# automorphism group (linear code)

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Let $\mathbb{F}_q$ be the finite field with $q$ elements. The group $\mathcal{M}_{n,q}$ of $n \times n$ monomial matrices with entries in $\mathbb{F}_q$ acts on the set $\mathfrak{C}_{n,q}$ of linear codes over $\mathbb{F}_q$ of block length $n$ via the *monomial transform*: let $M = (M_{ij})_{i,j=1}^n \in \mathcal{M}_{n,q}$ and $C \in \mathfrak{C}_{n,q}$ and set

$$C_M := \left\{ \left( \sum_{i=1}^n M_{i1} c_i, \dots, \sum_{i=1}^n M_{in} c_i \right) \mid (c_1, \dots, c_n) \in C \right\}.$$

This definition looks quite complicated, but since $M$ is , it really just means that $C_M$ is the linear code obtained from $C$ by permuting its coordinates and then multiplying each coordinate with some nonzero element from $\mathbb{F}_q$.

Two linear codes lying in the same orbit with respect to this action are said to be *equivalent*. The isotropy subgroup of $C$ is its *automorphism group*, denoted by $\mathrm{Aut}(C)$. The elements of $\mathrm{Aut}(C)$ are the *automorphisms* of $C$.

Sometimes one is only interested in the action of the permutation matrices on $\mathfrak{C}_{n,q}$. The permutation matrices form a subgroup of $\mathcal{M}_{n,q}$ and the resulting subgroup of the automorphism group $\mathrm{Aut}(C)$ of a linear code $C \in \mathfrak{C}_{n,q}$ is called the *permutation group*. In the case of binary codes, this doesn't make any difference, since the finite field $\mathbb{F}_2$ contains only one nonzero element.