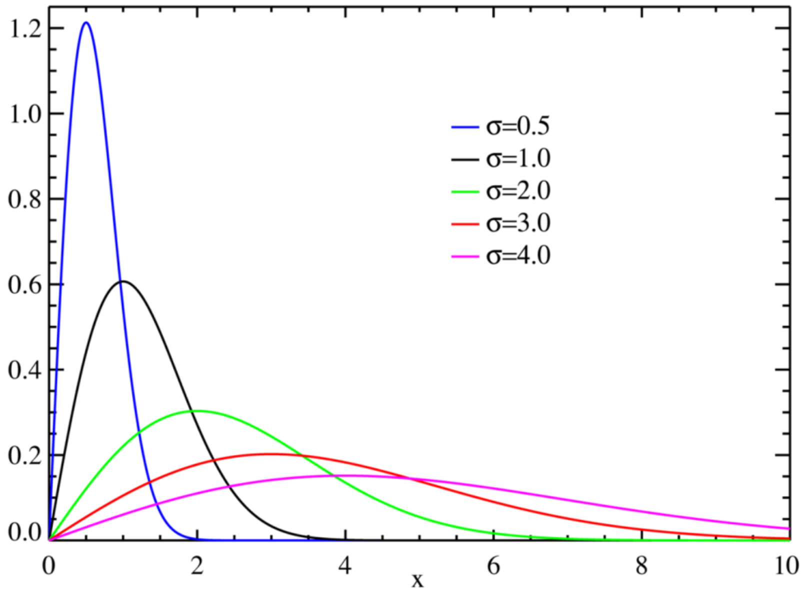
$$f_{Z_c Z_s}(x, y) = \frac{1}{2\pi\sigma_z^2} \exp[-\frac{x^2 + y^2}{2\sigma_z^2}],$$

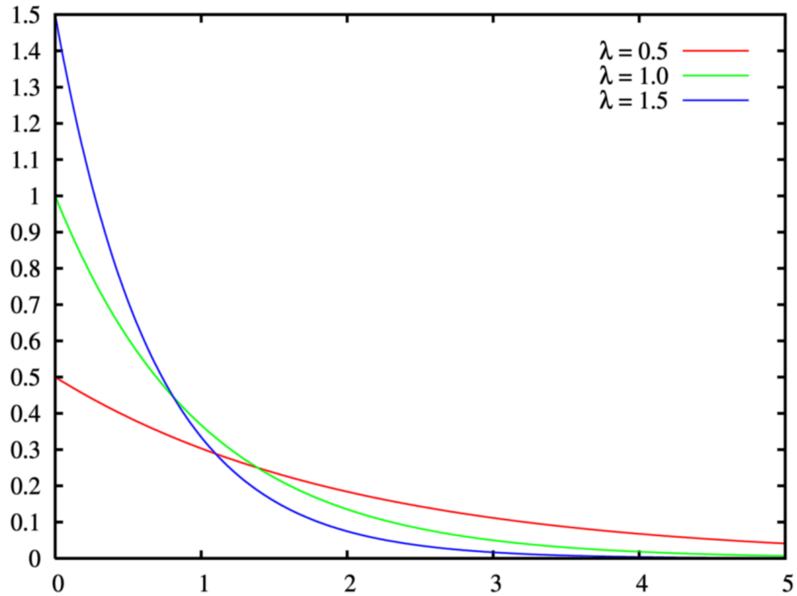
The amplitude fading, α, follows a Rayleigh distribution with parameter σ_z^2

$$f_\alpha(x) = \begin{cases} \frac{x}{\sigma_z^2} \exp(-\frac{x^2}{2\sigma_z^2}), & x \geq 0 \\ 0, & x < 0 \end{cases}$$

The phase distortion follows the uniform distribution over $[0, 2\pi]$,

Rayleigh, Chi, Exp, Chi-square





$Z = X_I + jX_Q : CN(0, \sigma^2),$

$X_I,\ X_Q$: $i.i.d.$ Gaussian, $N(0, \sigma^2)$

$X = \sqrt{X_I^2 + X_Q^2}, \quad X : \text{Rayleigh}(\sigma^2)$

X: also chi χ_2 with 2 degrees of freedom

$p_X(x) = \left[x / \sigma^2 \right] e^{-x^2/2\sigma^2},$

$\overline{X} = \sigma\sqrt{\pi/2}, \text{var} : \sigma_X^2 = \sigma^2 \left[2 - (\pi/2) \right]$

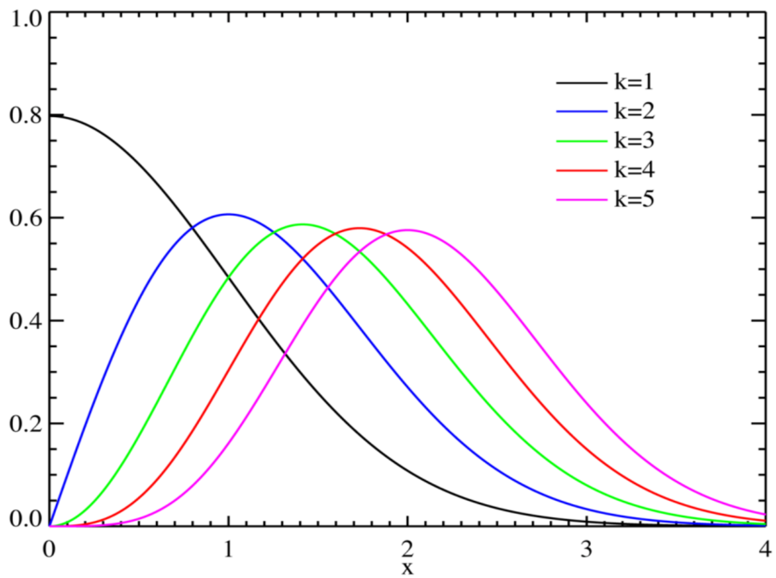
$Y = X_I^2 + X_Q^2, \quad Y : \text{Exponential } (\lambda)$

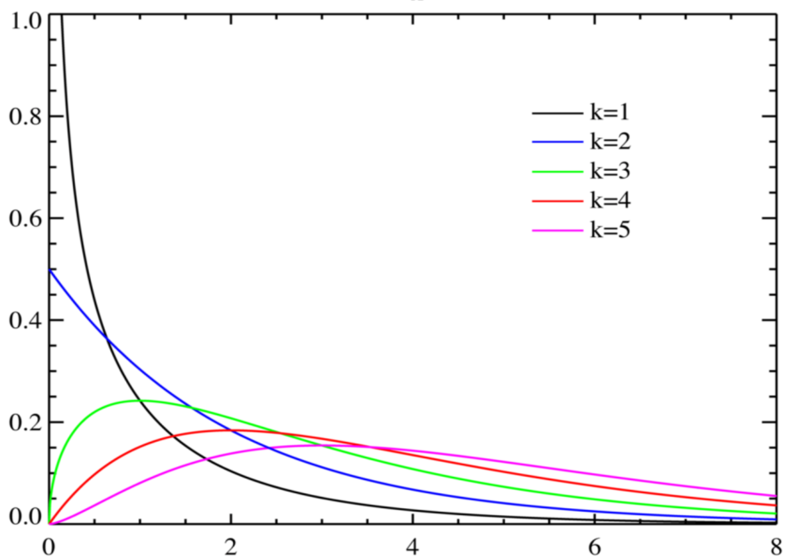
Y: also chi-square χ_2^2

$p_Y(y) = \left[2\sigma^2 \right]^{-1} e^{-y/2\sigma^2}, \lambda = \left[2\sigma^2 \right]^{-1}$

$\overline{Y} = 2\sigma^2, \text{var} : \sigma_Y^2 = 4\sigma^4$

Gaussian, Chi, Chi-square





$X_i,\ i=1,2,\ldots,k$: independent *Gaussian*: $N(\mu_i,\sigma_i^2)$

$$X=\sqrt{\sum_{i=1}^{k}\left[\frac{X_i-\mu_i}{\sigma_i}\right]^2} : \chi_K$$

$$p_X(x)=\left[2^{k/2-1}\Gamma(k/2)\right]^{-1}x^{k-1}e^{-x^2/2},$$

$$E\left\{X^n\right\}=2^{n/2}\,\Gamma\left(\left[k+n\right]/2\right)\big/\Gamma(k/2),$$

$$Y=\sum_{i=1}^{k}\left[\frac{X_i-\mu_i}{\sigma_i}\right]^2 :\ \chi_K^2$$

$$p_Y(y)=\left[2^{k/2}\Gamma(k/2)\right]^{-1}y^{k/2-1}e^{-y/2},$$

$$\overline{Y}=k,\ \mathrm{var}:\sigma_Y^2=2k$$

Gaussian, Chi, Chi-square

$X_i,\ i=1,2,\dots,K$: i.i.d. $zero-mean$ Gaussian: $N(0,\sigma^2)$

$X=\sqrt{\sum_{i=1}^{K} X_i^2}$: chi χ_K with K degrees of freedom

$p_X(x)=\left[2^{K/2-1}\sigma^K\Gamma(K/2)\right]^{-1} x^{K-1}e^{-x^2/2\sigma^2}, E\left\{X^n\right\}=\left[2\sigma^2\right]^{n/2}\Gamma\left(\left[K+n\right]/2\right)\Big/\Gamma(K/2),$

case K=2: Rayleigh, $p_X(x)=\sigma^{-2}xe^{-x^2/2\sigma^2}, E\left\{X\right\}=\sqrt{\pi\sigma^2/2}, E\left\{X^2\right\}=\left[2\sigma^2\right]$

$Y=\sum_{i=1}^{K} X_i^2$: chi-square χ_K^2 with K degrees of freedom

$p_Y(y)=\left[2^{K/2}\sigma^K\Gamma(K/2)\right]^{-1} y^{K/2-1}e^{-y/2\sigma^2}, \overline{Y}=K\sigma^2, \mathrm{var}:\sigma_Y^2=2K\sigma^4$

case K=2: exponential, $p_Y(y)=\left[2\sigma^2\right]^{-1}e^{-y/2\sigma^2}, \overline{Y}=2\sigma^2, \mathrm{var}:\sigma_Y^2=4\sigma^4$

$\Gamma(u)=\int_0^\infty t^{u-1}e^{-t}dt, u>0, \Gamma(u+1)=u\Gamma(u), \Gamma(1/2)=\sqrt{\pi},$

$\Gamma(2)=\Gamma(1)=1,\quad \Gamma(n)=(n-1)!$ for n : integer >1

Small-Scale Multipath Fading: Rician Fading (LOS propagation) case

$$Z(t) = Z_c(t) - jZ_s(t) + \Gamma(t)$$

$\Gamma(t) = \alpha_0(t)e^{-j\theta_0(t)}$ is the deterministic LOS component

$$f_\alpha(x) = \underbrace{\frac{x}{\sigma_z^2}\exp\left(-\frac{x^2}{2\sigma_z^2}\right)}_{\text{Rayleigh}} \cdot \underbrace{\exp\left\{-\frac{\alpha_0^2}{2\sigma_z^2}\right\} \cdot I_0\left(\frac{\alpha_0 x}{\sigma_z^2}\right)}_{\text{modifier}}$$

α_0^2: power of the LOS component

$$= \frac{x}{\sigma_z^2}\exp\left(-\frac{x^2+\alpha_0^2}{2\sigma_z^2}\right) I_0\left(\frac{\alpha_0 x}{\sigma_z^2}\right), \quad x \geq 0,$$

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp(x\cos\theta)\,d\theta.$$

zero-order modified Bessel function of the first kind

$$K \triangleq \frac{\text{Power of the LOS component}}{\text{Total power of all other scatterers}} = \frac{\alpha_0^2}{2\sigma_z^2}.$$

K = 0: Rayleigh

K $\rightarrow\infty$: no fading

Ricean

$X_I,\ X_Q$: *independent* Gaussian with same variance, $N(\mu_i,\sigma^2), i=I,Q$

$X = \sqrt{X_I^2 + X_Q^2}$: Ricean (σ^2),

$$p_X(x) = \left[x/\sigma^2\right] I_0(sx/\sigma^2) e^{-\left(x^2+s^2\right)/2\sigma^2},$$

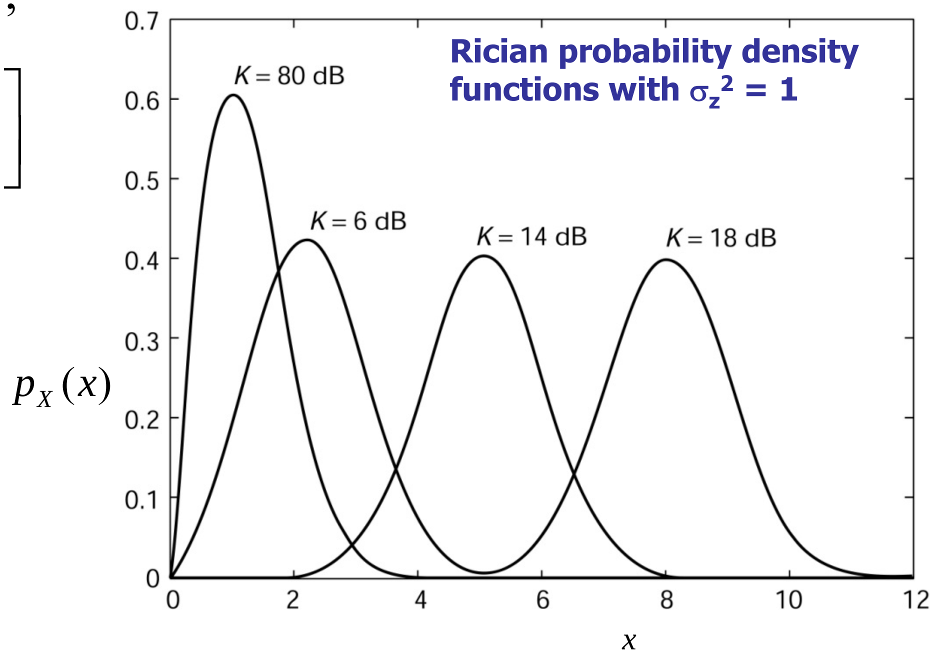
$$s = \sqrt{\mu_I^2 + \mu_Q^2}, \kappa = s^2/2\sigma^2, E\{X^2\} = 2\sigma^2 + s^2,$$

$$E\{X\} = e^{-\kappa/2}\sqrt{\frac{\pi\sigma^2}{2}}\left[(1+\kappa)I_0(\frac{\kappa}{2}) + \kappa + I_1(\frac{\kappa}{2})\right]$$

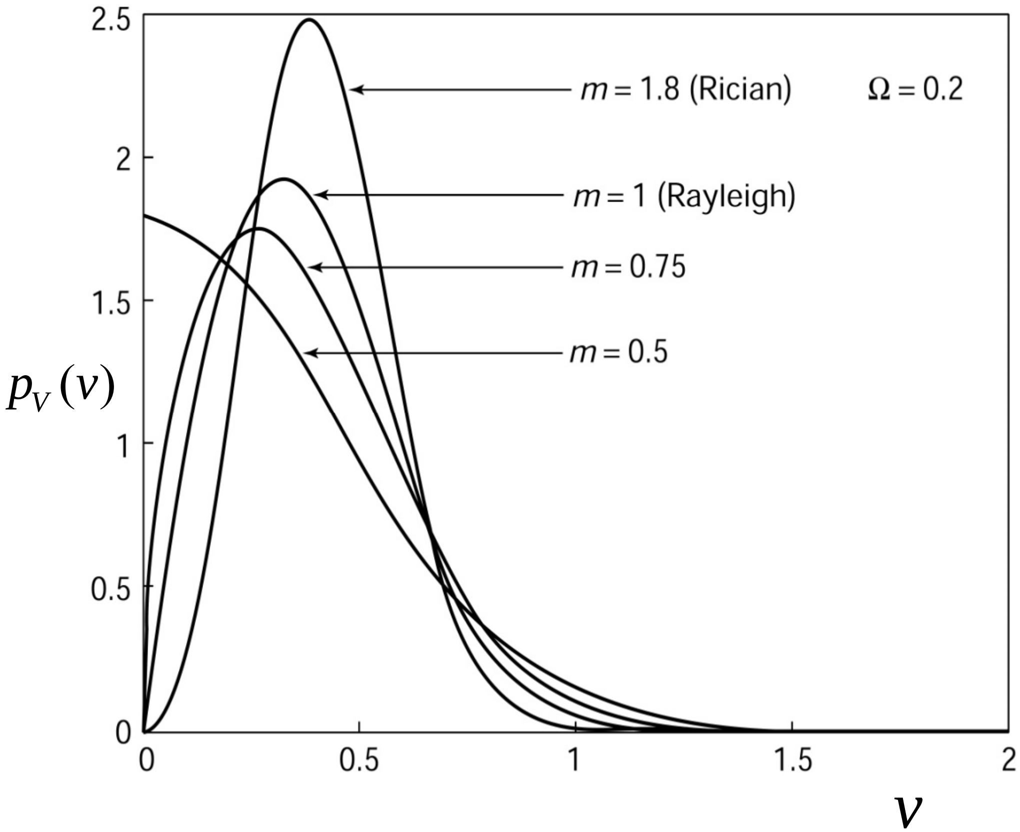
Bessel function of the 1st kind and order *a*:

$$I_a(y) = \sum_{k=0}^{\infty} (y/2)^{a+2k} / [\Gamma(a+k+1)k!]$$

$$\rightarrow I_0(y) = \sum_{k=0}^{\infty} \left[y^k/(2^k k!)\right]^2$$



Nakagami m-distribution:



V: Nakagami m-distributed:

$$p_V(v) = \left[2\left(m/\Omega\right)^m / \Gamma(m)\right] v^{2m-1} e^{-mv^2/\Omega}$$

$$\Omega = E\{V^2\}, E\{[V^2 - \Omega]^2\} = \Omega^2 / m$$

$m \geq 0.5$: fading figure,ratio of moments.

$$E\{V\} = (\sqrt{\Omega / m})\Gamma(m + 0.5) / \Gamma(m)$$

$$\Gamma(u) = \int_0^\infty t^{u-1} e^{-t} dt, u > 0,$$

$$\Gamma(u+1) = u\Gamma(u), \Gamma(1/2) = \sqrt{\pi},$$

$$\Gamma(2) = \Gamma(1) = 1,$$

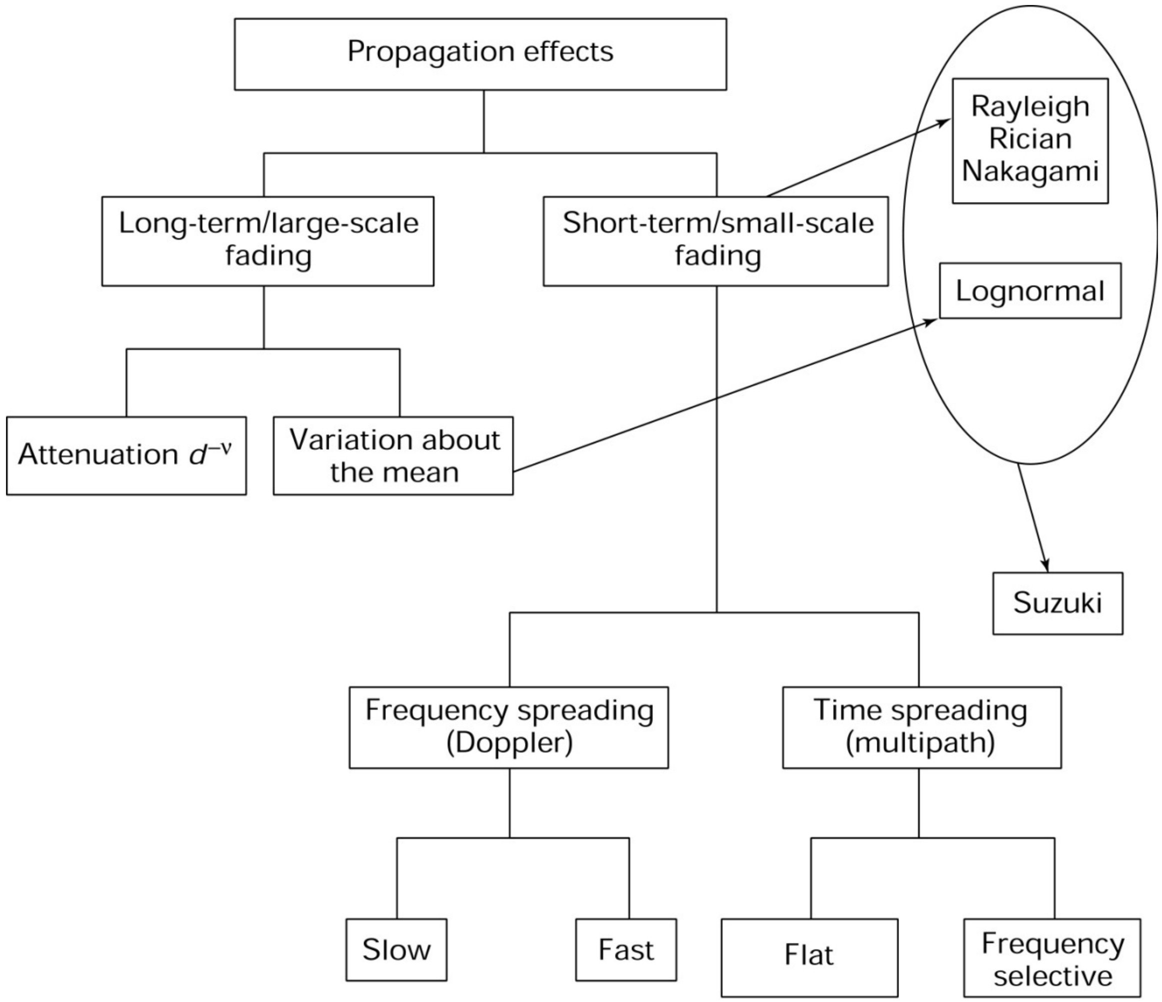
$\Gamma(n) = (n-1)!$ for n : integer > 1

$m = 1 \rightarrow X$: Rayleigh, $p_X(x) = \sigma^{-2} x e^{-x^2/2\sigma^2}$

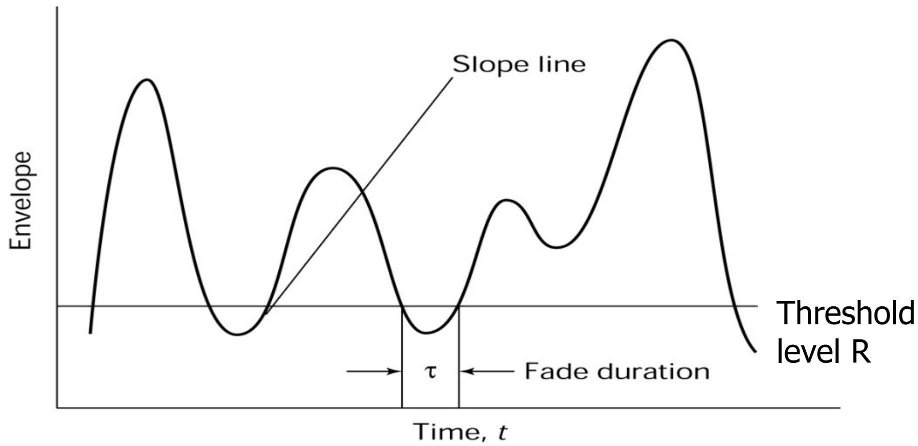
$$E\left\{X^2\right\} = \left[2\sigma^2\right], E\left\{X\right\} = \sqrt{\pi\sigma^2 / 2}$$

- pdf converges to a delta function for increasing *m*.
- matched empirical results for short wave ionospheric propagation.

attenuation and fading



LEVEL CROSSINGS & FADE DURATION



fade duration:

- a user is in continuous outage since the actual SNR (γ) is below the threshold level R required to maintain a maximum BER
- can be derived from level crossing rate of fading process
- for Rayleigh fading,
  - Inversely proportional to Doppler frequency
  - Dependent on margin

$z(t) = |r(t)|$: stationary and ergodic,

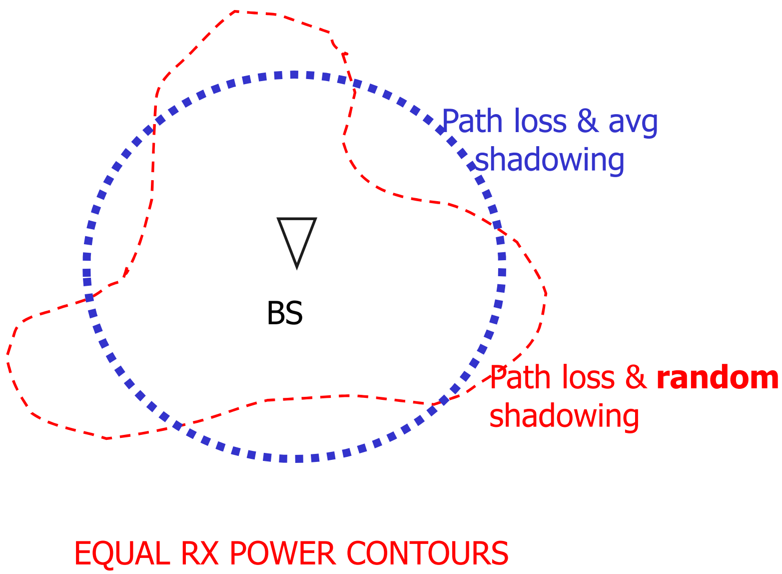
$\bar{z} = E\{|r(t)|\}$, threshold R, i.e., $z(t) < R$: outage, fade

t_i: fade duration, for large observation time T, $\Pr\{z(t) < R\} = \left[\sum_i t_i\right] / T$

$$\text{average fade duration: } \bar{t}_R = \frac{(e^{(R/\bar{z})^2} - 1)}{(R/\bar{z})\, f_d \sqrt{2\pi}}$$

Outage Probability and Cell Coverage Area

- **Outage**: received power below given minimum required for acceptable performance.
- **cell coverage area** : expected percentage of area within a cell that has received power above a given minimum required for acceptable performance.
- circular cells for path loss only,
- amoeba cells for path loss & shadowing as tradeoff between coverage and interference
- Cell coverage area increases as shadowing variance decrease



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