Pregunta 1:

Demuestre que la función de autocovarianza $\gamma_X(t+h,t)$ de un MA(1) es la siguiente:

First-Order Moving Average or MA(1) Process

Consider the series defined by the equation

$$X_t = Z_t + \theta Z_{t-1}, \quad t = 0, \pm 1, \ldots, \tag{1.4.1}$$

where $\{Z_t\} \sim \mathrm{WN}\left(0, \sigma^2\right)$ and θ is a real-valued constant. From (1.4.1) we see that $EX_t = 0$, $EX_t^2 = \sigma^2(1+\theta^2) < \infty$, and

$$\gamma_X(t+h,t) = \begin{cases} \sigma^2\left(1+\theta^2\right), & \text{if } h = 0, \\ \sigma^2\theta, & \text{if } h = \pm 1, \\ 0, & \text{if } |h| > 1. \end{cases}$$

Pregunta 2: Ejercicio 1.4 de Brockwell and Davis (2016).

1.4 Let $\{Z_t\}$ be a sequence of independent normal random variables, each with mean 0 and variance σ^2, and let a, b, and c be constants. Which, if any, of the following processes are stationary? For each *stationary* process specify the mean and autocovariance function.

a. $X_t = a + bZ_t + cZ_{t-2}$

b. $X_t = Z_t \cos(ct) + Z_{t-1} \sin(ct)$

c. $X_t = Z_0 \cos(ct)$

d. $X_t = Z_t Z_{t-1}$

Pregunta 3: Ejercicio 1.5 de Brockwell and Davis (2016).

1.5 Let $\{X_t\}$ be the moving-average process of order 2 given by

$$X_t = Z_t + \theta Z_{t-2},$$

where $\{Z_t\}$ is WN(0, 1).

a. Find the autocovariance and autocorrelation functions for this process when $\theta = 0.8$.

b. Compute the variance of the sample mean $(X_1+X_2+X_3+X_4)/4$ when $\theta = 0.8$.

c. Repeat (b) when $\theta = -0.8$ and compare your answer with the result obtained in (b).

Pregunta 4: Ejercicio 1.8 de Brockwell and Davis (2016).

1.8 Let $\{Z_t\}$ be IID $\mathrm{N}(0,1)$ noise and define

$$X_t = \begin{cases} Z_t, & \text{if } t \text{ is even} \\ \left(Z_{t-1}^2 - 1\right)/\sqrt{2}, & \text{if } t \text{ is odd} \end{cases}$$

a. Show that $\{X_t\}$ is $\mathrm{WN}(0,1)$ but not $\mathrm{iid}(0,1)$ noise.

b. Find $E\left(X_{n+1} \mid X_1, \ldots, X_n\right)$ for n odd and n even and compare the results.

Pregunta 5: Ejercicio 1.21 de Shumway and Stoffer (2016)

1.21

a. Simulate a series of $n = 500$ moving average observations as in *Example 1.9* and compute the sample ACF, $\hat{\rho}(h)$, to lag 20. Compare the sample ACF you obtain to the actual ACF, $\rho(h)$. [Recall *Example 1.20*.]

b. Repeat part (a) using only $n = 50$. How does changing n affect the results?