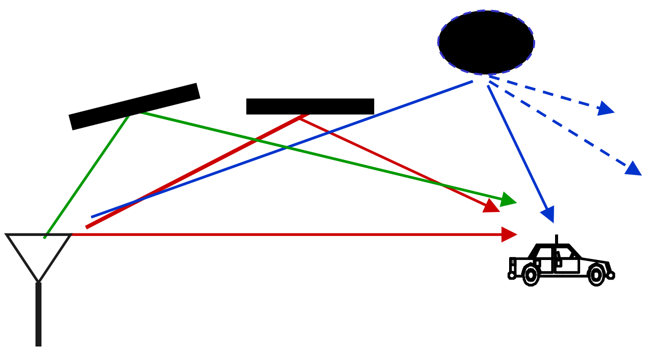
ORTHOGONAL FREQUENCY-DIVISION MULTIPLEXING (OFDM) for broadband transmission over frequency-selective fading channels

Frequency-Selective Fading



- Sampled baseband-equivalent noise-free received signal:

$$r[m] = r(m/W) = \sum_{k=1}^{L} h_m[k]x[m-k] + w[m],$$

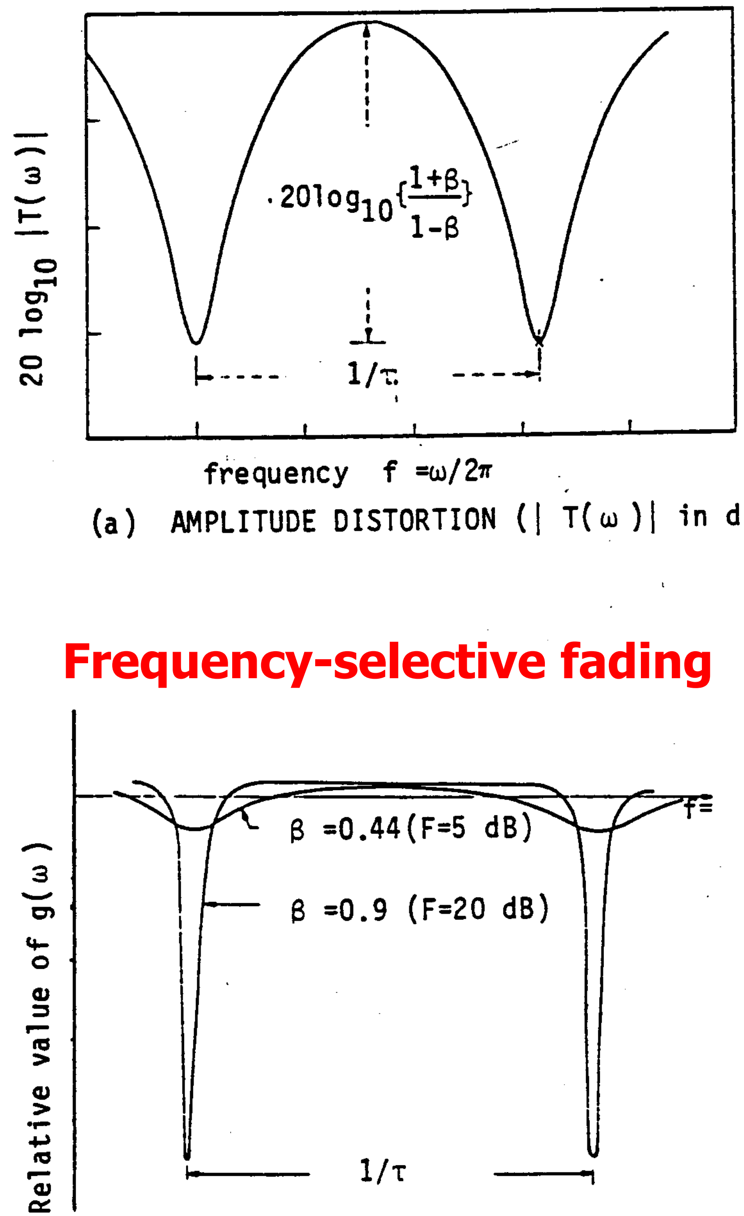
$w[m]$: zero-mean complex Gaussian noise

$h_m[k]$: complex, random (resolvable) tap

- W: double-sided signal bandwidth, sampling rate: 1/W
- Delay spread T_m=L/W>1/W: multiple taps (resolvable paths), frequency-selective fading
- Delay spread exceeding a symbol time causes Inter-Symbol Interference (ISI):
  - ISI is self interference,
  - Increasing signal power increases ISI power
  - ISI leads to irreducible error floor

EXAMPLE OF 2-PATH MODEL

At receiver, the received signal is
$r(t)= x(t) + \beta\, x(t-\tau)$
where $x(t)$: the main path
β : relative level between the main (strong) and reflected paths
$\tau=2d_0/c$: relative time delay between the main and reflected path,
Channel transfer function $T(\omega) = 1 + \beta e^{-j\omega\tau}$
Amplitude distortion:
$|T(\omega)|^2= 1 + \beta^2 + 2\beta\cos \omega\tau= 1 + \beta^2 + 2\beta\cos \omega\tau$
phase distortion:
$\Phi(\omega) = \tan^{-1} [\beta\sin \omega\tau/(1 + \beta\cos \omega\tau)]$
group delay distortion $g(\omega) = d\Phi/d\omega$
$g(\omega)=\beta\tau(\beta+\cos \omega\tau)/(1 + \beta^2 + 2\beta\cos\omega\tau)$



(a) AMPLITUDE DISTORTION (| T(ω)| in dB)

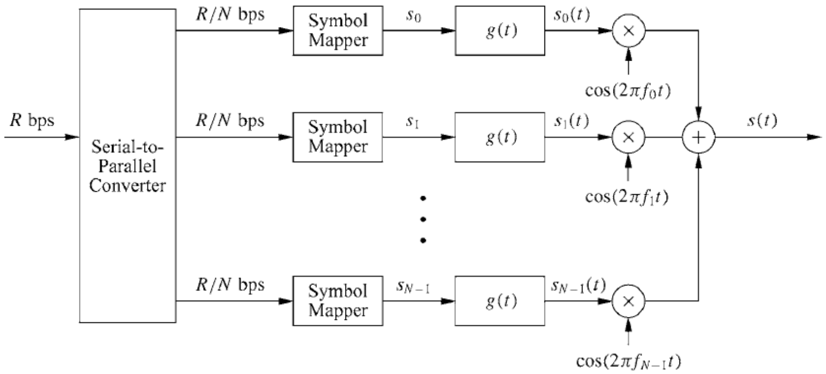
Frequency-selective fading

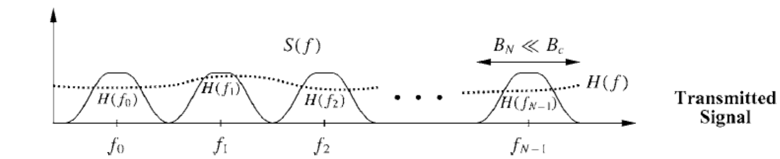
(b) GROUP DELAY DISTORTION (g(ω))

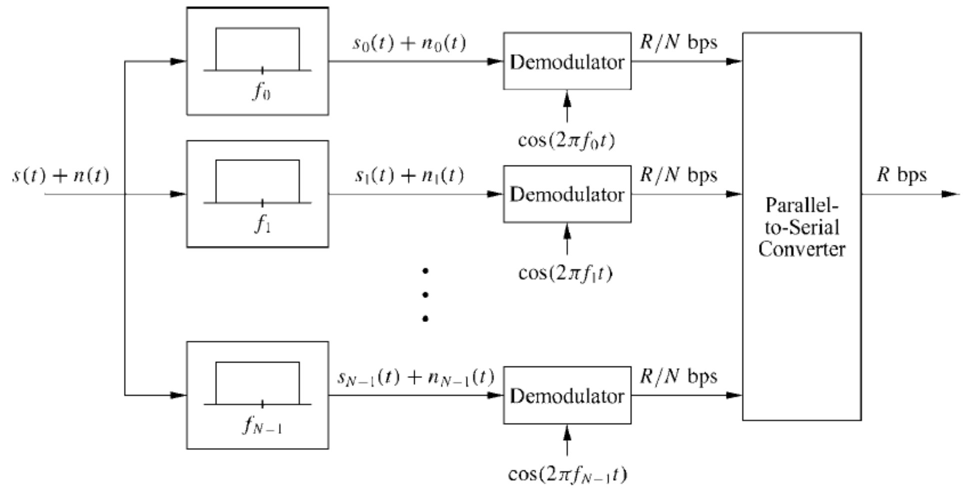
Equalization in single-carrier Tx

- Rx equalization and Tx Pre-distortion
- Frequency-domain equalization: linear, FFT+EQ+IFFT
- Time-domain equalization: Linear equalization using FIR filter, DFE, or combined
- Criterion for coefficient selections:
  - Zero-forcing to eliminate ISI ($\rightarrow$ increase noise)
  - MMSE (balances noise increase with ISI removal)
  - Others, e.g., Minimum error rate (?)
- Possibility of inefficient use of power due to deep fades
- Channel must be learned through training and tracked during data transmission:
  - complex at high data rates,
  - poor performance in fast-changing channels

Multicarrier Modulation with non-overlapping sub-channels







Transmitter:

- Breaks data into N substreams
- Substream modulated onto separate carriers
  - Substream bandwidth is B/N for B total bandwidth
  - $B/N<B_c$ implies flat fading on each subcarrier (no ISI)

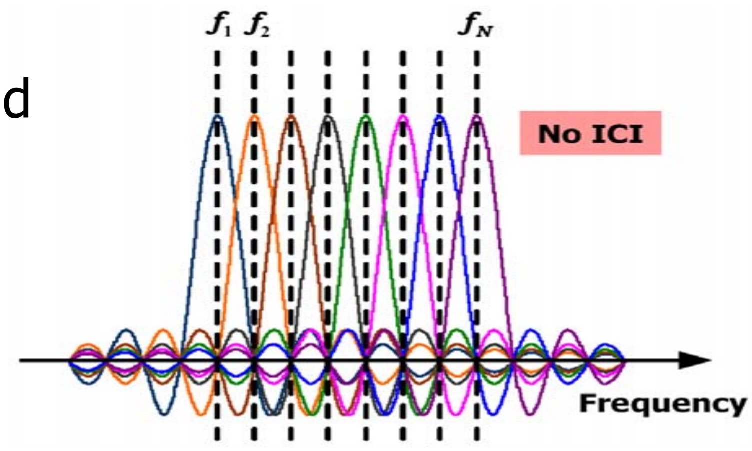
Receiver: Each substream is passed through a narrowband filter (to remove the other substreams), demodulated, and combined via a parallel-to-serial converter to form the original data stream.

this scheme of multicarrier modulation

- can be spectrally inefficient.
- needs near-ideal (and hence expensive) lowpass filters to maintain the orthogonality of the subcarriers at the receiver.
- requires N independent modulators and demodulators.

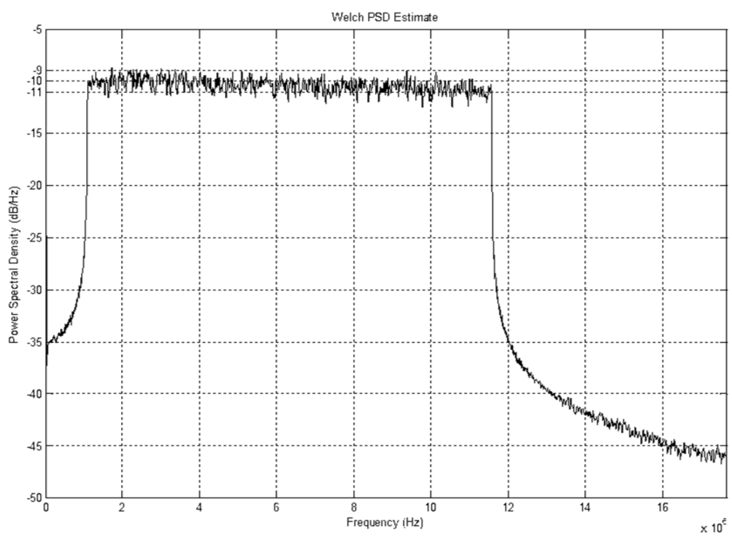
Orthogonal Frequency Division Multiplexing:

- Break data stream into lower-rate substreams modulated onto N narrowband (Δf) flat-fading subchannels with symbol time-interval T_s and required total bandwidth B:
  - Δf << coherence BW $\rightarrow$ T_s >> delay spread



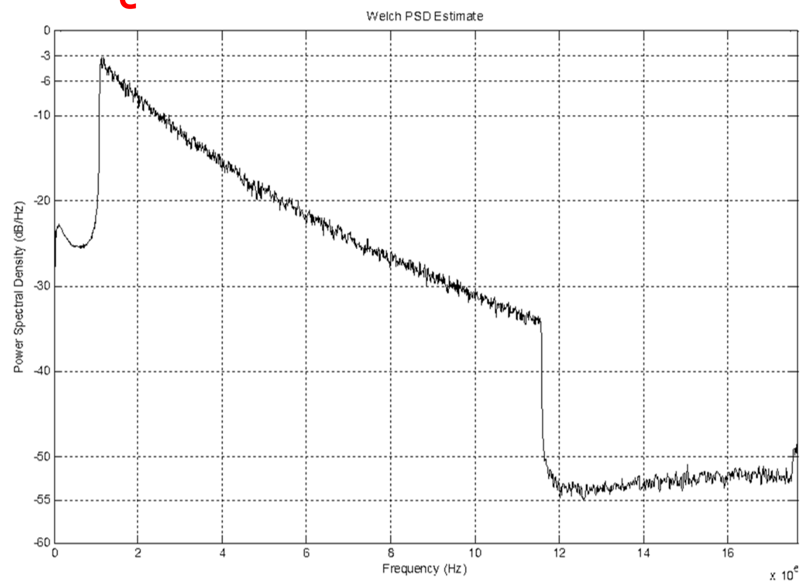
- Substreams must be separable in receiver
  $\rightarrow$ subcarrier orthogonality must be preserved
- Non-overlapped sub-channels:
  $\Delta f \geq (1+\varepsilon)/T_s \rightarrow B \geq N(1+\varepsilon)/T_s$
- OFDM with overlapped sub-channels: $\Delta f = 1/T_s \rightarrow B = N/T_s$
- OFDM implementation based on efficient IFFT (Tx)/ FFT (Rx)

EXAMPLES OF OFDM SPECTRA



FREQUENCY FLAT FADING

FREQUENCY-SELECTIVE FADING



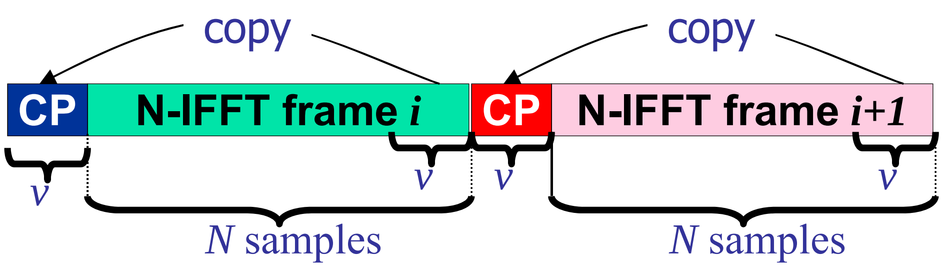
time-dispersive channel can cause

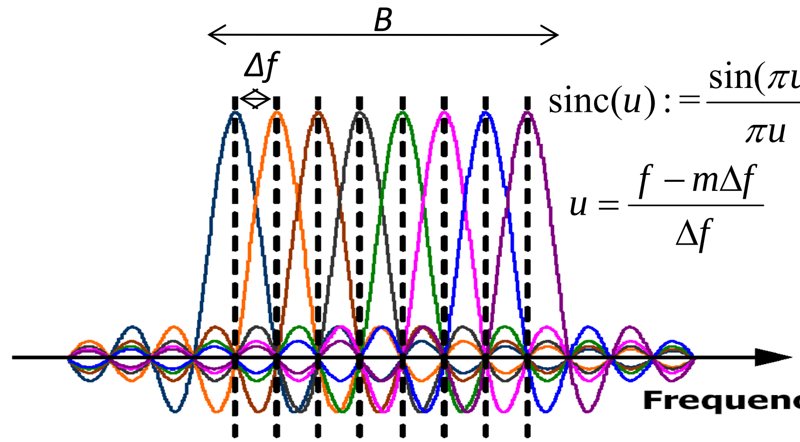
- inter-symbol interference (ISI)

and

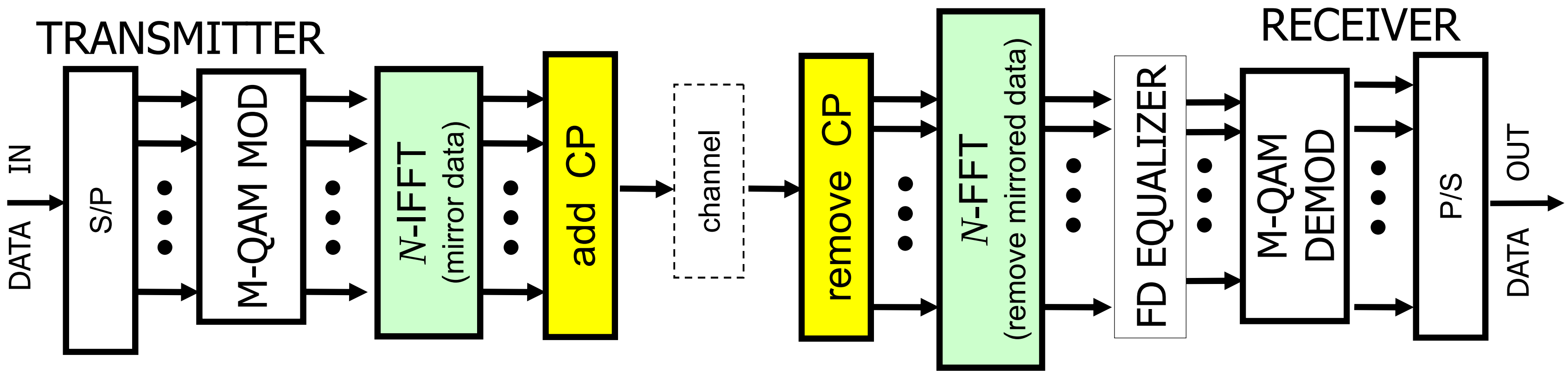
- loss of subcarrier orthogonality in OFDM, resulting in inter-carrier interference (ICI).

OFDM Transceiver





$$\mathrm{sinc}(u) := \frac{\sin(\pi u)}{\pi u}, \quad u = \frac{f - m\Delta f}{\Delta f}$$

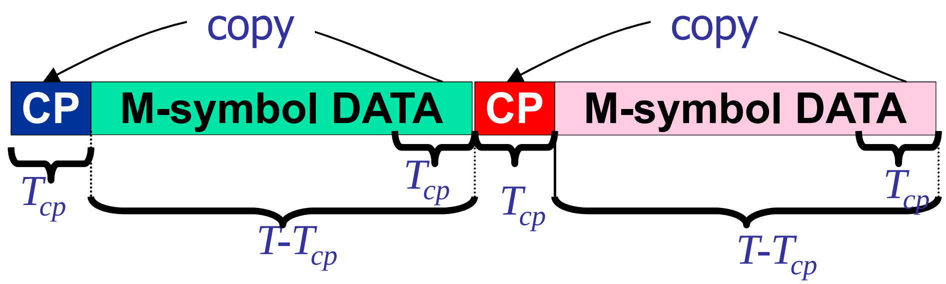


- Transmitters and Receivers are perfectly synchronized
- The fading is slow enough for the channel to be considered constant during one symbol interval

  underspread channels (delay spread << coherence time):

  Doppler spread <<Δf << coherence BW $\Leftrightarrow$ coherence time >> T_s >> delay spread
- Cyclic prefix (CP) prefix is a copy of the last part of the OFDM symbol to be *removed at* the receiver before the demodulation

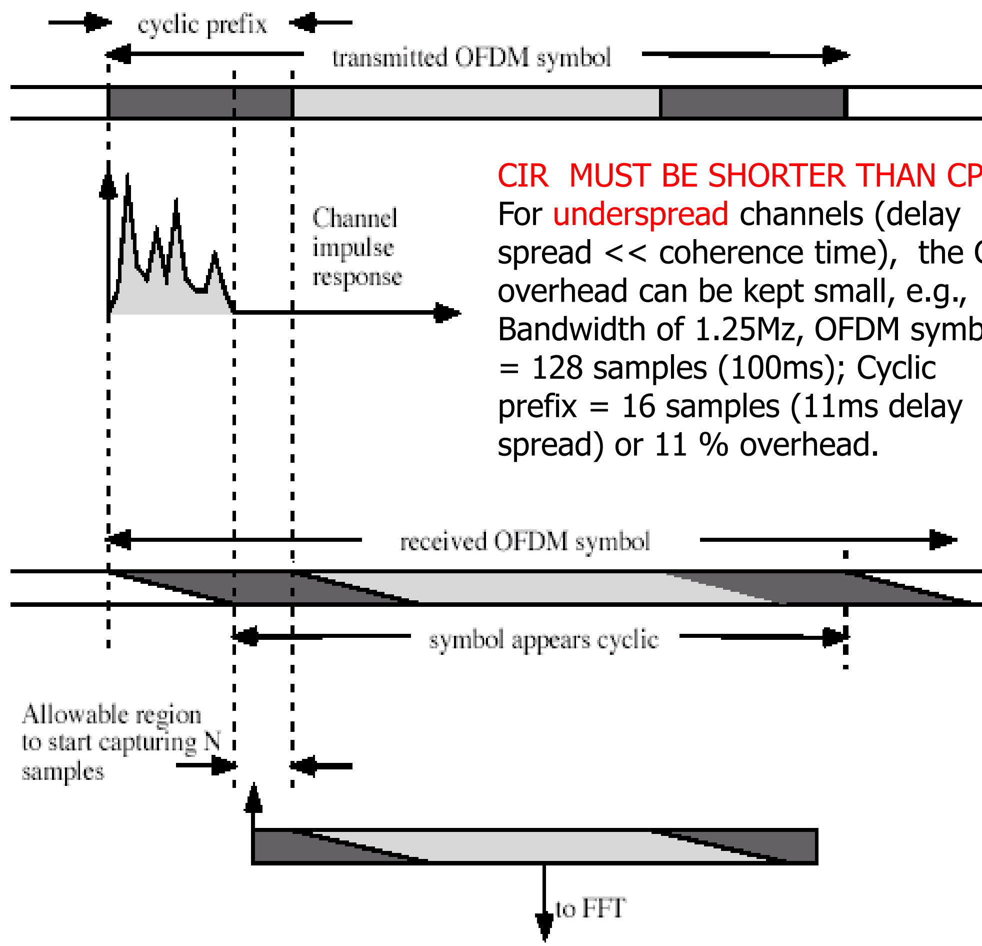
CP to remove ISI & ICI



- Selection of CP length must consider delay spread:

  channel impulse response (CIR) length< CP length
- CP avoids ISI because it acts as a guard space between successive OFDM symbols
- CP avoids ICI by converting the *linear* convolution with the CIR into a *cyclic* (or *circular*) convolution.
- a *cyclic* convolution in the time domain translates into a *scalar* multiplication in the frequency domain:

  → the subcarriers remain orthogonal and there is no ICI.



CIR MUST BE SHORTER THAN CP:
For underspread channels (delay spread << coherence time), the CP overhead can be kept small, e.g., Bandwidth of 1.25Mz, OFDM symbol = 128 samples (100ms); Cyclic prefix = 16 samples (11ms delay spread) or 11 % overhead.

Cyclic Prefix & Effect of dispersive channel

- For every block of N symbols, $\mathbf{d} = [d[0], d[1], \ldots, d[N-1]]^T$, add a prefix of length $L-1$ (at least) to create an input block as

$$\mathbf{x} = [\underbrace{d[N-L+1], \ldots, d[N-1]}_{\text{cyclic prefix}}, d[0], d[1], \ldots, d[N-1]]^T$$

- The output of the channel (assumed to be time-invariant)

$$y[m] = \sum_{l=0}^{L-1} h_l x[m-l] + z[m], \quad m = 1, 2, \ldots, N+L-1$$

- ISI appears in the first $L-1$ symbols: the receiver ignores them and only consider the last N output symbols

$$y[m] = \sum_{l=0}^{L-1} h_l d[(m-L-l) \text{ modulo } N] + z[m], \quad m \in [L, N+L-1]$$

- Input/output model $\mathbf{y} = \mathbf{d} \otimes \mathbf{x} + \mathbf{z}$

  where $\mathbf{y} = [y[L], \ldots, y[N+L-1]]^T$, $\mathbf{h} = [h_0, \ldots, h_{L-1}, 0, \ldots, 0]^T$ also of length N, $\mathbf{z} = [z[L], \ldots, z[N+L-1]]^T$ is a vector of i.i.d. complex Gaussian variables and $\otimes$ denotes the cyclic convolution

IDFT/DFT with Cyclic Convolution

- The output of the channel (assumed to be time-invariant)

$$\begin{aligned}\mathrm{DFT}(\mathbf{y}) &= \mathrm{DFT}(\mathbf{h} \otimes \mathbf{d}) + \mathrm{DFT}(\mathbf{z}) \\ \tilde{y}_n &= \underbrace{\sqrt{N}\mathrm{DFT}(\mathbf{h})_n}_{\tilde{h}_n} \cdot \underbrace{\mathrm{DFT}(\mathbf{d})_n}_{\tilde{d}_n} + \tilde{w}_n = \tilde{h}_n \cdot \tilde{d}_n + \tilde{w}_n\end{aligned}$$

where

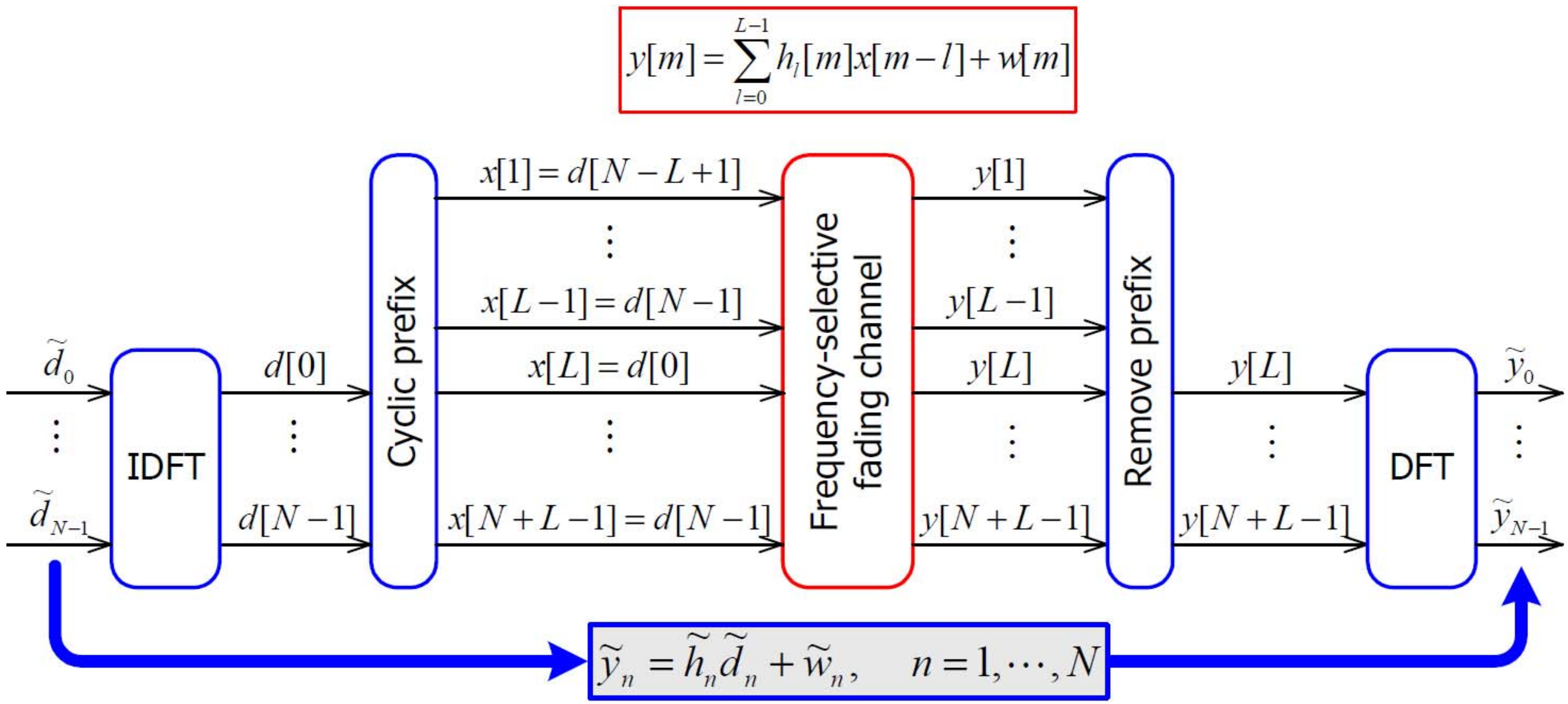
$$\tilde{d}_n = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} d[m] \exp\left(-\frac{j2\pi nm}{N}\right), \quad \tilde{h}_n = \sum_{l=0}^{L-1} h_l \exp\left(-\frac{j2\pi nl}{N}\right)$$

Review:

- $\otimes$: circular convolution defined by

- $DFT\{\{x(n)\} \otimes \{y(n)\}\} = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} x(m) y\big((n-m) \bmod N\big) W^{kn}$
- $\sum_{m=0}^{N-1} x(m) \sum_{n=0}^{N-1} y((n-m) \bmod N) W^{kn} = \sum_{m=0}^{N-1} x(m) Y(k) W^{km} = X(k)Y(k)$

Implementation of OFDM



$$\tilde{h}_n=\sum_{l=0}^{L-1}h_l\exp\left(-\frac{j2\pi nl}{N}\right)$$ channel frequency response at $f=nW/N$

Input/Output with OFDM

- With IDFT, DFT, and Cyclic Prefix, the transmission of OFDM signals over frequency-selective channels becomes the transmission over *N* orthogonal and frequency-flat channels
- With $\tilde{h}_n$ known at the Rx, perform a 1-tap equalizer

$$\tilde{y}_n/\tilde{h}_n = \tilde{d}_n + \tilde{w}_n/\tilde{h}_n$$

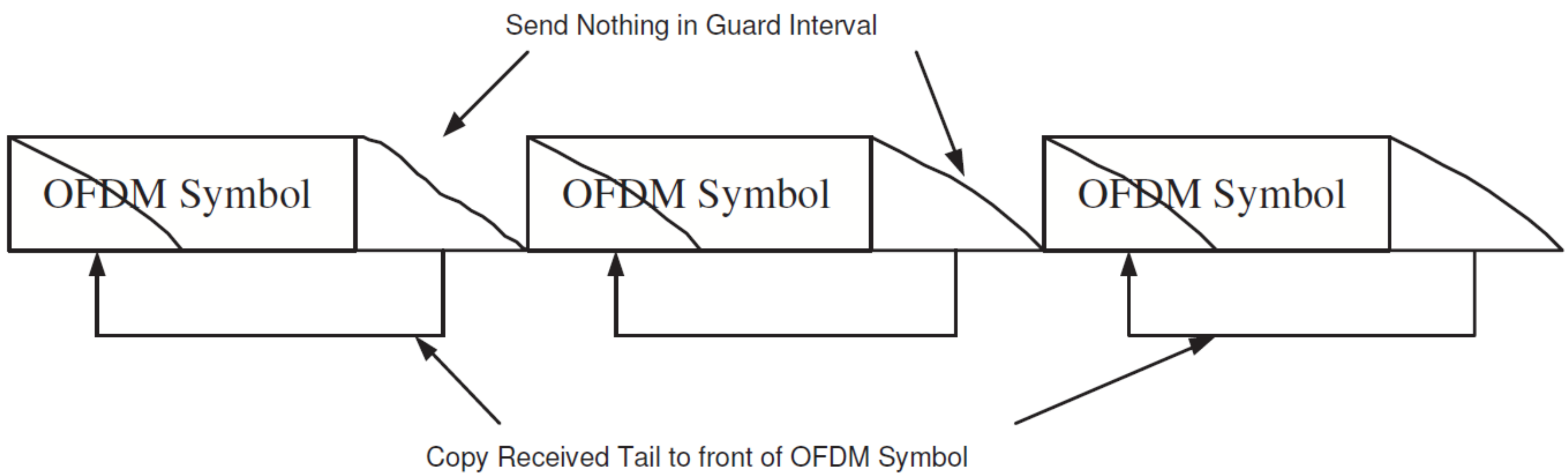
- Simple receiver design to detect $\tilde{d}_n$
- The simplicity of OFDM is achieved at a cost of under-utilizing two resources: bandwidth (or data rate) and power
  - The cyclic prefix occupies an amount of time which cannot be used to transmit data. This loss is a fraction of L/(N+L) of the total time
  - A fraction of L/(N+L) of the average power is allocated to the cyclic prefix cannot be used for transmit data
- Zero-padding?

CP & OFDM Block Length N

- Cyclic prefix introduces overhead that reduces bandwidth and power efficiencies. To minimize the overhead, N should be as large as possible.
- For a Tx BW of W, the sub-carrier spacing $\Delta f = W/N$
- Coherence BW $\approx 1/T_d$, T_d: delay spread $\Rightarrow$ frequency-selective fading.
- For frequency-flat fading, keep $\Delta f = W/N <<$ Coherence BW $\approx 1/T_d$ or $N >> WT_d$
- Doppler spread $\approx 1/T_c$, T_c: coherence time $\Rightarrow$ time-selective fading.
- channel model is desired to be approximately slowly time-varying channel over an OFDM symbol, i.e., $T_s = 1/W << T_c$: coherence time
- Doppler spread introduces uncertainty in the frequency of the received signal. $\Rightarrow$ keep Doppler spread $\approx 1/T_c << \Delta f = W/N$, or $N << WT_c$.
- In other words, the largest value of N must satisfy $WT_d << N << WT_c$ and the channel must have delay spread, $T_d << T_c$ coherence time, i.e., underspread channel

Zero-padding

- To save power due to cyclic prefix, a zero signal can be used



- However, due to the abrupt transition in the signal, using a zero guard interval introduce harmonics that are difficult to filter in the overall signal. Furthermore, the cyclic prefix can be used for timing and frequency acquisition in wireless applications

Example 1: Throughput of IEEE 802.11a

- Channel bandwidth: $W = 20\ \mathrm{MHz}$
- 64 subcarriers, outer 12 are zeroed to reduce adjacent channel interference, 4 are used for channel estimation
- Useful OFDM subcarriers: 48
- Samples per OFDM symbol time: 80 (including 16 for cyclic prefix)
- Sampling rate: $T_s = 1/W$
- The error correction code is a convolutional code with one of three possible coding rate: 1/2, 2/3, and 3/4
- Possible modulation schemes: BPSK, QPSK, 16-QAM, 64-QAM

Example 1: Throughput of IEEE 802.11a

A. Bandwidth of each channel?

$$W_N = \frac{W}{N} = \frac{20\text{ MHz}}{64} = 312.5\text{ kHz}$$

B. Maximum delay spread that ISI can still be removed?

$$T_s = 1/W, T_d < 16 \times T_s = 16/20\text{MHz} = 0.8\mu s$$

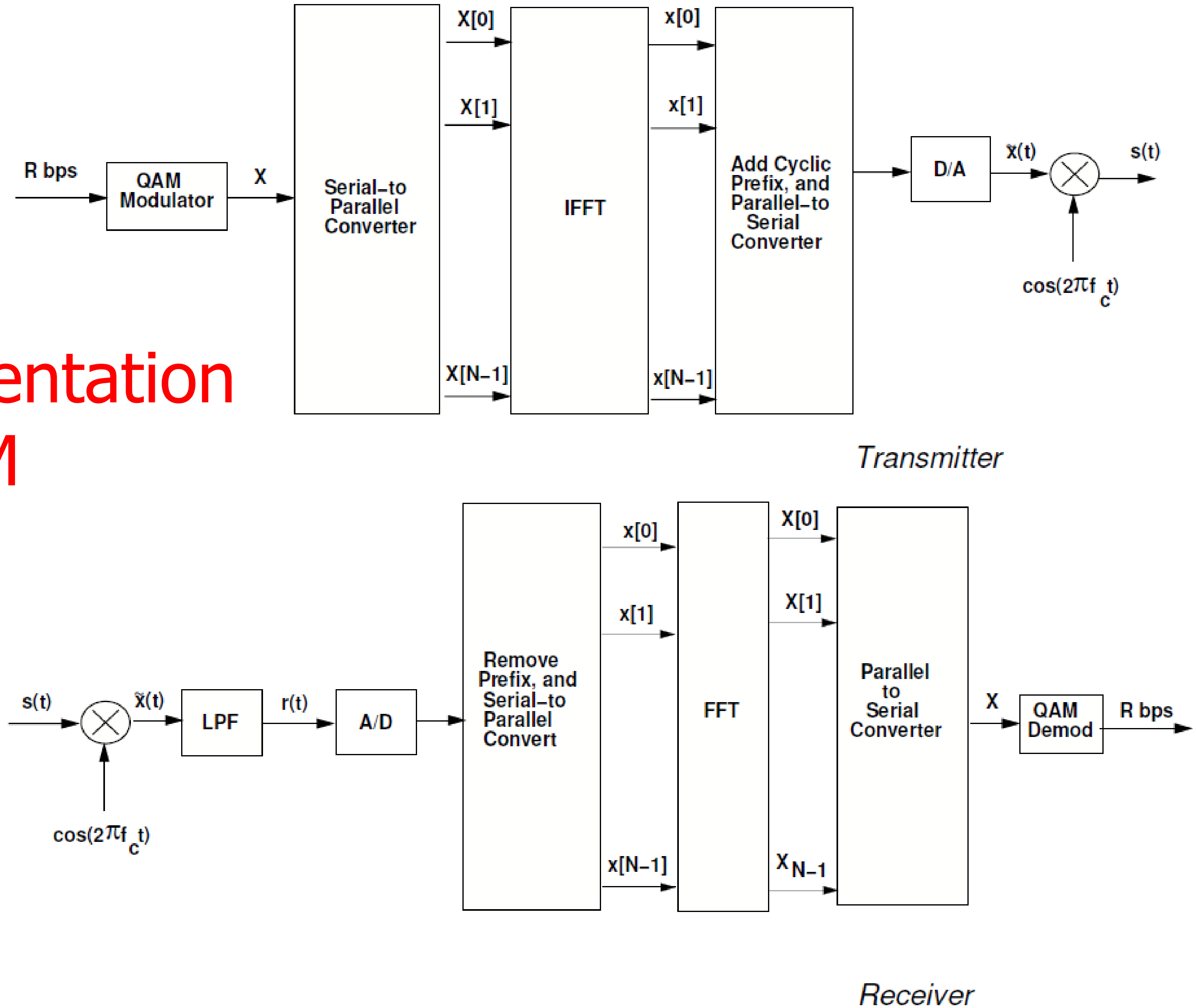
C. Calculate min data rate with BPSK, coding rate = ½?

$$R_{\min} = 48 \times \frac{1}{2} \times 1 \times \frac{20 \times 10^6}{80} = 6\text{ Mbps}$$

D. Calculate max data rate with 64-QAM, coding rate = ¾?

$$R_{\max} = 48 \times \frac{3}{4} \times 6 \times \frac{20 \times 10^6}{80} = 54\text{ Mbps}$$

Implementation of OFDM



Peak-to-Average Power Ratio (PAPR) in OFDM

$$\text{PAPR} = \frac{\max_t |x(t)|^2}{E_t\{\,|x(t)|^2\,\}} \quad \text{(continuous-time signal)}$$

$$\text{PAPR} = \frac{\max_t |x[n]|^2}{E_n\{\,|x([n]|^2\,\}} \quad \text{(discrete-time signal)}$$

- In general, the PAPR depends on the pulse shape used in the modulation and does not generally lead to a simple analytical formulas
- For simplification, we look at PAPR associated with the discrete-time signal. Consider the time domain samples output from the IFFT:

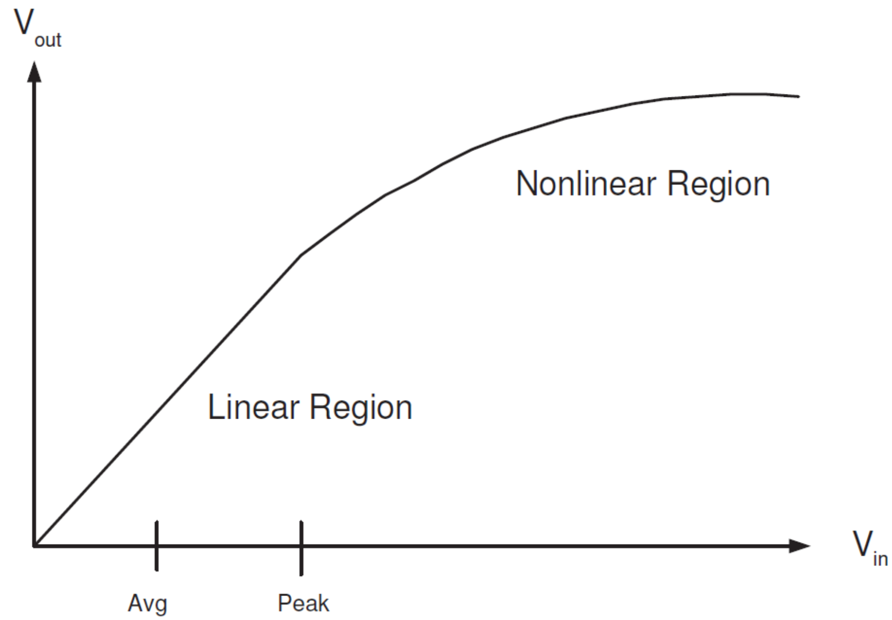
$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] \mathrm{e}^{-j2\pi kn}, \quad 0 \le n \le N-1$$

- If N is large, the Central Limit Theorem is applicable, x[n] are zero-mean complex Gaussian random variables. The Gaussian approximation for IFFT outputs is generally quite accurate for a reasonably large number of subcarriers (N≥64)
- The probability that the PAPR exceeds a threshold P_o is

$$\Pr(\text{PAPR} \ge P_0) = 1 - (1 - \mathrm{e}^{-P_0})^N$$

PAPR

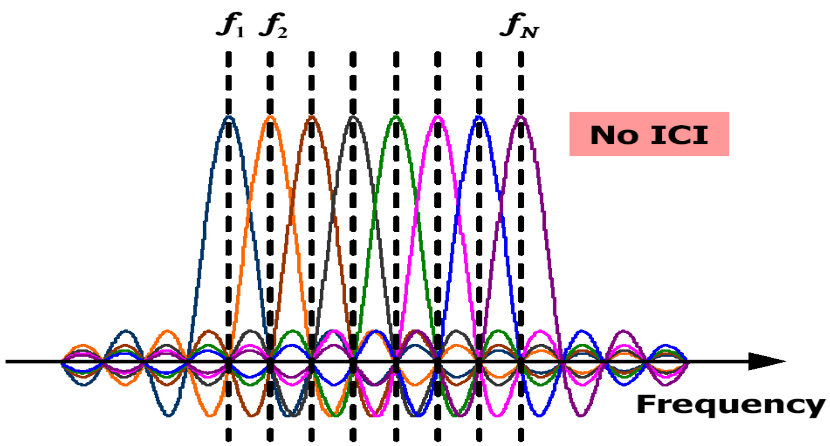
- A low PAPR allows the transmit power amplifier to operate efficiently, whereas a high PAPR forces the transmit power amplifier to a large backoff in order to ensure linear amplification of the signal

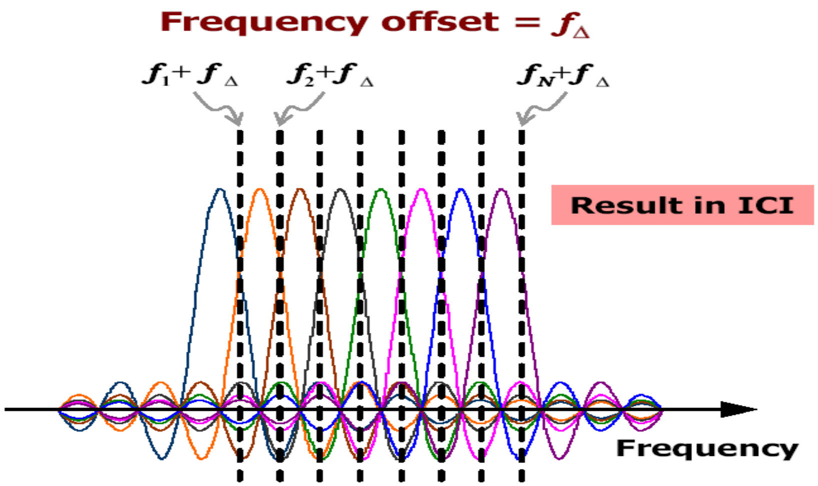


- It is desirable to have the peak and average values be as close as possible to have the amplifier operate at the maximum efficiency
- PAPR grows with the number of subcarriers, N.
- A large PAPR is an important penalty that must be paid for large
- Methods to reduce or tolerate PAPR of OFDM signals include: clipping OFDM signal above some threshold, peak cancellation with complimentary signal, allowing non-linear distortion from the power amplifier, and special coding techniques

Synchronization in OFDM

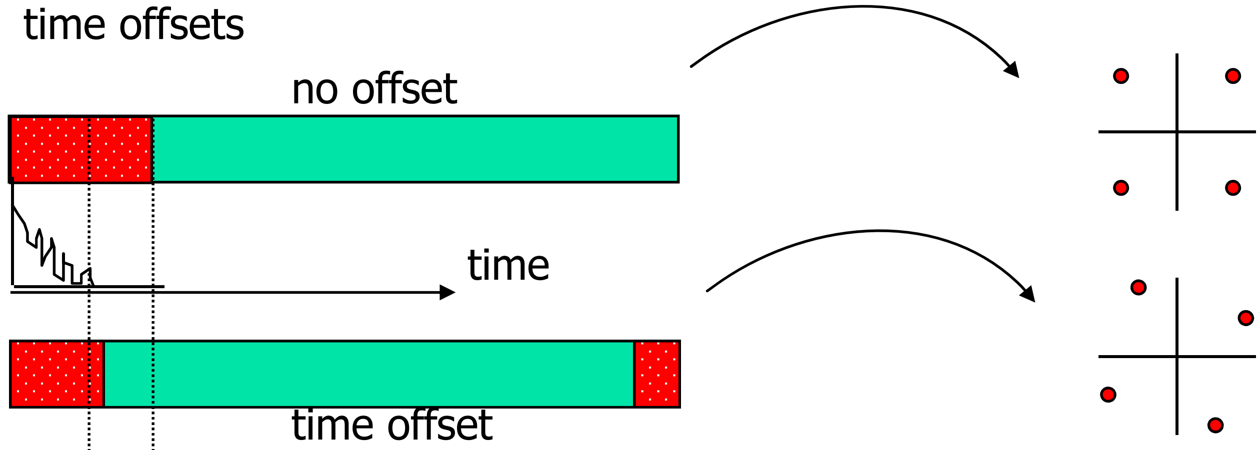
- Effect of symbol timing offset
  - If different users' transmitted signals are not time-aligned, ISI & ICI appear at FFT outputs of desired user.
- Effect of frequency offset
  - The amplitudes of FFT outputs are reduced.
  - One user's frequency misalignment causes the subcarriers to loose their orthogonality resulting in ICI.



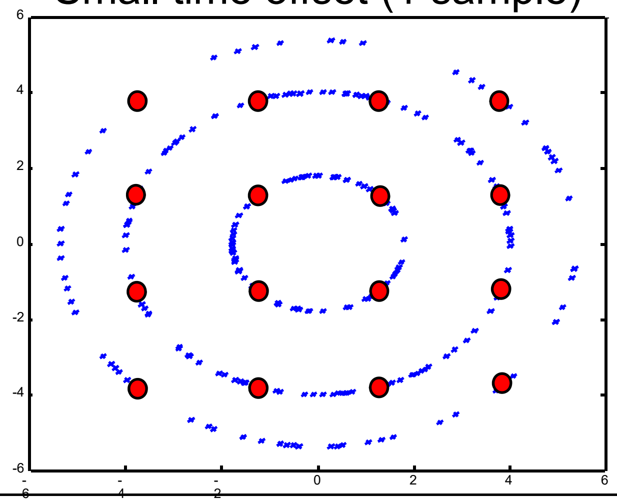


Effects of time offset

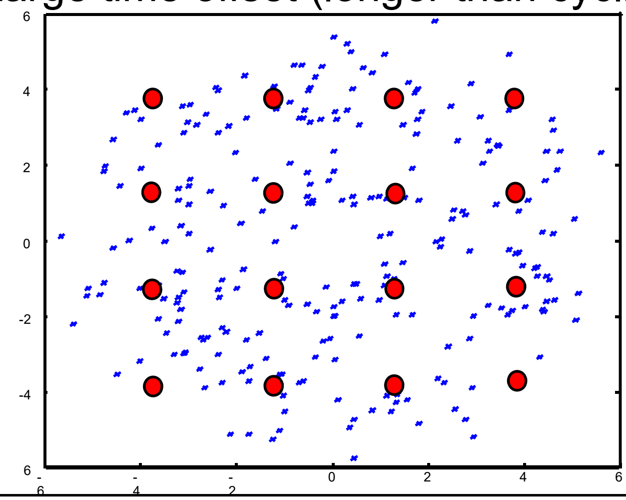
- SNR-degradation is not graceful
- The cyclic prefix and the channel estimator provide some immunity to small time offsets



Small time offset (1 sample)



Large time offset (longer than cyclic prefix)



OFDM: Intercarrier Interference

- Frequency and timing offset causes interference between carriers
- Mitigated by reducing N and non-rectangular pulse shaping
- OFDM symbol time limited by channel coherence time so that inter-carrier spacing (Δf) >> Doppler spread.

Frequency-selective Channel: Channel Capacity & Water-filling Power allocation

channel: $h(\tau) \leftrightarrow H(f) = \mathrm{F}_\tau\{h(\tau)\} = \lim_{\Delta f \to 0, M \to \infty} \sum_{m=-M}^{+M} H(m\Delta f) u(f - m\Delta f)$

AWGN with zero mean and variance $\mathrm{N_o}$, Tx power constraint: $\int_{-B}^{+B} p(f)df \le P_{av}$

channel capacity: C= $\lim_{\Delta f \to 0, M \to \infty} \sum_{m=-M}^{+M} \log_2\left[1 + \frac{p(m\Delta f)|H(m\Delta f)|^2}{N_0}\right]$ b/s/Hz

max C= $\int_{-B}^{+B} \log_2\left[1 + \frac{p(f)|H(f)|^2}{N_0}\right] df$ b/s/Hz subject to $\int_{-B}^{+B} p(f)df \le P_{av}$

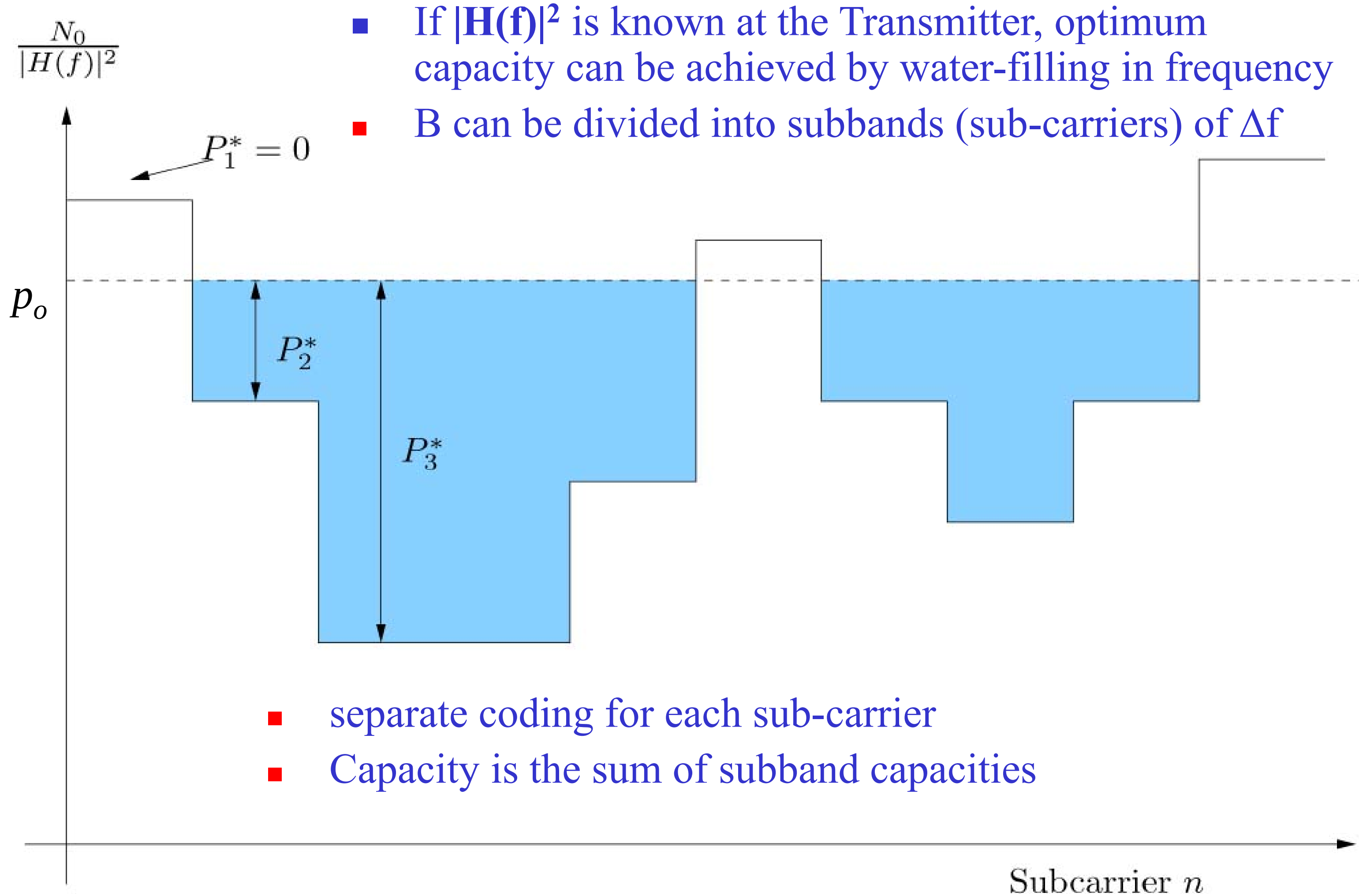
Lagrange multiplier technique: $\int_{-B}^{+B} \left\{ \log_2\left[1 + \frac{p(f)|H(f)|^2}{N_0}\right] - \lambda p(f) \right\} df$

solution: $\left[p(f) + \frac{N_0}{|H(f)|^2}\right]^{-1} + \lambda = 0 \to p(f) = \max\left\{0, \left[p_0 - \frac{N_0}{|H(f)|^2}\right]\right\}$

where p_0 is a constant, adjusted to satisfy $\int_{-B}^{+B} p(f)df \le P_{av}$

Waterfilling in Frequency Domain

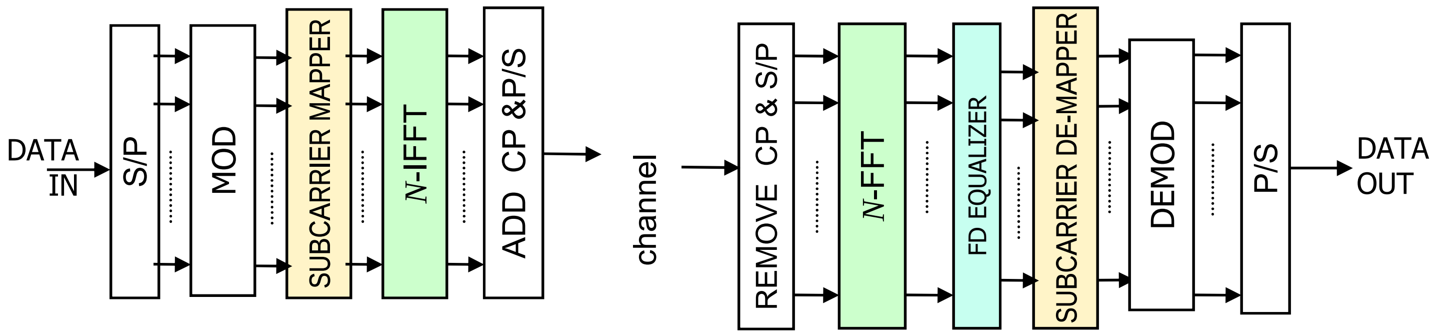
- If $|\mathbf{H(f)}|^2$ is known at the Transmitter, optimum capacity can be achieved by water-filling in frequency
- B can be divided into subbands (sub-carriers) of Δf



- separate coding for each sub-carrier
- Capacity is the sum of subband capacities

OFDMA

- Multiuser version of OFDM
- Multiple access is achieved by assigning subsets of subcarriers to individual users



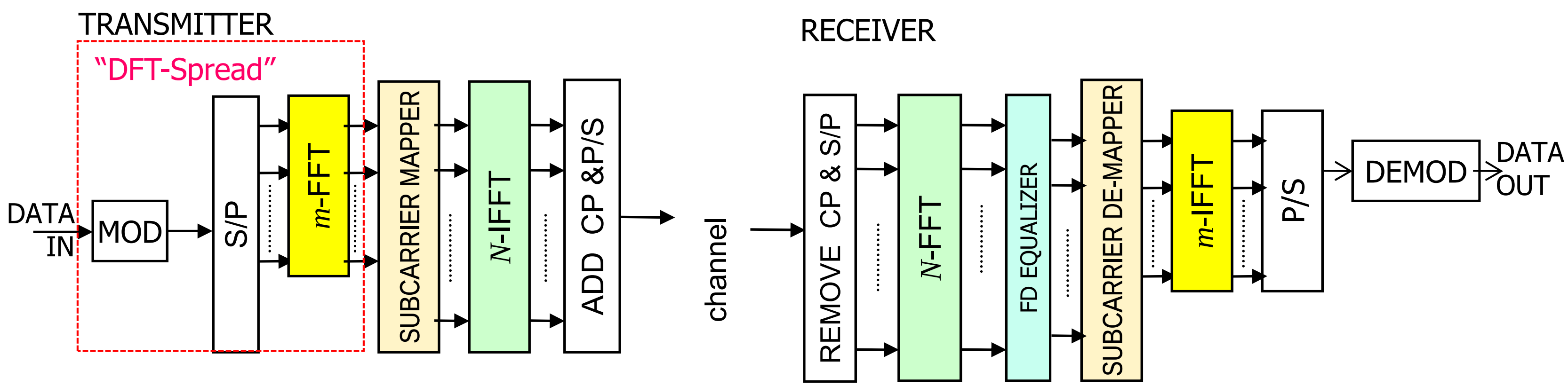
- Inherit advantages of OFDM
- Offer frequency diversity and multiuser diversity by spreading the carriers all over the used spectrum and subcarrier allocation
- Possible to fill free radio frequency band in cognitive radio networks

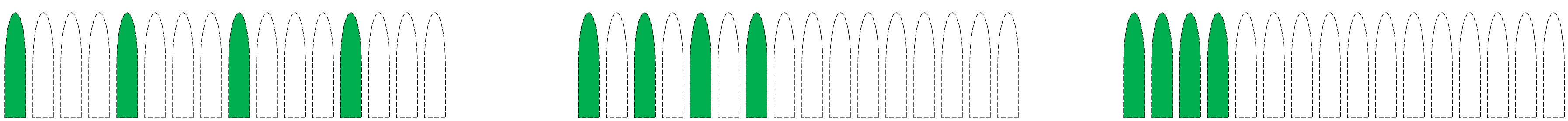
OFDMA for LTE Downlink

- Downlink:
  - Power-amplifier efficiency less critical at base-station side
  - Avoid excessive user-terminal receiver complexity
- OFDM for robust broadband transmission to enable frequency-domain (in addition to time-domain) adaptation to channel conditions and spectrum scenarios, e.g., scheduling on a 1ms x 180kHz basis
  - (Robustness to time dispersion can also be achieved with single-carrier transmission with Rx frequency-domain equalization: SC-FDE)
- Multi-layer transmission to provide spatial multiplexing gain, and hence very high data rates and high spectrum efficiency

SC-FDMA

- Single-carrier FDMA: Utilizes single carrier modulation, DFT-spread orthogonal frequency multiplexing, and frequency domain equalization
- Robust to the issue of PAPR



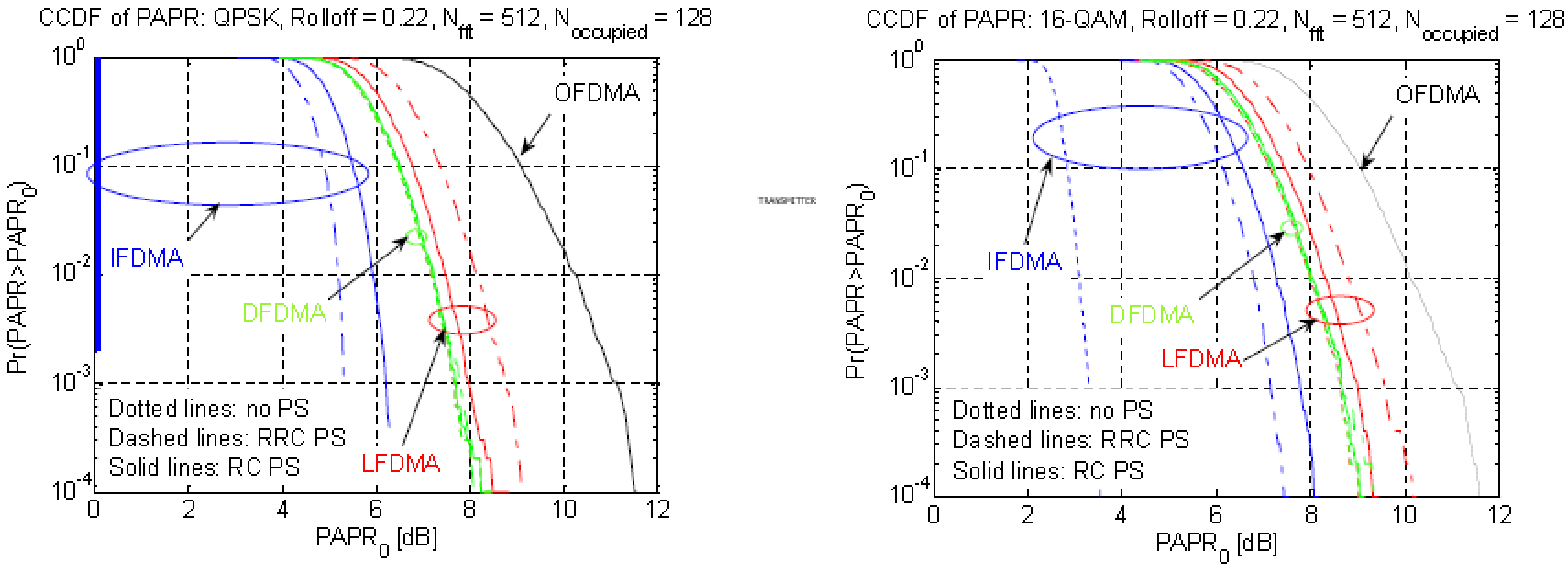


Interleaved FDMA Distributed FDMA Localized FDMA

SC-FDMA for LTE Uplink

- High power-amplifier efficiency is critically important for UEs
- Receiver complexity is more affordable for base-station
- Single-carrier FDMA for low-PAPR→ high power-amplifier efficiency (operating closer to saturation point)
  - Reduce UE power consumption and cost
  - Improve cell-edge performance, uplink coverage and capacity
- Multiple access among UEs is made possible by assigning different UEs, different sets of non-overlapping Fourier-coefficients (subcarriers). This is achieved at the transmitter by inserting (prior to IFFT) silent Fourier-coefficients (at positions assigned to other UEs), and removing them on the receiver side after the FFT

PAPR Characteristics of SC-FDMA

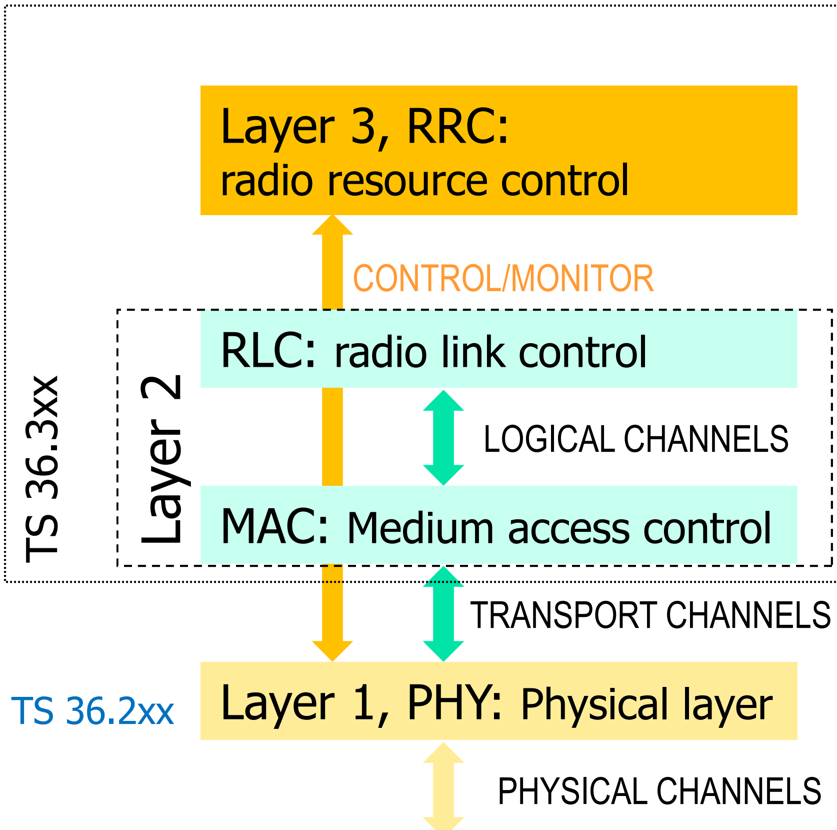


(a) QPSK **(b) 16-QAM**

* Monte Carlo simulations (Number of iterations: > 104)
* Time domain pulse shaping with 8-times oversampling
* N_{fft}: number of total subcarriers = FFT size
* $N_{occupied}$: number of occupied subcarriers = data block size
* RC: raised-cosine, RRC: root raised-cosine
* Rolloff factor of 0.22

H. G. Myung, J. Lim, and D. J. Goodman, "Peak-to-Average Power Ratio of Single Carrier FDMA Signals with Pulse Shaping," IEEE PIMRC ’06, Helsinki, Finland, Sep. 2006

LTE protocol structure



LTE Standard Specifications:
http://www.3gpp.org/ftp/Specs/html-info/36-series.htm

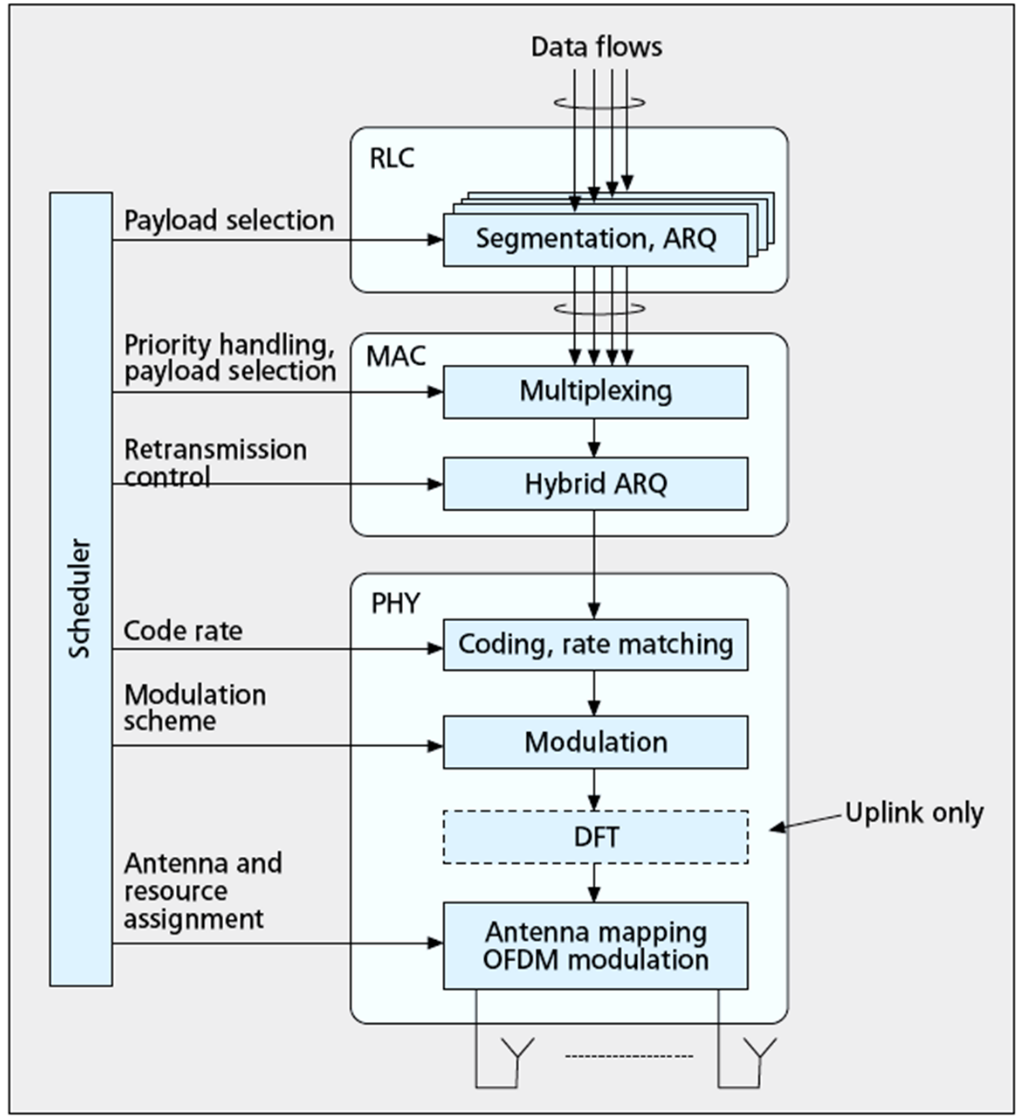
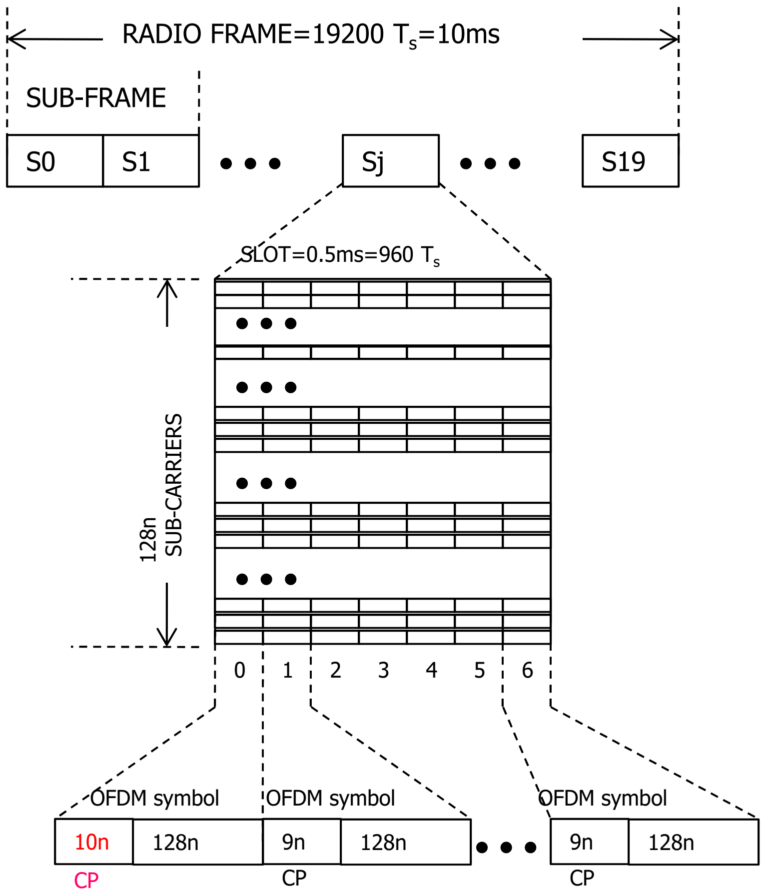


Figure 1. *LTE protocol structure (simplified).*
Astely, D.; Dahlman, E.; Furuskar, A.; Jading, Y.; Lindstrom, M.; Parkvall, S., "LTE: the evolution of mobile broadband", *IEEE Communications Magazine*, vol. 47, no. 4, April 2009, pp. 44–51.

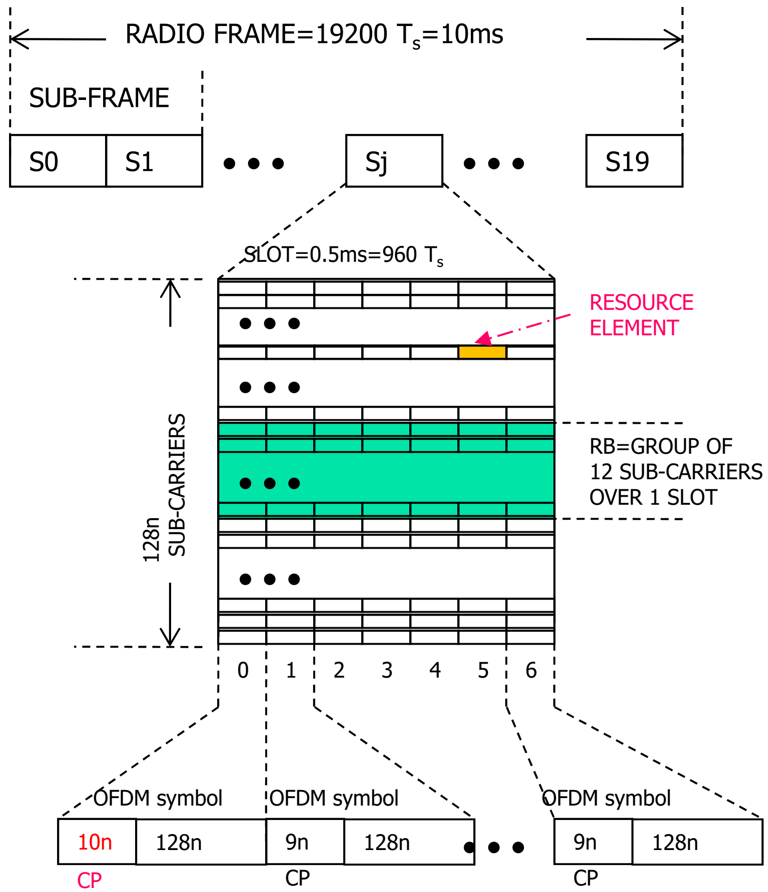
LTE OFDM Frame



For long delay dispersion : 6 OFDM symbols/slot with CP=16.67μs=32n in each OFDM symbol

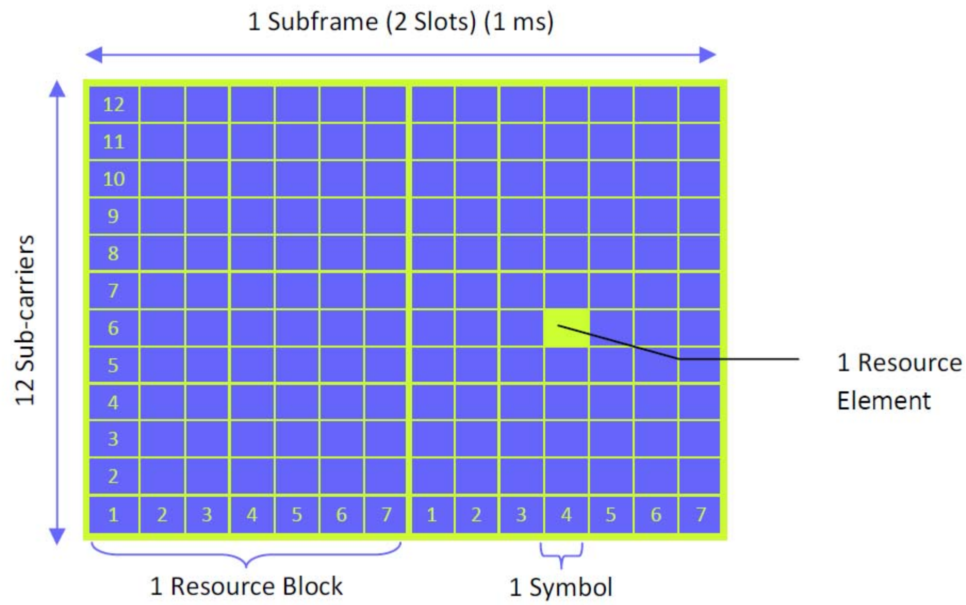
- Slot: 0.5 ms
- TTI (Time Transmit Interval): minimum 2 slots=1ms, concatenation possible, related to the interleaving depth
- DL parameters from 25.814 (Aug. 06)
- Number of OFDM symbols per slot: 7 (normal, short CP) or 6 (extended, long CP)
- Sub-carrier spacing 15 kHz (7.5-kHz subcarrier spacing considered for MBMS-dedicated channels, more sensitive to frequency offset).
- Transmission BW: 1.25nMHz
- n=1, 2, 4, 8, 12, 16
- Sampling frequency: 1.92nMHz
- FFT size (128n)
- Two cyclic prefix (CP) lengths depending on the delay dispersion characteristics:
  - normal, short CP (5.21/4.69μs) for urban and high data rate application
  - Extended, long CP (16.67μs) for multi-cell broadcast (MBMS) and very-large-cell scenarios (e.g., rural and low data rate), reduced bandwidth efficiency.

LTE Resource Block



For long delay dispersion : 6 OFDM symbols/slot with CP=16.67μs=32n in each OFDM symbol

- radio frame structure types: FDD, TDD.
- Resource Block (RB): 12 subcarriers of 12x15 kHz=180 kHz over a time-slot
- Sub-carrier scheduling:
  - Allocated sub-carriers in a RB are not necessarily adjacent
  - Each UE is allocated its individual best sub-carriers to achieve high channel capacity



Example 2: Throughput of LTE

- Channel bandwidth: $W = 20 \text{ MHz}$
- FFT size: 2048
- Useful (occupied) OFDM subcarriers: 1200
- Subcarrier spacing: $\Delta f = 15 \text{ kHz}$
- LTE frame: 10ms, subframe: 1ms, 2 slots/subframe, 7 OFDM symbol/slot
- 64-QAM

- With 4x4 MIMO, support 4 streams. What is the maximum downlink throughput in LTE? (assuming 1/4 rate loss due to coding rate and control signaling)