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# Handling informative dropout in longitudinal analysis of health-related quality of life:

Application on data from the oesophageal cancer clinical trial  
PRODIGE5/ACCORD17

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ISOQOL, Dublin

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## Health-related quality of life (HRQoL) in cancer clinical trials

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- ▶ Health-related quality of life = **PROs** used to measure the clinical benefits of new therapies

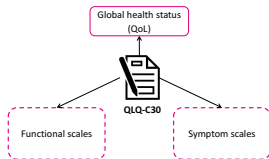
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- ▶ In clinical trials = more and more used as a **Secondary or a Co-Primary endpoint**
  - ↔ HRQoL assessed using questionnaires at different visit times (treatment and follow-up)

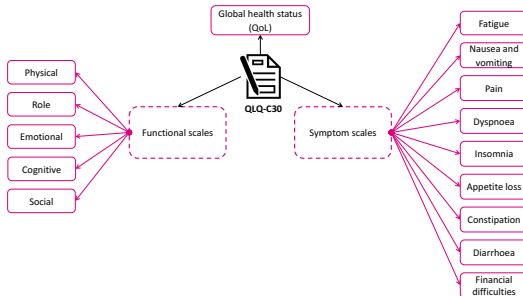
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- ▶ In clinical trials = more and more used as a **Secondary or a Co-Primary endpoint**
  - ↪ HRQoL assessed using questionnaires at different visit times (treatment and follow-up)
- ▶ Europe : **EORTC QLQ-C30** = 5 functional scales + 9 symptom scales + a global health status
  - ↪ **Continuous score** from 0 to 100 for each scale
  - ↪ To model score **evolution over time**



## Longitudinal analysis of the HRQoL score in clinical trials

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### Linear Mixed Model

For patient  $i$  and observation  $j$ , the score is as follows:

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where  $\begin{pmatrix} b_0 \\ b_1 \end{pmatrix} \sim \mathcal{N}(0, \Sigma)$  with  $\Sigma = \begin{pmatrix} \sigma_{b_0}^2 & \sigma_{b_0 b_1} \\ \sigma_{b_0 b_1} & \sigma_{b_1}^2 \end{pmatrix}$  and  $\varepsilon_{ij} \sim \mathcal{N}(0, \sigma^2)$

- **Fixed effects:**  $\beta^T X_{ij}$  model the **average trajectory**
- **Random effects:**  $b_i^T Z_{ij}$  represent the **individual deviation** from the average

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### Coefficients interpretation

- $\beta_1$ : **slope** in the standard arm
- $\beta_3$ : **interaction effect** = difference of the score evolution in the experimental arm compared to the standard arm



# Illustration on the PRODIGE5/ACCORD17 clinical trial

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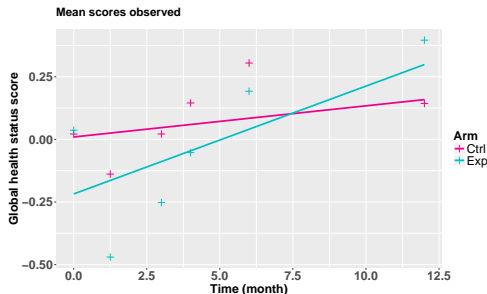
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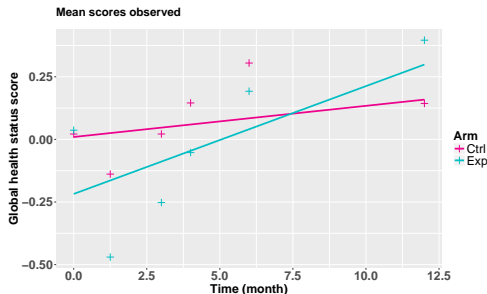
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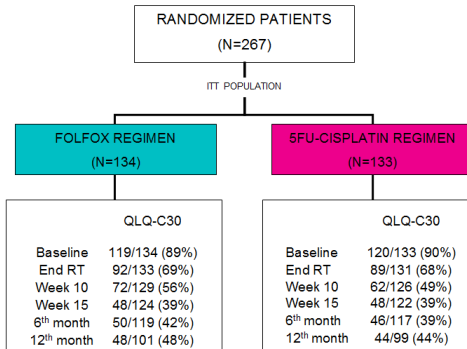
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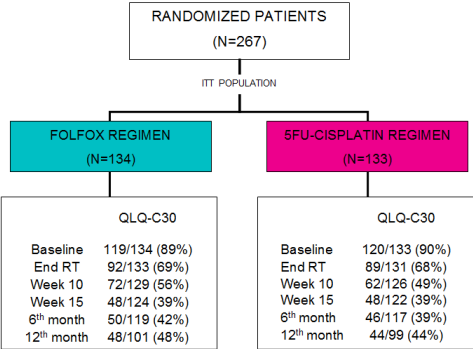


- ▶ **HRQoL improvement** over time ( $\beta_1 = 0.012$ ,  $p = 0.307$ )  
 $\hookrightarrow$  **positive slope**
- ▶ **HRQoL improvement** more important in the experimental arm ( $\beta_3 = 0.031$ ,  $p = 0.076$ )  
 $\hookrightarrow$  **positive arm effect**

# PRODIGE5/ACCORD17 clinical trial: compliance



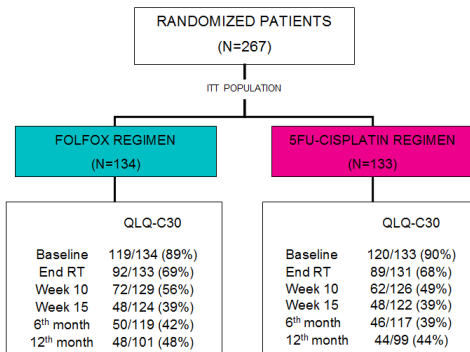
# PRODIGE5/ACCORD17 clinical trial: compliance



## Issue

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## Issue

- ▶ Estimations of the Linear Mixed Model are robust only if missing data are non informative.
- ↪ The **objective** was to explore two types of models accounting for potentially informative missing data.



## Patterns of missing data

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Two patterns of missing data:

Patient	Visit 1	Visit 2	Visit 3	Visit 4	Visit 5
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- **Intermittent** missing data  $\Rightarrow$  score is missing but subsequent score data can be observed

Patient	Visit 1	Visit 2	Visit 3	Visit 4	Visit 5
1	66.7	×	66.7	50.0	50.0
2	33.3	41.7	×	×	33.3
3	×	75.0	×	83.3	50.0

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Two patterns of missing data:

- ▶ **Intermittent** missing data  $\Rightarrow$  score is missing but subsequent score data can be observed
- ▶ **Monotone** missing data = **dropout**  $\Rightarrow$  no observations are made after a certain time point
  - $\hookrightarrow$  Generally corresponds to patient's dropout

Patient	Visit 1	Visit 2	Visit 3	Visit 4	Visit 5
1	66.7	×	66.7	50.0	50.0
2	33.3	41.7	×	×	33.3
3	×	75.0	×	83.3	50.0
4	83.3	100	66.7	66.7	×
5	91.7	25.0	25.0	×	×
6	66.7	66.7	×	×	×
7	50.0	×	×	×	×

# Mechanisms of missing data

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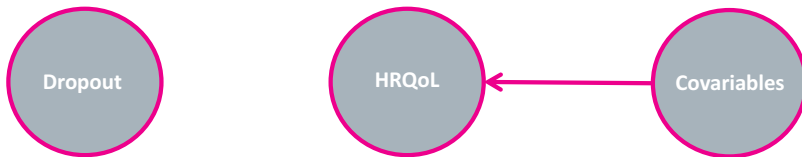
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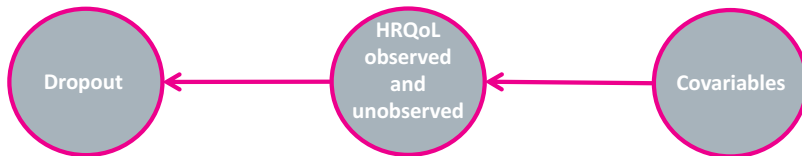
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↔ Missingness depends on the observed score

# Mechanisms of missing data

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## Missing Completely at Random (MCAR)

↪ Missingness is unrelated to the score

## Missing at Random (MAR)

↪ Missingness depends on the observed score

## Missing Not at Random (MNAR)

↪ The missingness is likely to be related to the **observed and unobserved** score

*Typically, if a patient does not complete a questionnaire because he/she is too tired, resulting in a missing score for the scale « fatigue », it is informative missing data.*

# Accounting for informative dropout in longitudinal analysis of the HRQoL score

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Model jointly the score  $Y_i$  and the dropout variable  $D_i$

**Two decompositions of the joint distribution** are possible according to the conditioning:

$$f_{\theta\psi}(Y_i, D_i) = \left\{ \right.$$

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We are interested in :

1. Distribution of **the score  $Y_i$  given the dropout  $D_i$**   $\times$  marginal distribution of the dropout  
 $\hookrightarrow$  **Pattern Mixture Model**

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 $\hookrightarrow$  **Pattern Mixture Model**
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# Pattern Mixture Model: Definition

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## Definition

$$f_{\theta\psi}(Y_i, D_i) = f_{\theta}(Y_i \mid D_i) \times f_{\psi}(D_i)$$

⇒ **A mixture of models** for the score given the dropout weighed by the dropout distribution

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▶ Multinomial model for  $f(D_i)$ :

▶ Mixed Model for  $f(Y_i \mid D_i)$ :

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To define two models :

- ▶ **Multinomial model for  $f(D_i)$ :** The probability of each dropout after time  $j$  is simply estimated by the **proportion** of patients which drop out after time  $j$
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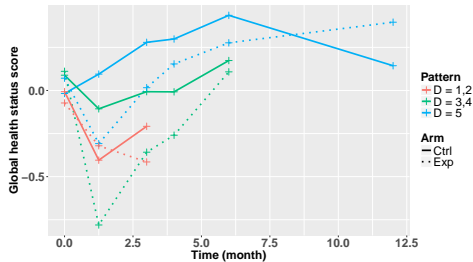
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# Pattern-Mixture Model: Application

Mean scores observed



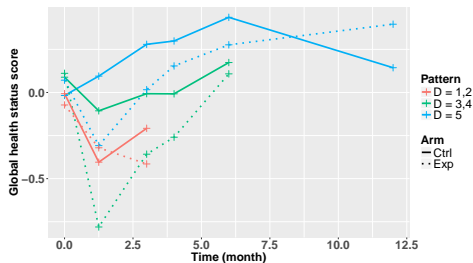
First step: define 3 patterns

- ▶ Early dropout
- ▶ Medium dropout
- ▶ Late dropout



# Pattern-Mixture Model: Application

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► Early dropout

$$\pi^{(1,2)} = 0.34$$

► Medium dropout

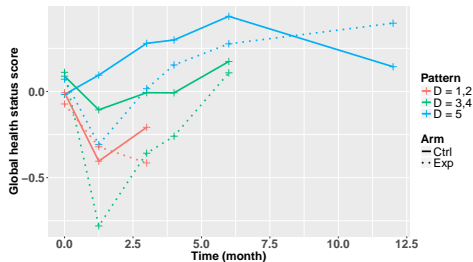
$$\pi^{(3,4)} = 0.30$$

► Late dropout

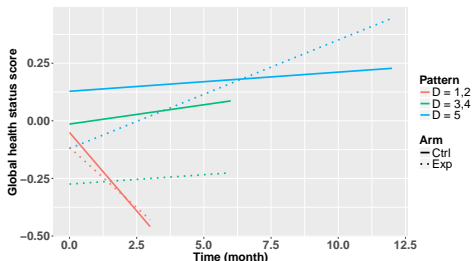
$$\pi^{(5)} = 0.36$$

# Pattern-Mixture Model: Application

Mean scores observed



Predicted mean score in each pattern



## First step: define 3 patterns

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## Second step: Mixed Models

We model the score in each pattern

$$(Y_{ij} \mid D = 1, 2) = X_{ij}\beta^{(1,2)} + Z_{ij}b_i + \varepsilon_{ij}$$

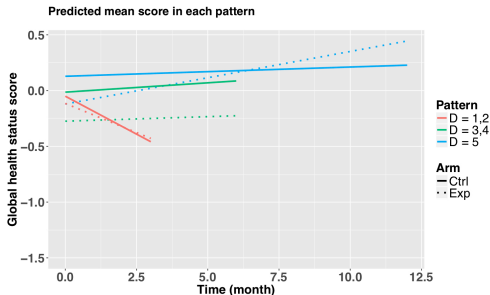
$$(Y_{ij} \mid D = 3, 4) = X_{ij}\beta^{(3,4)} + Z_{ij}b_i + \varepsilon_{ij}$$

$$(Y_{ij} \mid D = 5) = X_{ij}\beta^{(5)} + Z_{ij}b_i + \varepsilon_{ij}$$

# Pattern Mixture Model: Results

## Slope and interaction effect in each pattern

	<i>D</i> = 1,2		<i>D</i> = 3,4		<i>D</i> = 5	
	Est. (SE)	p	Est. (SE)	p	Est. (SE)	p
$\beta_1$	-0.136 (0.108)	0.182	0.017 (0.036)	0.817	0.008 (0.014)	0.545
$\beta_3$	0.032 (0.137)	0.958	-0.009 (0.054)	0.379	0.039 (0.019)	0.047



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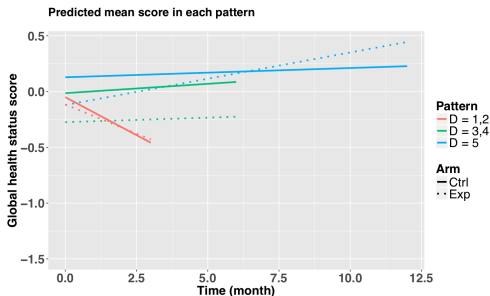
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Compare 2 treatments  $\Rightarrow$  Marginal parameters that is, irrespective of the pattern

Estimations in each pattern are **weighted** by the probability of dropout in each pattern:

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$\hookrightarrow$  **Extrapolation** = the score has the same trajectory after the dropout, that is, at times for which there is no available data.



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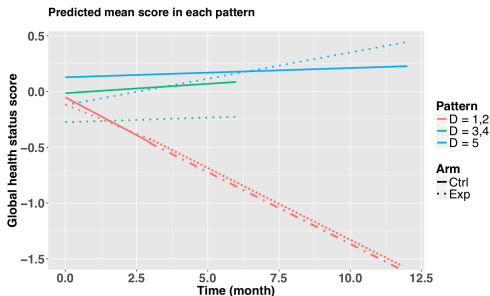
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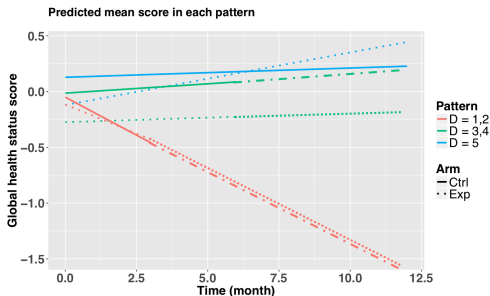
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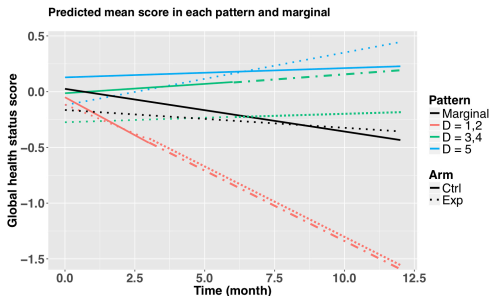
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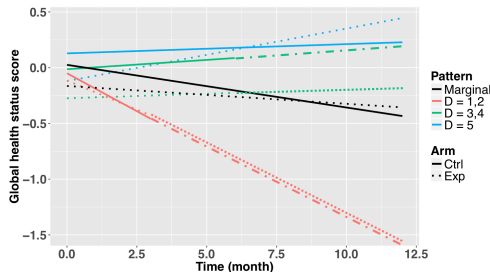
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## Pattern Mixture Model

Predicted mean score in each pattern and marginal





## Selection Model: Definition

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### Definition

$$Y_i = (Y_i^{\text{mis}}, Y_i^{\text{obs}})$$

$$f_{\theta\psi}(Y_i, D_i) = f_{\theta}(Y_i) \times f_{\psi}(D_i \mid Y_i)$$

⇒ the dropout  $D_i$  depends directly on the score  $Y_i$  observed

↔ **To test** the missingness mechanism

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$\Rightarrow$  the dropout  $D_i$  depends directly on the score  $Y_i$  observed **and unobserved**.

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$$Y_{ij} = \beta^T X_{ij} + b_i^T Z_{ij} + \varepsilon_{ij}$$

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► **Logistic regression for  $f(D_i \mid Y_i)$  :** The conditionnal probability for dropout at occasion  $j$  is:

$$\text{logit } \mathbb{P}(D_i = j \mid D_i \geq j, Y_{ij}, Y_{i,j+1}) = \psi_0 + \psi_1 Y_{ij} + \psi_2 Y_{i,j+1}$$

where  $Y_{i,j+1}$  is the unobserved current score

↔ **To test** the missingness mechanism

## Selection Model: Results

---

### Testing the $\psi$ parameters ( $\mathcal{H}_0 : \psi = 0$ )

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Selection Model			
	Estimation	SE	$p$
$\psi_0$	-0.063	0.072	0.383
$\psi_1$	0.001	0.087	0.990
$\psi_2$	0.001	0.117	0.992
$\beta_1$	0.019	0.027	0.486
$\beta_3$	0.035	0.032	0.266

- The assumption of **completely random dropout** is acceptable testing the  $\psi$  parameters.

## Summarising the results of the 3 models

	Selection Model			Linear Mixed Model			Pattern Mixture Model		
	Estimation	SE	p	Estimation	SE	p	Estimation	SE	p
$\beta_1$	0.019	0.027	0.486	0.012	0.012	0.307	-0.038	0.038	0.318
$\beta_3$	0.035	0.032	0.266	0.031	0.017	0.076	0.022	0.049	0.649
$\psi_0$	-0.063	0.072	0.383	—	—	—	—	—	—
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$\psi_2$	0.001	0.117	0.992	—	—	—	—	—	—

In comparison with the Linear Mixed Model:

► **Selection Model:**

- ↪ Estimations of the fixed effects are **similar** to the Linear Mixed Model
- ↪ Missing data are non **informative** according to the test of the  $\psi$  parameters

► **Pattern Mixture Model:**

- ↪ **Differences:** negative slope and a trend to a deterioration
- ↪ Caused by the **extrapolation** step

**These conclusions are valid only if the model assumptions are valid.**

## Conclusion

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All models for non-informative data require the analyst to make **strong assumptions**:

### Selection Model



Model directly the quantity of interest (the marginal score)



Test missingness mechanism (MNAR/MAR/MCAR)



Untestable = **normality assumption**

↔ score observed and *unobserved* is assumed to be normally distributed

### Pattern Mixture Model



To obtain the marginal score ⇒ extrapolation of linear trends



Description of the HRQoL according to the dropout pattern



Untestable = **extrapolation**  
↔ quality of life of a patient evolves in the same way after dropout

Both of these methods assume **untestable assumptions**