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conditional entropy

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Definition (Discrete) Let $(\Omega, \mathcal{F}, \mu)$ be a discrete probability space, and let X and Y be discrete random variables on Ω.

The conditional entropy $H[X|Y]$, read as "the conditional entropy of X given Y," is defined as

$$H[X|Y] = -\sum_{x \in X} \sum_{y \in Y} \mu(X = x, Y = y) \log \mu(X = x | Y = y) \tag{1}$$

where $\mu(X|Y)$ denotes the conditional probability. $\mu(Y = y)$ is nonzero in the discrete case

Discussion The results for discrete conditional entropy will be assumed to hold for the continuous case unless we indicate otherwise.

With $H[X, Y]$ the joint entropy and f a function, we have the following results:

$$H[X|Y] + H[Y] = H[X, Y] \tag{2}$$
$$H[X|Y] \leq H[X] \qquad \text{(conditioning reduces entropy)} \tag{3}$$
$$H[X|Y] \leq H[X] + H[Y] \qquad \text{(equality iff } X, Y \text{ independent)} \tag{4}$$
$$H[X|Y] \leq H[X|f(Y)] \tag{5}$$
$$H[X|Y] = 0 \iff X = f(Y) \qquad \text{(special case } H[X|X] = 0\text{)} \tag{6}$$
$$\tag{7}$$

The conditional entropy $H[X|Y]$ may be interpreted as the uncertainty in X given knowledge of Y. (Try reading the above equalities and inequalities with this interpretation in mind.)