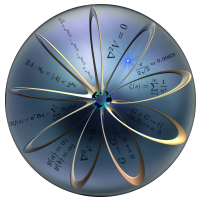

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Hartley function

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Definition

The *Hartley function* is a of uncertainty, introduced by Hartley in 1928. If we pick a sample from a finite set A uniformly at random, the revealed after we know the is given by the Hartley function

$$H(A) := \log_b(|A|).$$

If the base of the logarithm is 2, then the uncertainty is measured in bits. If it is the natural logarithm, then the is nats. It is also known as the Hartley entropy.

Remark:

The Hartley function is a special case of Shannon's entropy. Each element in the sample space A is associated with probability $p = 1/|A|$. For an element $\omega \in A$, the Hartley of the event $\{\omega\}$ is $-\log(p) = \log(|A|)$, which is constant over $\omega \in A$. The average over the whole sample space is thus also equal to $\log(|A|)$.

Characterization

The Hartley function only depends on the number of elements in a set, and hence can be viewed as a function on natural numbers. Rényi showed that the Hartley function in base 2 is the only function mapping natural numbers to real numbers that

1. $H(mn) = H(m) + H(n)$ (),
2. $H(m) \leq H(m+1)$ (monotonicity), and
3. $H(2) = 1$ (normalization).

Condition 1 says that the uncertainty of the Cartesian product of two finite sets A and B is the sum of uncertainties of A and B. Condition 2 says that a larger set has larger uncertainty.