ECE 6200 HWK 2 Revision

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21 October 2025

1 Introduction

This file contains the revision for problem 3 of homework 2 for the ECE 6200 course.

2 Question 3

Example 2.1. Continue building up on the Satellite Azimuth Control case study in Lecture 3. Obtain a combined transfer function from the power amplifier input $V_p(s)$ to the antenna azimuth output $\theta_o(s)$. Also obtain a combined state-space model (state and output equations) with $u(t) = v_p(t)$ as the input and $y(t) = \theta_o(t)$ as the output. You can use the existing choice of state variables as used for the subsystems in the lecture. Reproduce the MATLAB/Simulink simulations carried out in the class. Attach your MATLAB code/Simulink diagram.

2.0.1 Combine the Subsystems

From $V_p(s)$ or the power amplifier input, through motor and load to $\theta_o(s)$, we get:

$$G(s) = G_{PA}(s) \cdot G_{ML}(s)$$

Next, we will substitute:

$$G(s) = \frac{K_A}{s + a_A} \cdot \frac{K_m}{s(s + a_m)}$$

Then, let's combine our constants to get:

$$G(s) = \frac{K}{s(s + a_A)(s + a_m)}, \qquad \text{where} K = K_A K_m$$

We can observe from this transfer function that it is a *third-order transfer function* with one integrator and two real poles. Now, our combined transfer function is defined as:

$$\frac{\theta_o(s)}{V_p(s)} = \frac{K_A K_m}{s(s + a_A)(s + a_m)}$$

Now, if we include the preamplifier or K_p and the feedback potentiometer K_{pot} later in the closed-loop analysis, they will multiply in series:

$$G_{open}(s) = K_p K_{pot} \frac{K_A K_m}{s(s + a_A)(s + a_m)}$$

We do not need this, so we will stop before K_p.

2.0.2 Choose the State Variables

So we will use our familiar notation used in Lecture 3. Let $x_1 = \theta_o$ be our antenna azimuth angle, $x_2 = \dot{\theta}_o$ be our angular velocity, and x_3 be our power-amp internal state (armature current / internal voltage). We will derive a cascade realization.

From our transfer function:

$$G(s) = \frac{K}{s(s + a_A)(s + a_m)}$$

we will then multiply both sides by our denominator and obtain:

$$s(s + a_A)(s + a_m)\Theta_o(s) = KV_p(s)$$

By expanding, we obtain:

$$(s^3 + (a_m + a_A)s^2 + a_m a_A s)\Theta_o(s) = KV_p(s)$$

Finally, we apply the inverse Laplace Transform (time-domain):

$$\theta_o^{(3)} + (a_m + a_A)\ddot{\theta}_o + a_m a_A \dot{\theta}_o = Kv_p(t)$$

Therefore, we have now obtained our state variables.

2.0.3 Obtain State Equations

Let

$$x_1 = \theta_o, \quad x_2 = \dot{\theta}_o, \quad x_3 = \ddot{\theta}_o,$$

Then, it will follow that:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = -(a_m + a_A)x_3 - a_m a_A x_2 + Ku \end{cases}$$

with $u(t) = v_p(t)$, $y = x_1$. Next, we need to obtain our state-space matrices for this problem:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a_m a_A & -(a_m + a_A) \end{bmatrix} \mathbf{Ax} + \begin{bmatrix} 0 \\ 0 \\ K \end{bmatrix} \mathbf{Bu}, \qquad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{Cx}, \quad \mathbf{D} = 0$$

Then, it must be the case that:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a_m a_A & -(a_m + a_A) \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ K \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

Therefore, we have now effectively obtained our state-space model.

2.0.4 Example Numerical Substitution

We will implement this section into MATLAB.

```
% Problem 3: Antenna Azimuth Contol -  MATLAB Implementation

% Parameters

K = 100; % Preamplifier Gain
Kpot = 1/pi; % Potentiometer Gain

%State-Space Model

A = [0 1 0; 0 -1.71 2.083;0 0 -100];
B = [0; 0; 100];
C = [0.1 0 0];
D = 0;

sys_dt= ss(A,B,C,D,1)
sys_dt
```

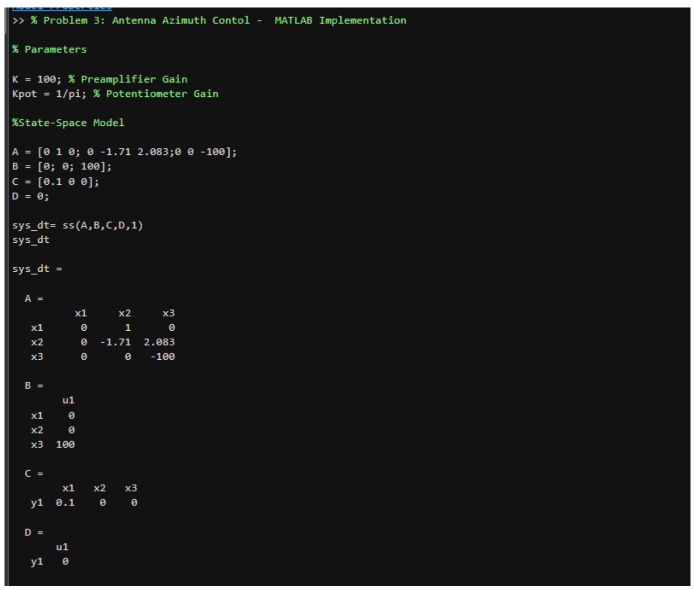


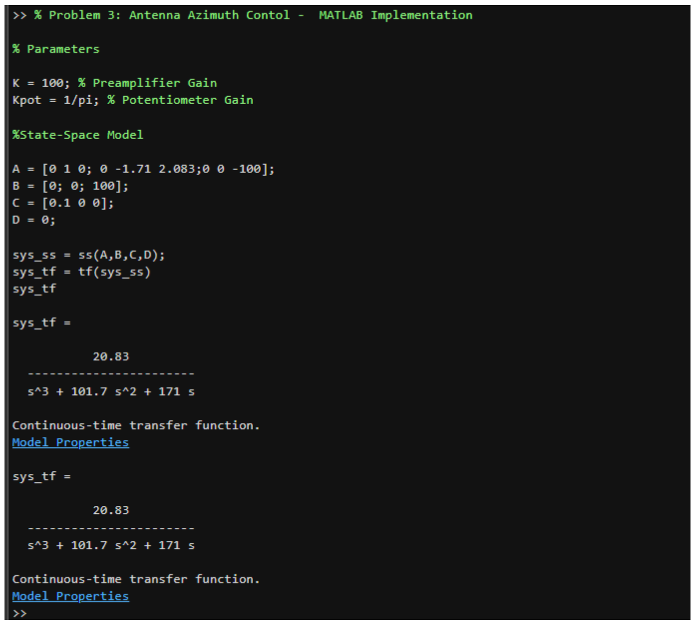
Figure 1: State Space Matrices Output

Below is with the transfer function:

```
% Problem 3: Antenna Azimuth Contol -  MATLAB Implementation

% Parameters

K = 100; % Preamplifier Gain
Kpot = 1/pi; % Potentiometer Gain

%State-Space Model
```

```
A = [0 1 0; 0 -1.71 2.083;0 0 -100];
B = [0; 0; 100];
C = [0.1 0 0];
D = 0;

sys_ss = ss(A,B,C,D);
sys_tf = tf(sys_ss)
sys_tf =
```

Now, this is the output of the above code:



```
>> % Problem 3: Antenna Azimuth Contol -  MATLAB Implementation

% Parameters

K = 100; % Preamplifier Gain
Kpot = 1/pi; % Potentiometer Gain

%State-Space Model

A = [0 1 0; 0 -1.71 2.083;0 0 -100];
B = [0; 0; 100];
C = [0.1 0 0];
D = 0;

sys_ss = ss(A,B,C,D);
sys_tf = tf(sys_ss)
sys_tf

sys_tf =

           20.83
  ----------------------
  s^3 + 101.7 s^2 + 171 s

Continuous-time transfer function.
Model Properties

sys_tf =

           20.83
  ----------------------
  s^3 + 101.7 s^2 + 171 s

Continuous-time transfer function.
Model Properties
>>
```

Figure 2: Transfer Function Output

$$\boxed{G(s) = \frac{20.83}{s^3 + 101.7s^2 + 171s}}$$

Therefore, this proves that the combined state-space model and transfer function are the same, which is what the lecture shows.

2.1 Graphing in MATLAB

```
%% Problem 3: Antenna Azimuth Control - MATLAB Implementation

% Parameters
Kp   = 100;        % Preamplifier gain
Kpot = 1/pi;       % Potentiometer (feedback) gain

```

```
% State-Space Model (from lecture constants)
A = [0   1      0; 0 -1.71   2.083; 0   0    -100];
B = [0; 0; 100];
C = [0.1 0 0];
D = 0;

% Create State-Space System (open-loop plant)
sys_ss = ss(A,B,C,D);

% Verify Transfer Function equivalence
sys_tf = tf(sys_ss);
disp('Combined Transfer Function G(s) = theta_o(s)/V_p(s):');
sys_tf

% Close the loop with Kp (preamplifier) and Kpot (sensor)
sys_cl = feedback(Kp*sys_ss, Kpot);   % Negative feedback

% Impulse and Step Responses
figure;
impulse(sys_cl);
grid on;
title('Closed-Loop Impulse Response');

figure;
step(sys_cl);
grid on;
title('Closed-Loop Step Response');
```

Let us now see how the closed-loop step and impulse responses look graphed:

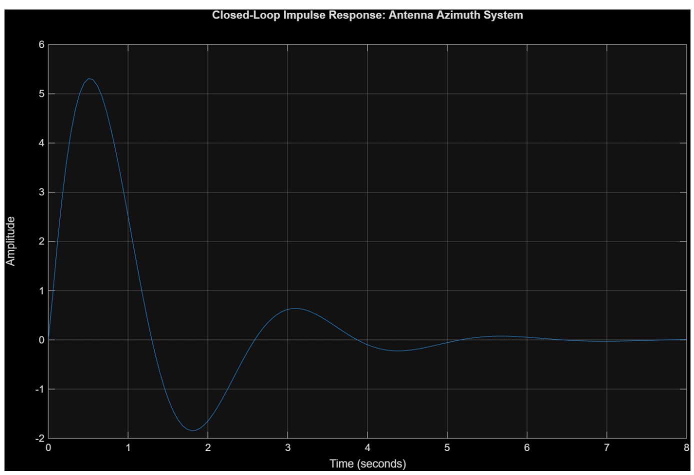


Figure 3: Closed-Loop Impulse Response

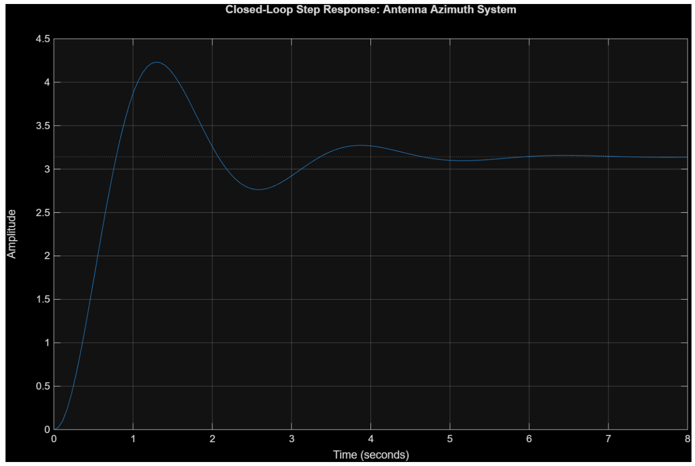


Figure 4: Closed-Loop Step Response