Diversity Techniques

Outline:

- Performance of M-ary Modulation (recap)
- Diversity
- Diversity Techniques at Receiver
- Diversity Techniques at Transmitter
- Summary

Performance of M-ary Digital Modulation in an AWGN Channel with coherent receiver (a quick review)

- Consider AWGN channel with $h = 1, \quad n \sim \mathcal{CN}(0, N_0), \quad \mathbb{E}[|x|^2] = E_s$

$$y = hx + n$$

UNION BOUND: $P_s \le (M-1)Q\left(\sqrt{d_{\min}^2/(2N_0)}\right), 2Q\left(x\sqrt{2}\right) = erfc\left(x\right)$

where: $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du = \frac{1}{\pi}\int_0^{\pi/2} e^{-x^2/(2\sin^2\theta)} d\theta$ to simplify calculations (by Laplace transform) [J. Craig, 91]

M-QAM,M-PSK, (Linear): more spectrally efficient than M-FSK (Nonlinear)

$$s(t) = \sum_n a_n g(t - nT_s)\cos(2\pi f_c t) - \sum_n b_n g(t - nT_s)\sin(2\pi f_c t)$$

Performance:

$P_s \approx A_M Q\left(\sqrt{B_M^2 (E_s / N_0)}\right), E_s / N_0 = P/(f_s N_0) = SNR$ (f_s: Nyquist BW)

e.g., $squared\ M - QAM: \quad A_M = 2(1 - M^{-1/2}), B_M^2 = 3/\left(M - 1\right)$

Performance of M-ary Modulation in AWGN channel

- BER approximation
  - Gray mapping: 1 symbol error=1 bit error $P_b \approx P_s / \log_2(M)$
  - Symbol energy equally divided among bits $E_b \approx E_s / \log_2(M)$

$$P_b \approx \hat{\alpha}_M \cdot Q\left(\sqrt{\frac{\hat{\beta}_M E_b}{N_0}}\right) \qquad \hat{\alpha}_M = \alpha_M / \log_2(M), \quad \hat{\beta}_M = \beta_M \cdot \log_2(M)$$

α_M, β_M depend on M and approximation

- Chernoff bound: $Q(\sqrt{2x}) < \exp(-x)$

- SER/BER over AWGN $P_s(\gamma_s)$, $P_b(\gamma_b)$ decay exponentially with SNR

$$\gamma_s = E_s/N_0, \quad \gamma_b = E_b/N_0$$

Performance of M-ary Digital Modulation in an AWGN Channel

binary, antipodal signaling: $M=2,\ P_b = P_e = \frac{1}{2} erfc\left[\sqrt{\frac{E_b}{N_0}}\right]$

$E_S = (\log_2 M) E_b$

Union bound: $P_e \le \frac{1}{2}(M-1) erfc\left[\frac{d}{2\sqrt{N_0}}\right]$

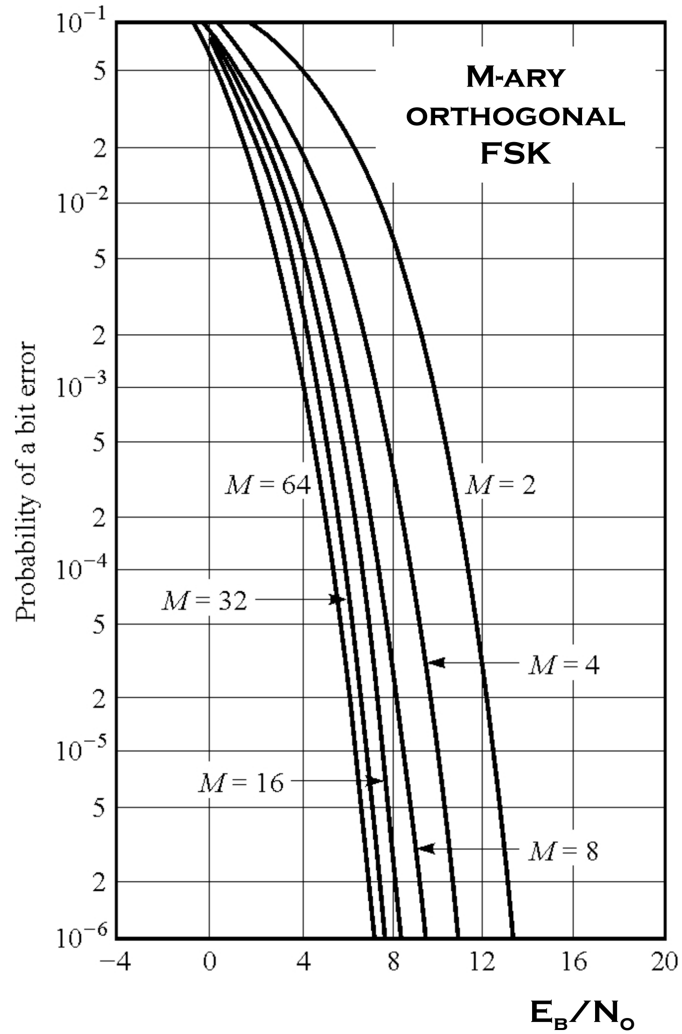
M-ary ASK: $P_e \approx erfc\sqrt{\frac{3}{M^2-1}\frac{E_S}{N_0}},\ d = \sqrt{\frac{12}{(M^2-1)}E_S}$

M-ary PSK: $P_e \approx erfc\left[\sin\frac{\pi}{M}\sqrt{\frac{E_S}{N_0}}\right], d_{min} = \sqrt{E_S}.\sin \pi/M$

squared M-ary QAM: $P_{e,M-aryQAM} \approx 2P_{eASK} \approx 2\left(1-\frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3}{2(M-1)}\frac{E_S}{N_0}}\right)$

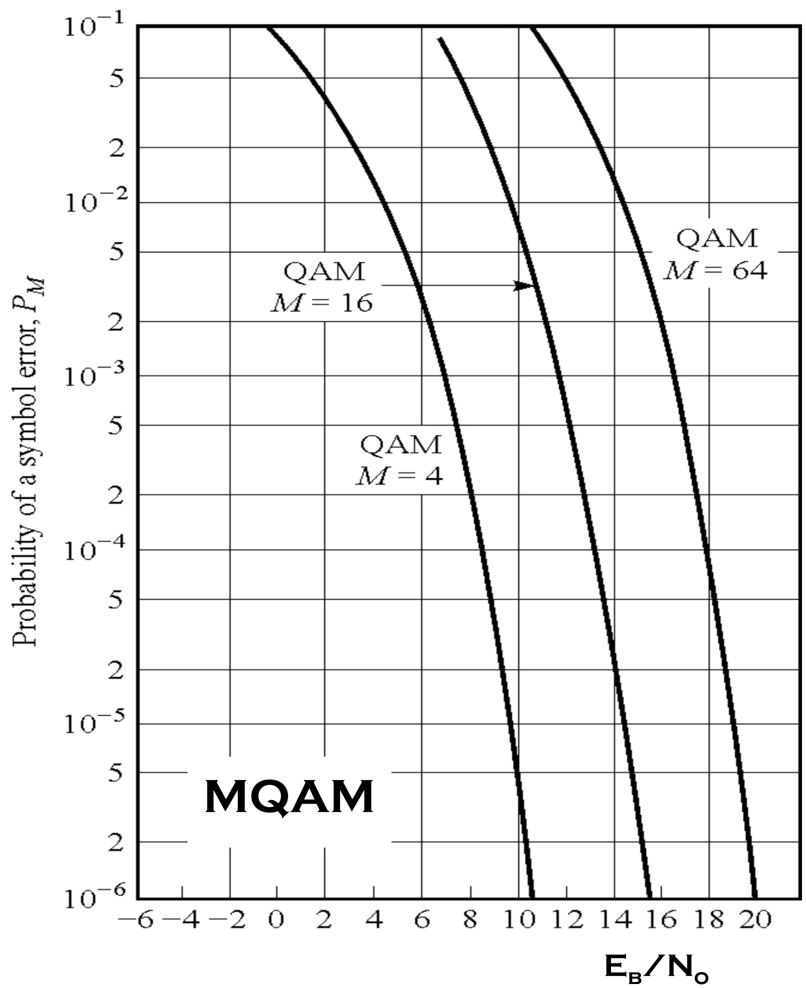
Orthogonal FSK: $P_e \le \frac{1}{2}(M-1) erfc\sqrt{\frac{E_S}{2N_0}}$

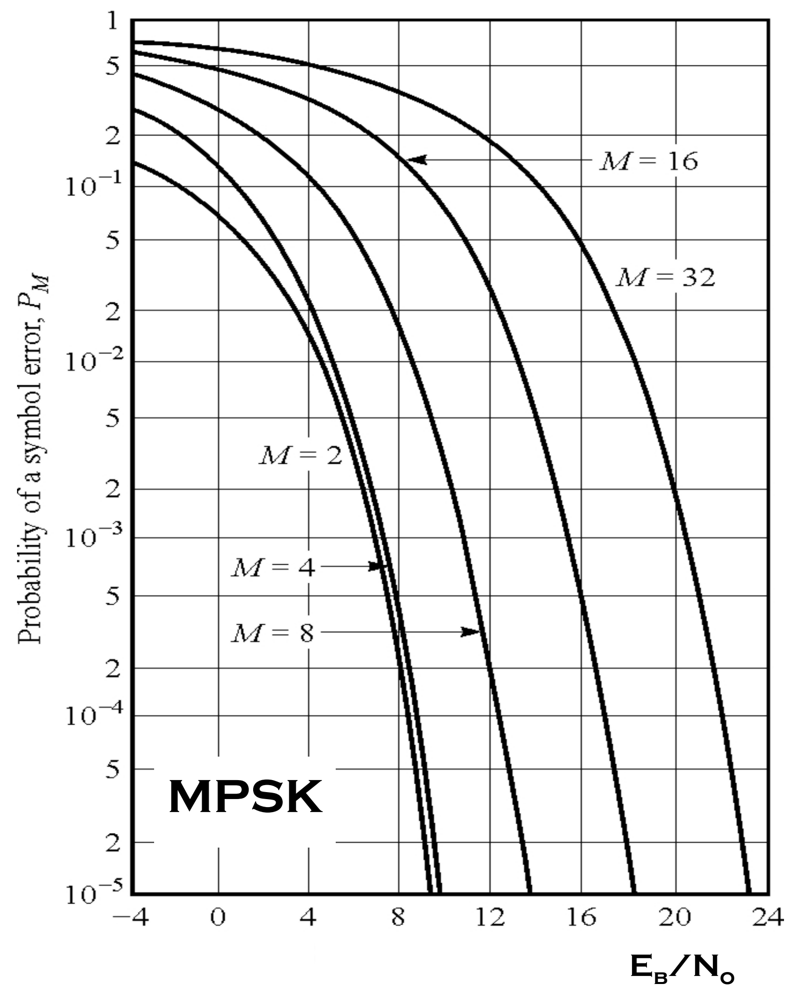
M-ary orthogonal FSK signaling schemes are power-efficient but not bandwidth-efficient.



Performance in AWGN: PROBABILITY OF SYMBOL ERROR

error probability decays exponentially in SNR in the AWGN channel





M-QAM, M-PSK: BW-efficient but not power-efficient
For M>8, M-QAM outperforms M-PSK

Performance of M-ary Modulation in Fading channel

- With fading, instantaneous SNR is random

$$\gamma_s = E_s|h|^2/N_0, \quad \gamma_b = E_b|h|^2/N_0,$$

- Error performance for given realization $P_s(\gamma_s),\ P_b(\gamma_b)$ also random
- Average SER/BER $\bar{P}_s = \mathbb{E}[P_s(\gamma_s)], \quad \bar{P}_b = \mathbb{E}[P_b(\gamma_b)]$

- Rayleigh fading $h \sim \mathcal{CN}(0,1)$
  - BPSK:

$$\bar{P}_b = \mathbb{E}[Q(\sqrt{2\gamma_b})] = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right] \approx \frac{1}{4\bar{\gamma}_b}$$

  - M-QAM:

$$\bar{P}_s \approx \mathbb{E}[\alpha_M Q(\sqrt{\beta_M \gamma_s})] = \frac{\alpha_M}{2}\left[1 - \sqrt{\frac{\bar{\gamma}_s \beta_M}{2+\bar{\gamma}_s \beta_M}}\right] \approx \frac{\alpha_M}{2\bar{\gamma}_s \beta_M}$$

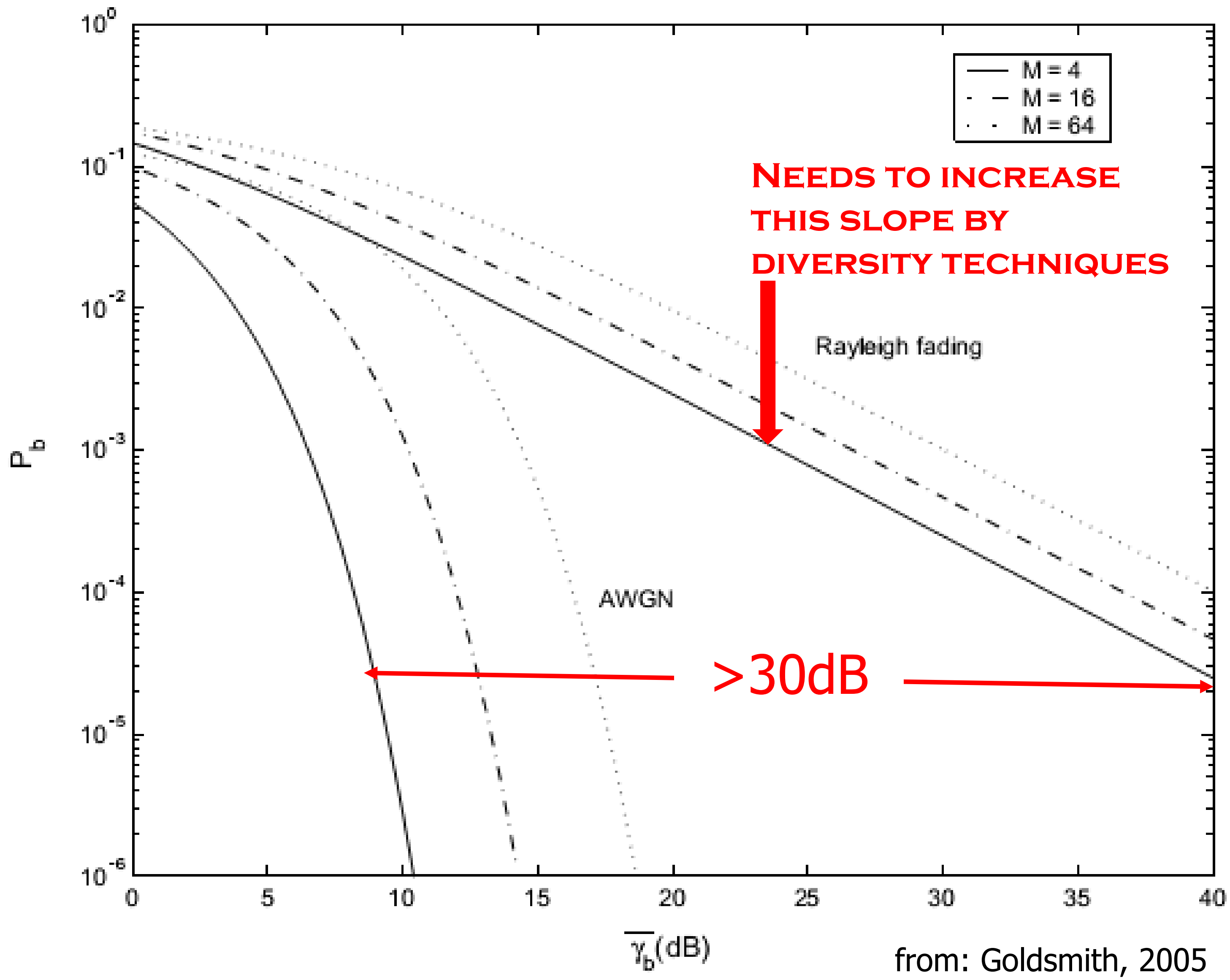
where α_M, β_M depend on M and approximation,
and

$$\bar{\gamma}_s = E_s \cdot \mathbb{E}[|h|^2]/N_0, \quad \bar{\gamma}_b = E_b \cdot \mathbb{E}[|h|^2]/N_0$$

Why poor performance?

- Due to randomness of channel gain
- When $\gamma_s \gg 1$,
  - Conditional error probability small since Q() decays rapidly
  - Constellation pts separation larger than standard deviation of noise
- When $\gamma_s \ll 1$ (**deep fade**),
  - Conditional error probability large
  - Constellation pts separation smaller than standard deviation
    - Deep fade event: $\gamma_s < 1$
    - Prob. of deep fade: $\mathbb{P}\{\gamma_s < 1\} = 1 - \exp(-1/\bar{\gamma}_s)$
    - At high SNR: $\mathbb{P}\{\gamma_s < 1\} \approx 1/\bar{\gamma}_s$
- At high SNR, error events most often occur because of deep fade and not because of large noise

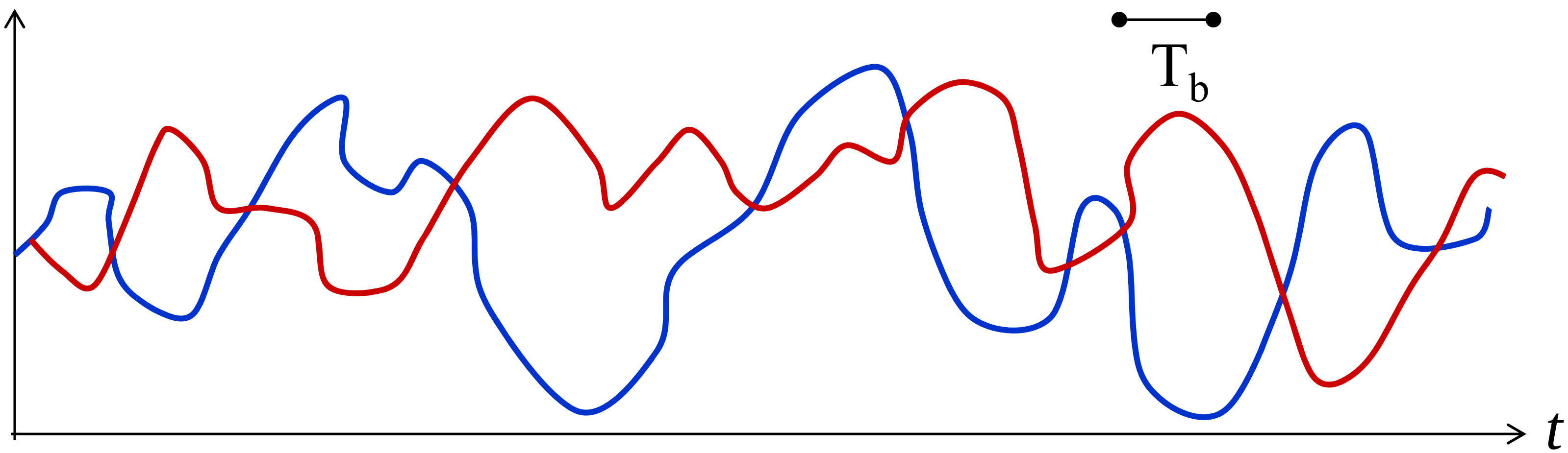
BER/SER decay only inversely with SNR



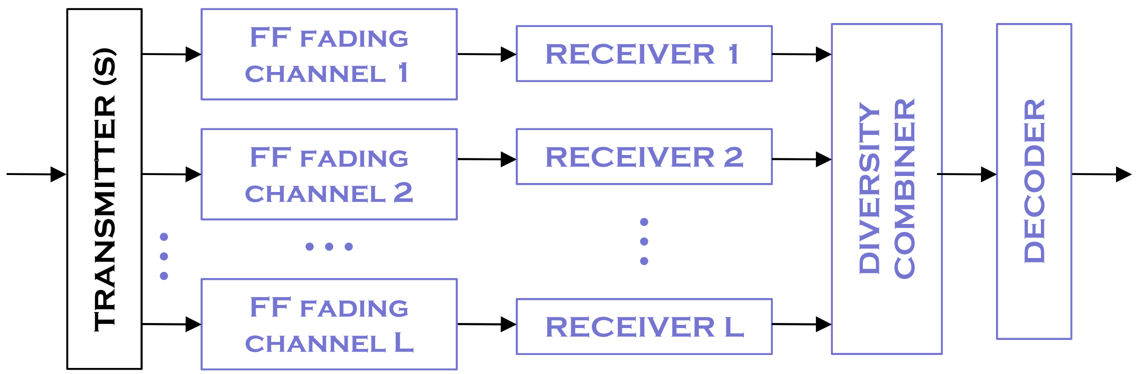
from: Goldsmith, 2005

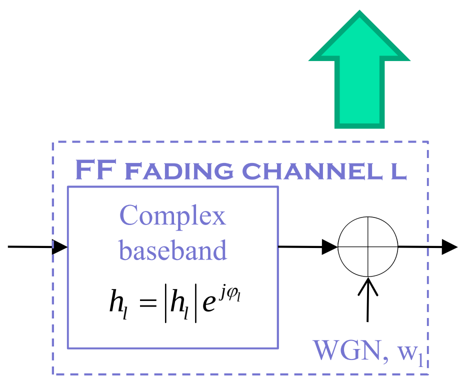
Diversity

- Single path suffers from deep fading
- Multiple independent paths unlikely to fade simultaneously



Diversity approach for frequency-flat fading channels

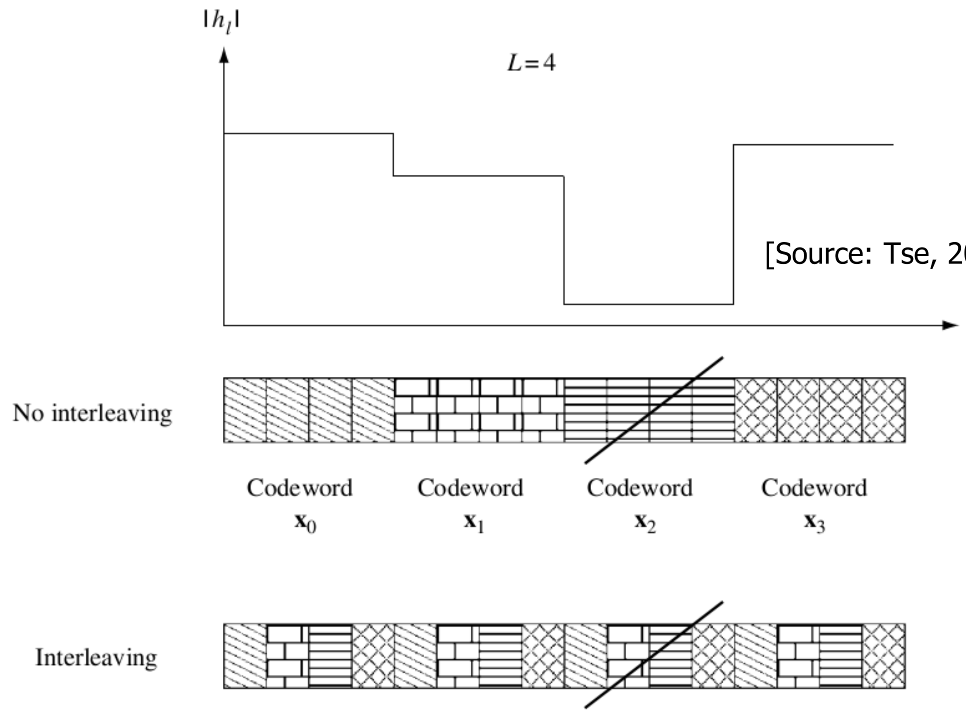




- Multiple independent paths (or channels) unlikely to fade simultaneously
- ⇒ Diversity techniques:
  - Send the same signals over independent fading paths obtained by diversity in time, space, frequency, …
    - ⇒ reduced possibility of all paths in deep fading simultaneously
  - Combine paths to mitigate fading effects: exploit the diversity in an efficient manner to *combat fading effectively.*

Time Diversity

- Dispersing information signals over multiple time intervals
- Repetition coding: Transmit the same signal repeatedly over multiple coherence times (time separation > coherence time)
  $\rightarrow$ Bandwidth inefficiency!!
- Error Control Coding: Much more sophisticated scheme
- coding and interleaving



[Source: Tse, 2005]

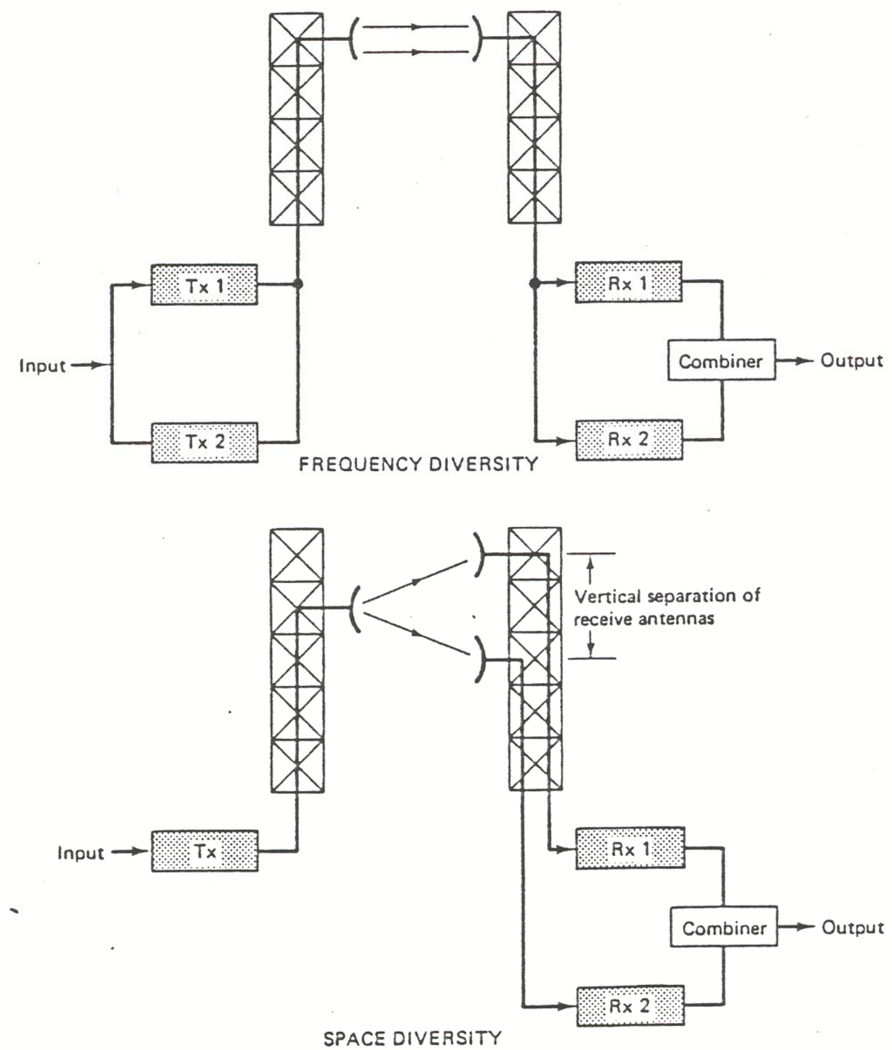
Diversity: Frequency, Space

Frequency diversity:

- Transmit same info over different subcarriers
- frequency separation > coherence bandwidth

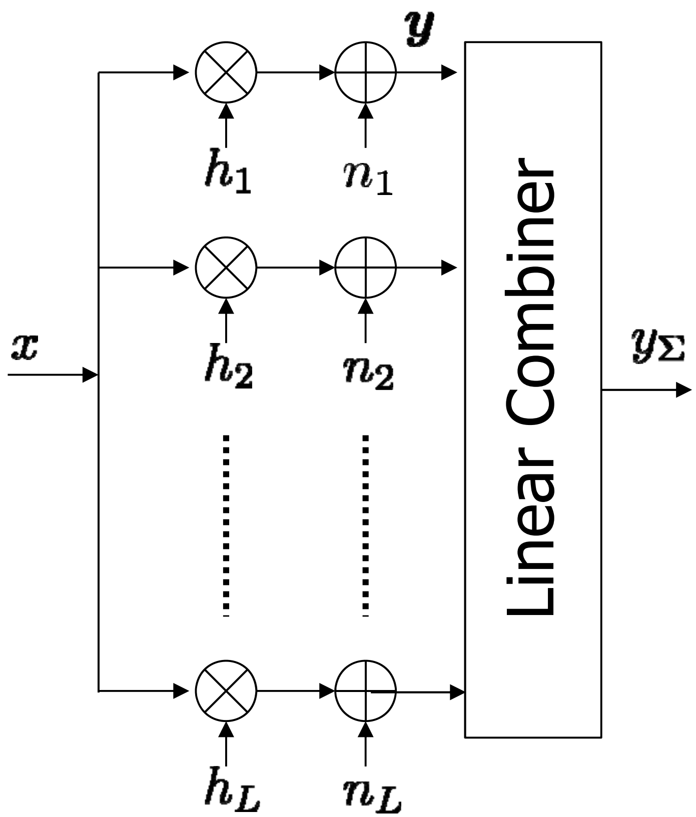
Space diversity:

- Transmit/receive from multiple antennas
- Sufficient antenna separation to achieve uncorrelated channel gains, e.g., about half wavelength, $\lambda/2$, for a Rayleigh fading



Diversity Techniques at Receiver

- Transmitter sends same signal over L independent fading paths obtained by diversity in time, space, frequency (**repetition coding**)



$$\boldsymbol{y} = \boldsymbol{h}x + \boldsymbol{n}$$

$$\boldsymbol{y} = [y_1, \ldots, y_L]^\top$$
$$\boldsymbol{h} = [h_1, \ldots, h_L]^\top = [r_1 e^{j\theta_1}, \ldots, r_L e^{j\theta_L}]^\top$$
$$\boldsymbol{n} = [n_1, \ldots, n_L]^\top \sim \mathcal{CN}(\boldsymbol{0}, N_0 \boldsymbol{I}_L)$$

- Most combiners are **linear** (weighted sum of **branches**)

$$y_\Sigma = (\boldsymbol{\alpha}^\top \boldsymbol{h}) \cdot x + (\boldsymbol{\alpha}^\top \boldsymbol{n})$$
$$\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_L]^\top$$

Diversity Techniques at Receiver

- Can select one or combine multiple branches
  - To add coherently, combining requires **co-phasing** $\alpha_i = a_i e^{-j\theta_i}$
- Combiner SNR

$$\gamma_\Sigma = \frac{|\boldsymbol{\alpha}^\top \boldsymbol{h}|^2 E_s}{\|\boldsymbol{\alpha}\|^2 N_0}$$

- We hope that γ_Σ has better distribution than $\gamma_l = [|h_l|^2 E_s]/N_0$
- SER

$$\bar{P}_s = \mathbb{E}[P_s(\gamma_\Sigma)] = \int P_s(\gamma) f_{\gamma_\Sigma}(\gamma)\, d\gamma$$

- Performance gains?

Receiver Diversity: selection combining (SC)

- **SC**: Select fading path with largest SNR $\gamma_l = [|h_l|^2 E_s]/N_0$

$$l^\star = \operatorname{argmax}_{1 \leq l \leq L} \quad \gamma_l$$

$$\boldsymbol{\alpha}_{\mathrm{SC}} = [0, \ldots, \underset{\substack{\uparrow \\ l^\star}}{1}, \ldots, 0]^\top \quad \Rightarrow \quad \gamma_\Sigma = \gamma_{l^\star}$$

- CDF $\quad F_{\gamma_\Sigma}(\gamma) = \mathbb{P}[\gamma_\Sigma < \gamma] = \mathbb{P}[\max(\gamma_1, \ldots, \gamma_L) < \gamma] = \prod_{l=1}^{L} F_{\gamma_l}(\gamma)$

- If all branches equally distributed

CDF: $F_{\gamma_\Sigma}(\gamma) = [F_\gamma(\gamma)]^L$

PDF: $f_{\gamma_\Sigma}(\gamma) = \dfrac{d\,[F_\gamma(\gamma)]^L}{d\gamma} = L[F_\gamma(\gamma)]^{L-1} f_\gamma(\gamma)$

SC over Rayleigh fading

received vector: $\mathbf{r}(k) = \mathbf{h}x(k) + \mathbf{n}(k),\ \mathbf{h} = [h_1, h_2, ..., h_L]^T$

Rayleigh channel: $h_l = |h_l| e^{j\varphi_l},\ l = 1, 2, .., L : i.i.d., |h_l|$:Rayleigh

$Y = |h_l|^2 > 0$: exponential, $\quad p_Y(y) = \left[2\sigma^2\right]^{-1} e^{-y/2\sigma^2},\ \overline{Y} = 2\sigma^2, \text{var}: \sigma_Y^2 = 4\sigma^4$

$2\sigma^2 = 1$ (normalized)

select the max $|h_*|$ and coherently demodulate:

$$\tilde{r}(k) = |h_*| x(k) + n_*(k),\ |h_*| = \max\{|h_l|, l = 1, 2, .., L\}$$

$$\text{cdf: } \Pr\left\{|h_*|^2 \le y\right\} = \Pr\left\{\bigcap_{l=1}^{L} |h_l|^2 \le y\right\} = \left[\int_0^y \left[2\sigma^2\right]^{-1} e^{-x^2/2\sigma^2} dx\right]^L$$

$$p_{|h_*|^2}(y) = \frac{d \Pr\left\{|h_*|^2 \le y\right\}}{dy} = \frac{L}{2\sigma^2} e^{-y/2\sigma^2} \left[1 - e^{-y/2\sigma^2}\right]^{L-1},\ y \ge 0$$

No longer exponential

$SNR_{SC} = |h_*|^2 [E_s / N_o]$ as compared to non-diversity case: $SNR = |h_l|^2 [E_s / N_o]$

$$\text{BPSK: } P_{s||h_*|} = Q\left(\sqrt{2|h_*|^2 E_b / N_o}\right)$$

Average SNR: $E_s/N_o = \bar{\gamma}$

Instantaneous SNR: $\gamma = y(E_s/N_o) = y\bar{\gamma}$

$$\rightarrow \overline{P}_s = \left[L/(2\sigma^2)\right] \int_0^\infty Q\left(\sqrt{2yE_b / N_o}\right) e^{-y/2\sigma^2} \left[1 - e^{-y/2\sigma^2}\right]^{L-1} dy$$

solved by numerical integration

SC performance: Probability of deep fade event

- i.i.d. Rayleigh fading

CDF: $F_{\gamma_\Sigma}(\gamma) = [1 - \exp(-\gamma/\bar{\gamma})]^L$

No longer exponential

PDF: $f_{\gamma_\Sigma}(\gamma) = [L/\bar{\gamma}] \cdot [1 - \exp(-\gamma/\bar{\gamma})]^{L-1} \exp(-\gamma/\bar{\gamma})$

- Probability of deep fade event

$$\mathbb{P}\{\gamma_\Sigma < 1\} = F_{\gamma_\Sigma}(1) = [1 - \exp(-1/\bar{\gamma}_l)]^L \approx \frac{1}{\bar{\gamma}_l^L}$$

- Deep fade only if all channel gains are small
- BER BPSK

$$\bar{P}_b = \int Q(\sqrt{2\gamma}) f_{\gamma_\Sigma}(\gamma)\, d\gamma \qquad \text{(numerical evaluation)}$$

SC: array gain

Rayleigh channel: $h_l = |h_l| e^{j\varphi_l}, l = 1, 2, .., L : i.i.d., |h_l|$:Rayleigh

$Y = |h_l|^2 > 0$: exponential, $p_Y(y) = \left[2\sigma^2\right]^{-1} e^{-y/2\sigma^2}, \bar{Y} = 2\sigma^2$, var : $\sigma_Y^2 = 4\sigma^4$

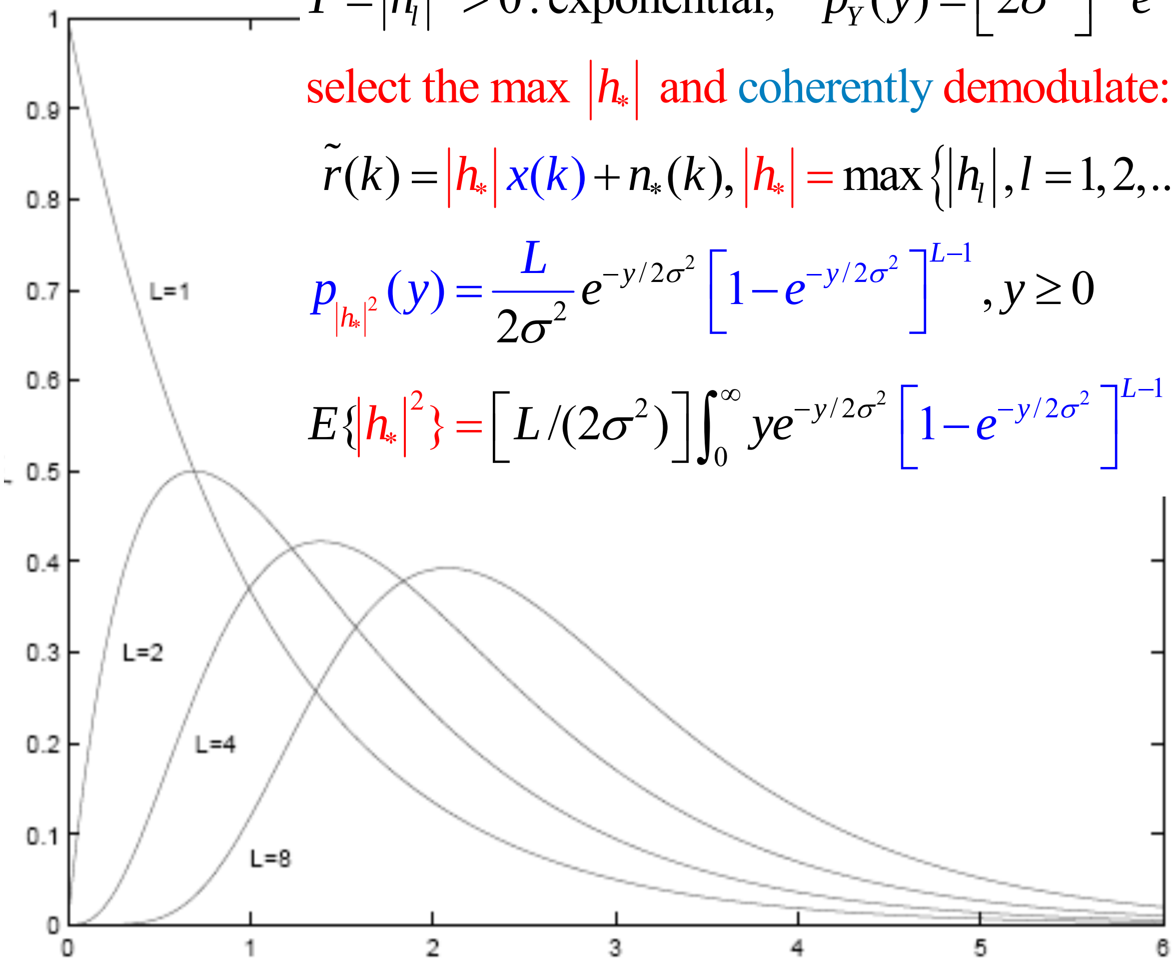
select the max $|h_*|$ and coherently demodulate:

$$\tilde{r}(k) = |h_*| x(k) + n_*(k), |h_*| = \max\left\{|h_l|, l = 1, 2, .., L\right\}$$

$$p_{|h_*|^2}(y) = \frac{L}{2\sigma^2} e^{-y/2\sigma^2} \left[1 - e^{-y/2\sigma^2}\right]^{L-1}, y \geq 0$$

$$E\{|h_*|^2\} = \left[L/(2\sigma^2)\right] \int_0^\infty y e^{-y/2\sigma^2} \left[1 - e^{-y/2\sigma^2}\right]^{L-1} dy = (2\sigma^2) \sum_{l=1}^{L} l^{-1} \geq (2\sigma^2)$$

$p_{|h_*|^2}(y)$

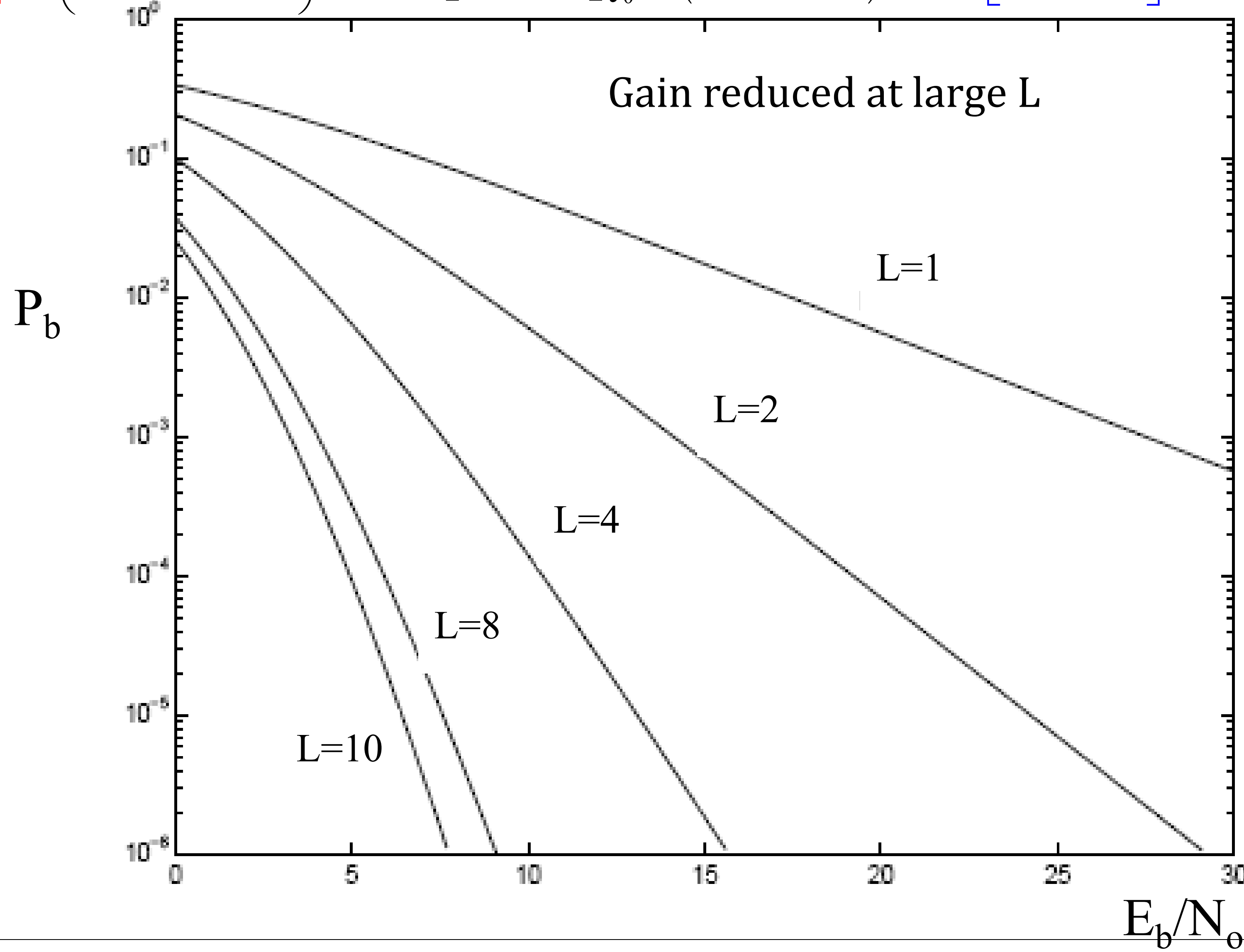


y

array gain: $\sum_{l=1}^{L} 1/l$ is reduced with large L

SC: performance of BPSK over Rayleigh fading

BPSK: $P_{s||h_*|} = Q\left(\sqrt{2|h_*|^2 E_b / N_o}\right) \rightarrow \overline{P}_s = \left[L/(2\sigma^2)\right]\int_0^\infty Q\left(\sqrt{2yE_b/N_o}\right)e^{-y/2\sigma^2}\left[1-e^{-y/2\sigma^2}\right]^{L-1} dy$



Array gain & Diversity gain

- **Array gain:** $$A_g = \frac{\mathbb{E}[\gamma_\Sigma]}{\mathbb{E}[\gamma_l]} = \frac{\bar{\gamma}_\Sigma}{\bar{\gamma}_l}$$
  - From coherent combining of multiple received signals
  - Applies to AWGN and fading channels
- **Diversity gain:**
  - More favorable distribution of γ_Σ
  - At large SNRs $\bar{P}_s \approx \underbrace{c}_{\text{Constant (depends on modulation/coding)}} \cdot \bar{\gamma}^{-\underbrace{d}_{\text{Diversity order (slope of BER vs. SNR)}}}$

    Constant (depends on modulation/coding)

    Diversity order
    (slope of BER vs. SNR)
  - Average BER decreases with average SNR^{-d}
  - Change in slope of BER
  - Applies only to fading channels
  - System is said to be **full-diversity** if d=L

Receiver Diversity: Maximal ratio combining (MRC)

- **MRC**: Co-phase all branches and add with optimal weights to maximize combiner output SNR

$$\alpha_{\mathrm{MRC}} = [a_1^\star e^{-j\theta_1}, \ldots, a_L^\star e^{-j\theta_L}]^\top \Rightarrow \gamma_\Sigma = \frac{E_s\left(\sum_{l=1}^{L} a_l^\star |h_l|\right)^2}{N_0\left(\sum_{l=1}^{L} (a_l^\star)^2\right)}$$

- Optimal weights solution to

$$\max_{a_l} \quad \frac{E_s\left(\sum_{l=1}^{L} a_l |h_l|\right)^2}{N_0\left(\sum_{l=1}^{L} a_l^2\right)}$$

- Intuitively, need to give higher weights to branches with higher SNR

MRC: Matched filter solution

- Solution (by partial derivatives or Cauchy-Schwarz inequality):

  Matched filter or maximal ratio combiner

$$\left.\begin{aligned} a_l^\star &= |h_l| \\ \alpha_l^\star &= |h_l| e^{-j\theta_l} \end{aligned}\right\} \boldsymbol{\alpha}_{\mathrm{MRC}}^\top = \boldsymbol{h}^\dagger$$

Hermitian

- In general when branches do not have equal noise variance

$$\alpha_l^\star = \frac{|h_l|}{\sqrt{N_l}} e^{-j\theta_l}$$

- Combiner SNR: sum of SNRs on each branch

$$\gamma_\Sigma = \frac{\|\boldsymbol{h}\|^2 E_s}{N_0} = \sum_{l=1}^{L} \gamma_L$$

MRC: array gain

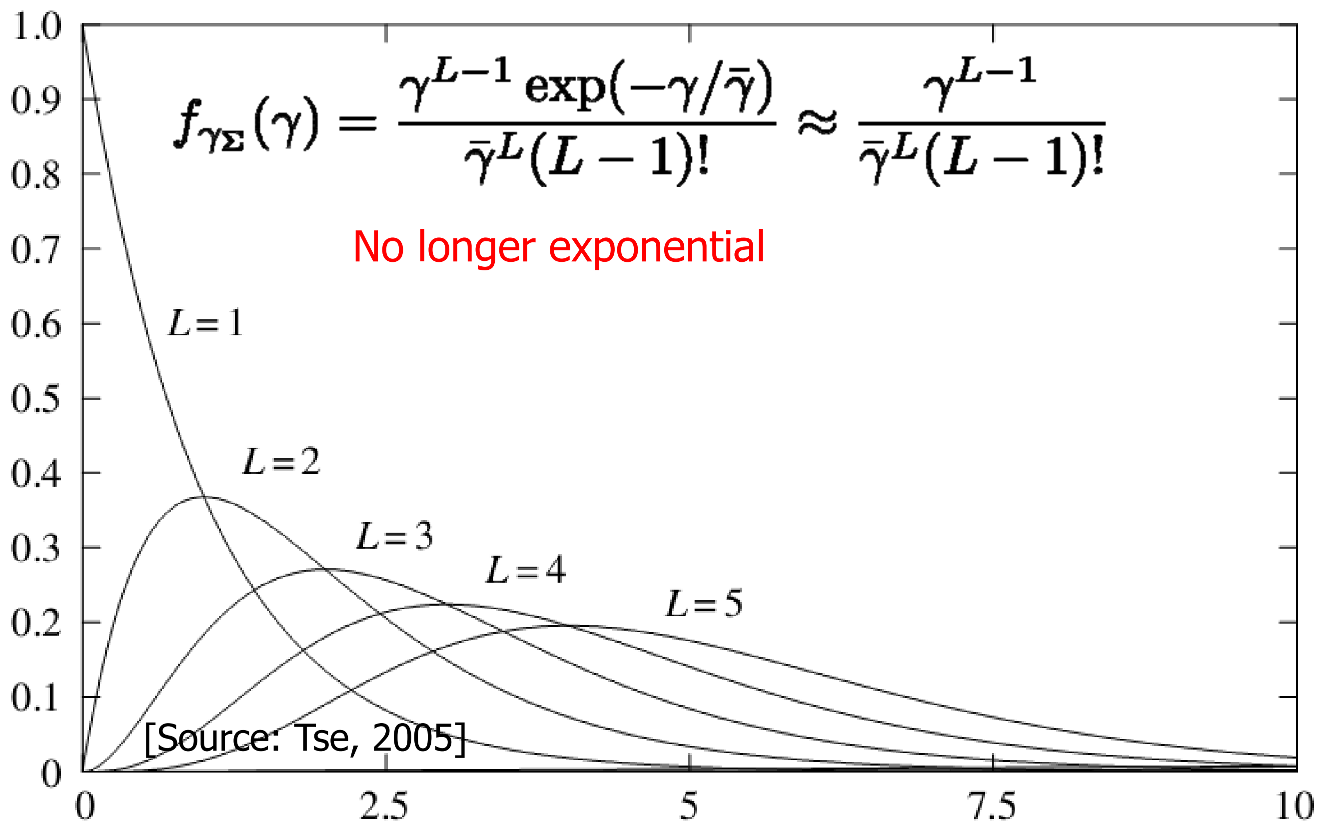
- Average SNR

$$\bar{\gamma}_{\Sigma} = \sum_{l=1}^{L} \mathbb{E}[\gamma_L] = L \cdot \bar{\gamma}$$

  - Array gain increases linearly with L (unlike SC)
- PDF: convolution of branches PDFs if independent

MRC in iid Rayleigh fading channels

- PDF: sum of exponential RVs (chi-square with 2L degrees of freedom or Erlang distributed)



[Source: Tse, 2005]

$$f_{\gamma_\Sigma}(\gamma) = \frac{\gamma^{L-1}\exp(-\gamma/\bar{\gamma})}{\bar{\gamma}^L (L-1)!} \approx \frac{\gamma^{L-1}}{\bar{\gamma}^L (L-1)!}$$

No longer exponential

- Probability of fading event: $\mathbb{P}\{\gamma_\Sigma < 1\} \approx \int_0^1 \frac{\gamma^{L-1}}{\bar{\gamma}^L (L-1)!} = \frac{1}{\bar{\gamma}^L \cdot L!}$

MRC & its Performance in Rayleigh Channels

received vector: $\mathbf{r}(k) = \mathbf{h}x(k) + \mathbf{n}(k)$, $\mathbf{h} = [h_1, h_2, ..., h_L]^T$

Rayleigh channel: $h_l = |h_l| e^{j\varphi_l}$, $l = 1, 2, .., L : i.i.d.$, $|h_l|$:Rayleigh

$Y = |h_l|^2 > 0$: exponential, $\quad p_Y(y) = \left[2\sigma^2\right]^{-1} e^{-y/2\sigma^2}$

$\overline{Y} = 2\sigma^2$, var : $\sigma_Y^2 = 4\sigma^4$

select the max $|h_*|$ and coherently demodulate and combine with optimum weights, $\mathbf{h}^H$:

matched filter: $\tilde{r}(k) = \mathbf{h}^H \mathbf{r}(k) = \|\mathbf{h}\|^2 x(k) + w(k)$, $w(k) = \mathbf{h}^H \mathbf{n}(k) : Gaussian(0, \|\mathbf{h}\|^2 N_o / 2)$

MAX OUTPUT SNR

instantaneous $SNR_{MRC} = \left[\|\mathbf{h}\|^2\right]\left[E_s / N_o\right]$ as compared to non-diversity case: $SNR = |h_l|^2 \left[E_s / N_o\right]$

$Y = \|\mathbf{h}\|^2$, $p_Y(y) = \left[2^L \sigma^{2L} (L-1)!\right]^{-1} y^{L-1} e^{-y/2\sigma^2}$, $\overline{Y} = 2L\sigma^2$, var : $\sigma_Y^2 = 4L\sigma^4$

BPSK: $P_{s|\|h\|} = Q\left(\sqrt{2\|\mathbf{h}\|^2 E_b / N_o}\right)$

$$\rightarrow \overline{P}_s = \left[2^L \sigma^{2L} (L-1)!\right]^{-1} \int_0^\infty Q\left(\sqrt{2yE_b / N_o}\right) y^{L-1} e^{-y/2\sigma^2} dy$$

$$\overline{P}_s = \left[\frac{1-\gamma}{2}\right]^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} \left[\frac{1+\gamma}{2}\right]^l, \gamma = \sqrt{\frac{2\sigma^2 \left[E_b / N_o\right]}{1 + 2\sigma^2 \left[E_b / N_o\right]}}$$

MRC performance: with BPSK

- BER

$$\bar{P}_b = \int Q(\sqrt{2\gamma}) f_{\gamma_\Sigma}(\gamma)\, d\gamma = \left(\frac{1-\mu}{2}\right)^L \sum_{l=0}^{L-1} \binom{L-1+l}{l} \left(\frac{1+\mu}{2}\right)^l$$

where $\mu = \sqrt{\dfrac{\bar{\gamma}}{1+\bar{\gamma}}}$

- At high SNRs $\dfrac{1+\mu}{2} \approx 1, \quad \dfrac{1-\mu}{2} \approx \dfrac{1}{4\bar{\gamma}}$

$$\bar{P}_b \approx \frac{1}{(4\bar{\gamma})^L} \sum_{l=0}^{L-1} \binom{L-1+l}{l} = \binom{2L-1}{L} \frac{1}{(4\bar{\gamma})^L}$$

Full diversity

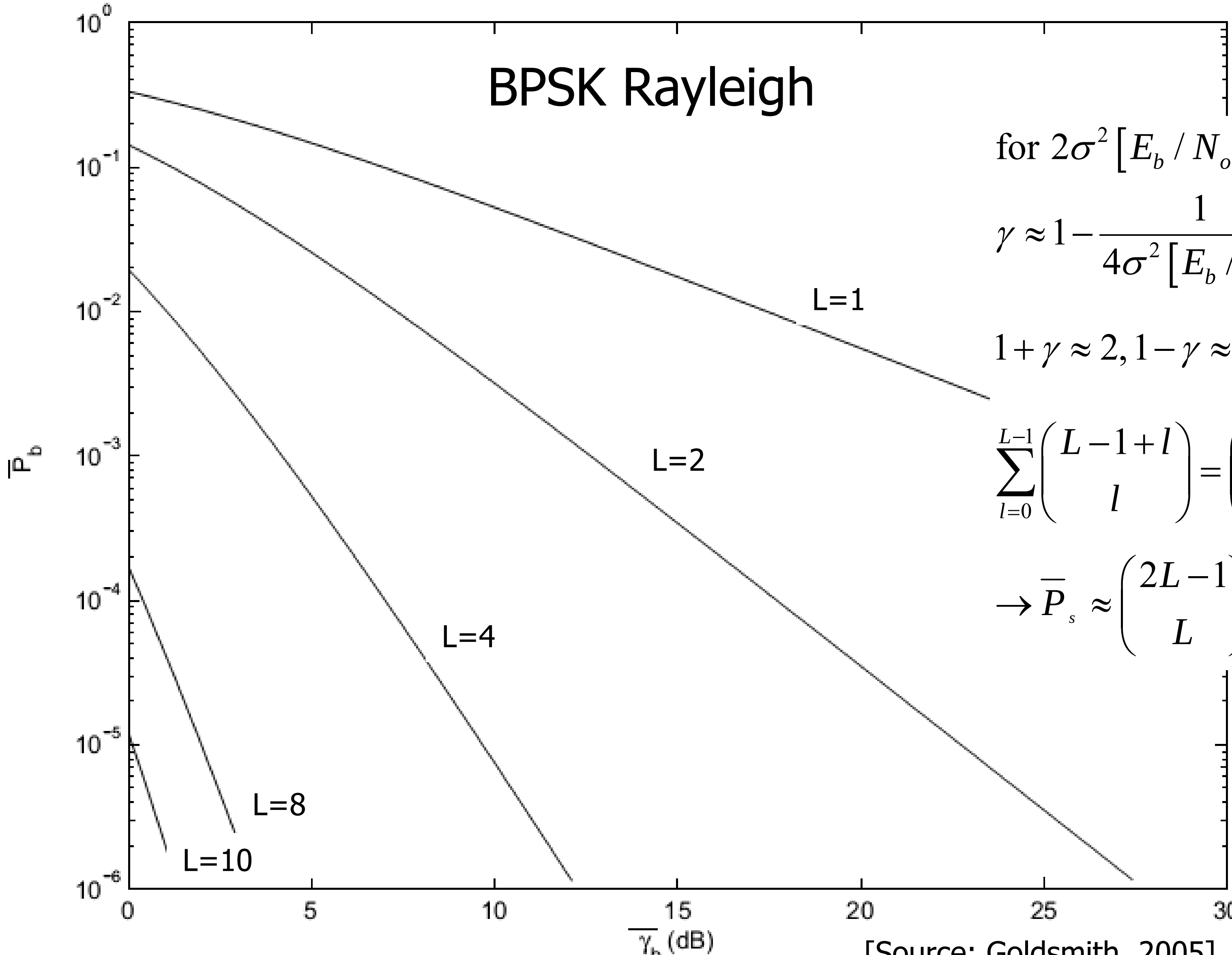
MRC performance: general case

- BER general

$$
\begin{aligned}
\bar{P}_s &= \mathbb{E}[\alpha_M Q(\sqrt{\beta_M \gamma_\Sigma})] \\
&\leq \mathbb{E}[\alpha_M \exp(-\beta_M \gamma_\Sigma / 2)] \quad \text{(Chernoff bound)} \\
&= \mathbb{E}[\alpha_M \exp(-\beta_M [\gamma_1 + \ldots + \gamma_L]/2)] \\
&= \alpha_M \prod_{l=1}^{L} \frac{1}{1 + \beta_M \bar{\gamma}_l / 2} \\
&\approx \alpha_M \left(\frac{\beta_M \bar{\gamma}}{2} \right)^{-L} \quad \text{(if i.i.d.)}
\end{aligned}
$$

- Full diversity of L

MRC performance: with BPSK



$$\text{for } 2\sigma^2\left[E_b/N_o\right] >> 1,$$

$$\gamma \approx 1 - \frac{1}{4\sigma^2\left[E_b/N_o\right]},$$

$$1+\gamma \approx 2,\ 1-\gamma \approx \frac{1}{4\sigma^2\left[E_b/N_o\right]},$$

$$\sum_{l=0}^{L-1}\binom{L-1+l}{l} = \binom{2L-1}{L}$$

$$\rightarrow \overline{P}_s \approx \binom{2L-1}{L}\left[\frac{1}{4\sigma^2\left[E_b/N_o\right]}\right]^L$$

[Source: Goldsmith, 2005]

Receiver Diversity: Equal-gain combining (EGC)

- **EGC**: Co-phase all branches and add with equal weights

$$\alpha_{\text{EGC}} = [e^{-j\theta_1}, \ldots, e^{-j\theta_L}]^\top \quad \Rightarrow \quad \gamma_\Sigma = \frac{E_s}{LN_0}\left(\sum_{l=1}^{L} |h_l|\right)^2$$

- In general, PDF and CDF does not exist in closed-form

- <u>Example</u>: i.i.d. Rayleigh fading
  - Average SNR

$$\bar{\gamma}_\Sigma = \frac{E_s}{LN_0}\sum_{l=1}^{L}\sum_{l=1}^{k}\mathbb{E}[|h_l||h_k|] = \bar{\gamma}\left(1 + \frac{\pi}{4}[L-1]\right)$$

EGC & its Performance in Rayleigh Channels

received vector: $\mathbf{r}(k) = \mathbf{h}x(k) + \mathbf{n}(k)$,

Rayleigh channel: $h_l = |h_l| e^{j\varphi_l}$, $l = 1, 2, .., L : i.i.d., |h_l|$:Rayleigh

$Y = |h_l|^2 > 0$: exponential, $p_Y(y) = \left[2\sigma^2\right]^{-1} e^{-y/2\sigma^2}, \overline{Y} = 2\sigma^2$, var : $\sigma_Y^2 = 4\sigma^4$

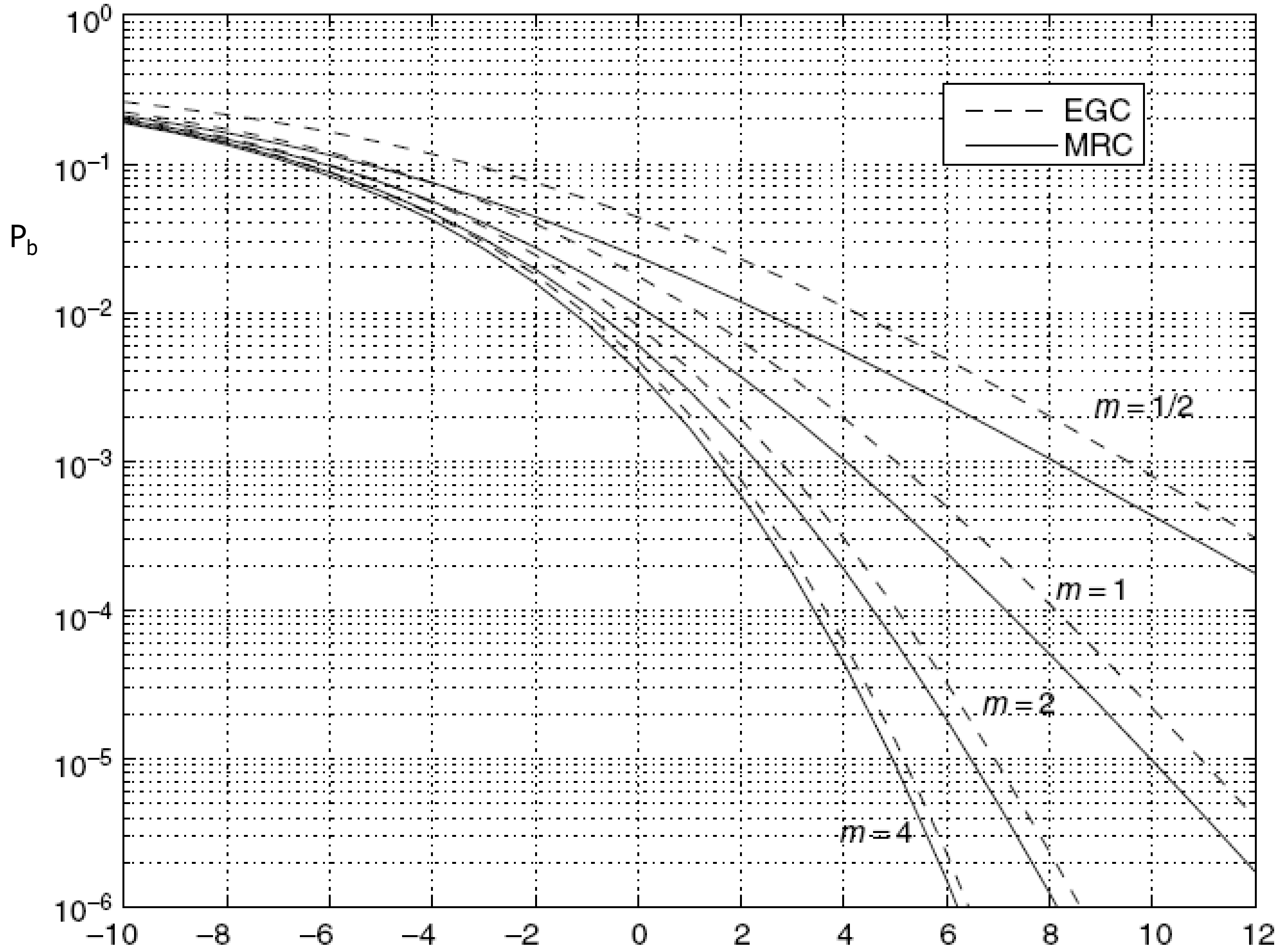
Coherently demodulate and combine with *equal* weights: $\mathbf{\Phi_h} = [e^{-j\varphi_1}, e^{-j\varphi_2}, 2, .., e^{-j\varphi_L}]$

$$\tilde{r}(k) = \mathbf{\Phi_h}\mathbf{r}(k) = h_{sum}x(k) + w(k),\ h_{sum} = \left[\sum_{l=1}^{L} |h_l|\right], w(k) = \mathbf{\Phi_h}\mathbf{n}(k) : Gaussian(0, LN_o/2)$$

$SNR_{EGC} = \left[h_{sum}^2 / L\right]\left[E_s / N_o\right]$ as compared to non-diversity case: $SNR = |h_l|^2 \left[E_s / N_o\right]$

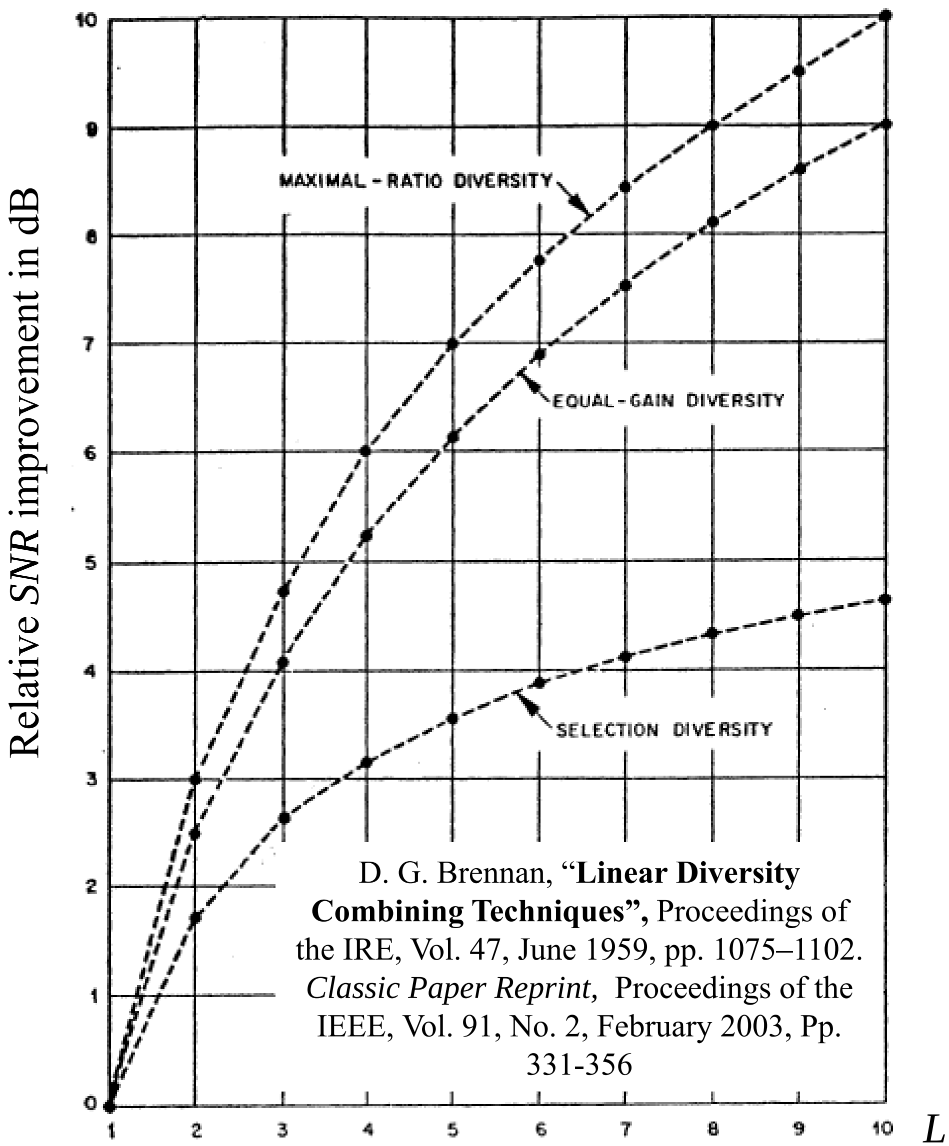
BPSK: $P_{s|h_{sum}} = Q\left(\sqrt{2h_{sum}^2 E_b / N_o}\right)$

BPSK over Nakagami-m channels with MRC and EGC (L = 4).



From Marvin K. Simon and Mohamed-Slim Alouini, *Digital Communication over Fading Channels, 2nd Edition,* John Wiley & Sons, 2005

EGC and Performance



D. G. Brennan, "**Linear Diversity Combining Techniques**", Proceedings of the IRE, Vol. 47, June 1959, pp. 1075–1102. *Classic Paper Reprint*, Proceedings of the IEEE, Vol. 91, No. 2, February 2003, Pp. 331-356

- The received signals from the L diversity branches are coherently combined with equal weights
- The receiver does not need the information of $||\mathbf{h}||$
- Performance is worse than that of MRC (about 1 dB), but much better than SC for large L

COMBINING TECHNIQUES

Selective combining:

- The receiver monitors the SNR of the received signal from each diversity branch, and, selects *only* the Rx signal corresponding to the highest SNR *for detection;*
- Simple but low performance.

Equal gain combining:

- Signals from the L diversity branches are *coherently* and *weighted equally*
- Complexity: receiver needs to estimate/know *only* the *phase* distortion introduced by each branch so that signals can be combined coherently.
- *Performance: good, better than SC but worse than MRC* (quite close, 1 dB of power penalty)

Maximal ratio combining:

- remove *the phase* distortion introduced by each branch so that signals can be combined coherently.
- select the weighting factor amplitude proportional to branch amplitude:

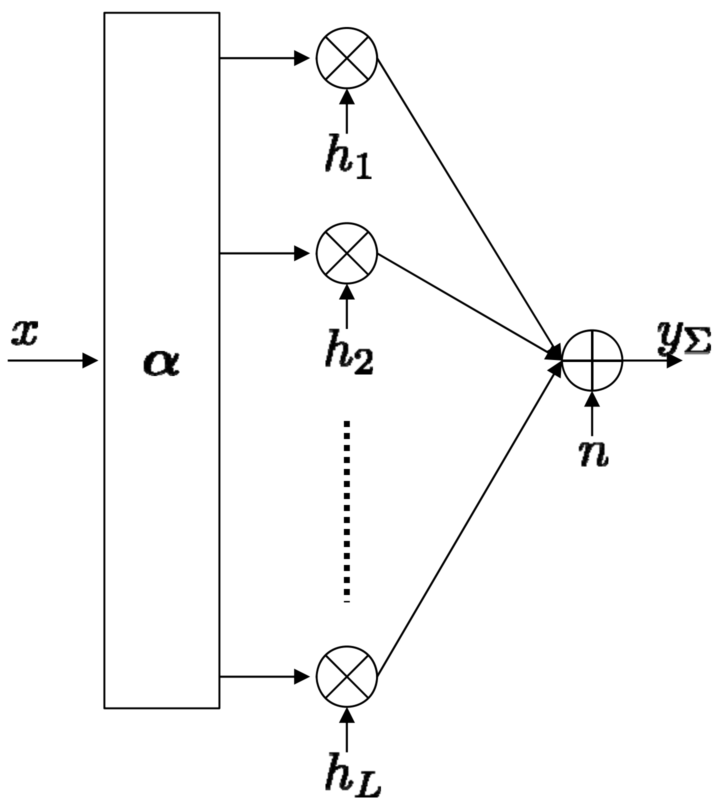
  larger branch amplitude yields higher SNR; hence more weight should be put on the corresponding received signal (with better quality).
- maximal ratio combining achieves the best transmission performance at the cost of receiver complexity, i.e., the receiver needs to estimate/know both branch amplitude and phase

Performance/complexity tradeoff

	SC	EGC	MRC
Complexity	Simplest (co-phasing not required)	Only need to estimate phase	Highest (need to track phase and SNR)
Performance	Worst	Much better than SC and worse than MRC (close, 1dB penalty)	Best (much better than SC)

Diversity Techniques at Transmitter

- Receiver obtains same signal from L independent fading paths



$$y_\Sigma = \sum_{l=1}^{L} (\alpha_l x) \cdot h_l + n$$
$$= (\boldsymbol{\alpha}^\top \boldsymbol{h}) \cdot x + n$$

- To keep power constraint

$$\sum_{l=1}^{L} |\alpha_l|^2 = 1$$

Diversity Techniques at Transmitter

- Output SNR

$$\gamma_\Sigma = \frac{|\boldsymbol{\alpha}^\top \boldsymbol{h}|^2 E_s}{N_0}$$

- If CSI available at transmitter, similar to receiver diversity
- Example: MRC

$$\boldsymbol{\alpha}_{\mathrm{MRC}}^\top = \boldsymbol{h}^\dagger / \|\boldsymbol{h}\| \quad \Rightarrow \quad \gamma_\Sigma = \frac{\|\boldsymbol{h}\|^2 E_s}{N_0} = \sum_{l=1}^{L} \gamma_L$$

  - Still sum of SNRs on each branch (same behavior)

- Same follows for EGC and SC

Diversity Techniques at Transmitter

- How about CSI unknown at transmitter?
- Consider i.i.d. Rayleigh fading $h \sim \mathcal{CN}(0,1)$

$$\boldsymbol{\alpha}_{\mathrm{MRC}}^{\top} = \underbrace{[1/\sqrt{L}, \ldots, 1/\sqrt{L}]}_{L} \Rightarrow y_{\Sigma} = \frac{1}{\sqrt{L}} \left(\sum_{l=1}^{L} h_l \right) x + n$$

- Since $[\sum_{l=1}^{L} h_l/\sqrt{L}] \sim \mathcal{CN}(0,1)$, same distribution as no diversity
- Need smarter way
  - i.e., space-time codes (will be covered later in the term)

Summary

- Performance of M-ary Modulation
  - BER over AWGN exponentially decaying (good)
  - BER over fading linearly decaying (bad)
  - Why? Deep fading
- Diversity
  - Send multiple independent copies
  - Improve BER slope
  - Time, frequency or space

Summary

- Diversity Techniques at Receiver
  - Array and diversity gains
  - SC (select best path, simple, diminishing gain)
  - MRC (optimal weights, sum of SNRs, best)
  - EGC (equal weights, close to MRC)
- Diversity Techniques at Transmitter
  - If CSIT, same as receiver techniques

References

- A. Goldsmith, *Wireless Communications,* Cambridge University Press, 2005, Chapter 4.
- Tse, P. Viswanath, *Fundamentals of Wireless Communication,* Cambridge University Press, 2005, Chapter 5
- and materials from various sources