Solutions for Multiagent Pursuit-Evasion Games on Communication Graphs: Finite-Time Capture and Asymptotic Behaviors

Victor G. Lopez, *Student Member, IEEE*, Frank L. Lewis, *Fellow, IEEE*, Yan Wan, *Senior Member, IEEE*, Edgar N. Sanchez, *Senior Member, IEEE*, and Lingling Fan

***Abstract*—In this paper, the multiagent pursuit-evasion (MPE) games are solved in order to obtain optimal strategic policies for all players. In these games, multiple pursuers attempt to intercept multiple evaders who try to avoid capture. A graph-theoretic approach is employed to study the interactions of the agents with limited sensing capabilities, such that distributed control policies are obtained for every agent. Furthermore, the minimization of performance indices associated with the goals of the agents is guaranteed. Nash equilibrium among the players is obtained by means of optimal policies that use the solutions of the Hamilton–Jacobi–Isaacs (HJI) equations of the game. Minmax strategies are also proposed to guarantee a security-level performance when the solutions of the HJI equations for Nash equilibrium do not exist. Scenarios for finite-time capture and for asymptotic rendezvous are analyzed, and emergent behaviors are obtained by means of modifications of the proposed general-case performance indices. The containment control results are shown to be special cases of the solutions of the MPE games.**

***Index Terms*—Differential games, graph theory, multiagent systems, pursuit-evasion games.**

I. Introduction

PURSUIT-EVASION games are one of the most interesting and widely studied interactions in multiagent systems. Their applications include aircraft control and missile guidance in military implementations, as well as civilian applications such as collision avoidance designs in intelligent transportation systems, wireless sensor networks, and sport/game strategies. Animal behavior in hunting scenarios can also be studied using differential game analysis. Thus, the agents in pursuit-evasion games can be autonomous mobile robots, unmanned air vehicles, spacecraft, wireless sensors, or living organisms.

The single-pursuer single-evader game has been studied in detail since R. Isaacs' development of strategic policies for both players [1]. The single-pursuer single-evader case is a zero-sum game that can be solved using an extension of the well-known Bellman equations, known as the Hamilton–Jacobi–Isaacs (HJI) equations [2]. A closed-form solution of the game was obtained by Bryson and Ho [3]. Extensions of these results have been obtained for the case of two pursuers versus one evader [4], and for the multiple-pursuer single-evader case [5]–[9]. In [7], a pursuit-evasion game between aircraft is solved by differentiation of a particular value function. The discrete-time multiple-pursuer single-evader game is solved in [8]. In [9], a thorough survey about the studies on multiple-pursuer single-evader games is presented.

In recent years, modern interacting multiagent systems have motivated the study of general multiple-pursuer multiple-evader games. In [10], the pursuit-evasion game with two multiagent teams was analyzed with a distributed hybrid approach using time potential fields. In [11], conditions to guarantee capture or evasion are determined by defining level-set functions as objectives for the players. Suboptimal strategies are studied in [12] by decomposing the game into multiple two-player games. The possibility of the players being connected in a communication graph is considered in [13], where the pursuit-evasion game is formulated as a connectivity control problem.

In the pursuit-evasion game, performance functions are used to represent the interest of the pursuers in reaching the evaders' position and the interest of the evaders in avoiding capture. These individual performance indices are then used to define Nash equilibrium and other solution concepts for the game [2], [3], [14].

The mathematical techniques to analyze the behavior of multiagent systems with limited sensing capabilities have been developed in the field of graph theory. The most studied interaction for multiagent system control using graph theory is consensus [15]; some results in this paper can be seen as a generalization of the consensus-seeking protocol. Multiagent system controllers

Manuscript received January 30, 2019; accepted June 21, 2019. Date of publication July 3, 2019; date of current version April 23, 2020. This work was supported in part by the Office of Naval Research under Grant N00014-17-1-2239 and Grant N00014-18-1-2221, and in part by the U.S. National Science Foundation under Grant CNS-1714519, Grant CNS-1730675, and Grant ECCS-1839804. Recommended by Associate Editor M. K. Camlibel. *(Corresponding author: Victor G. Lopez.)*

V. G. Lopez and Y. Wan are with the Electrical Engineering Department, University of Texas at Arlington, Arlington, TX 76010 USA (e-mail: victor.lopezmejia@mavs.uta.edu; yan.wan@uta.edu).

F. L. Lewis is with the UTA Research Institute, University of Texas at Arlington, Fort Worth, TX 76118 USA (e-mail: lewis@uta.edu).

E. N. Sanchez is with the CINVESTAV Unidad Guadalajara, Guadalajara, Jalisco 45091, Mexico (e-mail: sanchez@gdl.cinvestav.mx).

L. Fan is with the School of Electronic and Information Engineering, Beijing Jiaotong University, Beijing 10044, China (e-mail: fanlingling_bjtu@hotmail.com).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TAC.2019.2926554

rely on the implementation of distributed algorithms on which each agent computes his feedback controller using only the state information it can perceive from his neighbors [16]. Stability analysis of multiagent systems in graphs has been studied in [17] and [18].

The general multiagent pursuit-evasion (MPE) games present some interesting challenges in designing coordinated strategies among all the pursuers to accomplish the goal of capturing the evaders. This requires the development of strategies that combine the tools in both differential game theory and cooperative control theory to solve the distributed pursuit-evasion game. Cooperation among agents has been well studied [19], [20], and their extensions when considering a well-defined minmax performance index were recently developed using a differential game approach [21].

In this paper, distributed control strategies are designed to solve the MPE games on graphs. The contributions of this paper are summarized as follows:

1) Three communication graphs are introduced to represent, respectively, the interactions among the pursuers, interactions among the evaders, and interactions between pursuers and evaders. This formulation allows us to analyze different behaviors between the agents by varying their objectives with respect to each graph. Unlike other papers in the literature [13], we allow the graphs to have directed links.
2) For the first time, different game-theoretic solution concepts are analyzed for MPE games. First, conditions for Nash equilibrium with respect to individual performance indices are stated. Then, a minimum guaranteed performance is proposed for the first time for MPE games with dynamical systems, using minmax strategies. Minmax strategies provide robust control policies that, furthermore, reduce the complexity of the HJI equations to be solved. Most of the results in the literature find solutions to the MPE games without determining their nature as Nash equilibria, minmax, or any other solution concept [7]–[13].
3) A novel analysis of emergent behaviors, obtained by making particular modifications to the cost functions of the agents, is presented. Finite-time interception and asymptotic behaviors are studied as variations of the individual goals of an agent with respect to his teammates and his opponents.
4) The containment control problem with static and moving leaders [22], [23] is solved as a special case of the MPE games.

This paper is organized as follows. The different graphs and variables that are used throughout the paper are defined in Section II. MPE games on directed graphs are formulated in Section III, where Nash equilibrium and minmax strategies are studied as possible solutions of the games. In Section IV, an algorithm for target selection is proposed, and conditions for finite-time capture are stated. Asymptotic behaviors are analyzed in Section V. Finally, the proposed control policies are tested via simulation studies in Section VI.

II. Preliminaries

This section presents the basic notions and definitions required to analyze the MPE game on graphs.

A. Three Graphs for MPE Game Interactions

The distributed MPE game involves players with limited sensing range; that is, each agent is able to measure only the position of his closest neighbors. In this section, three different graphs are proposed to analyze the MPE game problem.

Define the graph $G_p = (V_p, E_p)$ with the N pursuers as the set of nodes $V_p = \{v_{p1}, \dots, v_{pN}\}$ and a set of edges $E_p \subseteq V_p \times V_p$. Let a_{ik} be the connectivity weights of graph G_p, with $a_{ik} = 1$ if $(v_{pk}, v_{pi}) \in E_p$ and $a_{ik} = 0$ otherwise. Define the in-degree of pursuer i as $d_i^p = \sum_{k=1}^N a_{ik}$ and the in-degree matrix of the graph $D_p = \mathrm{diag}\{d_i^p\}$. The weighted adjacency matrix is $\mathcal{A}_p = [a_{ik}]$. Finally, define the graph Laplacian matrix $L_p = D_p - \mathcal{A}_p$.

Similarly, the graph $G_e = (V_e, E_e)$ represents the interactions among the M evaders as the nodes $V_e = \{v_{e1}, \dots, v_{eM}\}$. The graph weights are b_{jl}, with $b_{jl} = 1$ if $(v_{el}, v_{ej}) \in E_e$, and $b_{jl} = 0$ otherwise. The in-degree of evader j is $d_j^e = \sum_{l=1}^M b_{jl}$. Define the matrices $D_e = \mathrm{diag}\{d_j^e\}$, $\mathcal{A}_e = [b_{jl}]$, and $L_e = D_e - \mathcal{A}_e$.

Finally, define the bipartite graph $G_{\mathrm{pe}} = (V_{\mathrm{pe}}, E_{\mathrm{pe}})$ that includes every player in the game, with the set of N pursuers as one partition and the set of M evaders as the other. Graph G_{pe} captures the information exchange among the agents in different partitions. Specifically, edge weights c_{ij} represent the knowledge of pursuer i about the position of evader j, with $c_{ij} = 1$ if pursuer i can measure evader j and $c_{ij} = 0$ otherwise. Similarly, e_{ji} stands for the knowledge of evader j about the position of pursuer i. The in-degree of pursuer i in graph G_{pe} is defined as $d_i^{\mathrm{pe}} = \sum_{j=1}^M c_{ij}$, and the in-degree of evader j is $d_j^{\mathrm{ep}} = \sum_{i=1}^N e_{ji}$. Define the pursuer-partition matrices $D_{\mathrm{pe}} = \mathrm{diag}\{d_i^{\mathrm{pe}}\}$ and $\mathcal{A}_{\mathrm{pe}} = [c_{ij}]$, and the evader-partition matrices $D_{\mathrm{ep}} = \mathrm{diag}\{d_j^{\mathrm{ep}}\}$ and $\mathcal{A}_{\mathrm{ep}} = [e_{ji}]$.

B. Local Errors and Dynamics

Consider a team of N dynamical agents, regarded as the pursuers, each with linear dynamics

$$\dot{x}_i = Ax_i + Bu_i, \qquad i = 1, \dots, N \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the position of pursuer i in the n-dimensional space of the game. Consider also a set of M agents as the evader team, with dynamics

$$\dot{y}_j = Ay_j + Bv_j, \qquad j = 1, \dots, M \tag{2}$$

where $y_j \in \mathbb{R}^n$ is the position of the jth evader.

The relevant distances between the agents are defined as follows. Let the local error variable δ_i be defined as the position difference of pursuer i with respect to his neighbors

$$\delta_i = \sum_{k=1}^N a_{ik}(x_i - x_k) + \sum_{j=1}^M c_{ij}(x_i - y_j). \tag{3}$$

The error dynamics can be found from (3), using the system dynamics (1) and (2), as

$$\dot{\delta}_i = A\delta_i + (d_i^p + d_i^{\mathrm{pe}})Bu_i - \sum_{k=1}^{N} a_{ik}Bu_k - \sum_{j=1}^{M} c_{ij}Bv_j. \quad (4)$$

Similarly, we define the local position error for evader *j* as

$$\varepsilon_j = \kappa \sum_{l=1}^{M} b_{jl}(y_j - y_l) - \sum_{i=1}^{N} e_{ji}(y_j - x_i) \quad (5)$$

where $\kappa > 0$ is a scalar gain, and the local error dynamics are

$$\dot{\varepsilon}_j = A\varepsilon_j + (\kappa d_j^e - d_j^{\mathrm{ep}})Bv_j - \kappa \sum_{l=1}^{M} b_{jl}Bv_l + \sum_{i=1}^{N} e_{ji}Bu_i. \quad (6)$$

We here define the center of gravity to facilitate the analysis developed in this paper. In particular, the center of gravity for pursuer *i*, $\bar{x}_{-i}$, is defined as the weighted positions of the neighbors of pursuer *i*

$$\bar{x}_{-i} = \frac{1}{d_i^p + d_i^{\mathrm{pe}}}\left(\sum_{k=1}^{N} a_{ik}x_k + \sum_{j=1}^{M} c_{ij}y_j\right). \quad (7)$$

Then, we can determine the center of gravity dynamics as

$$\begin{aligned}\dot{\bar{x}}_{-i} &= \frac{1}{d_i^p + d_i^{\mathrm{pe}}}\left[\sum_{k=1}^{N} a_{ik}(Ax_k + Bu_k) + \sum_{j=1}^{M} c_{ij}(Ay_j + Bv_j)\right] \\ &= A\bar{x}_{-i} + B\bar{u}_{-i}\end{aligned}$$

where

$$\bar{u}_{-i} = \frac{1}{d_i^p + d_i^{\mathrm{pe}}}\left(\sum_{k=1}^{N} a_{ik}u_k + \sum_{j=1}^{M} c_{ij}v_j\right). \quad (8)$$

We can now represent the local position error of pursuer *i* (3), and its dynamics (4), in terms of the center of gravity as follows:

$$\delta_i = (d_i^p + d_i^{\mathrm{pe}})(x_i - \bar{x}_{-i}) \quad (9)$$

$$\dot{\delta}_i = A\delta_i + (d_i^p + d_i^{\mathrm{pe}})Bu_i - (d_i^p + d_i^{\mathrm{pe}})B\bar{u}_{-i}. \quad (10)$$

Similarly, the pseudocenter of gravity for evader *j*, $\bar{y}_{-j}$, is defined based on the neighbors of evader *j* as

$$\bar{y}_{-j} = \frac{1}{kd_j^e - d_j^{\mathrm{ep}}}\left(\kappa \sum_{l=1}^{M} b_{jl}y_l - \sum_{i=1}^{N} e_{ji}x_i\right) \quad (11)$$

with dynamics

$$\begin{aligned}\dot{\bar{y}}_{-j} &= \frac{1}{\kappa d_j^e + d_j^{\mathrm{ep}}}\left[\sum_{l=1}^{M} b_{jl}(Ay_l + Bv_l) - \sum_{i=1}^{N} e_{ji}(Ax_i + Bu_i)\right] \\ &= A\bar{y}_{-j} + B\bar{v}_{-j}\end{aligned}$$

where

$$\bar{v}_{-j} = \frac{1}{\kappa d_j^e - d_j^{\mathrm{ep}}}\left(\sum_{l=1}^{M} b_{jl}v_l - \sum_{i=1}^{N} e_{ji}u_i\right). \quad (12)$$

Using this definition, the local position error of evader *j* (5) and its dynamics (6) can be written as

$$\varepsilon_j = (\kappa d_j^e - d_j^{\mathrm{ep}})(y_j - \bar{y}_{-j}) \quad (13)$$

$$\dot{\varepsilon}_j = A\varepsilon_j + (\kappa d_j^e + d_j^{\mathrm{ep}})Bv_j - (\kappa d_j^e + d_j^{\mathrm{ep}})B\bar{v}_{-j}. \quad (14)$$

III. Formulation and Solutions for MPE Games

This section presents the formal definition of MPE games. Conditions for asymptotic capture are analyzed. New definitions for Nash equilibrium and minmax strategies as main solution concepts for the MPE games are also introduced.

A. Definitions for MPE Games on Communication Graphs

Consider the following formulation for MPE games. Let the evaders have the objective of maximizing their distances from their neighboring pursuers. Moreover, the evaders also desire to maintain their group cohesion, that is, to stay close to their teammates. The justification for this objective is that, in many practical applications, the agents of a team may want to perform their tasks without losing contact with each other. Similarly, the pursuers desire to collectively intercept as many evaders as possible while remaining close to their teammates. In Section IV, we explore a different scenario where the agents do not have the group cohesion objective.

The goals of each pursuer can be represented by means of a scalar function $J_{pi}(\delta_i, u_i)$, regarded as the performance index of the game for pursuer *i*. The performance index of pursuer *i* can now be defined as

$$J_{pi} = \int_0^\infty \left[\delta_i^T Q_{pi}(\delta_i)\delta_i + u_i^T R_p(\delta_i)u_i\right] dt \quad (15)$$

where $Q_{pi}(\delta_i) = Q_{pi}^T(\delta_i) > 0$ and $R_p(\delta_i) = R_p^T(\delta_i) > 0$.

Pursuer *i* is thus concerned with the minimization of J_{pi}. From (9), the dependence of J_{pi} on the local position errors δ_i can be seen as the goal of pursuer *i* to minimize its distance with respect to the center of gravity of all his neighbors.

On the other side, evader *j* desires to minimize his cost represented by the performance index $J_{ej}(\varepsilon_j, v_j)$. Considering his goals of fleeing from the pursuers while remaining close to the evaders in his neighborhood, the performance index for evader *j* can be defined as

$$J_{ej} = \int_0^\infty \left[\varepsilon_j^T Q_{ej}(\varepsilon_j)\varepsilon_j + v_j^T R_e(\varepsilon_j)v_j\right] dt \quad (16)$$

where $Q_{ej}(\varepsilon_j) = Q_{ej}^T(\varepsilon_j) > 0$, $R_e(\varepsilon_j) = R_e^T(\varepsilon_j) > 0$, and ε_j is expressed in (13). Notice that the pseudocenter of gravity (11), used in (13), considers the opposite sign of the relative position of the pursuers such that minimization of the errors ε_j implies escaping from the pursuers, as desired.

The scalar κ in (5) can now be seen as the priority of the evaders to stay close to each other, against their drive to escape from the pursuers. The value of κ can be selected according to the objectives of the game.

Notice that the performance indices (15) and (16) as defined in this section formulate the game as an infinite-horizon problem. In Section IV, we present a modification of these functions to obtain finite-time capture.

Using these definitions, the MPE differential games on communication graphs are defined as follows.

Definition 1. (MPE game): Define the MPE game as

$$\begin{aligned} V_{pi}(\delta_i) &= \min_{u_i} J_{pi}(\delta_i, u_i) \\ V_{ej}(\varepsilon_j) &= \min_{v_j} J_{ej}(\varepsilon_j, v_j) \end{aligned} \tag{17}$$

where V_{pi} and V_{ej} are the values of the game for pursuer *i* and for evader *j*, respectively.

Pursuers and evaders must now determine the control policies u_i and v_j, respectively, that solve the MPE game (17).

To state the following definition, notice that the performance index of an agent depends not only on his own behavior, but also on the behavior of his neighbors. Let u_{-i} be the set of control policies of the pursuer neighbors of pursuer *i*, and v_{-i} represent the policies of all the evader neighbors of *i*. Thus, although we have defined functions $J_{pi}(\delta_i, u_i)$ and $J_{ej}(\varepsilon_j, v_j)$ in terms of the local variables, we can represent explicitly their dependence on neighbor policies as $J_{pi}(\delta_i, u_i) = J_{pi}(\delta_i, u_i, u_{-i}, v_{-i})$ for the pursuers, and $J_{ej}(\varepsilon_j, v_j) = J_{ej}(\varepsilon_j, v_j, u_{-j}, v_{-j})$ for the evaders. Using these expressions, Nash equilibrium is defined as follows.

Definition 2. (Nash equilibrium): Control policies u_i, $i = 1, \ldots, N$, and v_j, $j = 1, \ldots, M$, form a Nash equilibrium if the inequalities

$$\begin{aligned} J_{pi}\left(\delta_i, u_i^*, u_{-i}^*, v_{-i}^*\right) &\le J_{pi}\left(\delta_i, u_i, u_{-i}^*, v_{-i}^*\right) \\ J_{ej}\left(\varepsilon_j, v_j^*, u_{-j}^*, v_{-j}^*\right) &\le J_{ej}\left(\varepsilon_j, v_j, u_{-j}^*, v_{-j}^*\right) \end{aligned}$$

hold for all agents in the game.

In Nash equilibrium, every agent uses his optimal policy against the optimal policy of his neighbors. In the following section, Nash equilibrium is studied as the most important solution concept for the MPE games (17).

B. Nash Equilibrium for MPE Games

Given neighbor policies u_k and v_j, the optimal control policy of pursuer *i* can be obtained by means of the *i*th Hamiltonian function defined as

$$\begin{aligned} H_{pi} &= \delta_i^T Q_{pi}\delta_i + u_i^T R_p u_i + \dot{V}_{pi}(\delta_i) = \delta_i^T Q_{pi}\delta_i + u_i^T R_p u_i \\ &\quad + \nabla V_{pi}^T(\delta_i)\Bigg(A\delta_i + (d_i^p + d_i^{\mathrm{pe}})Bu_i \\ &\qquad - \sum_{k=1}^N a_{ik}Bu_k - \sum_{j=1}^M c_{ij}Bv_j\Bigg) \end{aligned}$$

where $V_{pi}(\delta_i)$ is the value function of the game for pursuer *i*, and the dynamics (4) has been used. The optimal policy of pursuer *i* minimizes H_{pi}, and is found to be

$$u_i^* = -\frac{1}{2}(d_i^p + d_i^{\mathrm{pe}})R_p^{-1}(\delta_i)B^T\nabla V_{pi}(\delta_i). \tag{18}$$

Using the same procedure, define the Hamiltonian function for evader *j* as

$$\begin{aligned} H_{ej} &= \varepsilon_j^T Q_{ej}\varepsilon_j + v_j^T R_e v_j + \dot{V}_{ej}(\varepsilon_j) = \varepsilon_j^T Q_{ej}\varepsilon_j + v_j^T R_e v_j \\ &\quad + \nabla V_{ej}^T(\varepsilon_j)\Bigg(A\varepsilon_j + (\kappa d_j^e - d_j^{\mathrm{ep}})Bv_j \\ &\qquad - \kappa\sum_{l=1}^M b_{jl}Bv_l + \sum_{i=1}^N e_{ji}Bu_i\Bigg) \end{aligned}$$

and the optimal policy for evader *j* is given by

$$v_j^* = -\frac{1}{2}(\kappa d_j^e - d_j^{\mathrm{ep}})R_e^{-1}(\varepsilon_j)B^T\nabla V_{ej}(\varepsilon_j). \tag{19}$$

The functions V_{pi} and V_{ej} are obtained as the solutions of the coupled HJI equations of the game

$$\begin{aligned} &\delta_i^T Q_{pi}\delta_i + u_i^{*T} R_p u_i^* \\ &\quad + \nabla V_{pi}^T\Bigg(A\delta_i + (d_i^p + d_i^{\mathrm{pe}})Bu_i^* \\ &\qquad - \sum_{k=1}^N a_{ik}Bu_k^* - \sum_{j=1}^M c_{ij}Bv_j^*\Bigg) = 0 \end{aligned} \tag{20}$$

and

$$\begin{aligned} &\varepsilon_j^T Q_{ej}\varepsilon_j + v_j^{*T} R_e v_j^* \\ &\quad + \nabla V_{ej}^T\Bigg(A\varepsilon_j + (\kappa d_j^e - d_j^{\mathrm{ep}})Bv_j^* \\ &\qquad - \kappa\sum_{l=1}^M b_{jl}Bv_l^* + \sum_{i=1}^N e_{ji}Bu_i^*\Bigg) = 0 \end{aligned} \tag{21}$$

for $i = 1, \ldots, N$ and $j = 1, \ldots, M$. Substituting control policies (18) and (19) in (20) and (21), we obtain

$$\begin{aligned} &\nabla V_{pi}^T A_{pi}^{cl} + \delta_i^T Q_{pi}\delta_i + \frac{1}{2}\sum_{k=1}^N a_{ik}(d_k^p + d_k^{\mathrm{pe}})\nabla V_{pi}^T BR_p^{-1}B^T\nabla V_{pk} \\ &\quad + \frac{1}{2}\sum_{j=1}^M c_{ij}(\kappa d_j^e - d_j^{\mathrm{ep}})\nabla V_{pi}^T BR_e^{-1}B^T\nabla V_{ej} = 0 \end{aligned} \tag{22}$$

and

$$\begin{aligned} &\nabla V_{ej}^T A_{ej}^{cl} + \varepsilon_j^T Q_{ej}\varepsilon_j + \frac{\kappa}{2}\sum_{l=1}^M b_{jl}(\kappa d_l^e - d_l^{\mathrm{ep}})\nabla V_{ej}^T BR_e^{-1}B^T\nabla V_{el} \\ &\quad - \frac{1}{2}\sum_{i=1}^N e_{ji}(d_i^p + d_i^{\mathrm{pe}})\nabla V_{ej}^T BR_p^{-1}B^T\nabla V_{pi} = 0 \end{aligned} \tag{23}$$

respectively, where the closed-loop matrices are

$$A_{pi}^{cl} = A\delta_i - \frac{1}{4}(d_i^p + d_i^{\mathrm{pe}})^2 B R_p^{-1} B^T \nabla V_{pi}$$

and

$$A_{ej}^{cl} = A\varepsilon_j - \frac{1}{4}\left(\kappa d_j^e - d_j^{\mathrm{ep}}\right)^2 B R_e^{-1} B^T \nabla V_{ej}.$$

The following theorem states the conditions in the MPE game to achieve Nash equilibrium as defined in Definition 2.

Theorem 1. (Nash equilibrium in MPE games): Let the pursuers with dynamics (1) and the evaders with dynamics (2) be connected in a communication graph topology with local errors (3) and (5). Let (18) and (19) be the control policies for pursuers *i* and evaders *j*, respectively, where the functions V_{pi} and V_{ej} are the solutions of the HJI equations (22) and (23), such that $V_{pi}(0) = V_{ej}(0) = 0$. Then, capture occurs in the MPE game (17) in the sense that dynamics (4) is stable. Moreover, game (17) is in Nash equilibrium, the value of the game for pursuer *i* is given by $V_{pi}(\delta_i(0))$, and the value of the game for evader *j* is $V_{ej}(\varepsilon_j(0))$.

Proof: To prove capture, select the function $V_{pi}(\delta_i(t))$ as a Lyapunov function candidate. Its derivative is given by

$$\begin{aligned}\dot{V}_{pi} = \nabla V_{pi}^T \dot{\delta}_i = \nabla V_{pi}^T &\left(A\delta_i + (d_i^p + d_i^{\mathrm{pe}}) B u_i^* - \sum_{k=1}^{N} a_{ik} B u_k^* \right.\\ &\left. - \sum_{j=1}^{M} c_{ij} B v_j^* \right)\\ = &- \delta_i^T Q_{pi} \delta_i - u_i^{*T} R_p u_i^*\end{aligned}$$

as (20) holds. Because the right-hand side of this equation is negative definite, we conclude that dynamics (4) is stable and every pursuer captures his neighbors.

To prove Nash equilibrium, notice that we can write the performance index (15) as

$$\begin{aligned}J_{pi} = &\int_0^\infty \left[\delta_i^T Q_{pi}(\delta_i)\delta_i + u_i^T R_p(\delta_i) u_i\right] dt + V_{pi}(\delta_i(0))\\ &+ \int_0^\infty \dot{V}_{pi}(\delta_i(t)) dt\end{aligned}$$

because $V_{pi}(\delta_i(\infty)) = V_{pi}(0) = 0$. For convenience, we omit the explicit dependence of matrices Q_{pi} and R_p on the local errors δ_i in the remaining procedure. Notice that, if Assumption 1 holds, then $\dot{V}_{pi}(\delta_i) = \nabla V_{pi}^T \dot{\delta}_i$ for any trajectory of $\delta_i(t)$. Thus, using (4) we get

$$\begin{aligned}J_{pi} = &\int_0^\infty \left[\delta_i^T Q_{pi}\delta_i + u_i^T R_p u_i\right] dt + V_{pi}(\delta_i(0))\\ &+ \int_0^\infty \nabla V_{pi}^T \left(A\delta_i + (d_i^p + d_i^{\mathrm{pe}}) B u_i \right.\\ &\left. - \sum_{k=1}^{N} a_{ik} B u_k - \sum_{j=1}^{M} c_{ij} B v_j \right) dt.\end{aligned}$$

Completing the squares, and using the fact that $(d_i^p + d_i^{\mathrm{pe}})\nabla V_{pi}^T B u_i = -2 u_i^{*T} R_p u_i$ with u_i^* in (18), yields

$$\begin{aligned}J_{pi} = &\int_0^\infty \left[\delta_i^T Q_{pi}\delta_i + u_i^T R_p u_i + u_i^{*T} R_p u_i^* - u_i^{*T} R_p u_i^*\right] dt\\ &+ V_{pi}(\delta_i(0)) + \int_0^\infty \nabla V_{pi}^T \left(A\delta_i - \sum_{k=1}^{N} a_{ik} B u_k \right.\\ &\left. - \sum_{j=1}^{M} c_{ij} B v_j \right) dt - 2\int_0^\infty u_i^{*T} R_p u_i dt\\ = &\int_0^\infty \left[\delta_i^T Q_{pi}\delta_i + (u_i - u_i^*)^T R_p (u_i - u_i^*) - u_i^{*T} R_p u_i^* \right.\\ &\left. + \nabla V_{pi}^T \left(A\delta_i - \sum_{k=1}^{N} a_{ik} B u_k - \sum_{j=1}^{M} c_{ij} B v_j\right)\right] dt\\ &+ V_{pi}(\delta_i(0)).\end{aligned}$$

As $-u_i^{*T} R_p u_i^* = -2 u_i^{*T} R_p u_i^* + u_i^{*T} R_p u_i^* = (d_i^p + d_i^{\mathrm{pe}})\nabla V_{pi}^T B u_i^* + u_i^{*T} R_p u_i^*$, we finally get

$$\begin{aligned}J_{pi} = &\int_0^\infty \left[\delta_i^T Q_{pi}\delta_i + u_i^{*T} R_p u_i^* + \nabla V_{pi}^T \left(A\delta_i + (d_i^p \right.\right.\\ &\left.\left. + d_i^{\mathrm{pe}}) B u_i^* - \sum_{k=1}^{N} a_{ik} B u_k - \sum_{j=1}^{M} c_{ij} B v_j\right)\right] dt\\ &+ \int_0^\infty (u_i - u_i^*)^T R_p (u_i - u_i^*) dt + V_{pi}(\delta_i(0)).\end{aligned}$$

As (20) holds, the first integral in this expression is equal to zero for neighbor policies $u_k = u_k^*$ and $v_j = v_j^*$. The remaining expression

$$J_{pi} = \int_0^\infty (u_i - u_i^*)^T R_p (u_i - u_i^*) dt + V_{pi}(\delta_i(0))$$

shows that control policy u_i^* minimizes the performance function of pursuer *i* against neighbor policies u_k^* and v_j^*, and his value of the game is $V_{pi}(\delta_i(0))$.

The same procedure can be performed to show that

$$J_{ej} = \int_0^\infty \left(v_j - v_j^*\right)^T R_e \left(v_j - v_j^*\right) dt + V_{ej}(\varepsilon_j(0))$$

for evader *j* and, therefore, control policy v_j^* minimizes J_{ej}. As these conditions hold for all agents in the game, Nash equilibrium is achieved. ■

Remark 1: Notice that if there exist matrices P_{pi} and P_{ej}, for all *i* and *j* in the game, such that the value functions V_{pi} and V_{ej} have the form

$$V_{pi}(\delta_i) = \delta_i^T P_{pi} \delta_i \tag{24}$$

and

$$V_{ej}(\varepsilon_j) = \varepsilon_j^T P_{ej} \varepsilon_j \tag{25}$$

then the control policies (18) and (19) take the form

$$u_i^* = -(d_i^p + d_i^{\mathrm{pe}})R_p^{-1}(\delta_i)B^T P_{pi}\delta_i \tag{26}$$

and

$$v_j^* = -(\kappa d_j^e - d_j^{\mathrm{ep}})R_e^{-1}(\varepsilon_j)B^T P_{ej}\varepsilon_j \tag{27}$$

respectively. In (26) and (27), the distributed property of the control policies is clear.

C. Guaranteed Performance in MPE Games: Minmax Strategies

In the general case, there may not exist a set of functions $V_{pi}(\delta_i)$ and $V_{ej}(\varepsilon_j)$ that solve the HJI equations (22) and (23) to provide distributed control policies as in (26) and (27). This is an expected result due to the limited knowledge of the agents connected in the communication graph. If pursuer *i* does not know the neighbors of his target, evader *j*, then he cannot determine *j*'s best response in the game and prepare his strategy accordingly.

Despite this inconvenience, we can expect agent *i* to determine a best policy for the information he has available from his neighbors. In this section, we let each agent prepare himself for the worst case scenario in the behavior of his neighbors. The resulting solution concept is regarded as a minmax strategy [14] and, as it is shown later, the corresponding HJI equations are generally solvable for linear systems and the resulting control policies are distributed. The following definition states the concept of minmax strategy employed in this paper.

Definition 3. (Minmax strategies): In an MPE game, the minmax strategy of pursuer *i* is given by

$$u_i^* = \arg\min_{u_i}\max_{u_{-i},v_{-i}} J_{pi} \tag{28}$$

and the minmax strategy for evader *j* is

$$v_j^* = \arg\min_{v_j}\max_{u_{-j},v_{-j}} J_{ej}. \tag{29}$$

The idea behind the minmax strategies in MPE games is that an agent prepares his best response assuming that his neighbors will attempt to maximize his performance index. As this is usually not the strategy followed by such neighbors during the game, every agent can expect to achieve a better payoff than his minmax value.

To determine the minmax strategy for pursuer *i*, we can redefine the performance index (15) and formulate a zero-sum game between agent *i* and a virtual target. Considering the center of gravity (7) as the target of agent *i*, we can define the performance index

$$J_{pi} = \int_0^\infty \left[\delta_i^T Q_{pi}\delta_i + u_i^T R_p u_i - \bar{u}_{-i}^T R_{-i}\bar{u}_{-i}\right] dt. \tag{30}$$

In order to define a meaningful weighting matrix R_{-i} in (30), consider the expressions (8) and (18), and select

$$R_{-i}^{-1} = \frac{1}{d_i^p + d_i^{\mathrm{pe}}}\left(\sum_{k=1}^N a_{ik}R_p^{-1} + \sum_{j=1}^M c_{ij}R_e^{-1}\right). \tag{31}$$

The solution of the zero-sum game is now determined by (18), where the value function V_{pi} is the solution of the HJI equation

$$\begin{aligned}\nabla V_{pi}^T A\delta_i + \delta_i^T Q_{pi}\delta_i - (d_i^p + d_i^{\mathrm{pe}})\nabla V_{pi}^T BR_p^{-1}B^T\nabla V_{pi}\\ + (d_i^p + d_i^{\mathrm{pe}})\nabla V_{pi}^T BR_{-i}^{-1}B^T\nabla V_{pi} = 0.\end{aligned} \tag{32}$$

If the value function has a quadratic form as in (24), then the control policy is expressed as in (26) and matrix P_{pi} is the solution of the HJI equation

$$\begin{aligned}&Q_{pi} + P_{pi}A + A^T P_{pi}\\ &\quad - (d_i^p + d_i^{\mathrm{pe}})^2 P_{pi}B(R_p^{-1} - R_{-i}^{-1})B^T P_{pi} = 0.\end{aligned} \tag{33}$$

It is observed that these policies are always distributed, in contrast to the policies based on the Nash solution given by (22).

Similarly, define the zero-sum game

$$J_{ej} = \int_0^\infty \left[\varepsilon_j^T Q_{ej}\varepsilon_j + v_j^T R_e v_j - \bar{v}_{-j}^T R_{-j}\bar{v}_{-j}\right] dt \tag{34}$$

with

$$R_{-j}^{-1} = \frac{1}{d_j^e + d_j^{\mathrm{ep}}}\left(\sum_{l=1}^M b_{jl}R_e^{-1} + \sum_{i=1}^N e_{ji}R_p^{-1}\right). \tag{35}$$

Assuming a quadratic value function (25), the minmax strategy for evader *j* is the control policy (27) where matrix P_{ej} solves the equation

$$\begin{aligned}&Q_{ej} + P_{ej}A + A^T P_{ej}\\ &\quad - (\kappa d_j^e - d_j^{\mathrm{ep}})^2 P_{ej}B(R_e^{-1} - R_{-j}^{-1})B^T P_{ej} = 0.\end{aligned} \tag{36}$$

The following theorem formalizes these results.

Theorem 2. (Minmax strategies): Let the agents with dynamics (1) and (2) use control policies (26) and (27), respectively. Moreover, let matrices P_{pi} and P_{ej} be the solutions of the Riccati equations (33) and (36). Then, policy (26) is the minmax strategy of pursuer *i* as defined in (28), and policy (27) is the minmax strategy of evader *j* as in (29).

Proof: The Hamiltonian function associated with the performance index (30) is $H_{pi} = \delta_i^T Q_{pi}\delta_i + u_i^T R_p u_i - \bar{u}_{-i}^T R_{-i}\bar{u}_{-i} + \dot{V}_{pi}$. For a quadratic value function (24), the optimal control policy for the pursuer is (26) and for its target is $\bar{v}_{-i}^* = -(d_i^p + d_i^{\mathrm{pe}})R_{-i}^{-1}B^T P_{pi}\delta_i$. Substituting these control policies in H_{pi} and equating to zero, we obtain the HJI equation (33). Following a similar procedure as in the proof of Theorem 1, and considering dynamics (10), we can complete the squares and write the performance index (30) as

$$\begin{aligned}J_{pi} &= \int_0^\infty \left[\delta_i^T Q_{pi}\delta_i + u_i^T R_p u_i - \bar{u}_{-i}^T R_{-i}\bar{u}_{-i}\right] dt + V_{pi}(\delta_i(0))\\ &\quad + \int_0^\infty \nabla V_{pi}^T\left(A\delta_i + (d_i^p + d_i^{\mathrm{pe}})Bu_i - (d_i^p + d_i^{\mathrm{pe}})B\bar{u}_{-i}\right) dt\\ &= \int_0^\infty \Big[(u_i - u_i^*)^T R_p(u_i - u_i^*) - (\bar{u}_{-i} - \bar{u}_{-i}^*)^T\\ &\quad \times R_{-i}(\bar{u}_{-i} - \bar{u}_{-i}^*)\Big] dt + V_{pi}(\delta_i(0)).\end{aligned}$$

Therefore, (26) with P_{pi} as in (33) is the minmax strategy of pursuer *i*. The same procedure can be used to prove that control policy (27) with P_{ej} as the solution of (36) is the minmax strategy for evader *j*. ■

Remark 2: Minmax strategies provide distributed control policies for the agents as long as there exist positive definite solutions P_{pi} and P_{ej} for (33) and (36), respectively. Equations (33) and (36) are algebraic Riccati equations, well known in linear optimal control theory. As such, they are known to have positive definite solutions if (A, B) is stabilizable, $(A, \sqrt{Q_{pi}})$ and $(A, \sqrt{Q_{ej}})$ are observable, and $R_p^{-1} - R_{-i}^{-1} > 0$ and $R_e^{-1} - R_{-j}^{-1} > 0$ [2].

IV. Target Selection and Finite-Time Capture

In the previous section, the evaders had the objective of maintaining their group cohesion while escaping from the pursuers. In this section, we modify the formulation of the game to obtain different behaviors among the agents. Let the evaders have the objective of increasing their distances with respect to each other, to force the pursuers to separate as well. Now, each pursuer must select a single target among the evaders in his neighborhood. Moreover, the control strategies of the agents are designed to achieve capture in finite time.

A. Target Selection by the Pursuers

Consider a game in which each pursuer can initially observe several evaders according to the communication graph topology. In most real-world applications, it can be impractical to expect each pursuer to chase many evaders simultaneously. Instead, each pursuer can select only one target among the evaders, disregarding the position of all other agents. If each pursuer targets a different evader, their objective of capturing as many evaders as possible is fulfilled.

In this section, we assume that the numbers of pursuers and evaders are the same, i.e., $N = M$. If there were more evaders, some of them would be able to escape unchallenged and therefore are not part of this analysis. If there were more pursuers, some of them would need to chase already-targeted evaders and their participation would be trivial.

Algorithm 1 presents a procedure for pursuer *i* to select a target among the evaders. Initially, the graph weights for pursuer *i* are set as $a_{ik} \geq 0$ and $c_{ij} \geq 0$. Algorithm 1 relies on setting $a_{ik} = 0$ for all pursuers *k*, and $c_{ij} = 0$ for all evaders but one. This one evader becomes the target of pursuer *i*.

In the ideal scenario, every pursuer targets its closest evader. If two pursuers have the same closest evader, additional criteria are needed to decide which pursuer will change its target. We propose to change the target of the pursuer with shortest distance to its second-closest evader. Thus, long distances between a pursuer and its target are avoided. Algorithm 1 generalizes this idea using an iterative procedure that eliminates the longest distances between the agents, until each pursuer is left with only one target.

Pursuer *i* is assumed to be unaware of the initial topologies in graphs G_p and G_{pe} except for the links that connect him to his neighbors. Thus, to perform Algorithm 1 pursuer *i* must assume that his neighbors have the same state information as himself. Let $\mathcal{N}_i$ be the set of neighbors of pursuer *i*. Pursuer *i* can define virtual (or assumed) connectivity weights for his neighbors as $\bar{a}_{kh} = 1$ if $k, h \in \mathcal{N}_i$ for pursuers *k* and *h*, and $\bar{c}_{kj} = 1$ if $k, j \in \mathcal{N}_i$ for pursuer *k* and evader *j*. In the following procedure, all agents are pursuer *i*'s neighbors.

Algorithm 1.

1. If $c_{ij} = 1$ for only one evader *j*, stop.
2. Determine the pursuer *k* and evader *j* such that $\rho_{\max,i} = \rho_{kj}$, and set $\bar{c}_{kj} = 0$.
3. If for some pursuer *k*, $\bar{c}_{kl} = 1$ for only one evader *l*, set $\bar{c}_{hl} = 0$ for all pursuers $h \neq k$ and $c_{il} = 0$ if $i \neq k$.
4. Go to Step 1.

Define $\rho_{ij} > 0$ as the distance between pursuer *i* and a neighboring evader *j*, i.e., $\rho_{ij} = c_{ij}\|x_i - y_j\|$. Define also $\rho_{kj} = \bar{c}_{kj}\|x_k - y_j\|$ for all pursuers and evaders $k, j \in \mathcal{N}_i$. Let $\rho_{\max,i}$ be the longest distance between any pursuer $k \in i \cup \mathcal{N}_i$ and his possible targets, that is, $\rho_{\max,i} = \max_{k,j}(\rho_{kj})$ for $k \in i \cup \mathcal{N}_i$, $j \in \mathcal{N}_i$. With these definitions, Algorithm 1 can now be used for pursuer *i* to select his target.

In Algorithm 1, pursuer *i* assigns targets to his teammates in order to discard those evaders for himself in benefit of the collective goals of the team. Notice that these virtual assignments may not correspond to the actual selection of targets of the other pursuers. Step 2 in Algorithm 1 indicates that, as long as there are other options, the longest distance between a pursuer and an evader must be avoided. Step 3 expresses that once a pursuer targets an evader, the other pursuers discard that evader as a possible objective. The following theorem shows that if every agent in the game is each other's neighbor, then target selection by Algorithm 1 minimizes the longest distance between a pursuer and his target.

Theorem 3: Consider the MPE game where every pursuer selects his target using Algorithm 1. If the graphs G_p and G_{pe} are complete, then every pursuer selects a different target. Moreover, this selection of targets minimizes the longest distance between a pursuer and his target.

Proof: If the graph topologies are complete, then all agents possess the same state information and each pursuer selects precisely the same target that was assigned to him by all his neighbors. By the construction of Algorithm 1, each pursuer selects a unique target.

Consider now pursuer *i* who travels the longest distance to capture his target. Let *i* change his selection of target such that his distance to travel is reduced. Pursuer *i* can only have discarded this closer target in Algorithm 1 because it was the only option for another pursuer *k* to chase. Pursuer *k* had selected his target either because his other options had the longest distances $\rho_{\max}$ or because other pursuers had previously selected those evaders. This consecutive change of targets must eventually lead to a pursuer left to chase an evader with a travel distance longer than the original longest distance produced by Algorithm 1. ■

Using Algorithm 1, every pursuer modifies his perception of the environment such that his local error measurement focuses only on his target; i.e., if pursuer *i* targets evader *j*, then $\delta_i = c_{ij}(x_i - y_j)$. Notice that the target selection procedure is also useful if the evaders change their objectives in the game and decide to separate from their teammates as well as from the pursuers.

Another practical consideration in MPE games consists in designing the agents to use a sustained control effort throughout the game, which allows achieving finite-time capture as studied in the following section.

B. Finite-Time Intercept

In practical applications, pursuers and evaders are expected to use their maximum effort to achieve their goals. In this case, a pursuer is able to intercept his target in finite time because his velocity is not decreased when approaching the target.

To generate this behavior, let the *i*th pursuer use Algorithm 1 to target only one evader, and use the control policy (26) with the matrix $R_p(\delta_i)$ selected as

$$R_p(\delta_i) = \|\delta_i\| \, r_p I \tag{37}$$

as long as $\delta_i \neq 0$, where r_p is a positive scalar and I is the identity matrix. Similarly, evader *j* uses the policy (27) with $R_e(\varepsilon_j)$ as

$$R_e(\varepsilon_j) = (d_j^e + d_j^{\mathrm{ep}}) \, \|\varepsilon_j\| \, r_e I \tag{38}$$

where r_e is a positive scalar, and d_j^e and d_j^{ep} are defined in Section II.

The following theorem shows that the selection of matrices (37) and (38) produces finite-time interception if the pursuer is allowed to use a greater control effort than the evader, and if the Lyapunov equation

$$P_{pi} A + A^T P_{pi} = -Q_s \tag{39}$$

with $Q_s \geq 0$, holds for matrix P_{pi}. Notice that (39) is solvable if the real parts of the eigenvalues of matrix *A* are nonpositive. Condition (39) is further studied in Section IV-C.

Theorem 4. (Finite-time capture in MPE games): Consider an MPE game (17) where pursuer *i* and evader *j* have dynamics (1) and (2), respectively, with a marginally stable system matrix *A*. Pursuer *i* selects evader *j* as his only target. Let the control policies be (26) and (27), with weight matrices (37) and (38). Let the gain matrix $P_{pi} = P_{pi}^{T\prime} P'_{pi}$ be such that the Lyapunov equation (39) holds. Then, finite-time capture occurs if

$$r_p^{-1} \, \|P_{pi}\| \geq r_e^{-1} \, \|P_{ej}\| \, . \tag{40}$$

Proof: Define the candidate Lyapunov function $V_L = (1/2)\|P'_{pi}\delta_i\|^2 = (1/2)\delta_i^T P_{pi} \delta_i$. Thus,

$$\begin{aligned}
\dot{V}_L &= \frac{1}{2}\delta_i^T \left(P_{pi} A + A^T P_{pi}\right) \delta_i - r_p^{-1} \|\delta_i\|^{-1} \delta_i^T P_{pi} B B^T P_{pi} \delta_i \\
&\quad - r_e^{-1} \|\varepsilon_j\|^{-1} \delta_i^T P_{pi} B B^T P_{ej} \varepsilon_j \\
&\leq - r_p^{-1} \|\delta_i\|^{-1} \delta_i^T P_{pi} B B^T P_{pi} \delta_i \\
&\quad - r_e^{-1} \|\varepsilon_j\|^{-1} \delta_i^T P_{pi} B B^T P_{ej} \varepsilon_j
\end{aligned}$$

because (39) holds. As $\delta_i^T P_{pi} B B^T P_{pi} \delta_i$ is a positive scalar, it is equal to its norm. Now,

$$\begin{aligned}
\dot{V}_L \leq & - r_p^{-1} \|\delta_i\|^{-1} \left\|\delta_i^T P_{pi} B B^T P_{pi} \delta_i\right\| \\
& + r_e^{-1} \|\varepsilon_j\|^{-1} \left\|\delta_i^T P_{pi} B B^T P_{ej} \varepsilon_j\right\| \\
\leq & - r_p^{-1} \|\delta_i\|^{-1} \left\|P'_{pi}\delta_i\right\| \left\|P'_{pi} B B^T\right\| \|P_{pi}\| \, \|\delta_i\| \\
& + r_e^{-1} \|\varepsilon_j\|^{-1} \|P'_{pi}\delta_i\| \left\|P'_{pi} B B^T\right\| \|P_{ej}\| \, \|\varepsilon_j\| \\
= & - \beta_i \, \|P'_{pi}\delta_i\| = -\sqrt{2}\beta_i \sqrt{V_L}
\end{aligned}$$

where

$$\beta_i = \left(r_p^{-1} \, \|P_{pi}\| - r_e^{-1} \, \|P_{ej}\|\right) \left\|P'_{pi} B B^T\right\| .$$

Clearly, if (40) holds, then $\dot{V}_L < 0$ and capture occurs. Furthermore, we can solve the differential equation $\dot{V}_L V_L^{-1/2} = -\sqrt{2}\beta_i$ to obtain $V_L^{1/2}(t) = (\sqrt{2}/2)\beta_i t + V_L^{1/2}(0)$. This shows that, if capture occurs, then V_L is equal to zero for a finite time *t*. ∎

C. Inverse Optimal Control for Finite-Time Capture

If matrix P_{pi} is selected first such that the Lyapunov equation (39) holds, then it is not directly obtained as the solution of the HJI equation (33). An optimality result can still be obtained if the performance index (30) is selected accordingly by means of inverse optimal control.

Theorem 5 shows a minmax inverse optimal result by selecting the matrix Q_{pi} in (30) as

$$Q_{pi} = Q_s + P_{pi} B R_p^{-1} B^T P_{pi} - P_{pi} B R_e^{-1} B^T P_{pi}. \tag{41}$$

Theorem 5: Let the pursuer *i* with dynamics (1) use control policy (26), where matrix P_{pi} is selected such that (39) holds for a given matrix $Q_s \geq 0$. Then, pursuer *i* uses his minmax strategy with respect to the performance index (30) with matrix Q_{pi} as in (41).

Proof: Substitute the matrix Q_{pi} in (41) into the HJI equation (33) to obtain

$$\begin{aligned}
& Q_s + P_{pi} B R_p^{-1} B^T P_{pi} - P_{pi} B R_e^{-1} B^T P_{pi} + P_{pi} A + A^T P_{pi} \\
& \quad - P_{pi} B R_p^{-1} B^T P_{pi} + P_{pi} B R_e^{-1} B^T P_{pi} = 0
\end{aligned}$$

and note that the equality holds because $P_{pi} A + A^T P_{pi} = -Q_s$. The proof is now completed as in the proof of Theorem 2. ∎

Although target selection and finite-time capture are studied for their broad range of practical implementations, interesting behaviors arise in the MPE games when the agents employ asymptotic strategies. These new scenarios are analyzed in the following section.

V. Extensions and Asymptotic Behaviors

This section is concerned with the analysis of asymptotic capture in MPE games. Rendezvous and containment control are studied as particular cases of this scenario.

A. Rendezvous or Asymptotic Capture

In this section, let the pursuers maintain their local errors as defined in (3), without the use of the target selection algorithm. Furthermore, let the matrices $R_p(\delta_i)$ and $R_e(\varepsilon_j)$ be constant throughout time, such that $R_p(\delta_i) = R_p$ and $R_e(\varepsilon_j) = R_e$. In the asymptotic version of the MPE games, capture occurs when all pursuers reach the position of all evaders.

Few definitions are required for the analysis performed in this section. Using the graph matrices defined in Section II-A, define the generalized Laplacian matrix *L* as

$$L = \begin{bmatrix} L_p + D_{\mathrm{pe}} & -\mathcal{A}_{\mathrm{pe}} \\ \mathcal{A}_{\mathrm{ep}} & \kappa L_e - D_{\mathrm{ep}} \end{bmatrix}. \quad (42)$$

Define also the matrix *K* as the block matrix

$$K = \begin{bmatrix} \mathrm{diag}(K_{pi}) & 0 \\ 0 & \mathrm{diag}(K_{ej}) \end{bmatrix} \quad (43)$$

where $K_{pi} = (d_i^p + d_i^{\mathrm{pe}})R_p^{-1}B^T P_{pi}$ for $i = 1, \ldots, N$, and $K_{ej} = -(\kappa d_j^e - d_j^{ep})R_e^{-1}B^T P_{ej}$ for $j = 1, \ldots, M$.

Theorem 6: states the conditions for capture relating the stability properties of the game with the three communication graph topologies employed in (42).

Theorem 6: Consider the MPE game (17) with system dynamics (1) and (2) and performance indices (15) and (16). Define matrix *K* as in (43). If there exist matrices P_{pi} and P_{ej} such that the value functions (24) and (25) satisfy the HJI equations (22) and (23) and the agents use control policies (26) and (27), then the eigenvalues of the matrix $[(I \otimes A) - (L \otimes B)K] \in \mathbb{R}^{n(N+M)}$ have all negative real parts, i.e.,

$$\mathrm{Re}\{\lambda_k\left((I \otimes A) - (L \otimes B)K\right)\} < 0 \quad (44)$$

for $k = 1, \ldots, n(M+N)$.

Proof: Define the vectors $\delta = [\delta_1^T, \ldots, \delta_N^T, \varepsilon_1^T, \ldots, \varepsilon_M^T]^T$ and $u = [u_1^T, \ldots, u_N^T, v_1^T, \ldots, v_M^T]^T$. Using the local error dynamics (4) and (6), we can write

$$\dot{\delta} = (I \otimes A)\delta + (L \otimes B)u \quad (45)$$

where $\otimes$ stands for the Kronecker product. Control policies (26) and (27) can be expressed as $u_i = -K_{pi}\delta_i$ and $v_j = -K_{ej}\varepsilon_j$, respectively, where matrices K_{pi} and K_{ej} are defined as for (43). Now we can write

$$u = -K\delta. \quad (46)$$

Substitution of (46) in (45) yields the global closed-loop dynamics

$$\dot{\delta} = \left[(I \otimes A) - (L \otimes B)K\right]\delta. \quad (47)$$

Theorem 1 shows that if matrices P_{pi} and P_{ej} satisfy (22) and (23), then the control policies (26) and (27) make the pursuers achieve capture. This is equivalent to state that the system (47) is stable, and thus the condition (44) holds. ■

The following behaviors are corollaries for Theorem 2.

B. L_2 Gain Bound

The L_2 gain bound in MPE games refers to the problem of determining a feedback policy $u_i(\delta_i)$ for pursuer *i* such that when $\delta_i(0) = 0$ and for all neighbor policies $u_k \in L_2[0, \infty)$ and $v_j \in L_2[0, \infty)$, the inequality

$$\int_0^T (\delta_i^T Q_{pi}\delta_i + u_i^T R_p u_i)dt \leq r_{-i}\int_0^T \|\bar{u}_{-i}\|\, dt \quad (48)$$

where $\bar{v}_{-i}$ is defined in (8), holds for a scalar $r_{-i} > 0$.

Corollary 1. (L2 gain bound): Let the conditions of Theorem 2 hold and let matrix R_{-i} in (30) be $R_{-i} = r_{-i}I$. Then, the L_2 gain of pursuer *i* is bounded above by the L_2 gain of his neighbors according to the inequality (48).

Proof: If $R_{-i} = r_{-i}I$, then the Hamiltonian function for pursuer *i* can be written as

$$H_{pi}(u_i, \bar{v}_{-i}) = \delta_i^T Q_{pi}\delta_i + u_i^T R_p u_i - r_{-i}\bar{u}_{-i}^T\bar{u}_{-i} + \dot{V}_{pi}(\delta_i). \quad (49)$$

Let V_{pi} be the solution to the HJI equation $H_{pi}(u_i^*, \bar{u}_{-i}^*) = 0$. Considering the policies (26) and $\bar{u}_{-i}^* = -(d_i^p + d_i^{\mathrm{pe}})R_{-i}^{-1}B^T P_{pi}\delta_i$, complete the squares in (49) to obtain

$$\begin{aligned} H_{pi} &= \delta_i^T Q_{pi}\delta_i + u_i^{*T} R_p u_i^* - \bar{u}_{-i}^{*T} R_{-i}\bar{u}_{-i}^* + \dot{V}_{pi} \\ &\quad + (u_i - u_i^*)^T R_p(u_i - u_i^*) - (\bar{u}_{-i} - \bar{u}_{-i}^*)^T R_{-i}(\bar{u}_{-i} - \bar{u}_{-i}^*) \\ &= (u_i - u_i^*)^T R_p(u_i - u_i^*) - (\bar{u}_{-i} - \bar{u}_{-i}^*)^T R_{-i}(\bar{u}_{-i} - \bar{u}_{-i}^*). \end{aligned}$$

Select $u_i = u_i^*$ to obtain $H_{pi} = -(\bar{u}_{-i} - \bar{u}_{-i}^*)^T R_{-i}(\bar{u}_{-i} - \bar{u}_{-i}^*) \leq 0$, which from (49) implies

$$\delta_i^T Q_{pi}\delta_i + u_i^T R_p u_i - r_{-i}\bar{u}_{-i}^T\bar{u}_{-i} + \dot{V}_{pi}(\delta_i) \leq 0. \quad (50)$$

Integrating the inequality (50), we get

$$\begin{aligned} &\int_0^T (\delta_i^T Q_{pi}\delta_i + u_i^T R_p u_i - r_{-i}\|\bar{u}_{-i}\|)dt \\ &\qquad + V_{pi}(\delta_i(T)) - V_{pi}(\delta_i(0)) \leq 0. \end{aligned}$$

Now, $\delta_i(0) = 0$ implies $V_{pi}(\delta(0)) = 0$. As $V_{pi}(\delta_i(T)) > 0$ for all $\delta_i(T)$, inequality (48) is directly obtained. ■

C. Containment Control

In this section, the MPE behaviors are related to the containment control problem [22]. Define the global vector of pursuers' positions as $x = [x_1^T, \ldots, x_N^T]^T$ and the global position vector of the evaders as $y = [y_1^T, \ldots, y_M^T]^T$. Corollary 2 shows that, for the special case of static evaders and taking the system matrix $A = 0$, the solution of the MPE games recovers the containment control results.

Corollary 2. (Containment with stationary evaders): Let the conditions of Theorem 2 hold, with system matrix $A = 0$ in (1) and (2). Select matrix $R_e = r_e I$ in performance index (34) and consider the matrix K_{pi} defined as for (43). Let the graph topologies be such that there is a directed path from at least one evader to each pursuer. In the limit $r_e \to \infty$, the convex hull of the evaders' positions, according to the expression

$$x = [(L_p + D_{\mathrm{pe}})^{-1}\mathcal{A}_{\mathrm{pe}} \otimes I]y \quad (51)$$

is an equilibrium set for the pursuer dynamics. Furthermore, if the matrix

$$(I \otimes B)\mathrm{diag}(K_{pi})\left[(L_p + D_{\mathrm{pe}}) \otimes I\right] \tag{52}$$

is nonsingular, then (51) is the only stable equilibrium set for the pursuer dynamics.

Proof: Notice that control policies (27) with $r_e \to \infty$ produce static evaders. Using control policies $u_i = -K_{pi}\delta_i$, the global pursuer dynamics is given by

$$\begin{aligned}\dot{x} = &-(I \otimes B)\,\mathrm{diag}(K_{pi})\left[(L_p + D_{\mathrm{pe}}) \otimes I\right] x \\ &+ (I \otimes B)\,\mathrm{diag}(K_{pi})\left[\mathcal{A}_{\mathrm{pe}} \otimes I\right] y.\end{aligned} \tag{53}$$

Substituting (51) in (53), and using the properties of the Kronecker product [25], we get $\dot{x} = 0$. Therefore, the points in (51) are equilibrium points for the pursuers. It is easy to prove that, if there is a directed path from at least one evader to each pursuer, all eigenvalues of $(L_p + D_{\mathrm{pe}})$ are positive, all the elements of $(L_p + D_{\mathrm{pe}})^{-1}\mathcal{A}_{\mathrm{pe}}$ are nonnegative, and matrix $(L_p + D_{\mathrm{pe}})^{-1}\mathcal{A}_{\mathrm{pe}}$ has all row sums equal to 1 [22]. This shows that (51) is the convex hull of the evaders' positions.

Now, let $\dot{x} = 0$ in (53). The resulting linear equation has (51) as its unique solution if matrix (52) is nonsingular. ■

Remark 3: From the definition of K_{pi}, notice that matrix (52) is nonsingular for the special case of single integrator dynamics, that is, if $B = I$.

Corollary 3 shows a similar result, allowing the evaders to be moving in a formation with a constant velocity.

Corollary 3. (Containment with moving evaders): Let the conditions in Corollary 2 hold, and let all the evaders move with constant velocities, $\dot{y}_j = \nu$, $j = 1, \ldots, M$. Control inputs

$$u_i = -(d_i^p + d_i^{\mathrm{pe}})R_p^{-1}(\delta_i)B^T P_{pi}\delta_i + \nu \tag{54}$$

make the pursuers converge to the convex hull of the evaders according to the expression (51) if matrix (52) is nonsingular.

Proof: Using control policies (54), the global pursuer dynamics is

$$\begin{aligned}\dot{x} = &-(I \otimes B)\,\mathrm{diag}(K_{pi})\left[(L_p + D_{\mathrm{pe}}) \otimes I\right] x + (\underline{1}_N \otimes \nu) \\ &+ (I \otimes B)\,\mathrm{diag}(K_{pi})\left[\mathcal{A}_{\mathrm{pe}} \otimes I\right] y\end{aligned} \tag{55}$$

where $\underline{1}_N \in \mathbb{R}^{N\times 1}$ is a vector of ones. The derivative of (55) is given by

$$\begin{aligned}\ddot{x} = &-(I \otimes B)\,\mathrm{diag}(K_{pi})\left[(L_p + D_{\mathrm{pe}}) \otimes I\right] \dot{x} \\ &+ (I \otimes B)\,\mathrm{diag}(K_{pi})\left[\mathcal{A}_{\mathrm{pe}} \otimes I\right](\underline{1}_M \otimes \nu)\end{aligned}$$

where $\underline{1}_M \in \mathbb{R}^{M\times 1}$. If $\dot{x} = [(L_p + D_{\mathrm{pe}})^{-1}\mathcal{A}_{\mathrm{pe}} \otimes I](\underline{1}_M \otimes \nu)$, then we get $\ddot{x} = 0$, and this result is unique if matrix (52) is nonsingular. Furthermore, by properties of the Kronecker product and of the graph matrices L_p, D_{pe}, and $\mathcal{A}_{\mathrm{pe}}$, $[(L_p + D_{\mathrm{pe}})^{-1}\mathcal{A}_{\mathrm{pe}} \otimes I](\underline{1}_M \otimes \nu) = \underline{1}_N \otimes \nu$. Substitute the derivative $\dot{x} = \underline{1}_N \otimes \nu$ in (55) to obtain

$$\begin{aligned}0 = &-(I \otimes B)\,\mathrm{diag}(K_{pi})\left[(L_p + D_{\mathrm{pe}}) \otimes I\right] x \\ &+(I \otimes B)\,\mathrm{diag}(K_{pi})\left[\mathcal{A}_{\mathrm{pe}} \otimes I\right] y.\end{aligned}$$

Again, (51) is the unique solution of this equation. ■

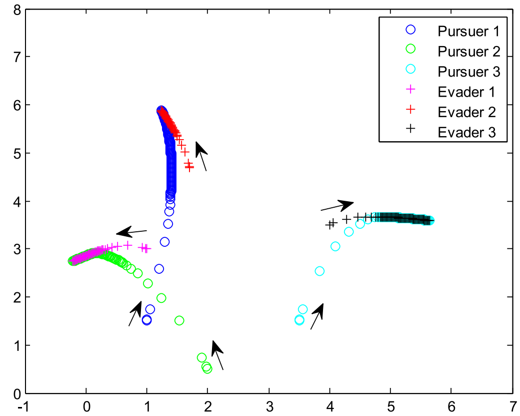

Fig. 1. MPE game with finite-time capture. The evaders separate from each other. The pursuers use Algorithm 1 to select a target.

VI. Simulation Results

Numerical simulations for finite-time intercept, asymptotic capture, and containment control are presented next.

A. Finite-Time Capture

Consider an MPE game with three pursuers and three evaders in $\mathbb{R}^2$ with single integrator dynamics, i.e., $A = 0$ and $B = I$ in (1) and (2), connected in a communication graph such that

$$L_p = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \quad L_e = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$\mathcal{A}_{\mathrm{pe}} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \mathcal{A}_{\mathrm{ep}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

A simulation of this system is performed, using performance indices (16) and (15) with $Q = 5I$, and input weighting matrices as in (37) and (38) with $r_p = 1$ and $r_e = 3$. Effectiveness of control policies (26) and (27) is tested. Furthermore, let the evaders try to separate from each other, besides escaping from the pursuers, to force the pursuers to separate as well. The pursuers use Algorithm 1 to select a target among the evaders. The result of this game is shown in Fig. 1. It can be noted that each pursuer achieves capture of its target.

B. Asymptotic Capture

Asymptotic minmax strategies are now tested via simulation for the same systems as in the simulation above. The evaders are now interested in maintaining their team cohesion and the pursuers do not select an individual target to chase. Select $R_p = I$ and $R_e = 3I$. To show the effect of the value of κ on the behavior of the agents, two cases are considered. First, a value of $\kappa = 3$ is taken. The result of this simulation is shown in Fig. 2, which shows that capture occurs, verifying the theoretical

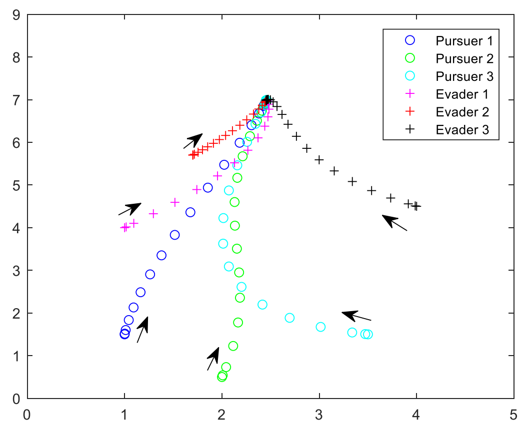

Fig. 2. Asymptotic behavior in an MPE game. A large κ makes the evaders attract each other. All agents converge into a single point.

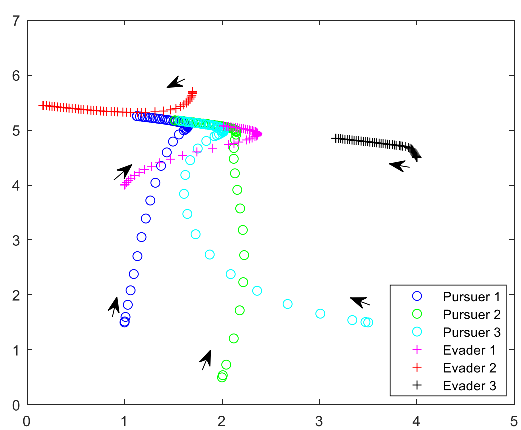

Fig. 3. Asymptotic behavior in an MPE game with small κ. The evaders have a low priority to remain together.

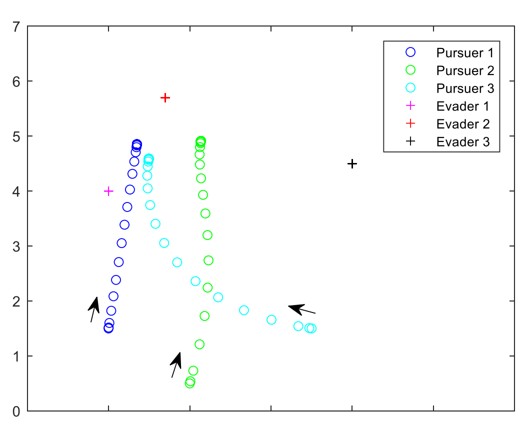

Fig. 4. Containment behavior of the agents. The evaders are static and the pursuers converge to their convex hull.

results. Then, a value of $\kappa = 1.2$ is used. This smaller value represents a lower priority of the evaders to remain together. Fig. 3 displays this result, where the evaders do not converge to a single point in the state space.

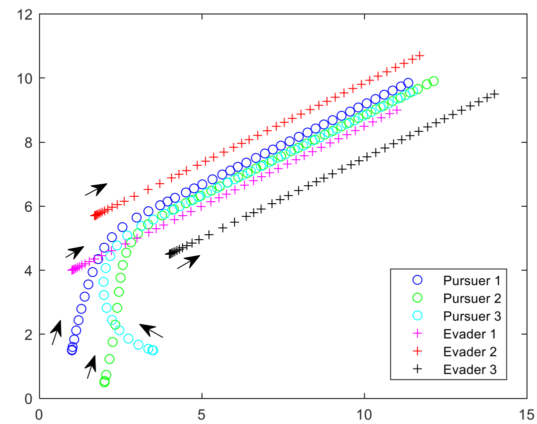

Fig. 5. Containment behavior with moving evaders. The evaders move in a formation with constant speed. The pursuers converge to their convex hull.

C. Containment Control

Using the same parameters as in the aforementioned asymptotic capture simulation, let the evaders remain static throughout the game. The behavior of the pursuers under these circumstances is displayed in Fig. 4. The pursuers can be seen converging to the convex hull of the positions of the evaders, as stated in Corollary 2.

Finally, let the evaders move in a formation with constant speeds, and let the pursuers use the control inputs (54). Take the vector $\nu = [2,\ 1]^T$ as the constant velocity of the evaders. Fig. 5 shows that the pursuers reach the convex hull of the positions of the evaders and maintain their positions thereafter.

VII. Conclusion

Performance of the players in MPE games on communication graphs was studied for performance indices that can include both cooperative and adversarial objectives among the agents. Nash equilibrium is guaranteed if the solutions of the coupled HJI equations exist. Otherwise, it is still possible to design the control policies of the agents to obtain a minimum guaranteed performance value, using minmax strategies. Conditions for capture were found to depend on the structure of the graph topologies and on different control design parameters, like the difference between the relative speeds of the teams or the inclination of the evaders of either staying together or escaping from the pursuers. Different emergent behaviors arise from changes in the goals of the players. Simulation plots show the differences between these behaviors, verifying the obtained results.

Acknowledgment

V. G. Lopez thanks the Mexican Council of Science and Technology (CONACYT) for its support.

References

[1] R. Isaacs, *Differential Games. A Mathematical Theory With Applications to Warfare and Pursuit, Control and Optimization*. New York, NY, USA: Wiley, 1965.

[2] F. L. Lewis, D. Vrabie, and V. L. Syrmos, *Optimal Control*, 3rd ed. Hoboken, NJ, USA: Wiley, 2012.

[3] A. Bryson and Y.-C. Ho, *Applied Optimal Control*. New York, NY, USA: Taylor & Francis, 1975.

[4] M. A. Foley and W. E. Schmitendorf, "A class of differential games with two pursuers versus one evader," *IEEE Trans. Autom. Control*, vol. 19, no. 3, pp. 239–243, Jun. 1974.

[5] E. Bakolas and P. Tsiotras, "Relay pursuit of a maneuvering target using dynamic Voronoi diagrams," *Automatica*, vol. 49, no. 9, pp. 2213–2220, Sep. 2012.

[6] W. Lin, Z. Qu, and M. A. Simaan, "Nash strategies for pursuit-evasion differential games involving limited observations," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 51, no. 2, pp. 1347–1356, Apr. 2015.

[7] J. S. Jang and C. J. Tomlin, "Control strategies in multi-player pursuit and evasion game," in *Proc. AIAA Guid., Navigation, Control Conf. Exhibit*, San Francisco, CA, USA, 2005, pp. 1–10.

[8] S. D. Bopardikar and F. Bullo, "On discrete-time pursuit-evasion games with sensing limitations," *IEEE Trans. Robot.*, vol. 24, no. 6, pp. 1429–1439, Dec. 2008.

[9] S. S. Kumkov, S. Le Menec, and V. S. Patsko, "Zero-sum pursuit-evasion differential games with many objects: Survey of publications," *Dyn. Games Appl.*, vol. 7, no. 4, pp. 609–633, Dec. 2017.

[10] M. M. Zavlanos and G. J. Pappas, "Distributed hybrid control for multiple-pursuer multiple-evader games," in *Proc. 10th Int. Conf. Hybrid Syst.: Comput. Control*, 2007, pp. 787–789.

[11] D. M. Stipanovic, A. Melikyan, and N. Hovakimyan, "Guaranteed strategies for nonlinear multi-player pursuit-evasion games," *Int. Game Theory Rev.*, vol. 12, no. 1, pp. 1–17, 2010.

[12] J. Ge, L. Tang, J. Reimann, and G. Vachtsevanos, "Suboptimal approaches to multiplayer pursuit-evasion differential games," in *Proc. AIAA Guid., Navigation, Control Conf. Exhibit*, Keystone, CO, USA, 2006, pp. 1–7.

[13] D. Li and J. B. Cruz, "Graph-based strategies for multi-player pursuit evasion games," in *Proc. 46th IEEE Conf. Decis. Control*, 2007, pp. 4063–4068.

[14] Y. Shoham and K. Leyton-Brown, *Multiagent Systems. Algorithmic, Game-Theoretic and Logical Foundations*. New York, NY, USA: Cambridge Univ. Press, 2008.

[15] W. Ren, R. W. Beard, and E. M. Atkins, "Information consensus in multivehicle cooperative control," *IEEE Control Syst.*, vol. 27, no. 2, pp. 71–82, Apr. 2007.

[16] F. L. Lewis, H. Zhang, K. Hengster-Movric, and A. Das, *Cooperative Control of Multi-Agent Systems. Optimal and Adaptive Design Approaches*. London, U.K.: Springer-Verlag, 2014.

[17] H. Zhang, F. L. Lewis, and Z. Qu, "Lyapunov, adaptive and optimal design techniques for cooperative systems on directed communication graphs," *IEEE Trans. Ind. Electron.*, vol. 59, no. 7, pp. 3026–3041, Jul. 2012.

[18] H. Zhang, Z. Li, Z. Qu, and F. L. Lewis, "On constructing Lyapunov functions for multi-agent systems," *Automatica*, vol. 58, pp. 39–42, Aug. 2015.

[19] Y. Cao, W. Yu, W. Ren, and G. Chen, "An overview of recent progress in the study of distributed multi-agent coordination," *IEEE Trans. Ind. Inform.*, vol. 9, no. 1, pp. 427–438, Feb. 2013.

[20] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1465–1476, Sep. 2004.

[21] K. G. Vamvoudakis, F. L. Lewis, and G. R. Hudas, "Multiagent differential graphical games: Online adaptive learning solution for synchronization with optimality," *Automatica*, vol. 48, no. 8, pp. 1598–1611, Aug. 2012.

[22] Z. Li, Z. Duan, W. Ren, and G. Feng, "Containment control of linear multi-agent systems with multiple leaders of bounded inputs using distributed continuous controllers," *Int. J. Robust Nonlinear Control*, vol. 25, no. 13, pp. 2101–2121, Sep. 2015.

[23] H. Liu, G. Xie, and L. Wang, "Necessary and sufficient conditions for containment control of networked multi-agent systems," *Automatica*, vol. 48, no. 7, pp. 1415–1422, Jul. 2012.

[24] W. Li, "A dynamics perspective of pursuit-evasion: Capturing and escaping when the pursuer runs faster than the agile evader," *IEEE Trans. Autom. Control*, vol. 62, no. 1, pp. 451–457, Jan. 2017.

[25] J. W. Brewer, "Kronecker products and matrix calculus in system theory," *IEEE Trans. Circuits Syst.*, vol. 25, no. 9, pp. 772–781, Sep. 1978.

[26] Z. Qu, *Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles*. New York, NY, USA: Springer-Verlag, 2009.

[27] Z. Qu and M. A. Simaan, "A design of distributed game strategies for networked agents," *IFAC Proc. Vol.*, vol. 42, no. 20, pp. 270–275, Sep. 2009.

Victor G. Lopez (S'17) received the B.S.E.E. degree from the Universidad Autonoma de Campeche, Campeche, Mexico, in 2010, and the M.S.E.E. degree, major in automatic control, from CINVESTAV, Mexico City, Mexico, in 2013. He is currently working toward the Ph.D. degree in automatic control at the University of Texas at Arlington, Arlington, TX, USA.

He was a Lecturer with the Western Technologic Institute of Superior Studies, Guadalajara, Mexico, in 2015. His research interests include cyber-physical systems, graphical games, distributed control, reinforcement learning, and robust control.

Frank L. Lewis (F'81) received the bachelor's degree in physics/electrical engineering and the M.S.E.E. degree from Rice University, Houston, TX, USA, in 1971, the M.S. degree in aeronautical engineering from the University of West Florida, Pensacola, FL, USA, in 1977, and the Ph.D. degree from Georgia Institute of Technology, Atlanta, GA, USA, in 1981.

He holds 7 U.S. patents, and has authored numerous journal special issues, 374 journal papers, and 20 books, some of which are used as university textbooks worldwide. His research interests include feedback control, intelligent systems, cooperative control systems, and nonlinear systems.

Dr. Lewis was the recipient of the Fulbright Research Award, NSF Research Initiation Grant, ASEE Terman Award, International Neural Network Society Gabor Award, U.K. Institute of Measurement & Control Honeywell Field Engineering Medal, IEEE Computational Intelligence Society Neural Networks Pioneer Award, AIAA Intelligent Systems Award, Outstanding Service Award from Dallas IEEE Section, and Texas Regents Outstanding Teaching Award for 2013. He was selected as Engineer of the Year by Fort Worth IEEE Section and listed in Fort Worth Business Press Top 200 Leaders in Manufacturing. He was the Founding Member of the Board of Governors of the Mediterranean Control Association, member of the National Academy of Inventors, Fellow of IFAC, Fellow of AAAS, Fellow of U.K. Institute of Measurement & Control, PE Texas, U.K. Chartered Engineer, UTA Distinguished Scholar Professor, UTA Distinguished Teaching Professor, and Moncrief-O'Donnell Chair at the University of Texas at Arlington Research Institute.

Yan Wan (SM'17) received the Ph.D. degree in electrical engineering from Washington State University, Pullman, WA, USA, in 2008, and then did postdoctoral training at the University of California, Santa Barbara, CA, USA.

She is currently an Associate Professor with the Electrical Engineering Department, University of Texas at Arlington, Arlington, TX, USA. From 2009 to 2016, she was an Assistant Professor and then an Associate Professor with the University of North Texas. She has authored or coauthored more than 130 publications and her research has led to successful technology transfer outcomes. Her research interests include the modeling, evaluation, and control of large-scale dynamical networks, cyber-physical system, stochastic networks, decentralized control, learning control, networking, uncertainty analysis, algebraic graph theory, and their applications to UAV networking, UAV traffic management, epidemic spread, complex information networks, and air traffic management.

Dr. Wan was the recipient of several prestigious awards, including the NSF CAREER Award, RTCA William E. Jackson Award, U.S. Ignite and GENI demonstration awards, IEEE WCNC Best Paper Award, UNT Early Career Award for Research and Creativity, and Tech Titan of the Future—University Level Award. She is currently the Vice President of IEEE ComSoc Fort Worth Chapter and Technical Committee Member of AIAA Intelligent Systems Society. She is also the Associate Editor for the *Transactions of the Institute of Measurement and Control*.

Edgar N. Sanchez (SM'93) received the B.S.E.E. degree, major in power systems, from Universidad Industrial de Santander, Bucaramanga, Santander, Colombia, in 1971, the M.S.E.E. degree, major in automatic control, from CINVESTAV-IPN, Mexico City, Mexico, in 1974, and the Dr. Ing. degree in automatic control from Institut Nationale Polytechnique de Grenoble, Grenoble, France, in 1980.

From 1971 to 1976, he was with different electrical engineering consulting companies in Bogota, Colombia. From January 1981 to November 1990, he was a Researcher with the Electrical Research Institute, Cuernavaca, Mexico. He was a Professor of Electrical Engineering graduate program, Universidad Autonoma de Nuevo Leon, Monterrey, Mexico, from December 1990 to December 1996. Since January 1997, he has been with CINVESTAV-IPN, Guadalajara, Mexico, as a Professor of Electrical Engineering graduate program. He has authored or coauthored 7 books and more than 150 technical papers in international journals and conferences. His research interests include neural networks and fuzzy logic as applied to automatic control systems.

Dr. Sanchez was the recipient of the USA National Research Council Award as a Research Associate with NASA Langley Research Center, Hampton, VA, USA (January 1985 to March 1987). He is also member of the Mexican National Research System (promoted to highest rank, III, in 2005), the Mexican Academy of Sciences, and the Mexican Academy of Engineering. He was also member of many international conferences (IPCs), both IEEE and IFAC ones.

Lingling Fan received the Ph.D. degree from the Beijing Jiaotong University, Beijing, China, in 2017.

She is currently a Lecturer with the School of Automation, Beijing Information Science and Technology University, Beijing, China. Her research interests include multiagent systems, pursuit-evasion, and data-driven control.