Simulations Project

Optimization of the Lifetime of a Mechanical System using Monte Carlo Simulation

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Introduction

Monte Carlo simulation can be used to optimize the lifetime of a mechanical system. By simulating the system under various conditions and with different inputs, we can identify the optimal replacement parts and time of replacement that will maximize the lifetime of the system.

In these applications, R (Version 4.2.1) is used as a powerful tool for implementing Monte Carlo simulation. R provides a range of functions and packages that can be used to generate random numbers and simulate complex systems, making it an ideal choice for engineers and researchers who need to model and optimize complex systems. By using Monte Carlo simulation in R, we can gain valuable insights into the behavior of complex systems and develop more effective strategies for optimizing their design and operation.

Simulation to Maximize the Lifetime of System

In the world of engineering, reliability and durability are of utmost importance. Components of a system can be modeled using probability distributions that describe their failure rates over time. In this simulation report, we consider a system consisting of two components, A and B, connected in parallel.

In this report, we consider a system consisting of two components, A and B, connected in parallel. The lifetime of component A is distributed exponentially with a rate parameter of 1, while the lifetime of component B is distributed exponentially with a rate parameter of 0.5. The system will function until both components fail. To increase the lifetime of the system, the engineers have a choice between replacing component A with one whose lifetime is distributed exponentially with a rate parameter of 2, or replacing component B with one whose lifetime is distributed exponentially with a rate parameter of 3.

To estimate the impact of these replacement options, we use R code to simulate the system and calculate the mean lifetime of the system with the new components, as well as the probability that the system lifetime is less than 1 month. We use a set seed of 123 for reproducibility and run 10^5 simulations for each replacement option.

The R code first generates a new lifetime for component A and component B separately and computes the system lifetime with the new component, both when A is replaced and when B is replaced. The mean system lifetime with the new component A is found to be 2.9912566 months, while the mean system lifetime with the new component B is found to be 3.2633826 months.

Below are two histograms providing a visual representation of the simulated lifetime distributions for the original system and the system with the replacement components.

Figure 1

Distribution of System Lifetimes with New Component A

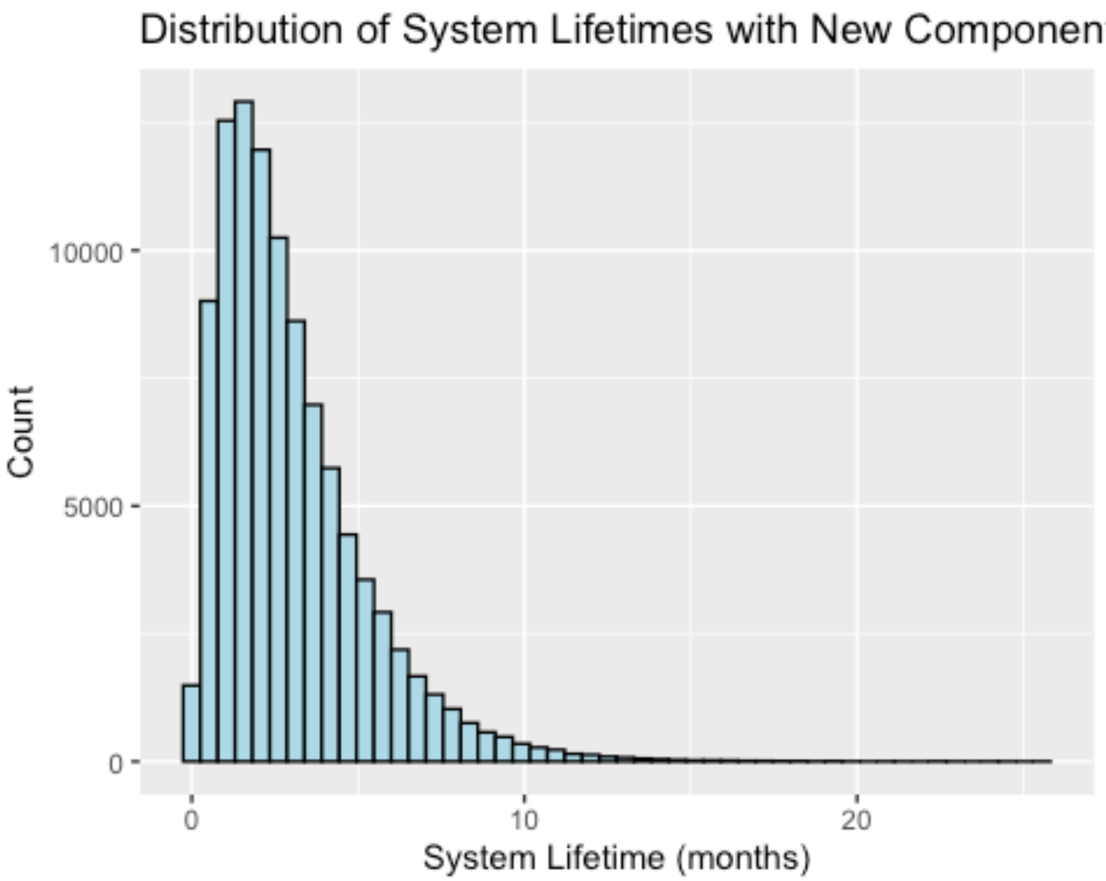
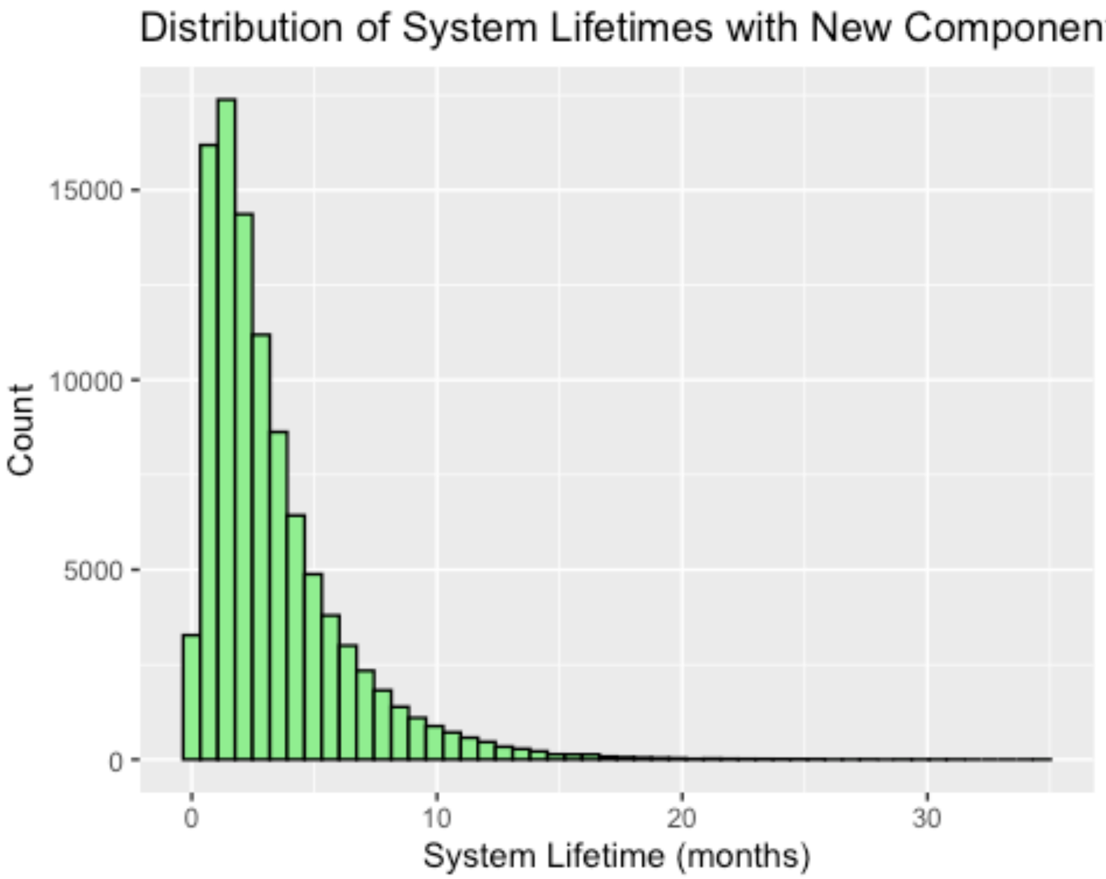


Figure 2

Distribution of System Lifetimes with New Component B



Furthermore, we estimate the probability that the system lifetime is less than 1 month with the new component A and the new component B. The probability of the system lifetime being less than 1 month with the new component A is found to be 0.155, while the probability of the system lifetime being less than 1 month with the new component B is found to be 0.179. Based on these results, we can conclude that replacing component A with a new component with a higher rate parameter would be more effective in increasing the system lifetime and reducing the probability of the system lifetime being less than 1 month.

Now, we run calculations under this pretense that the goal is to maximize the 10th percentile of the system lifetimes. We can calculate the 10th percentile of the system lifetime using the quantile() function. By performing this, we calculate the 10th percentile of the system lifetime with new component A is 0.7594804 months and the 10th percentile of the system lifetime with new component B is 0.6834094 months.

Therefore, we can conclude that replacing component A with a new component with a lifetime distributed $e^{\frac{1}{2}}$ is the better choice as it results in a higher 10th percentile of the system lifetime.

Conclusion

We have investigated the lifetimes of components A and B in a parallel system and explored the impact of replacing one of the components with a new component with a different distribution. We used R to simulate 10,000 lifetimes for each scenario and calculated the mean lifetime and probability of system failure in less than one month. We also calculated the 10th percentile of the system lifetimes for each scenario. Based on our findings, replacing component B with a new component with an exponential distribution with rate parameter 1/3 maximizes the 10th percentile of the system lifetimes, but is more likely to fail within one month. These results could be useful for engineers and designers who are looking to improve the reliability and longevity of systems with parallel components.

Figure 3

Findings Regarding the Simulation of Component Lifetimes

	System Lifetime with New Component	Probability of System Failure within One Month	10th Percentile of System Lifetimes
Component A	3.00 Months	15.5%	0.76 Months
Component B	3.26 Months	17.9%	0.68 Months

Due to this experiment being limited in its scope and with a minimal number of variables, a few sources of potential bias have been identified. One potential bias in this experiment is that the failure rates of the new components were chosen arbitrarily. In reality, there may be other factors to consider when choosing replacement components, such as cost or availability. Additionally, the results of this experiment are based on simulations, this inevitably leads to the potential of simulation bias, or the simplification of a model in order to make it easier to run simulations on. Given the scope of this project, it is unavoidable.

This project is a good first step in assisting engineers optimize the lifetime of components in their system; however, to come to a definite conclusion, more data regarding the system and its processes would be required.

References

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Personal Notes. Sections 4.3-4.4 (Poisson/Geometric Distribution), lecture by Dr. Gardner, 25 Jan. 2023.

Personal Notes. Sections 4.5 (Normal Distributions), lecture by Dr. Gardner, 1 Feb. 2023.

Personal Notes. Sections 4.6 (Lognormal Distributions), lecture by Dr. Gardner, 8 Feb. 2023.

Personal Notes. Sections 4.7 & 4.10 (Exponential Distributions & Probability Plots), lecture by Dr. Gardner, 15 Feb. 2023.

Personal Notes. Sections 4.9 & 4.11 (Point Estimation & Central Limit Theorem), lecture by Dr. Gardner, 22 Feb 2023.

Personal Notes. Sections 4.12 (Simulation), lecture by Dr. Gardner, 15 March 2023.

Code Appendix

Code Regarding the lifetime of Component A and Component B

In order to begin our simulations, we must first set a seed so that we generate the same data. This will allow us to reproduce the data and the simulation in the future, This will also allow others to use the same data we use. Along with this, we will set the number of simulations we plan on performing. In this case, 10,000.

```
set.seed(123) # set seed for reproducibility
n_sims <- 10^5 # number of simulations
```

Now, we will reproduce the situation provided in Example 4.81 to create a simulation of the full system working in parallel:

```
#simulate component A lifetime
a_life <- rexp(n_sims, rate = 1)
#
#simulate component B lifetime
b_life <- rexp(n_sims, rate = 0.5)
#
#simulate system lifetime
system_life <- pmax(a_life, b_life)
```

Next, we will generate N exponential random variables with a rate of $\frac{1}{2}$ which represent the lifetime of Component A. For Component B, we generate N exponential random variables with rate $\frac{1}{3}$.

We then calculate the new system lifetime vector using pmax function, taking the maximum of the new lifetime for component A and the original lifetime for component B.

```
#component A is replaced
 a_life_new <- rexp(n_sims, rate = 1/2) # new lifetime for component A
 system_life_a_new <- pmax(a_life_new, b_life) # system lifetime with
new component A
#
```

```r
#component B is replaced
 b_life_new <- rexp(n_sims, rate = 1/3) # new lifetime for component B
 system_life_b_new <- pmax(a_life, b_life_new) # system lifetime with
new component B
```

Now, we will calculate and print the mean system lifetimes when components A or B is replaced with a new one with different failure rate distributions. We use the variable mean_life_a_new to calculate the mean of system_life_a_new which is a vector of system lifetimes for the case where component A is replaced with a new one with an exponential distribution with a failure rate of $\frac{1}{2}$.We then do the same with mean_life_b_new which has a failure rate of $\frac{1}{3}$. We then print out a message that displays he mean system lifetime with the new component A and new component B.

```r
#calculate mean system lifetime with new component A
 mean_life_a_new <- mean(system_life_a_new)
 cat("Mean system lifetime with new component A:",
round(mean_life_a_new, 2), "months\n")

## Mean system lifetime with new component A: 2.99 months

#
#calculate mean system lifetime with new component B
 mean_life_b_new <- mean(system_life_b_new)
 cat("Mean system lifetime with new component B:",
round(mean_life_b_new, 2), "months\n")

## Mean system lifetime with new component B: 3.26 months
```

From this calculation, we are able to see that the mean system lifetime of new component A is shorter than the mean system lifetime of new component B.

To maximize the mean system lifetime, we should choose the component with the larger mean lifetime. From parts a and b, we see that replacing component B with a new component with a lifetime distributed as $e^{\frac{1}{3}}$ leads to a larger mean system lifetime. Therefore, we should choose to replace component B.

Now we must calculate and print the probability that the new system lifetime is less than one-month with the new components A and B. The first line of R code, calculates the proportion of system lifetimes that are less than 1 month when component A is replaced with a new one with an exponential distribution with a failure rate of $\frac{1}{2}$. The mean function calculates the proportion because system_life_a_new < 1 returns a logical vector where TRUE corresponds to lifetimes less than 1 month and FALSE corresponds to lifetimes greater than or equal to 1 month.

This is simply replicated for component B which has a failure rate of $\frac{1}{3}$.

```
#calculate probability of system lifetime less than 1 month with new
component A
 prob_less_than_1_a_new <- mean(system_life_a_new < 1)
 cat("Probability of system lifetime less than 1 month with new
component A:", round(prob_less_than_1_a_new, 3), "\n")

## Probability of system lifetime less than 1 month with new component
A: 0.155

#
#calculate probability of system lifetime less than 1 month with new
component B
 prob_less_than_1_b_new <- mean(system_life_b_new < 1)
 cat("Probability of system lifetime less than 1 month with new
component B:", round(prob_less_than_1_b_new, 3), "\n")

## Probability of system lifetime less than 1 month with new component
B: 0.179
```

Based on this calculation, with our goal of minimizing the probability that the system fails within a month, we should choose the component with the smaller probability of a short lifetime. From parts a and b, we see that replacing component A with a new component with a lifetime distributed as $e^{\frac{1}{2}}$ leads to a smaller probability of a short lifetime. Therefore, we should choose to replace component A.

Below, we calculate and print the 10th percentile of the system lifetime with the new components A and B. To do this, we use the quantile function to calculates the 10th percentile of the system lifetime with the new component A. The quantile function takes the vector of system lifetimes as the first argument and the quantile of interest (in this case, 0.1 or 10th percentile) as the second argument. This can be done for much systems interchangeably.

```
#calculate 10th percentile of system lifetime with new component A
 percentile_10_a_new <- quantile(system_life_a_new, 0.1)
 cat("10th percentile of system lifetime with new component A:",
round(percentile_10_a_new, 2), "months\n")

## 10th percentile of system lifetime with new component A: 0.76
months

#calculate 10th percentile of system lifetime with new component B
 percentile_10_b_new <- quantile(system_life_b_new, 0.1)
 cat("10th percentile of system lifetime with new component B:",
round(percentile_10_b_new, 2), "months\n")

## 10th percentile of system lifetime with new component B: 0.68
months
```

Based on the calculations, with our goal of maximizing the 10th percentile of the system lifetimes, we should choose the component with the smaller 10th percentile. From parts a and b, we see that replacing component A with a new component with a lifetime distributed as $e^{\frac{1}{2}}$ leads to a smaller 10th percentile of system lifetimes. Therefore, we should choose to replace component A.