

Homework 5: Graphical Models, MDPs

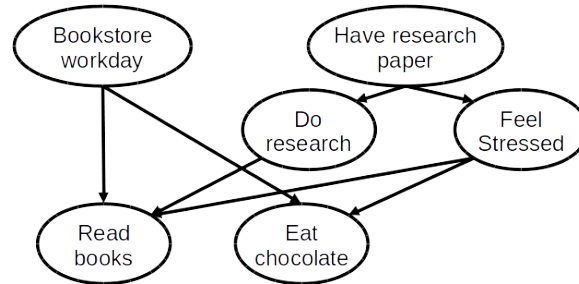
Introduction

There is a mathematical component and a programming component to this homework. Please submit your **tex, PDF, and Python files** to Canvas, and push all of your work to your GitHub repository. If a question requires you to make any plots, please include those in the writeup.

Bayesian Networks [7 pts]

Problem 1

In this problem we explore the conditional independence properties of a Bayesian Network. Consider the following Bayesian network representing a student's work. Each random variable is binary (true/false).



The random variables are:

- Have research paper: Does the student have a research paper?
- Do Research: Is the student doing research?
- Feel Stressed: Is the student feeling stressed?
- Bookstore Workday: Is the student working at the bookstore?
- Read Books: Is the student reading a book?
- Eat Chocolate: Is the student eating chocolate?

For the following questions, $A \perp B$ means that events A and B are independent and $A \perp B|C$ means that events A and B are independent conditioned on C. Use the concept of d-separation to answer the questions and show your work.

1. Is Bookstore Workday \perp Have research paper? If NO, give intuition for why.
2. Is Bookstore Workday \perp Have research paper | Read Books? If NO, give intuition for why.
3. Is Do Research \perp Eat Chocolate? If NO, give intuition for why.
4. Is Do Research \perp Eat Chocolate | Feel Stressed? If NO, give intuition for why.
5. Suppose the student has done some mindfulness exercises to avoid stress eating. Now, when they are stressed, they read (fun) books but don't eat chocolate. Draw the modified network.
6. For this modified network, is Do Research \perp Eat Chocolate? If NO, give an intuition why. If YES, describe what observations (if any) would cause them to no longer be independent.

Solution

Kalman Filters [7 pts]

Problem 2

In this problem, you will implement a one-dimensional Kalman filter. Assume the following dynamical system model:

$$\begin{aligned}z_{t+1} &= z_t + \epsilon_t \\x_t &= z_t + \gamma_t\end{aligned}$$

where z are the hidden variables and x are the observed measurements. The random variables ϵ and γ are drawn from the following Normal distributions:

$$\begin{aligned}\epsilon_t &\sim N(\mu_\epsilon, \sigma_\epsilon) \\ \gamma_t &\sim N(\mu_\gamma, \sigma_\gamma)\end{aligned}$$

where $\mu_\epsilon = 0$, $\sigma_\epsilon = 0.05$, $\mu_\gamma = 0$ and $\sigma_\gamma = 1.0$

You are provided with the observed data x and the hidden data z in `kf-data.csv`, and the prior on the first hidden state is $p(z_0) = N(\mu_p, \sigma_p)$ where $\mu_p = 5$ and $\sigma_p = 1$

1. The distribution $p(z_t|x_0...x_t)$ will be Gaussian $N(\mu_t, \sigma_t^2)$. Derive an iterative update for the mean μ_t and variance σ_t^2 given the mean and variance at the previous time step (μ_{t-1} and σ_{t-1}^2).
2. Implement this update and apply it to the observed data above (do not use the hidden data to find these updates). Provide a plot of μ_t over time as well as a $2\sigma_t$ interval around μ_t (that is $\mu_t \pm 2\sigma_t$). Does the Kalman filter “catch up” to the true hidden object?
3. Repeat the same process but change the observation at time $t = 10$ to $x_{t=10} = 10.2$ (an extreme outlier measurement). How does the Kalman filter respond to this outlier?
4. Comment on the misspecification of dynamical system model for these data. Based on the previous two parts, how does this misspecification affect the predictions?

Solution

Problem 3 (Explaining Away, 7 pts)

In this problem, you will carefully work out a basic example with the explaining away effect. There are many derivations of this problem available in textbooks. We emphasize that while you may refer to textbooks and other online resources for understanding how to do the computation, you should do the computation below from scratch, by hand.

We have three binary variables, rain r , grass-wet g , and sprinkler s . The conditional probability tables look like the following:

$$\begin{aligned}p(r = 1) &= 0.25 \\p(s = 1) &= 0.5 \\p(g = 1|r = 0, s = 0) &= 0 \\p(g = 1|r = 1, s = 0) &= .75 \\p(g = 1|r = 0, s = 1) &= .75 \\p(g = 1|r = 1, s = 1) &= 1\end{aligned}$$

1. You check on the sprinkler without checking on the rain or the grass. What is the probability that it is on?
2. You notice it is raining and check on the sprinkler without checking the grass. What is the probability that it is on?
3. You notice that the grass is wet and go to check on the sprinkler (without checking if it is raining). What is the probability that it is on?
4. You notice that it is raining and the grass is wet. You go check on the sprinkler. What is the probability that it is on?
5. What is the explaining away effect above?

Solution

- Name:
- Email:
- Collaborators:
- Approximately how long did this homework take you to complete (in hours):