APM 6334 - Latex Documentation of Final version 1

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1 Problem Description

MATLAB code for solving Poisson equation on the domain $\Omega = (0,1)^2$ using the 9-point stencil from question 1.

$$-\triangle u = f \quad \text{in } \Omega = (0,1)^2,$$
$$u = g \quad \text{on } \Gamma = \partial\Omega$$

2 The Usage of the code

The complete list of files:

1. fdmD9.m
2. exactD9_fct.m
3. rhsD9_fct.m

2.1 Refine

Input: l, helps create 2D mesh between 0 and 1 for both x and y. Increasing l, refines the mesh.

2.2 ExactD9_fct and rhsD9_fct

These functions contain the exact manufactured solutions and the source terms, respectively. Inputs:

- x: a vector of points where the evaluation happens (x component).
- y: a vector of points where the evaluation happens (y component).
- example: an integer to separate the different cases
  The existing test cases.

1. $u = sin(\pi x)sin(\pi y)$
2. $u = x(1-x)y^2(1-y)$

   To implement additional test cases, both of these files need to be modified.

- fourth_order (only for rhsD9_fct) parameter help you choose fourth-order or second-order
  Output: the evaluated function value(s). Could be a scalar or a vector, depending on the inputs x and y.

2.3 fdmD9

The main file to run.

Inputs:

- l = number of sub-intervals in the x direction and y direction.
- example: an integer to separate the different cases 1
- fourth_order (only for rhsD9_fct) parameters help you choose a fourth-order scheme or a second-order scheme. $fourth\_order = 0$ implies second order convergence scheme and $fourth\_order = 1$ implies 4th order convergence scheme.
- Two plots that is the exact and the numerical solution in different figures
- the infinite norm of the error

3 Possible improvements

1. The parameters of the pcg solver should be inputs.

4 Convergence tests

Demonstrate second-order and fourth-order convergence.

4.1 Second-order convergence

N	$\|\|u - u_h\|\|_\infty$	rate
10	0.0165577	
20	0.004119	2.0071
40	0.0010285	2.0018
80	2.5705e-04	2.0004
160	6.4257e-05	2.0001

$u(x, y) = sin(\pi x)sin(\pi y)$

4.2 Fourth-order convergence

N	$\|\|u - u_h\|\|_\infty$	rate
10	1.6397e-04	
20	1.0172e-05	4.0108
40	6.3457e-07	4.0027
80	3.9642e-08	4.0007
160	2.4770e-09	4.0004
$u(x,y) = sin(\pi x)sin(\pi y)$		