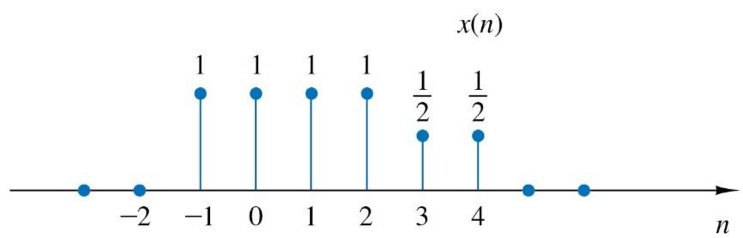
ECE 4020-5020: Digital Signal Processing

Homework 1 (100 pts.)

Due on Tuesday, February 3, 2026

Problem 1 (20 pts.):

Sketch the following signals corresponding to the original signal $x(n)$ shown below. Make sure to properly label amplitude and indices of the samples in your sketches.

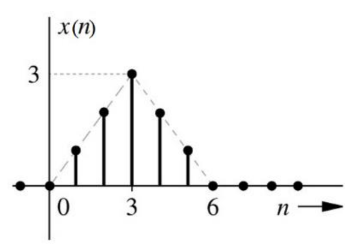


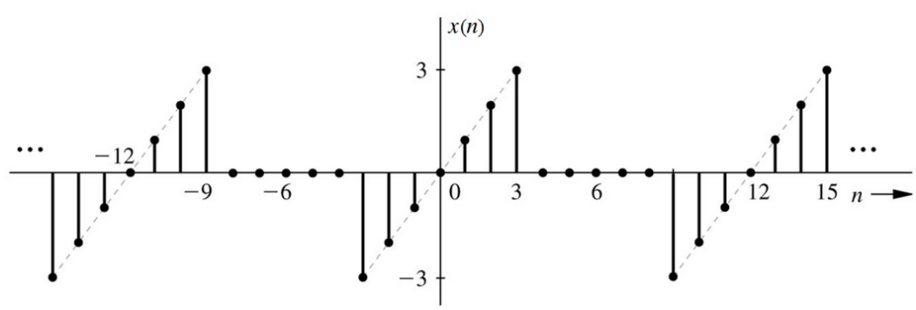
a. $x(n-2)$ b. $x(4-n)$ c. $x(n)u(2-n)$ d. $x(n-1)\delta(n-3)$ e. $x(n^2)$

*where the notation $u(n)$ represents unit-step and $\delta(n)$ represents unit-impulse.

Problem 2 (30 pts.):

a. Find the energy of the signal (shown left) and the power of the signal (shown right)





b. Determine an expression for the energy E of a converging discrete-time exponential signal $x(n) = a^n u(n)$, i.e., $|a| < 1$. Determine the power P if $|a| = 1$. For what range of $|a|$, the signal would neither be an energy nor a power signal.

Problem 3 (15 pts.):

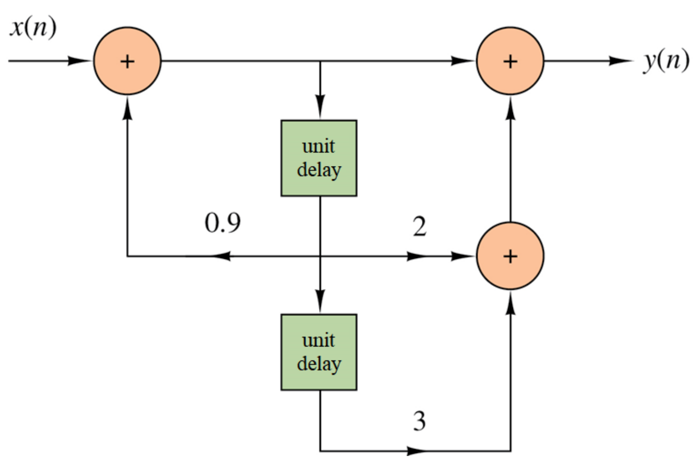
Determine the period N of the discrete sinusoids $\cos\left(\frac{\pi}{4}n\right)$, $\cos\left(\frac{4\pi}{17}n\right)$, and $\cos(0.8n)$. Also state how many sinusoidal envelops k are needed to constitute exactly one single period in each case.

Problem 4 (15 pts.):

a. Draw a block diagram of the discrete-time system given by the following difference equation

$$y(n) = -\frac{1}{4}y(n-1) - \frac{1}{16}y(n-2) + x(n)$$

b. Find the difference equation corresponding to the following block diagram



Problem 5 (20 pts.):

Determine the discrete-time system types (1) static or dynamic (2) linear or nonlinear (3) time-invariant or time-varying (4) causal or noncausal (5) stable or unstable

for each of the following discrete-time systems

a. $y(n) = \cos(x(n))$

b. $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

c. $y(n) = x(n)\cos(\omega_0 n)$

d. $y(n) = x(-n+2)$

ECE 5020 Homework I

Blaine Swieder

5 February 2026

1 Question 1

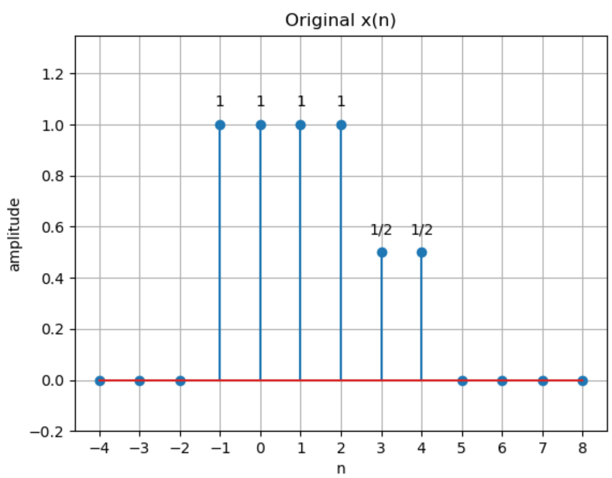


Figure 1: Corresponds with the original $x(n)$

Example 1.1. Sketch the following signals corresponding to the original signal $x(n)$ shown below. Make sure to properly label amplitude and indices of the samples in your sketches.

- $x(n-2)$
- $x(4-n)$
- $x(n)u(2-n)$
- $x(n-1)\delta(n-3)$
- $x(n^2)$

1.1 Part A

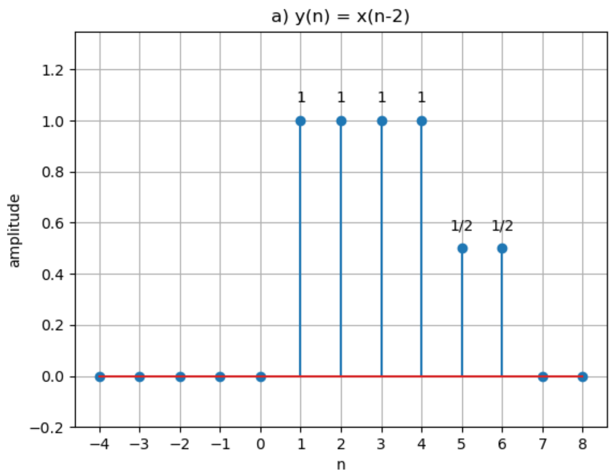


Figure 2: Graph of $x(n-2)$

For $x(n-2)$, our indices are $\{1, 2, 3, 4\}$ when our amplitude is 1 and our indices are $\{5, 6\}$ when our amplitude is $\frac{1}{2}$.

1.2 Part B

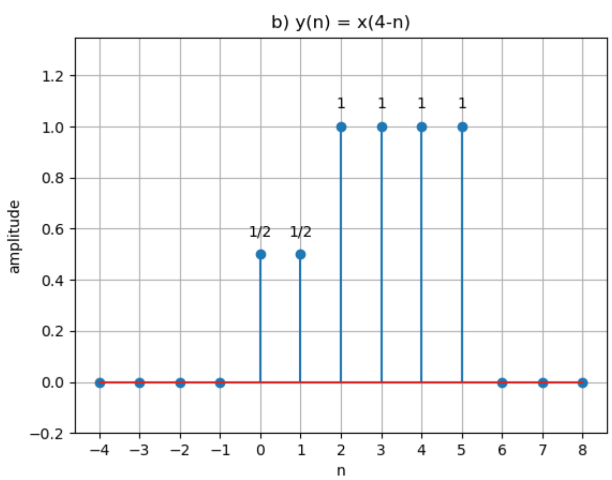


Figure 3: Graph of $x(4-n)$

For $x(4-n)$, our indices are $\{2,3,4,5\}$ when our amplitude is 1 and our indices are $\{0,1\}$ when our amplitude is $\frac{1}{2}$.

1.3 Part C

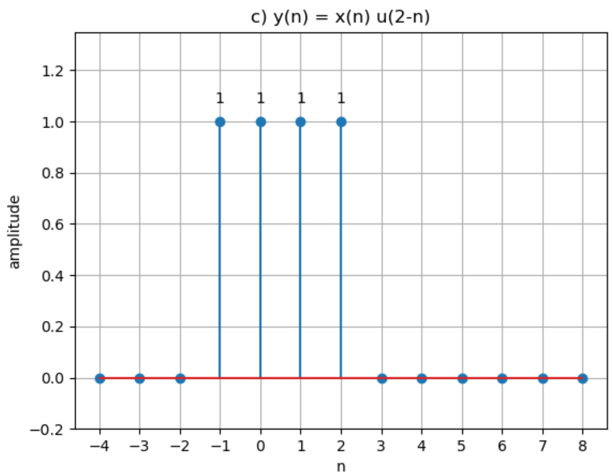


Figure 4: Graph of $x(n)u(2-n)$

For $x(n)u(2-n)$, our indices are $\{-1,0,1,2\}$ when our amplitude is 1 and our indices are $\emptyset$ when our amplitude is $\frac{1}{2}$.

1.4 Part D

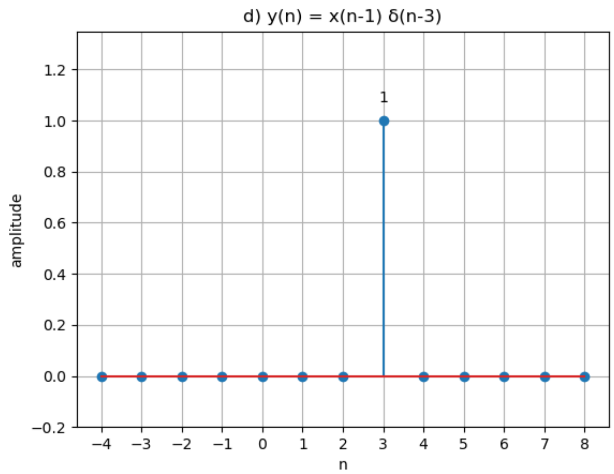


Figure 5: Graph of $x(n-1)\delta(n-3)$

For $x(n-1)\delta(n-3)$, our indices are $\{3\}$ when our amplitude is 1.

1.5 Part E

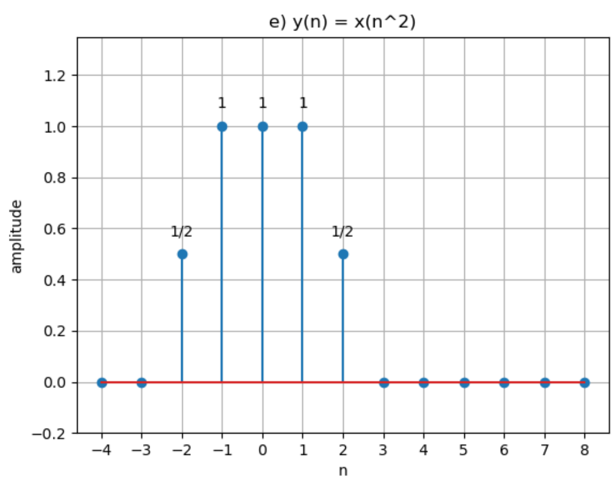


Figure 6: Graph of $x(n^2)$

For $x(n^2)$, our indices are $\{-1, 0, 1\}$ when our amplitude is 1 and our indices are $\{-2, 2\}$ when our amplitude is $\frac{1}{2}$.

2 Question 2

Example 2.1. Answer the following for question 2:

1. Find the energy of the signal (shown left) and the power of the signal (shown right).
2. Determine an expression for the energy E of a converging discrete-time exponential signal $x(n) = a^n u(n)$, i.e., $|a| < 1$. Determine the power P if $|a| = 1$. For what range of $|a|$, the signal would neither be an energy nor a power signal.

2.1 Part I

Example 2.2. Find the energy of the signal (shown left) and the power of the signal (shown right).

For the left signal, we have the values 1,2,3,2,1 (at $n = 1, 2, 3, 4, 5$, zeroes elsewhere):

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = 1^2 + 2^2 + 3^2 + 2^2 + 1^2 = 1 + 4 + 9 + 4 + 1 = 19$$

Thus, we get $E = 19$. And for the right signal, it is periodic with period $N = 12$, so one period is $n = 0 \ldots 11$. So,

- $x(1) = 1$, $x(2) = 2$, x(3) = 3
- $x(9) = -3$, x(10) = -2, x(11) = -1
- Others in the period are 0.

Hence, the average power for a periodic signal is:

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{12}(1^2 + 2^2 + 3^2 + (-3)^2 + (-2)^2 + (-1)^2) = \frac{1}{12}(14 + 14) = \frac{7}{3}$$

Therefore,

$$\boxed{P = \frac{7}{3}, E = 19}$$

2.2 Part II

Example 2.3. Determine an expression for the energy E of a converging discrete-time exponential signal $x(n) = a^n u(n)$, i.e., $|a| < 1$. Determine the power P if $|a| = 1$. For what range of $|a|$, the signal would neither be an energy nor a power signal.

When energy is $|a| < 1$:

$$E = \sum_{n=0}^{\infty} |a|^{2n} = \frac{1}{1 - |a|^2}, |a| < 1$$

Then,

$$\boxed{E = \frac{1}{1 - |a|^2} \text{ for } |a| < 1}$$

When $a = 1$ the power is:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |u(n)|^2 = \lim_{N \to \infty} \frac{N+1}{2N+1} = \frac{1}{2}$$

Thus,

$$\boxed{P = \frac{1}{2}, \text{ when a} = 1}$$

Now, for when $|a| > 1$, then there is neither energy nor power as energy diverges and power is infinite.

3 Question 3

Example 3.1. Determine the period N of the discrete sinusoids $\cos(\frac{\pi}{4}n)$, $\cos(\frac{4\pi}{17}n)$, $\cos(0.8n)$. Also state how many sinusoidal envelops k are needed to constitute exactly one single period in each case.

3.1 Part A

For $\cos(\frac{\pi}{4}n)$,

$$\frac{\pi}{4}N = 2\pi k \Rightarrow N = 8k \Rightarrow N_{min} = 8, k = 1$$

Therefore, there is one sinusoidal envelopes needed to constitute exactly one single period in this case.

$$\boxed{N = 8, k = 1}$$

3.2 Part B

For $\cos(\frac{4\pi}{17}n)$,

$$\frac{4\pi}{17}N = 2\pi k \Rightarrow N = \frac{17}{2}k$$

Our smallest integer N happens when $k = 2 \Rightarrow N = 17$. Therefore, there are 2 sinusoidal envelopes needed to constitute exactly one single period in this case.

$$\boxed{N = 17, k = 2}$$

3.3 Part C

For $\cos(0.8n)$, we see that $\frac{0.8}{2\pi}$ is not a rational number which implies that it is not periodic. Therefore it is aperiodic (aka no finite N), thus k is not defined.

4 Question 4

Example 4.1. Answer the following for question 4:

1. Draw a block diagram of the discrete-time system given by the following difference equation

$$y(n) = -\frac{1}{4}y(n-1) - \frac{1}{16}y(n-2) + x(n)$$

2. Find the difference equation corresponding to the following block diagram

4.1 Part A

See the paper I attached to my solutions set for number 4A.

4.2 Part B

From the given diagram we arrive at the following conclusions. Let $v(n)$ be the output of the left summer and we have the feedback $0.9v(n-1)$ into the left summer, thus:

$$v(n) = x(n) + 0.9v(n-1)$$

And the output is:

$$y(n) = v(n) + 2v(n-1) + 3v(n-2).$$

By eliminating v by using the z-transform, we obtain:

$$y(n) - 0.9y(n-1) = x(n) + 2x(n-1) + 3x(n-2)$$

or equivalently,

$$\boxed{y(n) = 0.9y(n-1) + x(n) + 2x(n-1) + 3x(n-2)}$$

5 Question 5

Example 5.1. Determine the discrete-time system types (1) static or dynamic (2) linear or non-linear (3) time invariant or time-varying (4) causal or noncausal (5) stable or unstable for each of the following discrete-time systems

1. $y(n) = \cos(x(n))$
2. $y(n) = \sum_{k=-\infty}^{n+1} x(k)$
3. $y(n) = x(n)\cos(\omega_0 n)$

4. $y(n) = x(-n + 2)$

Here are my answers to this question:

- For $y(n) = \cos(x(n))$, it is **static**(memoryless), **nonlinear**, **time-invariant**, **causal**, and is **stable** (output is always in $[-1, 1]$ for bounded input).
- For $y(n) = \sum_{k=-\infty}^{n+1} x(k)$, it is **dynamic** (uses many samples), **linear**, **time-invariant**, **non-causal** (depends on $x(n + 1)$), and is **unstable** (bounded input such as $x(k) = u(k)$ gives unbounded growth).
- For $y(n) = x(n)\cos(\omega_0 n)$, it is **static**, **linear**, **time-varying** (multiplier depends on n), **causal**, and is **stable** ($|y(n)| \leq |x(n)|$).
- For $y(n) = x(-n + 2)$, it is it is **dynamic**, **linear**, **time-varying** (time reversal breaks TI), **noncausal** (for some n, needs future input values), and is **stable** (bounded input stays bounded).