# On The Riemann Hypothesis

Brian Edward Croner

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## Acknowledgement

This is dedicated to the good people at Facebook, YouTube, and Google.

**Abstract**

We indirectly prove the Riemann Hypothesis by proving the Nicholas equivalency.

## 1 Introduction

The Riemann Hypothesis states that the nontrivial zeros of the Riemann Zeta Function have a real part of $\frac{1}{2}$. We examine the following equivalency [1] established by Nicholas [2]:

$$\frac{N_k}{\varphi(N_k)} > e^{\gamma} loglogN_k$$

where

- $\gamma$ is the Euler Mascheroni constant,
- $\varphi$ is the Euler totient function, and
- $N_k$ is the primorial of the first $k$ primes

The Nicholas equivalency states that if this relationship holds for all $k \geq 1$ then the Riemann Hypothesis is true.

## 2 Logic

We take the Möbius inversion [7] of $\varphi(N_k)$ on the left and we have:

$$\frac{N_k}{N_k \Sigma_{d|n} \frac{\mu(d)}{d}}$$

which reduces to:

$$\frac{1}{(1-\frac{1}{p_1})(1-\frac{1}{p_2})\cdot\cdot(1-\frac{1}{p_k})} > e^{\gamma} loglogN_k$$

We apply Merten's Theorem [3] [4]:

$$lim_{n\to\infty} \frac{1}{lnp_n} \prod_{k=1}^{\infty} \frac{1}{(1-\frac{1}{p_k})} = e^{\gamma}$$

To get:

$$log(p_k)e^{\gamma} = e^{\gamma} loglogN_k$$

On the right side, by the Chebyshev function [6] the log of the primorial [5] is:

$$ln(n\#) = \vartheta(n)$$

which asymptotically approaches $n$ for large values of $n$ [8], giving us:

$$log(p_k)e^{\gamma} > e^{\gamma} log(k)$$

Simplifying, we have:

$$\frac{log(p_k)}{log(k)} > 1$$

Which is true for all values of $k$, proving the Riemann Hypothesis.

## References


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