Example 1.9 and Example 1.10.

Example 1.9 Moving Averages and Filtering

We might replace the white noise series w_t by a *moving average* that smooths the series. For example, consider replacing w_t in Example 1.8 by an average of its current value and its immediate neighbors in the past and future. That is, let

$$v_t = \tfrac{1}{3}\left(w_{t-1} + w_t + w_{t+1}\right), \tag{1.1}$$

which leads to the series shown in the lower panel of Figure 1.8. Inspecting the series shows a smoother version of the first series, reflecting the fact that the slower

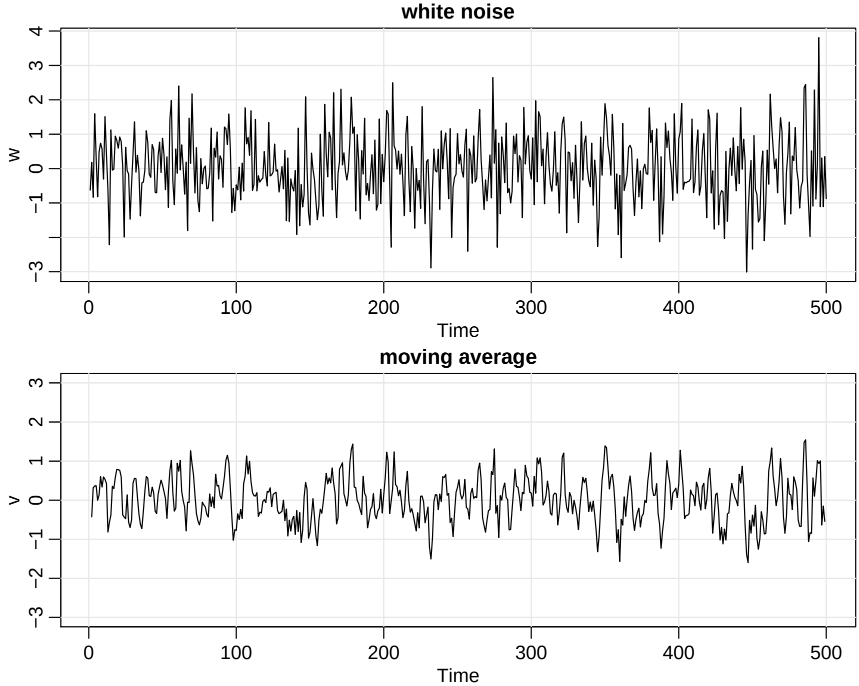

Fig. 1.8. *Gaussian white noise series (top) and three-point moving average of the Gaussian white noise series (bottom).*

oscillations are more apparent and some of the faster oscillations are taken out. We begin to notice a similarity to the SOI in Figure 1.5, or perhaps, to some of the fMRI series in Figure 1.6.

A linear combination of values in a time series such as in (1.1) is referred to, generically, as a filtered series; hence the command `filter` in the following code for Figure 1.8.

```
w = rnorm(500,0,1)                        # 500 N(0,1) variates
v = filter(w, sides=2, filter=rep(1/3,3))  # moving average
par(mfrow=c(2,1))
plot.ts(w, main="white noise")
plot.ts(v, ylim=c(-3,3), main="moving average")
```

The speech series in Figure 1.3 and the Recruitment series in Figure 1.5, as well as some of the MRI series in Figure 1.6, differ from the moving average series because one particular kind of oscillatory behavior seems to predominate, producing a sinusoidal type of behavior. A number of methods exist for generating series with this quasi-periodic behavior; we illustrate a popular one based on the autoregressive model considered in Chapter 3.

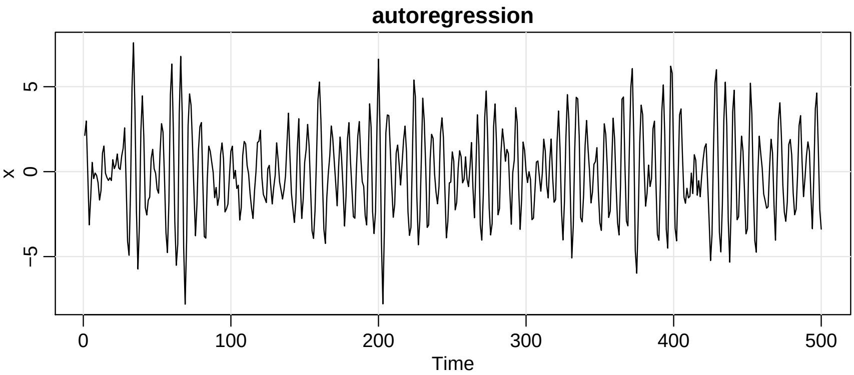

Fig. 1.9. *Autoregressive series generated from model* (1.2)*.*

Example 1.20 Stationarity of a Moving Average

The three-point moving average process of Example 1.9 is stationary because, from Example 1.13 and Example 1.17, the mean and autocovariance functions $\mu_{vt} = 0$, and

$$\gamma_v(h) = \begin{cases} \frac{3}{9}\sigma_w^2 & h = 0, \\ \frac{2}{9}\sigma_w^2 & h = \pm 1, \\ \frac{1}{9}\sigma_w^2 & h = \pm 2, \\ 0 & |h| > 2 \end{cases}$$

are independent of time t, satisfying the conditions of Definition 1.7.

The autocorrelation function is given by

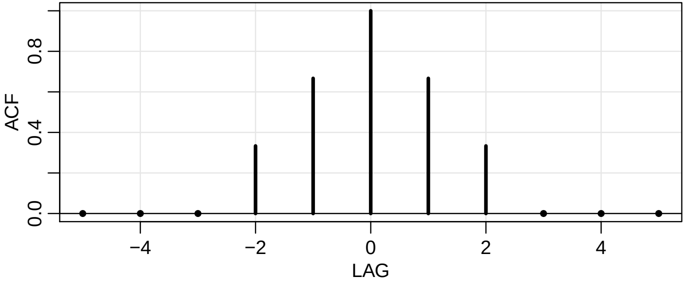

Fig. 1.12. *Autocorrelation function of a three-point moving average.*

$$\rho_v(h) = \begin{cases} 1 & h = 0, \\ \frac{2}{3} & h = \pm 1, \\ \frac{1}{3} & h = \pm 2, \\ 0 & |h| > 2. \end{cases}$$

Figure 1.12 shows a plot of the autocorrelations as a function of lag h. Note that the ACF is symmetric about lag zero.