CSC 6585 Homework VII

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1 Privacy Preserving Data Collection

1.1 Question

A company is conducting a survey and asking employees whether they have ever stolen from the company. To protect the employees' privacy and encourage truthful responses, the company uses a spinner model to provide plausible deniability.

Each employee is given a spinner with three equally probable outcomes: 1/3 "Yes," 1/3 "No," and 1/3 "Truth."

If the spinner lands on "Truth", the employee answers truthfully ("Yes" or "No").

If the spinner lands on "Yes" or "No", the employee responds according to what the spinner says, regardless of their true answer.

After collecting the responses from 300 employees, the company reports that 40% of the responses were "Yes."

Calculate the estimated true proportion of employees who have stolen from the company.

Hint: Let π represent the true proportion of employees who have stolen, and use the observed proportion (40%) to estimate π.

Show your work (obviously) and submit it in a PDF. You can scan handwritten notes, but please make sure the result is legible.

1.2 Answer

In order to solve this problem, we need to approximate the true proportion (which is π) of the employees who have stolen from the company based upon the observed data. Our goal is to understand how the spinner mechanism affects the responses.

The spinner outcomes are given:

- **Yes** ($\frac{1}{3}$ probability): The employee answers "Yes" regardless of the truth.

- **No** ($\frac{1}{3}$ probability): The employee answers "No" regardless of the truth.

- **Truth** ($\frac{1}{3}$ probability): The employee answers truthfully, i.e. Yes or No.

Next, we will now calculate the probability of a "Yes" response. The sum of the probabilities of each spinner outcome leads to a "Yes" outcome.

$$
\begin{aligned}
P(\text{"Yes"}) = {} & P(\text{Spinner lands on "Yes"}) \times 1 \\
& + P(\text{Spinner lands on "Truth"}) \times P(\text{Employee's true answer is "Yes"}) \\
& + P(\text{Spinner lands on "No"}) \times 0
\end{aligned}
$$

Then, it follows that:

$$
P(\text{"Yes"}) = \left(\frac{1}{3} \times 1 \right) + \left(\frac{1}{3} \times \pi \right) + \left(\frac{1}{3} \times 0 \right)
$$

Simplifying, we get:

$$
P(\text{"Yes"}) = \frac{1}{3} + \frac{\pi}{3}
$$

Since 40% of the responses were "Yes", it follows that:

$$0.40 = \frac{1}{3} + \frac{\pi}{3}$$

Now, we will solve for π. First, we subtract $\frac{1}{3}$ from both sides:

$$0.40 - \frac{1}{3} = \frac{\pi}{3}$$

Next, we calculate the left side:

$$0.40 - 0.3333 = 0.0667$$

Finally, we multiply both sides by 3:

$$0.0667 \times 3 = \pi$$

Thus, $\pi = 0.20$. Therefore, the estimated true proportion of employees who have stolen is 20%.

$$2 \quad + \frac{\pi}{3}$$