ECE 6200 Lecture Oct 28th

Blaine Swieder

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Definition 0.1. *The zero-input response of (5.4) or the equation* $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ *is* ***marginally stable*** *or stable in the sense of Lyapunov, if every finite initial state of* $\mathbf{x}_0$ *excites a bounded response. It is* ***asymptotically stable*** *if every finite initial state excites a bounded response, which, in addition, approaches* $\mathbf{0}$ *and* $t \to \infty$.

Theorem 0.1 (Theorems for Internal Stability)**.** *The following are Theorems for Internal Stability:*

1. *The equation* $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ *is marginally stable if and only if all eigenvalues of* $\mathbf{A}$ *have zero or negative real parts and those with zero real parts are simple roots of the minimal polynomial oof* $\mathbf{A}$.

2. *The equation* $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ *is asymptotically stable if and only if all eigenvalues of* $\mathbf{A}$ *have negative real parts.*

Remark. Every pole of the transfer matrix

$$\hat{\mathbf{G}}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

is an eigenvalue of $\mathbf{A}$.

Remark. For LTI Systems, Internal Stability implies BIBO Stability, but the converse is not true.

Remark. Notice that asymptotic stability is defined for the zero-input response, while BIBO stability is defined for zero-state response.

Marginal Stability is only used in the design of oscillators; however, for every other physical system is designed to be asymptotically stable / BIBO stable with some other considerations mentioned in later chapters.

0.1 Discrete-Time Case

Theorem 0.2. *These apply in the discrete-time case:*

1. *The equation* $\mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k]$ *is marginally stable if and only if all eigenvalues of* $\mathbf{A}$ *have magnitudes less than or equal to 1 and those equal to 1 are simple roots of the minimal polynomial of* $\mathbf{A}$.

2. *The equation* $\mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k]$ *is asymptotically stable if and only if eigenvalues of* $\mathbf{A}$ *have magnitudes less than 1.*