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weight enumerator

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Defines	complete weight enumerator
Defines	weight distribution
Defines	Hamming weight distribution

Let A be an alphabet and C a finite subset of A^*. Then the *complete weight enumerator* of C, denoted by cwe_C, is the polynomial in $|A|$ indeterminates X_a labeled by the letters of $a \in A$ with integer coefficients defined by

$$\mathrm{cwe}_C((X_a)_{a\in A}) := \sum_{c\in C} \prod_{a\in A} X_a^{\mathrm{wt}_a(c)},$$

where $\mathrm{wt}_a(c)$ is the a-weight of the string c.

If A is an abelian group, one defines the *Hamming weight enumerator* of C, denoted by we_C, as a polynomial in only two indeterminates X and Y:

$$\mathrm{we}_C(X,Y) := \mathrm{cwe}_C((X_a)_{a\in A})|_{\substack{X_0=X \\ X_a=Y \text{ if } a\neq 0}},$$

that is one distinguishes only between zero and the non-zero letters of the strings in C.

If C is a code of block length n, then both cwe_C and we_C are http://planetmath.org/node/657 of degree n. Therefore, one can set $Y = 1$ in we_C in this case without losing information. The resulting polynomial can be uniquely rewritten in the form

$$\mathrm{we}_C(X,1) = \sum_{i=0}^{n} A_i X^{n-i},$$

the sequence $A_0, \ldots A_n$ defining the *Hamming weight distribution*. Analogously, one can define more general weight distributions by setting all but one indeterminate in $\mathrm{cwe}_C((X_a)_{a\in A})$ equal to one.

Examples

- Let C be the ternary (that is $A = \mathbb{F}_3 = \{0,1,2\}$) linear code of block length 4 the vectors $(1,1,1,1)$, $(1,1,0,0)$ and $(1,0,1,0)$. Then

  $\mathrm{cwe}_C(X_0,X_1,X_2) = X_0^4+4X_0^2X_1^2+4X_0^2X_1X_2+4X_0^2X_2^2+4X_0X_1^2X_2+4X_0X_1X_2^2+X_1^4+4X_1^2X_2^2+$

  and

  $$\mathrm{we}_C(X,Y) = X^4 + 12X^2Y^2 + 8XY^3 + 6Y^4$$

  and the Hamming weight distribution is $1, 0, 12, 8, 6$.

- The Hamming weight enumerator of the full binary code of length n, $\mathbb{F}_2^n$, is simply given by $\mathrm{we}_{\mathbb{F}_2^n}(X,Y) = (X+Y)^n$, and the Hamming weight distribution is the n-th of Pascal's triangle.