

# The Impact of Geopolitical Conflicts on Trade, Growth, and Innovation

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## Abstract

Geopolitical conflicts have increasingly become a driver of trade policy. We study the dynamic costs of economic decoupling. Using a dynamic trade model, we show that the welfare losses from decoupling the global economy can be drastic, reaching as high as 12% over 20 years for some regions, with larger losses in lower-income countries. Two mechanisms are essential to capture these effects thoroughly: technological diffusion and input-output linkages, both of which magnify welfare losses.

*Keywords:* Innovation, International trade, international relations.

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The last decade has witnessed the beginning of a backlash against global trade integration. Political scientists conjecture that the emergence of China as a new superpower against the incumbent U.S. might lead to strategic competition between these countries —one in which geopolitical forces and the desire to limit interdependence take primacy over win-win international cooperation<sup>1</sup>. Rising support for populist and isolationist parties in many Western countries points in the same direction<sup>2</sup>. Additionally, the 2022 War in Ukraine and the subsequent strong retaliation of the European Union, the United States, and their allies against Russia suggest that the international economic order based on open markets and expanded globalization could be replaced by a more fragmented international economic system.

Using these facts as motivation, this paper aims to determine the potential effects of increased and persistent large-scale geopolitical conflicts on trade, economic growth, and technological innovation. Some of the adverse effects are well-known. Increased trade barriers decrease domestic welfare and gains from trade by shifting production away from the most cost-efficient producers and leaving households with a lower level of total consumption.

However, some of the main concerns of policymakers and practitioners regarding potentially detrimental effects of limiting trade are abstracted away in standard models. For instance, these models typically assume a fixed technology distribution for domestic firms, limiting gains from trade to static gains. This assumption renders it impossible to address one of the most important long-term consequences of continued geopolitical conflicts or a retreat from globalization —namely, reduced technology and know-how spillovers that may happen through trade.

In order to realistically assess the impact of trade conflicts on global innovation, we build a multi-sector multi-region general equilibrium model with dynamic sector-specific knowledge diffusion. We model the arrival of new ideas as a learning process for producers in each country-sector cluster. By engaging in international trade, i.e. importing intermediate inputs, domestic innovators have access to new sources of ideas, whose quality depends on the productivity of the countries and sectors from which they source intermediates.

This dynamic mechanism substantially alters the incentives in the face of trade conflicts. In a static setting, countries with large domestic markets are likely to have limited welfare losses if they cut trade ties with a foreign trade bloc. This is usually true even if this foreign trade bloc is of higher productivity and even more so if such a loss can be compensated by decreased trade costs with a third group of countries —as it would be the case with countries aligning in geopolitical blocs. This is no longer true in our model. If those countries lose access to high-productivity suppliers, they also forgo the idea-diffusion aspect of trade. As such, over time, the cumulative dynamic costs of trade conflicts become much larger, especially for countries away from the productivity frontier.

In our model, idea diffusion is mediated by the input-output structure of production, such that both sectoral intermediate input cost shares and import trade shares characterize the source distribution of ideas. Innovation is summarized by describing productivity in different sectors as evolving according to a trade-share weighted average of trade partners' sectoral productivities. This process is controlled by a parameter that determines

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<sup>1</sup>See [Wei \(2019\)](#) and [Wyne \(2020\)](#) for a review of the debate among Chinese and American scholars about the shift in foreign policies toward each other.

<sup>2</sup>For evidence of the impact of trade shocks on the rise of populist parties to power, see [Colantone and Stanig \(2018\)](#).

the speed of diffusion of ideas, which we calibrate using historical data on output growth.<sup>3</sup>

Our approach implies that the strength of ideas diffusion is a function of the strength of input-output linkages in production. Productivity thus evolves endogenously in each sector as a by-product of micro-founded market decisions —i.e., an externality that market agents affect with their behavior but do not take it into account when making decisions. In this framework, the outbreak of large-scale trade conflicts will have spillover effects on the future path of sectoral productivities of all countries. Changes in trade costs induce trade diversion and creation, which, in turn, impact productivity dynamics in a way that is not internalized by agents.

After characterizing the model, we solve it recursively and use it to perform policy experiments in the context of heightened geopolitical conflicts. We explore the potential impact of a “decoupling of the global economy,” a scenario under which technology systems would diverge in the global economy. We divide the global economy into a Western bloc and an Eastern bloc based on differential scores in foreign policy similarity. In doing so, we provide the first set of estimates for dynamic losses of economic decoupling.

We provide four sets of estimates. First, we simulate increased trade costs arising from geopolitical circumstances, which increase frictions prohibitively if one country wants to trade with another one outside its bloc. Second, we simulate a scenario of a global increase in tariffs, in which all countries move from a cooperative tariff setting in the context of the WTO to a non-cooperative tariff setting<sup>4</sup>. Third, we explore the potential effect of moving one of the regions from the Western Bloc to the Eastern Bloc. Fourth, we limit decoupling to electronic equipment, the sector displaying so far the most decoupling policies. These four policy experiments are essential to analyze the impact of decoupling, the difference between different ways to decouple (with resource-dissipating iceberg trade costs or rent-generating tariffs), the role of technology spillovers in the model by analyzing bloc membership, and decoupling in the sector most scrutinized. To limit the already large number of policy experiments, we focus on the hypothetical scenario of a complete decoupling into a Western and Eastern Bloc. Hence, we do not explore a scenario with a “neutral” bloc.

Our analysis leads to five main findings. First, we show that the projected welfare losses for the global economy from a decoupling scenario can be drastic, as large as 12% in some regions, and are largest in the lower-income regions because they suffer the most from reduced technology spillovers from richer areas. Second, the described size and pattern of welfare effects are specific to the model with diffusion of ideas. In a dynamic setting with diffusion of ideas, welfare losses are larger and display more variation. Third, if one of the middle-income regions, Latin America and the Caribbean (LAC), would switch from the higher-income Western bloc to the lower-income Eastern bloc, its welfare costs of decoupling would be significantly higher. This experiment illustrates that policymakers in low- and middle-income countries would face difficult decisions if decoupling were to intensify. Fourth, the welfare costs of decoupling only in electronic equipment, the sector where decoupling is already taking place, would be much smaller

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<sup>3</sup>Trade costs are calibrated targeting observed trade shares as in new quantitative trade models applying exact hat algebra, whereas initial productivity is calibrated based on actual labor productivity data. With this approach and the chosen calibration of the ideas diffusion parameter, we stay close to observed data. As such, baseline values to which counterfactual experiments are applied are identical to actual values, ensuring that the impact of counterfactual experiments is not distorted.

<sup>4</sup>Nicita, Olarreaga and Silva (2018) estimate an average increase in global tariffs of 32 percentage points. For simplicity, we use this average number as a reference and we assume that countries in different blocs raise tariffs against countries in the other bloc by this average amount.

than under full decoupling, albeit sizable, ranging from 0.4–1.9%. Finally, a multi-sector framework exacerbates diffusion inefficiencies induced by trade costs relative to a single-sector one and due to differences in either trade costs, unit costs and/or productivities between sectors in a country’s trading partner; we explore this issue both through theory and simulations<sup>5</sup>.

We make five main contributions to the literature. First, we build a multi-sector model of the global economy with Bertrand competition, profits, and technology spillovers that can be solved recursively, allowing us to assess realistic trade policy experiments. Second, we analyze idea diffusion inefficiencies in a multi-sector framework both analytically and numerically. We show analytically that such inefficiencies come from differences in trade costs and unit costs between sectors and numerically that such inefficiencies tend to be larger in a multi-sector framework. Third, we calibrate the strength of the diffusion of technologies through trade with a tight fit between simulated and historical GDP growth rates, which is appropriate for counterfactual simulation. Fourth, we examine the long-run effects of real-world policy experiments related to the decoupling of the global economy. Last, we draw insights from the Political Science literature to incorporate geopolitical conflicts into a workhorse trade model.

Our model builds on the work that evaluates the impact of trade on innovation and shows that trade openness can increase the level of domestic innovation, particularly in the single-sector model of [Buera and Oberfield \(2020\)](#). Compared to previous work, we present a recursive model with input-output linkages as in [Caliendo and Parro \(2015\)](#); calibrate the strength of the diffusion of ideas to target historical GDP growth rates across all regions; and explore diffusion inefficiencies in a multi-sector setting.

In our model, productivity growth is driven by an autonomous component and a diffusion component, which is a function of intermediate trade linkages. Other models such as [Cai, Li and Santacreu \(2022\)](#) in the spirit of [Eaton and Kortum \(1999\)](#) assume that technology diffusion is an autonomous process with the strength of diffusion calibrated to data on patent citations. Since we calibrate the strength of diffusion to target GDP growth controlling for labor and capital growth, in our model trade plays an important role in productivity catch-up. As such, our approach constitutes an upper bound for the potential welfare losses associated with trade decoupling through less diffusion of ideas. Furthermore, our results focus on the adjustment costs over the counterfactual transition path after some exogenous shock rather than a long-run balanced growth path.

**Related Literature.** Our paper is closely related to the literature that adds dynamics to trade models by incorporating knowledge diffusion channels. This topic goes back to [Eaton and Kortum \(1999\)](#), who developed a multi-country dynamic model in which firms innovate by investing in research and development (R&D) and knowledge diffuses, after some lag, to other markets. In this model, diffusion operated in a relatively stylized way, was not explicitly tied to trade linkages, and was set up to ultimately reach all countries<sup>6</sup>.

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<sup>5</sup>Before conducting simulations with the multi-sector, multi-region model calibrated to real-world data, we explore the discrepancy between actual and optimal levels of idea diffusion. This comparison shows that to maximize the total diffusion of ideas, trade shares must be at their optimal point *in every sector*.

<sup>6</sup>In differentiated varieties of trade models, knowledge diffusion shows up in papers like [Romer \(1990\)](#), [Rivera-Batiz and Romer \(1991\)](#), and [Grossman and Helpman \(1989\)](#). In the text, we focus on papers that incorporate knowledge diffusion into Ricardian models, which is the class of models to which our model belongs.

More recently, Alvarez et al. (2013) combined the Eaton and Kortum (2002) Ricardian model of trade with an idea diffusion process first presented by Kortum (1997). Importantly, the authors conjectured that the diffusion process is proportional to the quality of managers of firms whose products reach a given destination market. Ideas flow from one market to another in proportion to the trade linkages between them. Therefore, impediments to trade have not only static but also dynamic costs —as they decrease knowledge diffusion.

From a theoretical perspective, our work is related to Cai, Li and Santacreu (2022) and Deng and Zhang (2023), who develop multi-sector dynamic trade models of knowledge diffusion. Cai, Li and Santacreu (2022) extends Eaton and Kortum (1999) to a multi-sector model of trade, innovation, and knowledge diffusion with lag-diffusion dynamics, exploring how the welfare gains from trade are affected by knowledge diffusion through their impact on changes in comparative advantage. There are three main differences between their work and ours. First, our model emphasizes the nexus between trade and idea diffusion, whereas Cai, Li and Santacreu (2022) models technology spillovers as being independent of the amount of trade. Additionally, while they calibrate knowledge spillovers with data on patent citation, we calibrate the strength of the diffusion of ideas based on the fit between actual and simulated historical GDP growth rates. Third, the papers have a different focus: we focus on policy questions and explore how the effect of potential trade policy changes are affected by the inclusion of ideas diffusion in the model, while they highlight how patterns of comparative advantage change with technology spillovers.

Deng and Zhang (2023) integrate Buera and Oberfield (2020) into a Levchenko and Zhang (2016) multi-sector framework, finding strong convergence in comparative advantage, dynamic gains from trade about 1/3 larger than static gains, and identifying central players in technology diffusion. There are two main differences with our work. First, Deng and Zhang (2023) employ a different way to calibrate the model, following Levchenko and Zhang (2016) to estimate trade costs and productivity parameters. We instead infer trade costs and productivity based on observed data and target GDP growth rates with the diffusion of ideas parameter. Second, they explore issues like convergence in comparative advantage and central players in technology diffusion in a backward-looking non-recursive model, whereas we explore the costs of geopolitical decoupling and the repercussions of bloc membership besides studying the inefficiencies of ideas diffusion in a multi-sector recursive framework fit for policy experiments.

Our work is also related to the literature on the costs of economic decoupling. We share in common with Eppinger et al. (2021) and Felbermayr, Mahlkow and Sandkamp (2023) that we also use a Ricardian model with input-output linkages, as in Caliendo and Parro (2015). The former distinguishes between trade costs for intermediate and final goods (as in Pol Antràs and Davin Chor (2018)) to simulate a decoupling in global value chains. The latter simulates a set of scenarios for East-West decoupling by increasing trade costs in all sectors, and shutting down cross-bloc trade. We differ in that we add dynamics and knowledge diffusion to that framework, extending Buera and Oberfield (2020) to a multi-sector framework.

Attinasi, Boeckelmann and Meunier (2023) calibrate a Baqaee and Farhi (2024) model of trade and economic networks to a scenario of economic decoupling. Like us, they divide the world into geopolitical blocs using foreign policy similarity indices and simulate decoupling through an increase of iceberg trade costs. However, they focus on short-term static rigidities and abstract away from knowledge diffusion. Instead, we estimate long-

run dynamic losses.

**Organization.** This paper is organized as follows. In Section 1 we present the model, detailing production, demand, and consumption of the global economy. We also describe the dynamic evolution of productivities in different regions and sectors. In Section 2 we describe the discrepancy between the actual and optimal diffusion of ideas in a multi-sector framework. In Section 3 we describe how we estimate the structural parameters governing productivities and technology diffusion. In Section 4 we discuss the calibration of the remaining objects and underpin the examined policy experiments. In Section 5 we present the results of our main policy experiments and some alternative simulations. Finally, we conclude in Section 6 summarizing the key takeaways.

## 1 Environment

Consider a global economy with  $N$  countries  $s, d \in \mathcal{D} := \{1, 2, \dots, N\}$ . In each region, there are multiple industries  $i \in \mathcal{I} := \{1, 2, \dots, I\}$ . Time is discrete and indexed by  $t \in \mathcal{T}$ . We denote a trade flow from source region  $s$  to destination country  $d$  in industry  $i$  at period  $t$  using subscripts  $sdi, t$ .

**Households** In each region  $d$  and each period  $t$ , there is a representative household who inelastically supplies a mass  $L_{d,t}$  of labor at a competitive wage  $w_{d,t}$ . They own aggregate capital stock  $K_{d,t}$ , receive lump-sum transfers  $T_{d,t}$ , and own the profits from each industry,  $\Pi_{di,t}$ . Gross national income is determined by labor income  $w_{d,t}L_{d,t}$ , capital income  $r_{d,t}K_{d,t}$ , lump-sum transfers  $T_{d,t}$ , and industry profits  $\Pi_{di,t}$ . The representative agent maximizes consumption by buying a composite good  $C_{d,t}$ :

$$\max_{\{C_{di,t}\}_{i \in \mathcal{I}}} C_{d,t} \equiv \sum_{i \in \mathcal{I}} (C_{di,t})^{\kappa_{di}} \quad \text{where} \quad \sum_{i \in \mathcal{I}} \kappa_{di} = 1 \quad (1)$$

where the coefficients  $\kappa_{di}$  denote Cobb-Douglas consumption shares over composite goods produced by industry  $i$ . The ideal price index in country  $d$  at period  $t$  is given by:  $P_{d,t} = \prod_{i \in \mathcal{I}} (P_{di,t}/\kappa_{di})^{\kappa_{di}}$ , where  $P_{di,t}$  are prices of sectoral composite goods. Demand for each sectoral composite good is given by:  $C_{di,t} = \kappa_{di} Y_{d,t} / P_{di,t}$ , with  $Y_{d,t} = w_{d,t}L_{d,t} + r_{d,t}K_{d,t} + T_{d,t} + \sum_{i \in \mathcal{I}} \Pi_{di,t}$ .

**Assemblers of composite goods** In each industry, assemblers costlessly combine infinitely many intermediate varieties  $\omega \in [0, 1]$  using technology:

$$Y_{di,t} = \left[ \int_{[0,1]} y_{di,t}(\omega)^{\frac{\sigma_i-1}{\sigma_i}} d\omega \right]^{\frac{\sigma_i}{\sigma_i-1}} \quad (2)$$

where  $y_{di,t}(\omega)$  is the quantity an assembler of country  $d$  in industry  $i$  demands of variety  $\omega$ . Optimal demand for each variety is decreasing in relative prices, with this relationship modulated by the elasticity of substitution  $\sigma_i$ :

$$y_{di,t}(\omega) = \left( \frac{p_{di,t}(\omega)}{P_{di,t}} \right)^{-\sigma_i} Y_{di,t} \quad (3)$$

with the price of composite commodity  $i \in \mathcal{I}$  satisfying:

$$P_{di,t} = \left[ \int_{[0,1]} p_{di,t}(\omega)^{1-\sigma_i} d\omega \right]^{\frac{1}{1-\sigma_i}} \quad (4)$$

**Producers of intermediate varieties** There are many producers of different varieties  $\omega$  of each industry  $i$  of country  $d$ . Variety producers have individual productivities  $z_{di}(\omega)$  and combine uses of labor  $\ell_{di,t}(\omega)$ , capital  $k_{di,t}(\omega)$ , and composite goods  $M_{dij,t}(\omega)$  to produce output  $y_{d,t}^i(\omega)$ , the output of intermediate variety  $\omega$ :

$$y_{d,t}^i(\omega) = z_{d,t}^i(\omega) (\ell_{di,t}(\omega)^{\psi_{di}} k_{di,t}(\omega)^{1-\psi_{di}})^{\gamma_{di}} \prod_{j \in \mathcal{I}} M_{dij,t}(\omega)^{(1-\gamma_{di})\gamma_{dij}}, \quad \sum_{i \in \mathcal{I}} \gamma_{dij} = 1 \quad (5)$$

$\gamma_{di}$  is the value added share,  $\psi_{di}\gamma_{di}$  is the labor share,  $(1 - \psi_{di})\gamma_{di}$  is the capital share, and  $1 - \gamma_{di}$  is the intermediate use cost share. Cost minimization implies that the cost of the unit input bundle,  $c_{di,t}$ , is a function of the prices of factors of production, and prices of composite goods:

$$c_{di,t} = \Theta_{di} \left( w_{d,t}^{\psi_{di}} r_{d,t}^{1-\psi_{di}} \right)^{\gamma_{di}} \prod_{j \in \mathcal{I}} P_{d,j,t}^{(1-\gamma_{dij})\gamma_{dij}} \quad (6)$$

where  $\Theta_{di} \equiv \gamma_{di}^{-\gamma_{di}} \psi_{di}^{-\psi_{di}\gamma_{di}} (1 - \psi_{di})^{-(1-\psi_{di})\gamma_{di}} (1 - \gamma_{di})^{-(1-\gamma_{di})} \prod_{j \in \mathcal{I}} \gamma_{dij}^{-(1-\gamma_{dij})\gamma_{dij}}$ .

**International Trade** Trade occurs through the demand for varieties used as inputs of composite goods. Intermediate goods are subject to iceberg trade costs, meaning that for one unit of good  $\omega$  produced in  $s$  to arrive in  $d$ ,  $u_{sdi} \geq 1$  units need to be produced. Furthermore, trade can be subject to ad-valorem tariffs  $1 + t_{sdi} \geq 1$ . The combined trade cost parameter then becomes  $\tau_{sdi} \equiv u_{sdi}(1 + t_{sdi})$ . The cost of supplying variety  $\omega$  of commodity  $i$  produced in source region  $s$  and delivered to region  $d$  is, then:  $\tau_{sdi} c_{si,t}/z_{si,t}(\omega)$ .

Assemblers in destination region  $d$  will only buy variety  $\omega$  from the source with the lowest landed price. Hence, the firm with the lowest price of a variety captures the entire market for that variety.

Producers engage in Bertrand competition, as in [Bernard et al. \(2003\)](#). In this structure, the lowest cost supplier is constrained to not charge in excess of the second lowest cost supplier. Hence, price will be equal to either the optimal monopolist price or to the marginal cost of the second most efficient competitor, whichever is lowest. More formally, for each country, order firms  $k = [1, 2, \dots]$  such that  $z_{si,t}^1(\omega) > z_{si,t}^2(\omega), \dots$ . If the lowest-cost provider of the variety  $\omega$  to country  $d$  is a producer from country  $s$ , the price in  $d$  satisfies:

$$p_{di,t}(\omega) = \min \left\{ \underbrace{\frac{\sigma_i}{\sigma_i - 1} \underbrace{\frac{\tau_{sdi} c_{si,t}}{z_{si,t}^1(\omega)}}_{\text{optimal monopolist price}}, \underbrace{\frac{\tau_{sdi} c_{si,t}}{z_{si,t}^2(\omega)}}_{\text{MC of 2nd most productive firm from } s}, \min_{n \neq s} \underbrace{\frac{\tau_{ndi} c_{ni,t}}{z_{ni,t}^1(\omega)}}_{\text{MC of most productive firm from other countries}} \right\} \quad (7)$$

Following Eaton and Kortum (2002) (EK), we take  $z_{si,t}(\omega)$  to be the realization of an i.i.d. random variable. Productivity is distributed according to a Type II Extreme Value Distribution (Fréchet):

$$F_{si,t}(z) = \exp\{-\lambda_{si,t}z^{-\theta_i}\} \quad (8)$$

As in a standard EK model, the location parameter,  $\lambda_{si,t}$ , describes the productivity of region  $s$  in sector  $i$  and thus determines its absolute advantage. A higher  $\theta_i$  implies less variability in productivity and thus lower potential for diversification according to comparative advantage. Given this assumption and the fact that assemblers source the cheapest possible variety, expenditure in goods coming from  $s$  to  $d$  in industry  $i$  is:

$$X_{sdi,t} = \frac{\lambda_{si,t}(\tau_{sdi}c_{si,t})^{-\theta_i}}{\Phi_{di,t}} X_{di,t} \equiv \pi_{sdi,t} X_{di,t} \quad (9)$$

where  $X_{di,t}$  is total expenditure in industry  $i$  at country  $d$ ,  $X_{sdi,t}$  is expenditure in  $d$  on goods coming from  $s$  in the same industry;  $\pi_{sdi,t}$  is the probability that the lowest cost supplier is from  $s$ ; and  $\Phi_{sdi,t} \equiv \sum_{n \in \mathcal{D}} \lambda_{ni,t}(\tau_{ndi}c_{ni,t})^{-\theta_i}$ . Aggregate price levels can be expressed as  $P_{di,t} = \Gamma_i \Phi_{di,t}^{-\frac{1}{\theta_i}}$ , where  $\Gamma_i$  is a constant<sup>7</sup>. Intermediate varieties produces realize a profit that is proportional to the total expenditure of destination countries:

$$\Pi_{si,t} = \frac{1}{1 + \theta_i} X_{sdi,t} = \frac{1}{1 + \theta_i} \sum_{d \in \mathcal{D}} \pi_{sdi,t} X_{di,t} \quad (10)$$

**Cross-sectional equilibrium** Our model is characterized by a sequence of cross-sectional equilibria satisfying equilibrium equations in each period  $t$ , consisting of equilibrium in product and factor markets. Product markets can be stated in terms of expenditure on final and intermediate goods. Total expenditure on final goods reflects purchases by households for final consumption, net of tariff payments to the destination government:

$$X_{si,t}^F = \sum_{d \in \mathcal{D}} \frac{\pi_{sdi,t}}{\tau_{sdi}} \kappa_{di} Y_{d,t}$$

Total expenditure on a given source-sector accounts for expenditure on final and intermediate goods,  $X_{si,t} \equiv X_{si,t}^F + X_{si,t}^M$ . Using that identity, we can express spending on intermediate goods as a function of the share of gross output in each industry  $j$  and destination country  $d$  used to purchase intermediates from industry  $i$  of source country  $s$ :

$$X_{si,t}^M = \sum_{d \in \mathcal{D}} \frac{\pi_{sdi,t}}{\tau_{sdi}} \sum_{j \in \mathcal{I}} \frac{\theta_j}{1 + \theta_j} (1 - \gamma_{di}) \gamma_{dij} X_{dj,t}$$

where the factor  $\frac{\theta_j}{1 + \theta_j}$  comes from the fact that the profit share of total expenditures in sector  $j$  is  $\frac{1}{1 + \theta_j}$ . Factor income can be expressed as a constant of total expenditures in a given sector:

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<sup>7</sup>Specifically,  $\Gamma_i \equiv \left[1 - \frac{\sigma_i - 1}{\theta_i} + \frac{\sigma_i - 1}{\theta_i} \left(\frac{\sigma_i}{\sigma_i - 1}\right)^{-\theta_i}\right] \Gamma\left(\frac{1 - \sigma_i + \theta_i}{\theta_i}\right)$ , where  $\Gamma(\cdot)$  is the Gamma function

$$w_{d,t}L_{d,t} = \sum_{i \in \mathcal{I}} \frac{\theta_i}{1 + \theta_i} \gamma_{di} \psi_{di} X_{di,t}, \quad r_{d,t}K_{d,t} = \sum_{i \in \mathcal{I}} \frac{\theta_i}{1 + \theta_i} \gamma_{di} (1 - \psi_{di}) X_{di,t}$$

while government tariff revenues  $R_{d,t}$ , which are used to finance transfers  $T_{d,t}$ , are a function of total expenditure in foreign goods:

$$T_{d,t} = R_{d,t} = \sum_{s \in \mathcal{D}} \sum_{i \in \mathcal{I}} \frac{t_{sdi}}{\tau_{sdi}} \pi_{sdi,t} X_{si,t}$$

and, as mentioned above, profits are  $\Pi_{di,t} = \frac{1}{1+\theta_i} \sum_{n \in \mathcal{D}} \pi_{dni,t} X_{ni,t}$ . Therefore, we can express national income as:

$$\begin{aligned} Y_{d,t} &= w_{d,t}L_{d,t} + r_{d,t}K_{d,t} + \sum_{i \in \mathcal{I}} \Pi_{di,t} + T_{d,t} \\ &= \sum_{i \in \mathcal{I}} \left( \frac{\theta_i}{1 + \theta_i} \gamma_{di} X_{di,t} + \frac{1}{1 + \theta_i} \sum_{n \in \mathcal{D}} \pi_{dni,t} X_{ni,t} + \sum_{s \in \mathcal{D}} \frac{t_{sdi}}{\tau_{sdi}} \pi_{sdi,t} X_{si,t} \right) \end{aligned}$$

where the first term in the brackets accounts for factor income, the second term account for profits in each destination market, and the third term accounts for tariff revenue from each source country. Expenditure on sector  $i$  of country  $s$  is the sum of expenditure on final and intermediate goods:

$$X_{si,t} = \sum_{d \in \mathcal{D}} \frac{\pi_{sdi,t}}{\tau_{sdi}} \left( \kappa_{di} Y_{d,t} + \sum_{j \in \mathcal{I}} \frac{\theta_j}{1 + \theta_j} (1 - \gamma_{dj}) \gamma_{dji} X_{djt} \right)$$

Finally, we allow for (possibly non-zero) trade imbalances through an exogenous region-specific net transfer  $D_{d,t}$  (positive for a trade deficit), so that total imports minus total exports equals  $D_{d,t}$ :

$$\sum_{i \in \mathcal{I}} \left( \sum_{s \in \mathcal{D} \setminus \{d\}} \pi_{sdi,t} X_{si,t} - \sum_{n \in \mathcal{D} \setminus \{d\}} \pi_{dni,t} X_{ni,t} \right) = D_{d,t}$$

Given initial values for the capital stock  $\{K_{d,t}\}$ , labor force  $\{L_{d,t}\}$  and productivities  $\{\lambda_{di,t}\}$ , the equations above, the definition of trade shares, and composite goods prices characterize the cross-sectional equilibrium at each period  $t$ . We collect the full set of equilibrium conditions in Appendix E to make them easily accessible for numerical computation in both levels and changes.

**Productivity evolution** We model idea diffusion through random matches between domestic and foreign managers<sup>8</sup>. Like [Buera and Oberfield \(2020\)](#), we assume that a manager draws new insights as a by-product of sourcing a basket of inputs.

We extend this framework to a model with diffusion of ideas in a multi-sector context and solve it recursively to assess the long-run effects of policy experiments. The idea diffusion mechanism is mediated by the input-output structure of production, such that

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<sup>8</sup>For a detailed review of this literature, see the comprehensive review chapter published by [Buera and Lucas \(2018\)](#). Seminal examples of this work include [Jovanovic and Rob \(1989\)](#) and [Kortum \(1997\)](#). More recently, [Alvarez et al. \(2013\)](#) explored how idea diffusion is intertwined with trade linkages.

both sector cost shares and import trade shares characterize the source distribution of ideas<sup>9</sup>.

**Assumption 1** (Idea formation). *New ideas are the transformation of two random variables, namely: (i) original insights  $o$ , which arrive according to a power law:  $O_{i,t}(o) = \Pr(O < o) = 1 - \alpha_{i,t}o^{-\theta_i}$ ; (ii) derived insights  $z'$ , drawn from a source distribution  $G_{di,t}(z)$ . After the realization of those two random variables, the new idea has productivity  $z = o(z')^{\beta_i}$ , where  $o$  is the original component of the new idea,  $z'$  is the derived insight, and  $\beta_i \in [0, 1]$  captures the contribution of the derived insights to new ideas in sector  $i$ . Local producers only adopt new ideas if their quality dominates the quality of local varieties. Therefore, for any period, the domestic technological frontier evolves according to<sup>10</sup>:*

$$F_{di,t+\Delta}(z) = \frac{F_{di,t}(z)}{\Pr\{\text{productivity} < z \text{ at } t\}} \times \underbrace{\left(1 - \int_t^{t+\Delta} \int \alpha_\tau z^{-\theta_i} (z')^{\beta_i \theta_i} dG_{di,\tau}(z') d\tau\right)}_{\Pr\{\text{no better draws in } (t, t+\Delta)\}}$$

Given Assumption 1, as in [Buera and Oberfield \(2020\)](#), for any  $t$ ,  $F_{di,t}(z)$  is Fréchet with location parameter  $\lambda_{di,t} = \int_{-\infty}^t \alpha_{i,\tau} \int (z')^{\beta_i \theta_i} dG_{di,\tau}(z') d\tau$  and scale parameter  $\theta_i$ , with the former evolving according to the following law of motion:

$$\dot{\lambda}_{di,t} = \alpha_{i,t} \int (z')^{\beta_i \theta_i} dG_{di,t}(z')$$

To further characterize the equation above, we need to define the source distribution. We assume that managers learn from their suppliers, such that  $G_{di,t}(z')$  is proportional to the sourcing decisions in the production of commodity  $i$  in country  $d$ . Productivity thus evolves endogenously as a by-product of sourcing decisions.

Additionally, we assume that insights take time to come to fruition. Rather than drawing insights from interactions with suppliers in the current period, we assume that insights take one period to materialize. Intuitively, we are assuming that entrepreneurs have to study their purchases for one period and only then draw insights. This assumption will be convenient because it will allow us to compute the law of motion for technology without relying on present-period trade shares. Therefore, we will be able to solve the model recursively and use it for counterfactual analysis of the long-run impact of policy experiments.

**Assumption 2** (Source Distribution from Intermediates). *The source distribution  $G_{di,t}(z') \equiv \sum_{j \in \mathcal{I}} \tilde{\gamma}_{dij} \sum_{s \in \mathcal{D}} H_{sjdi,t}(z')$ , where  $\tilde{\gamma}_{dij} = (1 - \gamma_{di})\gamma_{dij}$  is the cost share of sector  $j$  when producing good  $i$  in region  $d$ ; and  $H_{sjdi,t}(z')$  is the fraction of commodities for which the lowest cost supplier is a firm located in  $s \in \mathcal{D}$  with productivity weakly less than  $z'$ .*

Given the assumptions above, the resulting law of motion depends on an exogenous arrival rate  $\alpha_{i,t}$ , constant intermediate cost shares  $\gamma_{sdi}$  and (lagged) conditional distributions  $H_{sjdi,t}$ :

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<sup>9</sup>As mentioned earlier, our work is closely related to [Cai, Li and Santacreu \(2022\)](#), who extend [Kortum \(1997\)](#) to a multi-sector framework. We differ in that they model diffusion as happening separately from trade, rather than as a trade-externality.

<sup>10</sup>Here we simply use the fact that  $o = z(z'^{-\beta_i})$  and note that, given an insight  $z'$ , at any moment  $t$  the arrival rate of ideas of quality better than  $z$  is  $\Pr(O > o) = \Pr(O > z(z')^{-\beta_i}) = \alpha_t z^{-\theta_i} (z')^{\beta_i \theta_i}$ . We then integrate over all possible values of  $z'$ .

$$\dot{\lambda}_{di,t} = \alpha_{i,t} \sum_{j \in \mathcal{I}} \tilde{\gamma}_{dij} \sum_{s \in \mathcal{D}} \int z^{\beta_i \theta_i} dH_{sjdi,t}(z) \quad (11)$$

In the following section, we fully characterize the expression above, extending this class of idea diffusion models to a multi-sector framework. For now, it suffices to understand that  $\lambda_{di,t}$  evolves over time such that at each period  $t$  we can characterize a cross-sectional equilibrium.

**Factors of production evolution** The supply of the factors of production changes over time. They are perfectly mobile and thus have a uniform price across sectors. Using external projections for population, we impose exogenous growth rates for labor supply:  $L_{d,t+1} = (1 + g_{d,t})L_{d,t}$ <sup>11</sup>.

In our simulations, we take a simple approach to capital: within each region, the capital-labor ratio is fixed over time. That is,  $K_{d,t}$  grows one-for-one with  $L_{d,t}$ , so  $K_{d,t} = (K_{d,0}/L_{d,0})L_{d,t}$  (equivalently,  $K_{d,t}/K_{d,0} = L_{d,t}/L_{d,0}$ ).

**Dynamic Equilibrium** Given initial values for  $\{L_{d,0}\}, \{K_{d,0}\}, \{\lambda_{di,0}\}$ , we can characterize a sequence of cross-sectional equilibria. Given the exogenous labor growth rates, we determine  $\{L_{d,t}\}$ . We then set  $\{K_{d,t}\}$  to keep each region's capital-labor ratio constant. Given the outcome of the cross-sectional equilibrium, we compute intermediate input trade shares and update  $\{\lambda_{di,t+1}\}$  using equation (11). Iterating forward yields the counterfactual transition path under a change in trade costs.

Note that the dynamic equilibrium model, as others in this class, is a sequence of cross-sectional equilibria with transition equations connecting the state variables across periods. This makes the model computationally more tractable and leads to a more straightforward interpretation of the simulation results, as we focus on the counterfactual trajectory of technology given a change in trade costs. However, we abstract from intertemporal optimization and do not model endogenous saving, investment, or international capital flows. In particular, the capital stock path is imposed exogenously (via a constant capital-labor ratio) and does not respond to shocks.<sup>12</sup>.

## 2 Idea Diffusion in a Multi-sector Framework

**Main theoretical results** In this section, we state three main results. First, we work out the integral in equation (11) to extend the *learning from sellers* specification of Buera and Oberfield (2020) to a multi-sector framework. Then, we derive the destination sector elasticity with respect to productivity shocks in source sectors. Finally, we show that, under reasonable restrictions, a multi-sector framework will magnify idea diffusion.

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<sup>11</sup>These growth rates are based on projections from the United Nations, as described in the data section below.

<sup>12</sup>As a result, we do not capture potentially important current-account and capital-flow dynamics emphasized in the international finance literature. For example, standard open economy models with intertemporal optimization have generated counterfactual predictions on the direction of capital flows between developed and emerging countries in the 1990s and 2000s. Capital was flowing on net from emerging to developed economies instead of flowing to emerging economies with higher growth rates as predicted by standard models.

**Proposition 1.** [Law of Motion in a Multi-Sector Framework] Suppose  $\beta_{ij} = \beta_i \theta_i / \theta_j < 1$  for each  $i, j \in \mathcal{I}$  pair. Then, in the multi-sector multi-region economy described above, the law of motion for productivities is as follows:

$$\dot{\lambda}_{di,t} = \alpha_{i,t} \sum_{j \in \mathcal{I}} \Gamma(1 - \beta_{ij}) \tilde{\gamma}_{dij} \sum_{s \in \mathcal{D}} \pi_{sdj,t} \left( \frac{\lambda_{sj,t}}{\pi_{sdj,t}} \right)^{\beta_{ij}} \quad (12)$$

where  $\Gamma(\cdot)$  is the gamma function,  $\tilde{\gamma}_{dij} = (1 - \gamma_{di})\gamma_{dij}$  are cost shares, and  $\pi_{sdj,t}$  are intermediate input trade shares.

*Proof.* Appendix G. □

Above, diffusion of ideas is proportional to intermediate trade shares from source country-industry pairs,  $s, j$ , into destination country  $d$  and use industry  $i$ . Total diffusion incorporates intermediate cost-shares. Embedded in this specification is that whenever trade shares are small, producers are drawing insights disproportionately from the highest-productivity firms among those that serve a given destination market, i.e.  $\left( \frac{\lambda_{sj,t}}{\pi_{sdj,t}} \right)^{\beta_i \theta_i / \theta_j}$  will be higher.

In a multi-sector framework, the effective diffusion elasticity is  $\frac{\beta_i \theta_i}{\theta_j}$  rather than just  $\beta$ . Knowledge diffuses more strongly to the use sector  $i$  if the contribution of derived insights to new ideas is higher ( $\beta_i$  is larger); if the productivity distribution in source sector  $j$  has a thicker right tail ( $\theta_j$  is smaller); and if the average quality of original insights is lower ( $\theta_i$  is higher). A smaller  $\theta_j$  increases diffusion since it raises the likelihood that highly productive firms supply the domestic market, generating derived insights. A larger  $\theta_i$  increases diffusion because derived ideas are less relevant when the original insights are smaller and the average quality of the original insights  $\alpha_{i,t} / \theta_i$  decreases in  $\theta_i$ .

As in a single sector framework where diffusion elasticities need to be restricted to  $\beta < 1$ , in this multi-sector framework effective diffusion elasticities are restricted to  $\beta_{ij} < 1$ . One simple way to bridge that restriction is to impose homogeneous trade elasticities, with  $\theta_i = \theta_j$  for all  $i, j$ <sup>13</sup>. However, there is strong evidence that elasticities vary substantially across sectors (Caliendo and Parro, 2015; Ossa, 2014). In the following sections, we estimate  $\beta_{ij}$  directly and back out  $\beta_i$  from estimated trade elasticities.

**Corollary 1** (Pairwise Diffusion Elasticities). *Under the assumptions of Proposition 1, fix  $t$  and hold  $\{\alpha_{i,t}, \tilde{\gamma}_{dij}, \pi_{sdj,t-1}, \beta_{ij}\}$  constant. The instantaneous elasticity of the diffusion flows in destination  $(d, i)$  with respect to the knowledge stock in source  $(s, j)$  is*

$$\varepsilon_{di,sj,t} \equiv \frac{\partial \log \dot{\lambda}_{di,t}}{\partial \log \lambda_{sj,t}} = \omega_{di,sj,t} \times \beta_{ij},$$

where the diffusion-flow weight  $\omega_{di,sj,t} = \frac{\tilde{\gamma}_{dij} \pi_{sdj,t}^{\frac{1-\beta_{ij}}{1-\beta_{ij'}}} \lambda_{sj,t}^{\beta_{ij}}}{\sum_{j'} \tilde{\gamma}_{dij'} \sum_{s'} \pi_{s'dj',t}^{\frac{1-\beta_{ij'}}{1-\beta_{ij}}} \lambda_{s'j',t}^{\beta_{ij'}}}$ . Aggregating across sources yields the average instantaneous diffusion elasticity for receiver  $(d, i)$ :

$$\varepsilon_{di,t} \equiv \sum_{j \in \mathcal{I}} \omega_{di,j,t} \times \beta_{ij},$$

<sup>13</sup>This condition is analogous to what Deng and Zhang (2023) show in the context of Ricardian trade with perfect competition. In Appendix G we show that the same restriction holds when dealing with Ricardian trade with Bertrand competition. We account for this in our estimation, whereas they assume an ad-hoc exogenous parameter that cancels out the role of heterogeneous elasticities.

where  $\omega_{di,j,t} = \sum_s \omega_{di,sj,t}$  denotes the importance weight of source industry  $j$  for the destination cluster  $(d, i)$ .

Another feature of this multi-sector framework is that, for most applications, dynamic losses will be larger with multiple sectors. In the proposition below, we derive a sufficient condition for average diffusion in a multi-sector framework to be smaller than its single-sector counterpart, even if elasticities do not vary across sectors.

**Proposition 2** (Diffusion Losses with Multiple Sectors). *Suppose that, for some arbitrary  $t - 1$ ,  $\sum_{j \in \mathcal{I}} \gamma_{dij} \lambda_{sj,t-1} \leq \lambda_{s,t-1}$  for each  $s$  and  $d$  and both the effective diffusion elasticity  $\beta_i = \beta$  and the arrival rate are uniform  $\alpha_{i,t} = \alpha_t$  for each  $i \in \mathcal{I}$ . Then:*

$$\left( \sum_{i \in \mathcal{I}} \kappa_{di} \Delta \lambda_{d,t}^i \right)^{\text{Diffusion Optimum}} < (\Delta \lambda_{d,t})^{\text{Diffusion Optimum}} \quad (13)$$

and

$$\left( \sum_{i \in \mathcal{I}} \kappa_{di} \Delta \lambda_{d,t}^i \right)^{\text{Market Allocation}} \leq (\Delta \lambda_{d,t})^{\text{Market Allocation}} \quad (14)$$

*Proof.* Appendix G. □

The proposition above states that if aggregate productivities  $\lambda_{s,t-1}$  in each source country  $s$  are at least as large as the cost-weighted average of its sectoral productivities in every destination country  $d$ , then: (a) the maximum diffusion rate in a single-sector framework will be strictly larger than what happens in a multi-sector framework; and (b) diffusion of ideas under the market allocation in a single-sector framework is guaranteed to be weakly larger than what happens in a multi-sector framework.

Note that this result holds while holding the diffusion elasticity and arrival rates equal across sectors, meaning that it stems directly from modeling multiple sectors. Why is that the case? To gain some intuition, suppose that, for each sector  $j$ , we might either have trade maximizing diffusion, which we refer to as *diffusion optimum*<sup>14</sup> or trade shares observed under the cross-sectional equilibrium, which we refer to as *market allocation*. These satisfy, respectively:

$$(\pi_{sdj,t})^{\text{Diffusion Optimum}} = \frac{\lambda_{sj,t}}{\sum_{k \in \mathcal{D}} \lambda_{kj,t}}, \quad (\pi_{sdj,t})^{\text{Market Allocation}} = \frac{\lambda_{sj,t} (\tau_{sdj} c_{sj})^{-\theta_j}}{\sum_{k \in \mathcal{D}} \lambda_{kj,t} (\tau_{kdj} c_{kj})^{-\theta_j}}$$

The point that maximizes diffusion sets trade shares proportional to the source country productivity in the source industry,  $\lambda_{sj,t}$ , as a proportion of the global productivity stock,  $\sum_{k \in \mathcal{D}} \lambda_{kj,t}$ . Under the market allocation, price differences induce deviations from the diffusion maximizing point. Trade will be skewed toward source countries with relatively lower input and trade costs.

Consider what happens in some sector  $i$  of country  $h$  in a global economy that is fully symmetric, both across countries and sectors, but where trade costs are present. The strict concavity of the diffusion equation implies that idea diffusion is not uniform as

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<sup>14</sup>The diffusion optimum solves the following program:  $\max_{\{\pi_{sdj,t-1}\}}$   $\sum_{j \in \mathcal{I}} \gamma_{dij} \sum_{s \in \mathcal{D}} (\pi_{sdj,t-1})^{1-\beta} (\lambda_{sj,t-1})^\beta$  s.t.  $\sum_s \pi_{sdj,t} = 1$

$\pi_{hhi}$  varies. The optimal diffusion point is  $(\pi_{hhi})^{\text{Diffusion Optimum}} = \lambda_{hi}/(\lambda_{hi} + \lambda_{fi}) = 1/2$ . Under the market allocation, trade costs induce home bias such that domestic share is  $(\pi_{hhi})^{\text{Market Allocation}} = 1/(1 + \tau^{-\theta}) > 1/2$  and idea diffusion is below the optimal point. If trade costs increase and  $\tau \rightarrow \infty$ , the home country moves to autarky, and deviations from the optimal idea diffusion reach a maximum. We plot the optimal, actual trade, and autarky points along the ideas diffusion function for sector  $i$  on the left-hand side panel of Figure 1.

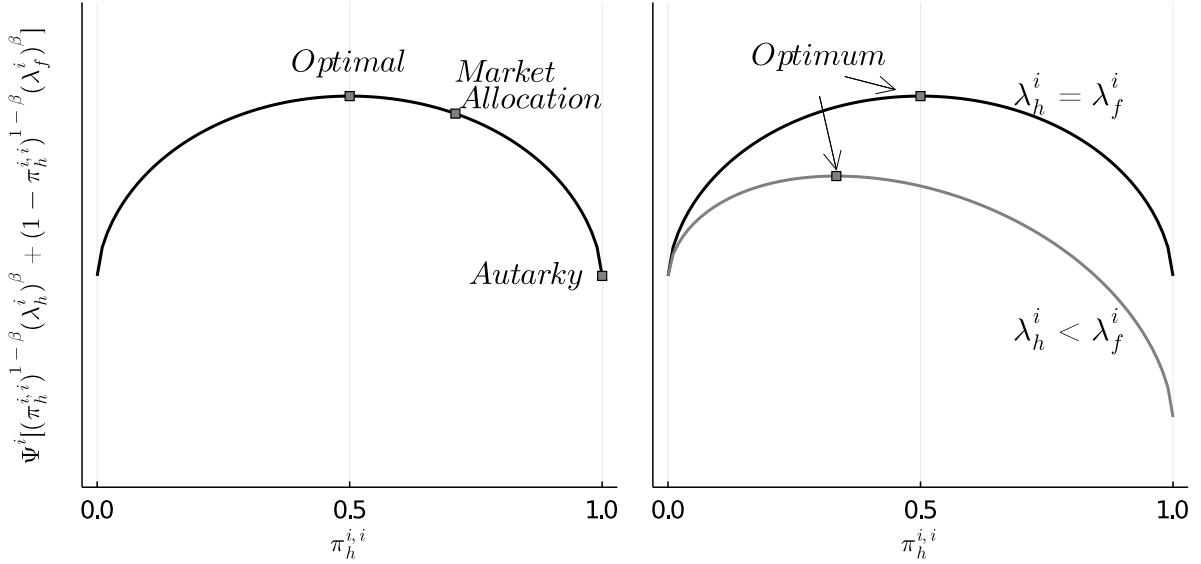


Figure 1: **Within sector idea diffusion functions in a two-by-two economy.** Both panels plot the idea diffusion functions for the home country in a two-by-two model within sector  $i$ :  $\kappa_i[(\pi_{hhi})^{1-\beta}(\lambda_{hi})^\beta + (1 - \pi_{hhi})^{1-\beta}(\lambda_{fi})^\beta]$ . The left-hand side panel shows the optimal, actual trade, and autarky points along the ideas diffusion function when countries are fully symmetric ( $\lambda_{hi} = \lambda_{fi}$ ). The right hand side panel plots the functions and diffusion optimal points for the cases when countries have identical productivities  $\lambda_{hi} = \lambda_{fi}$  and the home country is less productive  $\lambda_{hi} < \lambda_{fi}$ .

The right-hand side panel illustrates what happens when the home country has a lower productivity in sector  $i$ . The curve shifts down at the autarky point and the optimal solution moves to the left (smaller domestic trade share)<sup>15</sup>. When  $\lambda_{hi} < \lambda_{fi}$ , diffusion losses from high trade costs are higher. This highlights a key characteristic of this class of models: countries that are *less productive* in a given sector have *higher dynamic gains from trade*<sup>16</sup>.

Now, why does a multi-sector framework induce lower diffusion than its single-sector counterpart? The answer hinges on the strict concavity of the diffusion function (12) in trade shares and productivity terms. If one substitutes the solution for diffusion maximizing trade shares, as shown above, into the diffusion function, this will be proportional to:

<sup>15</sup>Formally, once countries are no longer symmetric, we need to make the following regularity condition to guarantee convergence to the autarky equilibrium:  $\lim_{\tau \rightarrow \infty} (\tau c_{fi})/c_{hi} = +\infty$ . Most models make this assumption either explicitly or implicitly.

<sup>16</sup>In fact, for any  $\pi_{hi} \in (0, 1]$ , the marginal change in diffusion as  $\pi_h^i$  increases will be increasing in a country's productivity. To see that, take  $\frac{\partial \Delta \lambda_{hi}}{\partial \pi_h^i} \propto (1 - \beta)[(\pi_{hi})^{-\beta}(\lambda_{hi})^\beta - (1 - \pi_{hi})^{-\beta}(\lambda_{fi})^\beta]$ , which is increasing in  $\lambda_{hi}$ .

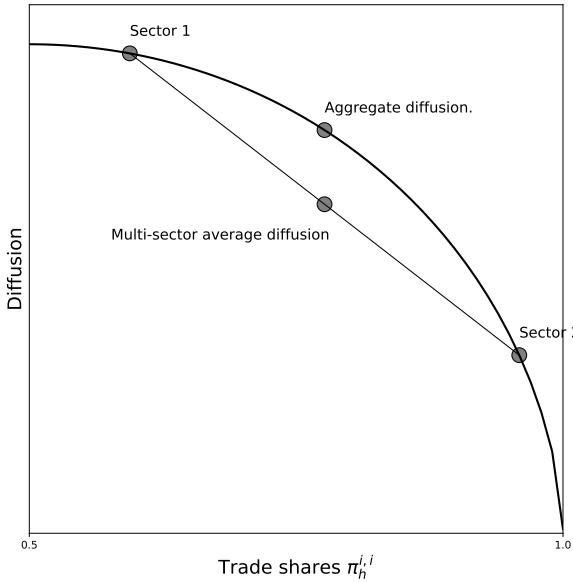
$$\begin{aligned}
& \underbrace{(\Delta\lambda_{d,t})^{\text{Diffusion Optimum}}}_{\text{single sector diffusion optimum}} \propto (1 - \gamma_d) \left( \sum_{s \in \mathcal{D}} \lambda_{s,t-1} \right)^{\beta} \\
& \underbrace{\sum_{i \in \mathcal{I}} \kappa_{di} (\Delta\lambda_{d,t}^i)^{\text{Diffusion Optimum}}}_{\text{average multi-sector diffusion optimum}} \propto \sum_{i \in \mathcal{I}} \kappa_{di} (1 - \gamma_{di}) \sum_{j \in \mathcal{I}} \gamma_{dij} \left( \sum_{s \in \mathcal{D}} \lambda_{sj,t-1} \right)^{\beta}
\end{aligned}$$

where  $(1 - \gamma_d) \equiv \sum_i \kappa_{di} (1 - \gamma_{di})$  is the intermediate goods share in a single sector framework, defined in a way to keep it consistent with its multi-sector analog. If aggregate productivities  $\lambda_{s,t-1}$  in each source country  $s$  are at least as large as the cost-weighted average of its sectoral productivities in every destination country  $d$  —i.e.,  $\lambda_{s,t-1} \geq \sum_{i \in \mathcal{I}} \kappa_{di} \sum_{j \in \mathcal{I}} \gamma_{dij} \lambda_{sj,t-1}$ , then:

$$\left( \sum_{s \in \mathcal{D}} \lambda_{s,t-1} \right)^{\beta} \geq \left( \sum_{j \in \mathcal{I}} \gamma_{dij} \sum_{s \in \mathcal{D}} \lambda_{sj,t-1} \right)^{\beta} > \sum_{j \in \mathcal{I}} \gamma_{dij} \left( \sum_{s \in \mathcal{D}} \lambda_{sj,t-1} \right)^{\beta}$$

The first inequality follows from the assumption and the fact that the function is increasing. The last inequality follows Jensen's Inequality. A restriction over the parameter space of sectoral productivities is sufficient to guarantee that a single-sector framework will lead to higher maximum diffusion than a multi-sector one.

By a similar reasoning, we can show that the allocation in the single-sector model under the market allocation will be greater than under a multi-sector framework. In Figure (2), we plot the multi-sector to single-sector diffusion ratio under the market allocation.



**Figure 2: Intuition: multiple sectors and strict concavity of the diffusion function.** The diffusion function is strictly concave in both  $\pi$  and  $\lambda$ . This figure shows an example of calculating diffusion in single-sector vs a multi-sector framework when trade shares differ across sectors. The same reasoning can be applied for when productivities vary across sectors, as long as  $\sum_{j \in \mathcal{I}} \gamma_{dij} \lambda_{sj,t-1} \leq \lambda_{s,t-1}$  for each  $s$  and  $d$ .

Intuitively, the more sectors' degrees of openness differ, the lower the level of diffusion in a multi-sector framework is compared to its single-sector counterpart<sup>17</sup>. The explanation again lies in the concavity of the diffusion function. By definition, aggregate trade shares are a convex combination of the sectoral trade shares. The higher the differences across sectors, the more the concavity of the diffusion function plays a role in inducing diffusion losses in a multi-sector framework.

The assumption stated in Proposition (2) guarantees the same condition on the productivity states. Since productivities are not directly observed and must be calibrated, the assumption stated in the proposition is then a sufficient restriction in the state space that ensures that the comparison satisfies Jensen's Inequality also in its second argument.

It rules out situations in which the calibrated aggregate productivity of a *source country* is not at least as large as the expenditure-weighted average in each *destination country* (because diffusion flows *from source to destination countries*)<sup>18</sup>. If this condition is satisfied, then we achieve the result in Proposition (2).

### 3 Estimation of Structural Parameters

This section describes our three-stage estimation procedure for the time-varying sectoral Fréchet location parameters  $\lambda_{si,t}$  and diffusion elasticities  $\beta_i$ . The overall approach is to estimate overall competitiveness from a gravity model and then net out the effects of unit costs, such that we are left with technology levels only.

#### 3.1 First stage: estimating gravity fixed effects with normalizations and constraints

Levchenko and Zhang (2016) estimate a gravity model in normalized trade-share form, controlling for exporter- and importer-specific components of technology and unit costs, as well as for bilateral trade frictions using symmetric observables such as distance, borders, and trade agreements. Following Waugh (2010), any asymmetry in bilateral trade costs is attributed to the exporter, so that exporter- and importer-specific terms are allowed to differ even though bilateral observables enter symmetrically.

In practice, this implies a gravity specification with time-varying exporter and importer fixed effects. The exporter fixed effect reflects exporter productivity and unit costs together with an average level of trade costs faced by exporters from a given country, while the importer fixed effect captures only the productivity and unit cost components and thus is the one used in further auxiliary analysis.

The identifying restriction is that asymmetric trade costs load on the exporter term rather than varying fully bilaterally or appearing along the importer side. Asymmetric

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<sup>17</sup>Figure (10) in the Appendix shows the ratio of single-sector to multi-sector diffusion across a continuum of domestic trade shares for two different sectors. In the upper left quadrant of Figure (10), sector  $i$  sources almost all of its inputs abroad, while sector  $-i$  buys most of its inputs domestically. Over the diagonal, the trade shares of the sectors are identical, and differences reach zero.

<sup>18</sup>This restriction does not require, however, aggregate productivities to be arbitrarily large. Considering a limiting case (which is not sustainable in equilibrium), helps us to place a natural upper bound. The most restrictive situation would be if some destination country  $d$  had the expenditure share  $\kappa_{di} = 1$  for some arbitrary  $i$  and the cost share  $\gamma_{dij} = 1$  for some arbitrary  $j$ . In that case, a sufficient (but not necessary) condition is that aggregate productivity states in source countries are at least as large as all sectoral productivities  $\lambda_{s,t} \geq \lambda_{s,t}^j$  for all  $j$ .

pairwise fixed effects are now the gravity literature's preferred approach to fully flexibly (cross-sectionally, at least) controlling out trade costs (Baier and Bergstrand, 2007; Head and Mayer, 2014). This lends itself to the following amended estimating equation:

$$\ln\left(\frac{X_{sdi,t}}{X_{ddi,t}}\right) = \underbrace{\ln\left(\lambda_{si,t}(c_{si,t})^{-\theta_i}\right)}_{\text{Exporter fixed effect}} - \underbrace{\ln\left(\lambda_{di,t}(c_{di,t})^{-\theta_i}\right)}_{\text{Importer fixed effect}} + \underbrace{\left(\ln(\tau_{sdi})^{-\theta_i}\right)}_{\text{Pair fixed effect}} \\ - \underbrace{\theta_i CU_{\{sd\}t} - \theta_i RTA_{\{sd\}t} - \theta_i Glob_{sdi,t}}_{\text{Bilateral observables}} - \underbrace{\theta_i \nu_{sdi,t}}_{\text{Error term}}. \quad (15)$$

Where, importantly, the pairwise fixed effect is designed to pick up all time-invariant trade costs. Additional time-varying bilateral observables can then also be included to control for trade cost changes that occur within-sample.

There are some important implications for estimation of (15). First, the exporter-year and importer-year fixed effects should be constrained to share the mirror vector of estimated values:  $\ln\left(\lambda_{si,t}(c_{si,t})^{-\theta_i}\right) \equiv \ln\left(\lambda_{di,t}(c_{di,t})^{-\theta_i}\right), \forall s = d$ . <sup>19</sup>

Separately, we also need to drop an appropriate set of fixed effects levels as reference categories. Unlike in many gravity applications, we need to be able to directly interpret the fixed effect estimates. The normalizations and reference levels applied to them are therefore important. In Levchenko and Zhang (2016), in the absence of any pairwise fixed effects, it is sufficient to drop one country's importer fixed effect (analogously one per year in a panel setting) and use it as the baseline for subsequent analysis. Once we add pair fixed effects, dropping these for domestic trade observations (i.e. setting  $\ln(\tau_{ddi}) = 0$ ) is standard and a natural assumption in a model focused primarily on international trade.

However, this still leaves a rank deficiency of  $N - 1$ , where  $N$  is the number of countries in the data. Consider the simplified version of the model<sup>20</sup>  $\eta_{sdi,t} = \xi_{si,t} - \xi_{di,t} + \gamma_{sd}$ . For a given  $c \neq$  reference country across all years, we can perturb  $\xi_{ci,t} \leftarrow \xi_{ci,t} + \delta$  on the exporter side but also  $\gamma_{cd} \leftarrow \gamma_{cd} - \delta$  and  $\gamma_{sc} \leftarrow \gamma_{sc} + \delta$ . Then, for any observation  $(s, d, t)$ :

- If  $s = c$ : the  $+\delta$  from  $\xi_{ci,t}$  cancels with the  $-\delta$  in  $\gamma_{cd}$ ;
- If  $d = c$ : the  $-\delta$  from  $-\xi_{ci,t}$  cancels with the  $+\delta$  in  $\gamma_{sc}$ ;
- Otherwise: no term shifts.

Thus in all cases, this yields the same predictor. In other words, there remain many different combinations of  $(\xi_{si,t}, \gamma_{sd})$  which would yield the same  $\eta_{sdi,t}$ , so there is no unique solution to our fixed effects system to pin down from the data.

This issue has a fairly intuitive interpretation here: the estimated fixed effects face an ambiguity as to where to attribute the average level of trade costs faced by a given country over the sample. They can be considered as part of the relevant country-year terms, as a shifter of the relevant pairwise terms, or spread in some proportion across both.

Given that we want to interpret the country-year terms as functions of technology and unit costs only, we need to anchor  $\gamma_{sd}$  in such a way that ensures it absorbs the average level of trade costs (and equivalently prevents any potential to directly compensate for perturbations in the country fixed effects). We achieve the required  $N - 1$  normalizations

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<sup>19</sup>This is not trivial to implement in standard high dimensional fixed effects packages but can be achieved by explicitly designing the model matrix.

<sup>20</sup>This issue also applies in the case where the fixed effects aren't constrained to mirror each other

by additionally setting  $\gamma_{s,\text{ref}} = 0, \forall s$ , where  $\text{ref}$  represents the reference country whose country-year fixed effects we are already dropping. This normalization pins down the full system of fixed effects to a unique set, but ensures the average level of trade costs can remain in the pairwise term. <sup>21</sup>

### 3.2 Second stage: estimating technology levels $\lambda$

From (15), the exporter-year fixed effect in sector  $i$  collects the (log) technology term and the unit-cost term raised to the trade elasticity:

$$\text{FE}_{si,t}^i \equiv \ln(\lambda_{si,t}(c_{si,t})^{-\theta_i}) - \ln(\lambda_{\text{ref},i,t}(c_{\text{ref},i,t})^{-\theta_i}).$$

With Cobb–Douglas production and (time- and country-) flexible cost shares we have:

$$c_{si,t} = \frac{1}{\lambda_{si,t}} w_{s,t}^{\psi_{si,t}\gamma_{si,t}} r_{s,t}^{(1-\psi_{si,t})\gamma_{si,t}} \prod_{j \in \mathcal{I}} P_{sj,t}^{(1-\gamma_{si,t})\gamma_{sij,t}},$$

so that, relative to a reference country  $\text{ref}$  in the same year:

$$\ln \frac{\lambda_{si,t}}{\lambda_{\text{ref},i,t}} = \text{FE}_{si,t}^i + \theta_i \left[ \psi_{si,t}\gamma_{si,t} \ln \frac{w_{s,t}}{w_{\text{ref},t}} + (1-\psi_{si,t})\gamma_{si,t} \ln \frac{r_{s,t}}{r_{\text{ref},t}} + \sum_j (1-\gamma_{si,t})\gamma_{sij,t} \ln \frac{P_{sj,t}}{P_{\text{ref},j,t}} \right]. \quad (16)$$

We assemble data on the cost components and shares as follows. Labour cost shares in value added  $\{\psi_{si,t}\}$  are inferred from OECD Trade in Employment (TiM) factor-share data. Value-added and intermediate input shares  $\{\gamma_{si,t}, \gamma_{sij,t}\}$  are computed from the OECD ICIO tables by sector and year and scaled by gross output. Wages  $w_{s,t}$  are constructed from TiM compensation and employment. The rental rate  $r_{s,t}$  is built from data in the Penn World Tables on real interest and depreciation rates.

For the intermediate-input price term  $\ln(P_{sj,t}/P_{\text{ref},j,t})$ , we again follow the approach in Levchenko and Zhang (2016). Let  $\pi_{ssj,t}$  denote the domestic expenditure share in sector  $j$  for country  $s$ , and  $\pi_{\text{ref},\text{ref},j,t}$  the corresponding share in the reference country.

With  $\text{FE}_{\text{ref},i,t}^i = 0$  by construction we have

$$\exp(\text{FE}_{si,t}^i) = \frac{\lambda_{si,t}}{\lambda_{\text{ref},i,t}} \left( \frac{c_{si,t}}{c_{\text{ref},i,t}} \right)^{-\theta_i} \equiv S_{si,t}^i.$$

Thus  $S_{si,t}^i$  is observed directly from the gravity first stage.

Relative sectoral prices can then be obtained from the adjusted domestic-share “double ratio”:

$$\frac{P_{sj,t}}{P_{\text{ref},j,t}} = \left( \frac{\pi_{ssj,t}/\pi_{\text{ref},\text{ref},j,t}}{S_{sj,t}^i} \right)^{1/\theta_j}, \quad (17)$$

We still need a set of values for  $\lambda_{\text{ref},i,t}$  to anchor the technology levels. Using a standard production function residual approach and KLEMS data provides an observed

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<sup>21</sup>This contrasts with an alternative normalization, often implemented by default in high dimensional fixed effects software, which is to enforce a sum-to-zero constraint on the pair fixed effect estimates. This would force each country’s average bilateral barrier to zero, pushing that average into the country-year terms and thereby mixing productivity/unit costs with trade costs in a way that would not be appropriate for further auxiliary analysis.

sectoral TFP index for the reference country,  $\Lambda_{it}^{\text{obs}}$ . However, this conflates the latent technology scale with access to foreign suppliers via the price index; we do not observe the full distribution of productivity draws (see, e.g., [Finicelli, Pagano and Sbracia \(2013\)](#); [Costinot, Donaldson and Komunjer \(2012\)](#)). The reference-country price index satisfies

$$P_{\text{ref},it}^{-\theta_i} = \sum_s \lambda_{si,t} (c_{si,t})^{-\theta_i} \tau_{s,\text{ref},i}^{-\theta_i} = \sum_s \exp(\text{FE}_{si,t}^i) \exp(\text{pair}_{s,\text{ref}}^i) \equiv \phi_{it},$$

where  $\text{pair}_{s,\text{ref}}^i$  captures time-invariant bilateral costs (normalized so that domestic pairs are zero). [Finicelli, Pagano and Sbracia \(2013\)](#) show that the latent technology scale for the reference country is obtained by removing this market-access term:

$$\lambda_{\text{ref},i,t} = \left( \frac{\Lambda_{it}^{\text{obs}\theta_i}}{\phi_{it}} \right)^{1/\theta_i}, \quad \phi_{it} = \sum_s \exp(\text{FE}_{si,t}^i) \exp(\text{pair}_{s,\text{ref}}^i).$$

Combining the reference technology with  $S_{si,t}^i$  and the cost ratio from (16), we obtain

$$\lambda_{si,t} = \left( \frac{\Lambda_{it}^{\text{obs}\theta_i}}{\phi_{it}} \right)^{1/\theta_i} \exp(\text{FE}_{si,t}^i) (c_{si,t}/c_{\text{ref},i,t})^{\theta_i}.$$

### 3.3 Third stage: estimating diffusion elasticities $\beta$

Let  $\Delta\lambda_{di,t} \equiv \lambda_{di,t} - \lambda_{di,t-1}$ . The law of motion implied by the model can be written as

$$\Delta\lambda_{di,t} = \alpha_{i,t} Q_{di,t-1}(\beta, \theta) + u_{di,t}, \quad (18)$$

where

$$Q_{di,t-1}(\beta, \theta) = \sum_j \tilde{\gamma}_{dij,t-1} \Gamma\left(1 - \frac{\beta_i \theta_i}{\theta_j}\right) \sum_s \pi_{sdj,t-1} \left( \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} \right)^{\frac{\beta_i \theta_i}{\theta_j}}.$$

Here  $\tilde{\gamma}_{dij,t-1}$  are input-cost shares for sector  $i$  in destination  $d$ , and  $\pi_{sdj,t-1}$  are bilateral expenditure shares on good  $j$ .

We compute  $\tilde{\gamma}_{dij,t}$  from the OECD ICIO as above, with bilateral shares  $\pi_{sdj,t}$  coming from summing sector- $j$  shipments from  $s$  to  $d$  across uses and normalizing by the destination-sector total. When countries are dropped in the estimation of  $\lambda$ , we renormalize  $\pi_{sdj,t-1}$  over the set of exporters  $s$  with observed  $\lambda_{sj,t-1}$  so that  $\sum_s \pi_{sdj,t-1} = 1$  within each  $(d, j, t)$  used in (18).

We estimate  $\beta_i$  by nonlinear least squares with  $\alpha_{i,t}$  profiled out. For any candidate vector  $\beta$  we compute  $Q_{di,t-1}(\beta, \theta)$  via (18) and set

$$\hat{\alpha}_{i,t}(\beta) \equiv \frac{\sum_d Q_{di,t-1}(\beta, \theta) \Delta\lambda_{di,t}}{\sum_d Q_{di,t-1}(\beta, \theta)^2}.$$

The objective is the sum of squared residuals,

$$\min_{\beta} \sum_{i,t} \sum_d \left[ \Delta\lambda_{di,t} - \hat{\alpha}_{i,t}(\beta) Q_{di,t-1}(\beta, \theta) \right]^2,$$

treating  $\{\theta_j\}$  as given.

### 3.4 Estimates and discussion

**Estimated technology levels.** We find substantial heterogeneity in estimated technology levels. Across regions, the median of  $\log \lambda_{si,t}$  spans several log points, and within each region we observe sizable cross-sector dispersion (Figure 3). This within-region heterogeneity further motivates our multi-sector approach: the technology gaps that matter for diffusion and welfare are not well summarized by country aggregates.

The cross-country ordering broadly aligns with development patterns, with countries like the United States and Germany appearing at the upper end. Across sectors, dispersion is similarly pronounced (Figure 4). The lowest estimated technology levels tend to occur in extractive and utilities sectors, whereas the highest are in several services and high-tech manufacturing sectors.

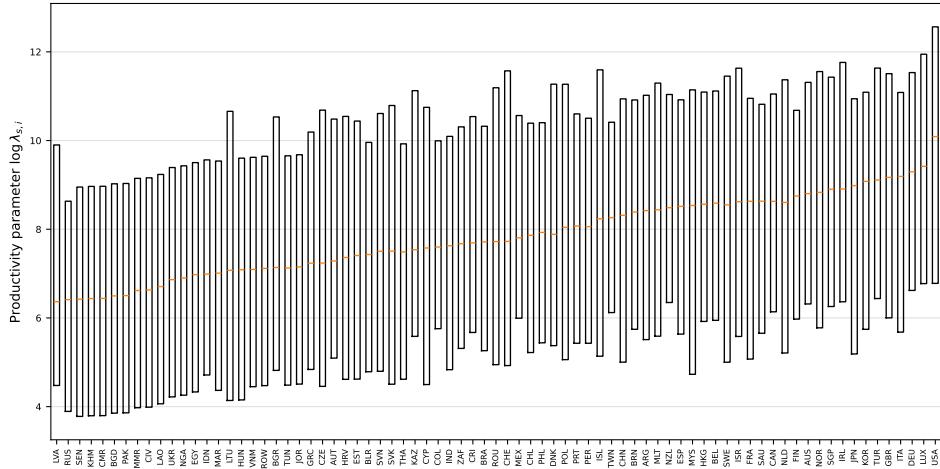


Figure 3: **Large dispersion within and across countries.** This figure summarizes the estimated technology levels  $\lambda_{si,t}$  across regions; boxes show the cross-sector distribution within each region.

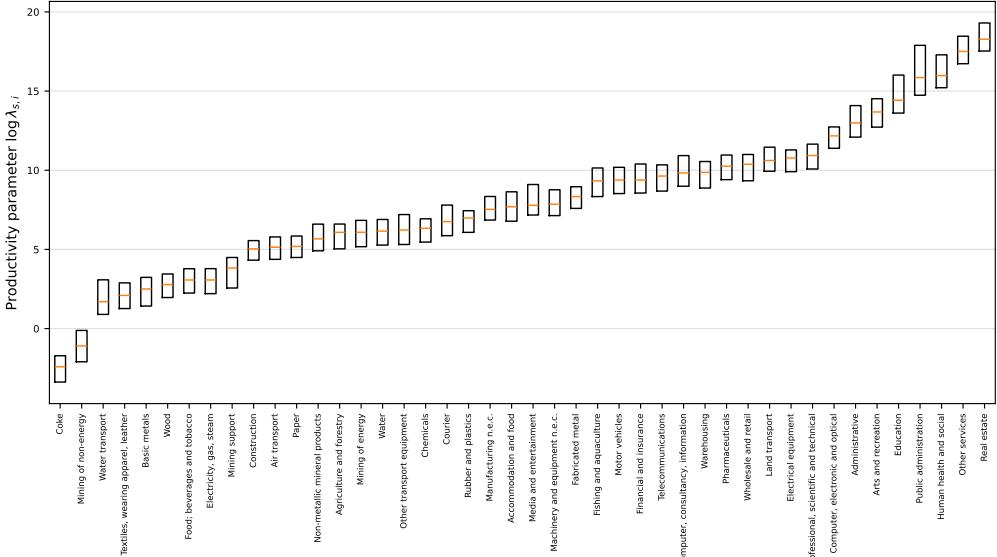


Figure 4: **Large dispersion across and within sectors.** This figure summarizes the estimated technology levels  $\lambda_{si,t}$  across sectors; boxes show the cross-country distribution within each sector.

As an external validation, Figure 5 compares our estimated technology index (scaled by  $1/\theta_i$  so that they reflect average productivity) with measures of sectoral labor productivity. The relationship is positive but far from mechanical, consistent with  $\lambda_{si,t}$  capturing technological differences rather than simply loading on observed factor costs.

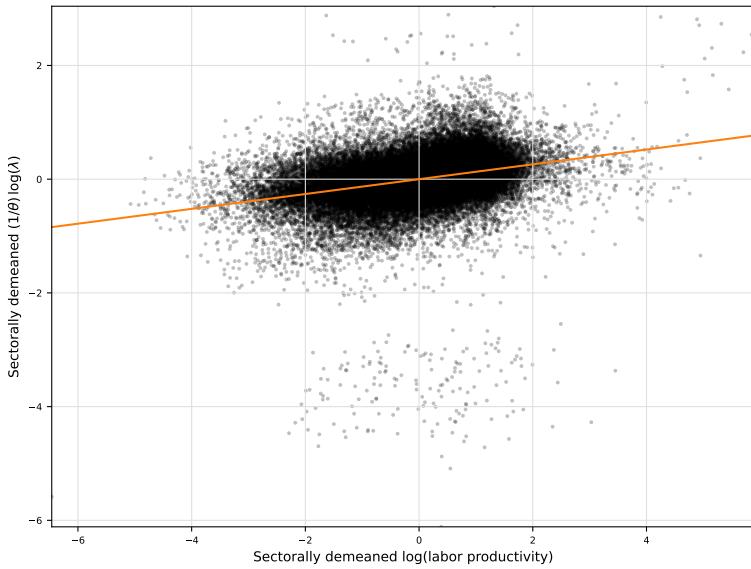


Figure 5: **Positive correlation with labor productivity.** Correlation between sectoral labor productivity and the estimated technology index (scaled by  $1/\theta_i$ ). The correlation coefficient is  $\rho = 0.29$ .

**Estimated diffusion elasticities.** Figure 6 reports the distribution of the implied pairwise elasticities  $\beta_{ij} \equiv \beta_i \theta_i / \theta_j$  by use sector  $i$ . Most sectors cluster around intermediate values (roughly between 0.2 and 0.4), but there is considerable heterogeneity: some sectors

like basic metals have values at effectively zero, implying no room for any conditional catchup at all, while the upper tail reaches towards 0.5 for sectors like construction where diffusion is strong. This heterogeneity implies that the innovation consequences of trade disruptions are highly sector-dependent.

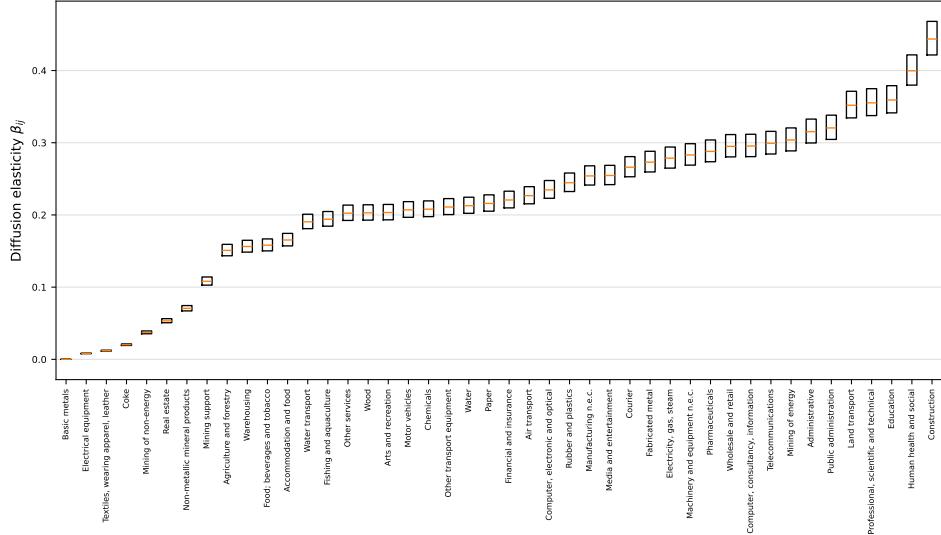


Figure 6: **Diffusion elasticities by use sector.** Distribution of implied pairwise elasticities  $\beta_{ij} \equiv \beta_i \theta_i / \theta_j$  by use sector  $i$ ; for each use sector  $i$ , the box summarizes dispersion across source sectors  $j$ .

**Pairwise diffusion elasticities.** To interpret the estimated diffusion elasticities in terms of bilateral exposure, we compute the pairwise diffusion elasticities

$$\varepsilon_{di,sj,t-1} \equiv \frac{\partial \log \Delta \lambda_{di,t}}{\partial \log \lambda_{sj,t-1}}.$$

Using (18) and defining  $\beta_{ij} \equiv \beta_i \theta_i / \theta_j$ , these elasticities decompose as

$$\varepsilon_{di,sj,t-1} = \beta_{ij} \tilde{\omega}_{di,sj,t-1}, \quad (19)$$

where  $\tilde{\omega}_{di,sj,t-1}$  is the weight of source sector  $(s, j)$  in the diffusion aggregator for destination sector  $(d, i)$ :

$$\tilde{\omega}_{di,sj,t-1} \equiv \frac{\tilde{\gamma}_{dij,t-1} \Gamma(1 - \beta_{ij}) \pi_{sdj,t-1}^{1-\beta_{ij}} \lambda_{sj,t-1}^{\beta_{ij}}}{\sum_{j'} \tilde{\gamma}_{dij',t-1} \Gamma(1 - \beta_{ij'}) \sum_{s'} \pi_{s'dj',t-1}^{1-\beta_{ij'}} \lambda_{s'j',t-1}^{\beta_{ij'}}}, \quad \sum_{s,j} \tilde{\omega}_{di,sj,t-1} = 1.$$

Intuitively, bilateral diffusion is larger when the relevant pair of sectors is intrinsically more diffusion-intensive (high  $\beta_{ij}$ ) and when the destination sector is more exposed to the source sector through input–output linkages and trade (high  $\tilde{\omega}_{di,sj,t-1}$ ). Figures 7 and 8 visualize these pairwise elasticities.

In the country-ordered representation (Figure 7), diffusion elasticities exhibit a pronounced block-diagonal pattern, reflecting strong within-country and closely-linked partner diffusion. Off-diagonal variation highlights that a subset of source countries accounts for a disproportionate share of idea inflows across destinations.

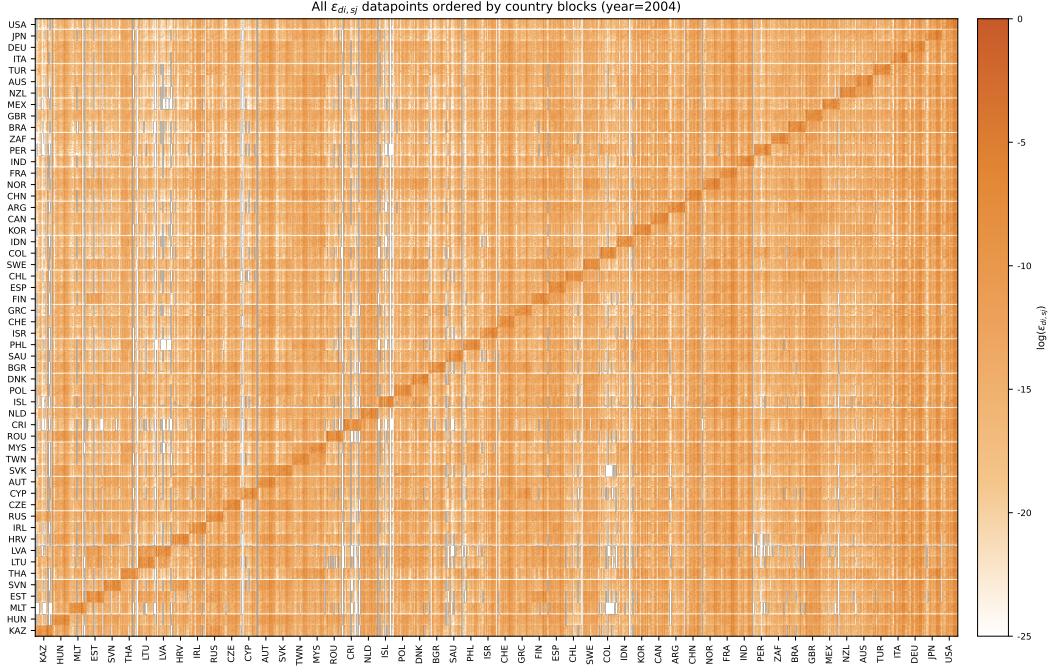


Figure 7: **Bilateral diffusion elasticities (by country).** Heatmap of  $\varepsilon_{di,sj}$  aggregated across sectors.

The industry-ordered heatmap (Figure 8) shows analogous heterogeneity across sector pairs: diffusion is unevenly distributed across the network, with some sectors systematically acting as more important sources and/or destinations in the diffusion process.

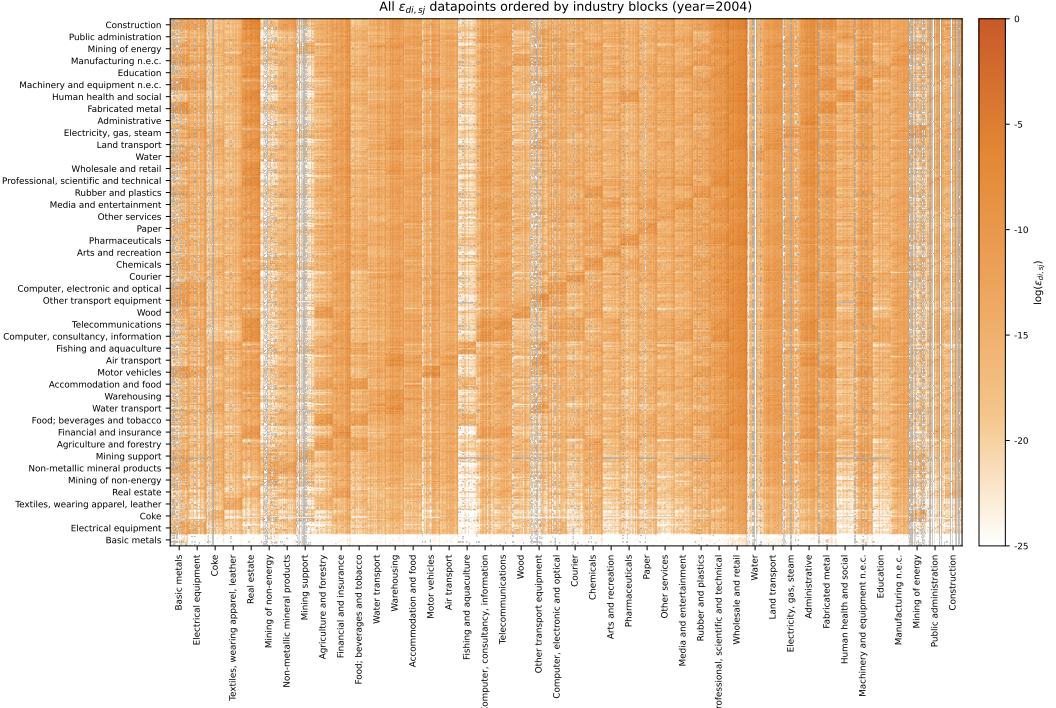


Figure 8: **Bilateral diffusion elasticities (by sector).** Heatmap of  $\varepsilon_{di,sj}$  aggregated across regions.

## 4 Calibration and Setup of Policy Experiments

In this section, we first outline the employed baseline data and remaining parameters. We then motivate and describe the detailed setup of the policy experiments.

### 4.1 Data and Calibration

**Data** The model is calibrated to the OECD Inter-Country Input-Output (ICIO) tables. We use the full ICIO country-sector resolution: 77 regions and 45 sectors, covering trade in both goods and services. All spending shares and cost shares are set equal to their counterparts in the ICIO benchmark input-output table, following the standard calibration logic in quantitative trade models (see, e.g., Dekle, Eaton and Kortum (2007)).

Population and labor supply evolve exogenously. We use United Nations population projections to construct growth rates for labor supply, and (as described in Section 1) we keep each region's capital-labor ratio constant over time.

The ICIO tables report value added but do not split it into labor and capital income. We therefore split primary income into labor and capital using OECD Trade in Employment (TiM) factor-share data on labor's share in value added at the country-sector(-year) level; when these factor shares are missing, we impute them using sector medians (and the overall median as a fallback).

**Rebalancing the data to incorporate profits.** The OECD ICIO tables do not report profit income in the sense required by our Bertrand model. Under Bertrand competition, profits in sector  $i$  and region  $s$  are pinned down as a constant share of the value of sales,

$$\Pi_{si} = \frac{1}{1 + \theta_i} X_{si},$$

where  $X_{si}$  denotes gross output (sales). We therefore construct a Bertrand-consistent benchmark by modifying the IO table so that the implied primary-income share can accommodate  $\Pi_{si}$  for every country-sector, while preserving the internal accounting identities of the table. Sectoral  $\theta_i$  are taken from Egger et al. (2021) and mapped to the ICIO sector classification (in our mapping  $\theta_i \in [3.18, 4.79]$ ).

We work directly with the IO table rather than factor payment splits. First, we define “primary income” as value added plus taxes less subsidies,

$$VA_{si}^{\sim} \equiv VA_{si} + TLS_{si},$$

and we do not treat  $TLS$  as a separate object thereafter. Second, within each country we reallocate  $VA^{\sim}$  across sectors so that each sector can accommodate the implied profit requirement  $\Pi_{si}$ , while holding total primary income by country fixed. Given gross output  $X_{si}$  and the reallocated  $VA_{si}^{\sim,\text{new}}$ , the accounting identity pins down the total intermediate bill,

$$INT_{si}^{\text{new}} = X_{si} - VA_{si}^{\sim,\text{new}}.$$

We implement this by scaling the entire column of intermediate purchases for each country-sector by a factor  $\zeta_{si}$ , i.e. multiplying all inputs used by buyer  $(i, s)$  by the common factor  $\zeta_{si} = INT_{si}^{\text{new}} / INT_{si}^{\text{old}}$ . This preserves the composition of intermediate inputs for that buyer (including bilateral import shares within intermediates). Finally, to ensure that each producer’s sales identity holds exactly (row sums equal  $X_{si}$ ), we use the inventories component of final demand (**INVNT**) as a residual balancing item; this adjustment is purely accounting and is not interpreted structurally.

With  $\theta_i$  from Egger et al. (2021), the implied distortion relative to the original ICIO table is moderate and concentrated: the total absolute reallocation of primary income across country-sectors is about 10.9% of total primary income, the intermediate-input scaling differs from unity by about 10.5% on average (intermediate-weighted), and the absolute inventory adjustments used to close producer sales identities sum to about 2.5% of world gross output.

Changes in inventories can be negative (inventory drawdowns), which does not correspond to “negative purchases from exporter  $n$ ” and can mechanically generate negative bilateral expenditure shares if treated as an ordinary final demand category. Because our model requires nonnegative expenditure shares, we remove inventories following the approach used in static quantitative trade applications (Costinot and Rodríguez-Clare, 2014). Concretely, for each destination country we (i) reassign positive inventory changes to household final demand and (ii) set inventories to zero. Holding the (Bertrand-adjusted) technical-coefficient matrix  $A \equiv Z \text{ diag}(X)^{-1}$  fixed, we recompute the inventory-free gross-output vector  $X^*$  from the Leontief system

$$(I - A)X^* = y^*,$$

where  $y^*$  is the resulting final-demand vector after removing inventories. We then reconstruct intermediate flows as  $Z^* = A \text{ diag}(X^*)$  and value added as  $VA^* = X^* - \mathbf{1}'Z^*$

(column-wise). All model calibration objects (including the single set of bilateral trade shares used for both intermediate and final uses, and the diffusion weights) are computed from the inventory-free benchmark  $(Z^*, Y^*, X^*)$ .

**Imputation of remaining parameter values.** The estimation procedure in Section 3 yields a panel of technology levels  $\{\lambda_{si,t}\}$  for country-sectors with the required factor data. For a small set of country-sectors (most notably the ROW aggregate), factor-share inputs are not available and the second-stage recovery of  $\lambda_{si,t}$  is therefore incomplete. We impute missing  $\lambda_{si,t}$  using a parsimonious relationship between estimated technologies and output per capita. Specifically, we estimate the within-(sector,year) regression  $\ln \lambda_{si,t} = \beta^\lambda \ln(\text{outputpc}_{s,t}) + \delta_{i,t} + \varepsilon_{si,t}$  on the set of observations with estimated  $\lambda_{si,t}$  and then use the fitted values to fill missing country-sectors, where  $\text{outputpc}_{s,t}$  is computed from ICIO gross output and UN population totals. We use the sectoral trade elasticities  $\theta_i$  from Egger et al. (2021) throughout this step (mapped to our 45 sectors). We take the diffusion elasticities  $\{\beta_i\}$  from the third-stage estimates in Section 3.

Buera and Oberfield (2020) set the growth rate of  $\alpha_t$  equal to the population growth rate of the US. We adopt the same heuristics and set the growth rate of the autonomous arrival rate of ideas  $\alpha_t$  at 1.18% per year, equal to the projected global population growth rate from 2021 to 2040.

## 4.2 Motivation and Setup of Policy Experiments

### 4.2.1 Motivation of Policy Experiments

Our main motivation for simulating large-scale trade conflicts is the possibility of a retreat from globalization due to a political backlash. Challenges to the international trade regime (and to globalization at large) might have once seemed like some circumstantial discontinuity in a long-run trend toward increasing openness. However, as we show below, political scientists argue that there is reason to believe that strategic geopolitical rivalries could trump economic gains—at least partially—in shaping the relationships of the U.S. and its allies with China and Russia and their allies.

There is ample evidence of substantial gains from trade openness, which can be as large as 50% of national income (Ossa, 2015) even in a static setting. At the same time, recent empirical evidence about frictions in local labor markets (Autor, Dorn and Hanson (2013); Dix-Carneiro and Kovak (2017)) highlights the distributional aspects of trade liberalization.

These concerns can translate into political grievances and may have contributed to an increase in the number of populist and isolationist parties in Western countries calling for less open trade policies (Colantone and Stanig, 2018). An increasing number of political parties use anti-globalization rhetoric to rally the support of constituents that have grievances against the distributional consequences of automation, structural change, offshoring, and trade opening, as shown in the review of the political science literature by Walter (2021).

A clear example of the shift in the consensus during the last decade is the trade conflict between the U.S. and China, which started under the first Trump Administration.<sup>22</sup> The

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<sup>22</sup>For a contemporaneous review of the policies implemented during that first term, see Bown and Irwin (2019). More recently, the renewed escalation of U.S. tariffs in the early part of the second Trump Administration illustrates that trade policy tools can be mobilized rapidly and at scale as instruments of economic statecraft. Gopinath and Neiman (2026) documents that statutory U.S. tariff rates rose sharply again in 2025 (to levels not seen in over a century), that implemented rates can diverge substantially

economic discourse shifted away from emphasizing the gains from trade to a framing of trade as a zero-sum game and to the use of national-security provisions of the international trade regime to engage in protectionist policy-making.

These geopolitical disputes are exemplified not only in the trade conflict between the U.S. and China but also in industry-specific policy changes, such as the U.S. government pressuring allies against allowing the participation of Chinese telecommunications companies in new infrastructure developments or limited cooperation in science and education between the two countries (Tang et al., 2021).

Wei (2019) provides a review of debates among Chinese scholars. Some Chinese analysts see an escalating and continuous conflict between China and the U.S. as a natural and “structural” development of a shifting international system that is moving from a unipolar (the U.S. being the only superpower) to a bipolar (China becoming a superpower on an equal footing to the U.S.) balance of power<sup>23</sup>. They tie a scenario of a continuous confrontation between the U.S. and China either to strategic geopolitical forces or to domestic political forces in America (Zhao, 2019).

In the West, political scientist Joseph S. Nye Jr. (Joseph Nye Jr., 2020) highlights that, while an abrupt decoupling between the U.S. and China is unlikely, both parties will try to decrease their (inter-)dependence with respect to each other’s actions, except where the costs of disengagement are too high to bear<sup>24</sup>. American policymakers and academics also motivate the conflict between China and the U.S. on geopolitical grounds. Although the tone of confrontation is stronger when coming from right-of-center policymakers and scholars<sup>25</sup>, both sides of the political spectrum in the U.S. discuss the readjustment of the economic relationship with China due to geopolitical concerns (Wyne, 2020).

The 2022 War in Ukraine and the global-scale retaliation against the Russian Federation that followed is another example of how geopolitical interests can take precedence over gains from economic integration. The escalation began in 2014, after Russia’s annexation of Crimea. The U.S. and its allies approved several rounds of sanctions against Russia, culminating in its expulsion from the G8. The confrontation reached another level in the aftermath of Russia’s invasion of Ukraine. Western countries imposed sanctions on banking transactions, froze foreign reserves, and closed the airspace for Russian planes. In March 2022, the G7 and the European Union revoked their recognition of Russia’s most-favored-nation status, opening the door for large tariff increases, and limited the operation of multinational corporations Russian subsidiaries<sup>26</sup>.

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from statutory rates due to exemptions and other frictions, and that tariff pass-through into U.S. import prices is close to one—implying that a large share of the costs are borne domestically.

<sup>23</sup>In the context above, the balance of power between functionally equivalent states (the “international structure”) provides incentives for strategic behavior by governments that try to maximize their power. We can interpret the unipolar or bipolar configurations as equilibria and disruptions between such equilibria as transition paths. This is known as the “structural realism” theory of international politics, developed by Kenneth Waltz (2010).

<sup>24</sup>Nye Jr. is mostly known for his joint work with Robert Keohane on “complex interdependence” during the post-World War II era (Keohane and Nye Jr, 2011). The authors focus their analysis on the creation of international rules and practices in a world in which the use of military force is very costly due to interdependence between multiple agents that engage both internationally and domestically. For instance, a great degree of trade integration increases the costs (and decreases the probability) of outright military conflict.

<sup>25</sup>See, for instance, the remarks of former White House Trade Council Director to Congress (Navarro, Peter, 2018) or a policy blueprint for decoupling by Scissors (2020), who is a scholar at the America Enterprise Institute, a right-of-center think tank.

<sup>26</sup>For a timeline of sanctions against Russia in the context of the War in Ukraine, see this web-

Like in the case of China, the relationship between Russia and the West can also be interpreted through the lens of a strategic game among great powers. Scholars argue about the geopolitical nature of the conflict (Mearsheimer, 2014) and the “reawakening” of geopolitics (Weber and Scheffer, 2022). Therefore, in either case, sanctions and trade conflicts fall within the larger backdrop of a strategic confrontation. These simultaneous conflicts can potentially be interpreted as a larger clash between the U.S. and its allies — a Western bloc; and Russia, China, and their allies — an Eastern bloc. As scholars have argued, we could be observing the emergence of a “China-Russia entente” (Lukin, 2021), which could lead to a “New Cold War” (Abrams, 2022).

We use these facts as motivation to apply our model to conduct hypothetical policy experiments of trade decoupling between East and West: namely, to simulate the effects of large-scale geopolitical conflicts between these blocs, in which players try to limit the level of interdependence between each bloc due to political drivers, even if that leads to economic costs.

The policy experiments focus on the potential effects of decoupling between an Eastern and Western Bloc, since this is most discussed scenario in policy discussions (Aiyar et al., 2023). We do not model the emergence of geopolitical conflict endogenously. The focus of this paper is on the potential consequences of economic decoupling.

#### 4.2.2 Setup of Policy Experiments

We are agnostic about the degree of future decoupling between East and West. Nonetheless, the fact that international relations scholars envisage disengagement as a real possibility underscores that estimating the potential economic consequences of decoupling is an important exercise. As our model highlights, changes in trade patterns and sourcing decisions have not only static effects, but also dynamic effects with respect to potential growth and innovation. Our policy experiments try to disentangle the static and dynamic costs of decoupling.

In order to develop the decoupling scenario, we classify countries as hypothetically belonging to either a Western or an Eastern bloc. These blocs are broadly defined, with some manual adjustment, through similarity in UN General Assembly voting records (Häge, 2011), with the United States and China as the centers of gravity of the Western and Eastern blocs, respectively.

Figure 9 illustrates the resulting classification. The Western bloc includes the United States, Canada, most of Europe, and U.S.-aligned partners such as Japan, South Korea, Australia, New Zealand, Israel, Saudi Arabia, Singapore, and the Philippines. The Eastern bloc includes China, Russia, India, Brazil, Egypt, Pakistan, and a set of economies in Southeast Asia and Sub-Saharan Africa. Latin America is split across blocs (e.g., Mexico, Argentina, Chile, Colombia, and Peru are classified as Western, while Brazil is classified as Eastern). Given our scenario is intended to be illustrative only, we do not build an explicitly non-aligned bloc (with the exception of the Rest of World aggregate, whose composition is highly heterogeneous and determined by data availability in the trade and input-output data). Endogenous bloc formation is beyond the immediate scope of this work.

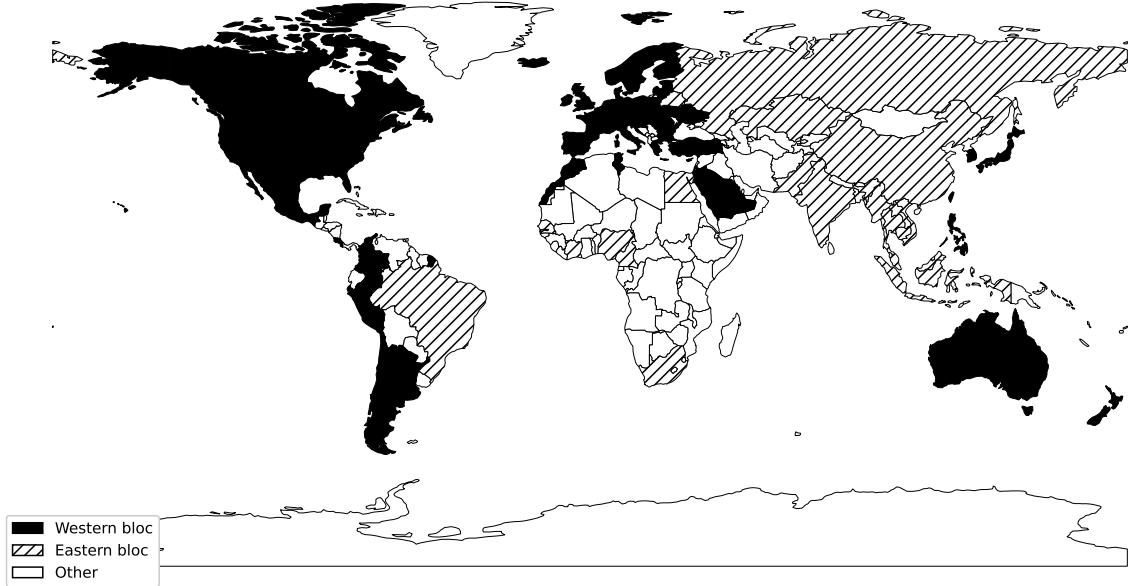


Figure 9: Illustrative geopolitical bloc assignment used in the decoupling scenarios. Countries are assigned to a Western bloc, an Eastern bloc, or “Other”, based on similarity in UN General Assembly voting records (Häge, 2011) relative to the United States and China, with some manual adjustments.

After classifying the countries into Eastern and Western geopolitical blocs, we increase iceberg trade costs  $\tau_{sd,t}^i$  by  $\sim 160$  percentage points and assume that the increases in trade costs are permanent. This represents a point where virtually all of the trade happens exclusively within each bloc, providing an important limiting case that can be useful for putting bounds on potential effects of decoupling.

Additionally, we explore a less hypothetical scenario: trade decoupling only in the electronic equipment sector. We perform a full decoupling between the blocs but restrict the increase in iceberg trade costs  $\tau_{sd,t}^i$  only to the electronic equipment sector. This scenario is motivated by U.S. and Chinese authorities being increasingly at loggerheads with each other in the technological arena.

One important example of this process has been the conflict involving Chinese telecom giant Huawei Technologies. Since 2019, American corporations have been banned from doing business with Huawei. In a similar move, the New York Exchange delisted China Unicom, China Mobile, and China Telecom. Despite legal challenges and a new administration, as of April 2021, neither decision has been reversed.

Additionally, the U.S. has been using its foreign policy arsenal to pressure allies to join them in limiting Chinese telecom companies’ reach. In particular, there is a desire to limit Chinese participation in 5G technology auctions, citing national security and privacy concerns<sup>27</sup>. So far, Australia, the United Kingdom and some European allies have chosen to ban or limit Chinese participation in technological auctions.

This conflict suggests that a large increase in trade costs between the U.S. allies and

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<sup>27</sup>North American Treaty Organization (NATO) researchers Kaska, Beckvard and Minárik (2019) review the arguments put forth from a Western national security perspective. This topic is extremely contentious and some Chinese commentators argue that the U.S. is using national security concerns as an excuse to implement industrial policy.

Chinese allies regarding technological equipment is a positive probability scenario in the future. In this case, decoupling would mean a near-total separation of the electronic equipment sectors of the two blocs.

Huawei and Google break of their business connections after the U.S. government sanctions against the Chinese corporation is a good illustration of what this separation could look like in real life. Huawei used Google’s *Android* ecosystem in their smartphones, which gave their users access to Google-approved updates and apps. After the ban issued by the Trump administration, however, Google announced it would comply with the U.S. government directives and Huawei was forced to shift away from Google software and design their own operating system *HarmonyOS*.

Since this separation is driven primarily by regulation rather than tariffs, it is appropriate to think of it as an increase in iceberg trade costs  $\tau_{sd,t}^i$  between blocs in the electronic equipment sector and so this is the scenario we will explore.

## 5 Main Results

We have four main scenarios. We simulate full decouple and tariff decouple, defined as explained above. We simulate either scenario both with and without diffusion of ideas, in order to assess the additional impact of the diffusion mechanism. After a discussion of the results of the full and tariff decouple scenarios, we compare the impact of decoupling on productivity in a multi-sector and single-sector setting. We finish this section with a description of the results of the two additional policy experiments that restrict decoupling to the electronics and equipment sector or change LAC from the Western bloc to the Eastern bloc.

In the results below we report the results of a comparison of the simulation results with and without policy experiments. We first simulate the dynamic world economy with no policy change, then do the same with the policy change, and report the long-run cumulative percentage difference between the two:  $\hat{x} = \sum_{t=p}^T (x'_t - x_t) / \sum_{t=p}^T x_t$ , where  $p$  is the date of the first policy change,  $x'_t$ ,  $x_t$  are the values of variable  $x$  with and without the policy change, respectively.

### 5.1 Full and Tariff Decouple

As expected, all scenarios show large negative impacts on cross-bloc trade after the introduction of the policy intervention. In the **full decouple** scenario, trade between the countries in the Western bloc and the Eastern bloc is virtually shut down, with imports and exports dropping by 98%. Those countries also shift a substantial part of their trade to the U.S., with trade flows increasing anywhere between 10 – 42% depending on the region and scenario. The domestic spending share in the U.S. increases by about 7%. The converse happens in the Eastern bloc but with larger dispersion across regions. Trade with the U.S. drops by 65 – 90% while trade with China increases by 9 – 60%. The domestic trade share in China increases by 3%. The **tariff decouple** scenario yields qualitatively similar results but with smaller magnitudes. We show the results by region and scenario in Figure 15.

[FIGURE 15 ABOUT HERE]

One of the reasons behind the asymmetry in trade decreases between blocs is the assumption of a fixed trade-balance-to-income ratio in all regions but one. This implies that

regions with large trade surpluses will shift proportionally less of their trade flows away from regions in other trade blocs in order to satisfy the fixed trade balance assumption.

Figure (16) shows that both the increases in iceberg trade costs (full decouple) and retaliatory tariff hikes (tariff decouple) induce substantial welfare decreases for all countries. The effects, however, are asymmetric. While welfare losses in the Western bloc range anywhere between  $-1\%$  and  $-8\%$  (median:  $-4\%$ ) in the Eastern bloc it falls in the  $-8\%$  to  $-12\%$  range (median:  $-10.5\%$ ).

[FIGURE 16 ABOUT HERE]

The underlying factor driving the divergence in results between the two blocs is a difference in the evolution of productivity, represented by the scale parameter of the Fréchet distribution of different sectors. Sourcing goods from high productive countries puts domestic managers in contact with better quality designs that inspire better ideas through innovation or imitation.

Importantly, the dynamics governed by equation (12) incorporate the input-output structure of production, such that domestic managers in each sector innovate in proportion to the quality and share of their inputs. Losing access to high quality designs does not only lead to static losses, but also to a lower level of future innovation, which implies larger dynamic losses. Additionally, the input-output structure of the model implies that cutting ties to innovative regions is particularly costly if the destination country sources many intermediate inputs from such regions prior to the policy change.

[FIGURE 17 ABOUT HERE]

For those reasons, in our policy experiments, countries in the Eastern bloc that currently have a lower level of productivity and have larger ties with innovative countries have larger losses. By looking at results in Figure 17, one can see the stark contrast between the differential evolution of  $\lambda_{di,t}$  for those countries in the Western bloc and those in the Eastern bloc. By cutting ties with richer and innovative markets such as OECD countries, destination countries such as China, India, and parts of Asia and Africa shift their supply chains toward lower-quality inputs, which, in turn, induces less innovation. By contrast, while countries in the Western bloc do suffer welfare losses, their innovation paths are virtually unchanged after decoupling, suggesting that nearly all of their losses are static, rather than dynamic.

There is large dispersion across both sectors and countries in differential productivity losses. The two most affected regions are India and the “rest of the Eastern bloc” region. Starting from a lower income level than China and Russia, those regions have a much slower productivity catch-up after severing trade ties with the West. Sectors with larger supply chain linkages to the West prior to the policy change, such as manufacturing in India, experience larger losses.

Among those regions in the Eastern bloc, differential productivity losses are larger in the manufacturing sectors ( $-1.5\%$  and  $-3\%$  with full decoupling and tariff decoupling, respectively; this includes **e1m**, **1mn**, and **hmn**) than in the services ( $-0.8\%$  and  $-1.6\%$ , respectively; **ots** **tas**) or primary ( $-0.5\%$  and  $-1\%$ , respectively; **pri**) sectors.

Finally, we address the contrast between the static effect (when the diffusion of ideas mechanism is shut down) and the dynamic effect. For the two poorer regions of the Eastern bloc, dynamic losses far outsize static losses, which can be explained by the loss of access to higher-quality inputs. In the right panel of Figure (16), we show the dynamic losses for each region.

In India, static welfare losses amount to  $1 - 2\%$  while dynamic losses range from  $7 - 10\%$ , depending on the decoupling scenario. Static losses to real income are small

because India is a relatively large country and its domestic trade share in the market equilibrium is large, which limits the range of goods affected by changes in terms of trade. However, because it is relatively poor, its losses in the diffusion of ideas version of the model are very large. By severing ties with the Western bloc, it limits the role of trade-induced innovation, which is a by-product of having access to high-quality suppliers.

By contrast, in Russia including dynamics leads only to small additional effects: welfare losses are very similar with or without the ideas diffusion mechanism. As explained above, this stems both from a higher income starting point and relatively limited input-output linkages with the West.

## 5.2 Diffusion Inefficiencies in a Multi-sector vs. a Single-sector Framework

In Section 2, we stressed that, except in knife-edge cases, within- and between-sector inefficiencies accumulate as the number of countries and sectors increase. The concavity of the diffusion process implies that *total* trade shares being at their optimal points is no longer sufficient for optimal diffusion. Optimal diffusion requires trade shares to be at their optimal points *in every sector*. This suggests that, in most cases, diffusion inefficiencies should increase with the number of sectors.

[FIGURE 18 ABOUT HERE]

Our numerical results confirm that theoretical intuition. Figure 18 contrasts the results of either the **full decouple** or the **tariff decouple** scenarios under the baseline specification presented in the previous section and an alternative simulation in which we collapse the model to a single-sector framework.

In both scenarios, countries in the Eastern bloc face higher cumulative diffusion inefficiencies (as measured by the reduction in the Fréchet parameters  $\lambda_{di,t}$ ) in a multi-sector framework. In fact, the single-sector dynamic productivity losses are outside the min-max range of the sectoral productivity changes for all countries in the Eastern bloc. These results underscore one important takeaway of this paper: modeling trade diffusion in a simplified single-sector framework can underestimate the level of dynamic losses induced by an increase in trade costs.

## 5.3 Consequences of Bloc Membership

In this section, we consider the consequences of moving one of the regions—Latin America and the Caribbean (LAC)—from the Western to the Eastern bloc. Intuitively, we expect that, by losing access to the highest productivity suppliers, LAC will experience less productivity growth. Nonetheless, the quantitative exercise allows us to have a sense of the magnitude induced by the change in group membership.

[FIGURE 19 ABOUT HERE]

Figure 19 compares the results of identical decoupling scenarios, simulating either *full decouple* or *tariff decouple*. The only difference is LAC bloc membership. As expected, most of the changes are concentrated in the LAC region. The left panel of Figure 19 shows that welfare losses in LAC are about 100 – 150% larger when it is included in the Eastern bloc, for both scenarios. The domestic trade share in LAC is virtually identical under both settings (with LAC in the Western or in the Eastern bloc), implying similar static welfare losses. This suggests that the increased losses from switching blocs stem almost entirely from dynamic losses.

Moving LAC to the Eastern bloc reduces the welfare losses of decoupling in India and China by about 2*p.p.* (16%) and 1*p.p.* (15%), respectively (results not reported). The reason is twofold. First, LAC has a higher income than India and the Rest of the Eastern bloc. All else equal, on average, its inclusion in the bloc raises average productivity and decreases dynamic losses. Second, lower tariff or iceberg trade costs between the Eastern bloc and LAC induce lower static losses for those countries.

The right-hand side panel of Figure 19 shows the differential productivity changes in the LAC region for different sectors. When LAC is included in the Western bloc, there are essentially no dynamic productivity losses in any sector: the evolution of the Fréchet Distribution scale parameter  $\lambda_{di,t}$  in the LAC Region behaves very similarly to a scenario with no policy changes.

In contrast, all sectors have dynamic productivity losses weakly greater than 1% when we simulate decoupling with LAC as part of the Eastern bloc. There is large sectoral heterogeneity. Under full decoupling, productivity losses range from 1% in Electronic Equipment (**elm**) to 0.4% in Business Services (**tas**). These differences are induced by input-output linkages.

This experiment underscores that the costs of decoupling might be unbearably high for low and middle-income countries that are excluded from the Western bloc. Many countries in Latin America and Africa benefit from increasingly large trade ties to China through both having larger market access and access to lower input costs. However, as the dynamic costs of severing ties with the West would be very high, and political leaders in those countries might have the incentive to keep an equidistant relationship between East and West, by preserving both mid-term gains from the relationship with China and longer term dynamic gains from having access to Western supply chains.

## 5.4 Electronic Equipment Decoupling

Finally, we compare our baseline scenario of *full decouple* in **all sectors** with a *full decouple restricted to the electronics equipment sector*. In both scenarios, we assume that the ideas diffusion mechanism works as described by equation (12) and we set  $\beta = 0.44$ , according to the calibration described before.

[FIGURE 20 ABOUT HERE]

Note that, due to the multi-sector structure of the model, an increase in iceberg trade costs in one particular sector potentially has an indirect effect in all sectors of the economy. The magnitude of such impact in a given sector can be split between a direct effect (proportional to input use from the **elm** sector as intermediates) and an indirect effect (proportional to the use of the **elm** sector in the production of intermediates inputs).

Results in Figure 20 show the productivity losses induced by policy changes represented by the evolution of the Fréchet Distribution scale parameter  $\lambda_{di,t}$  for those regions in the Eastern bloc. Contrasting the full decoupling in all sectors and one restricted to electronic equipment shows that, across all regions, productivity losses are substantially reduced and mostly restricted to the **elm** sector.

While there is some negative spillover effect to other sectors due to input-output linkages, particularly to business services (**tas**), these are very small for most regions. Regions such as Russia, which already had limited exposure to Western intermediate sourcing in the main scenario, see productivity losses go down to nearly zero across all sectors under the scenario that limits decoupling to the **elm** sector. China's losses in the **elm** sector are roughly similar to losses when decoupling happens in all products, but

other sectors are not substantially affected.

All other regions have non-negligible losses in the `elm` sector. The largest changes happen for India and the Rest of the Eastern bloc. Those regions have a lower productivity starting point and benefit proportionately more from exposure to higher-quality intermediate inputs. For that reason, full decoupling in all products leads to large differential losses in productivity in those regions. The more restricted full decoupling in `elm` scenario limits losses, since those are proportional to the use of Western electronic equipment as inputs in the production of other sectors.

[FIGURE 21 ABOUT HERE]

Changes in productivity map into changes in welfare, pictured in Figure 21. While welfare losses are substantial, ranging from 0.4–1.9%, they are very different in magnitude to the devastating results of a full decoupling in all products, in which losses range between 8 – 12%.

These results underline two important observations. First, the costs of sector-specific decoupling might be limited enough for this scenario to be feasible. Second, input-output structures play an important role in magnifying dynamic losses. Limiting decoupling to one specific sector tapers down indirect magnification effects that happen through the input-output network.

## 6 Conclusion

We build a multi-sector multi-region general equilibrium model with dynamic sector-specific knowledge diffusion in order to realistically investigate the impact of large and persistent geopolitical conflicts on global trade patterns, economic growth, and innovation. Canonical trade models typically start from a fixed technology assumption and thus miss a crucial source of gains from trade through the diffusion of ideas.

In our theoretical contribution, we show that large trade costs can lead to dynamic inefficiencies in sectoral knowledge diffusion. Furthermore, we show that in a multi-sector framework deviations from optimal knowledge diffusion happen both within and between sectors. Additionally, sectoral deviations accumulate, such that total trade shares being close to their aggregate optimal diffusion points is no longer sufficient to guarantee optimal diffusion. A takeaway of our theoretical discussion is that, as the number of sectors increases, so do the number of deviations from optimality and diffusion losses tend to be higher with multiple sectors.

We then use this toolkit to simulate the trade, innovation, and welfare effects of a potential retreat from globalization characterized by economic decoupling between the East and West, yielding three main insights. First, the projected welfare losses for the global economy of a decoupling scenario can be drastic, being as large as 12% in some regions, and are largest in the lower-income regions as they would benefit less from technology spillovers from richer areas. Second, the described size and pattern of welfare effects are specific to the model with diffusion of ideas. Without diffusion of ideas the size and variation across regions of the welfare losses would be substantially smaller. Third, a multi-sector framework exacerbates diffusion inefficiencies induced by trade costs relative to a single-sector one.

This has important implications for the role of the multilateral trading system. First, the current system with global trade rules guaranteeing open and free trade between all major players is especially important for the lowest-income regions. Second, if geopolitical

considerations would lead to a split of the big players into two blocs, it would be important that an institutional framework remains in place for smaller countries to keep open trade relations with both blocs, in particular for the lowest income regions.

The toolkit we have built is versatile and can be employed for many other research questions, in particular, focused on the analysis of policy. Future research could be extended in various directions. We mention two. First, there is ample empirical evidence for spillover effects from FDI, so the model could be extended with FDI and sales by multinational affiliates. Second, the framework with both technology spillovers and profits can be employed to analyze the economic effects of subsidy policies in different regions, an important policy topic in the multilateral trading system.

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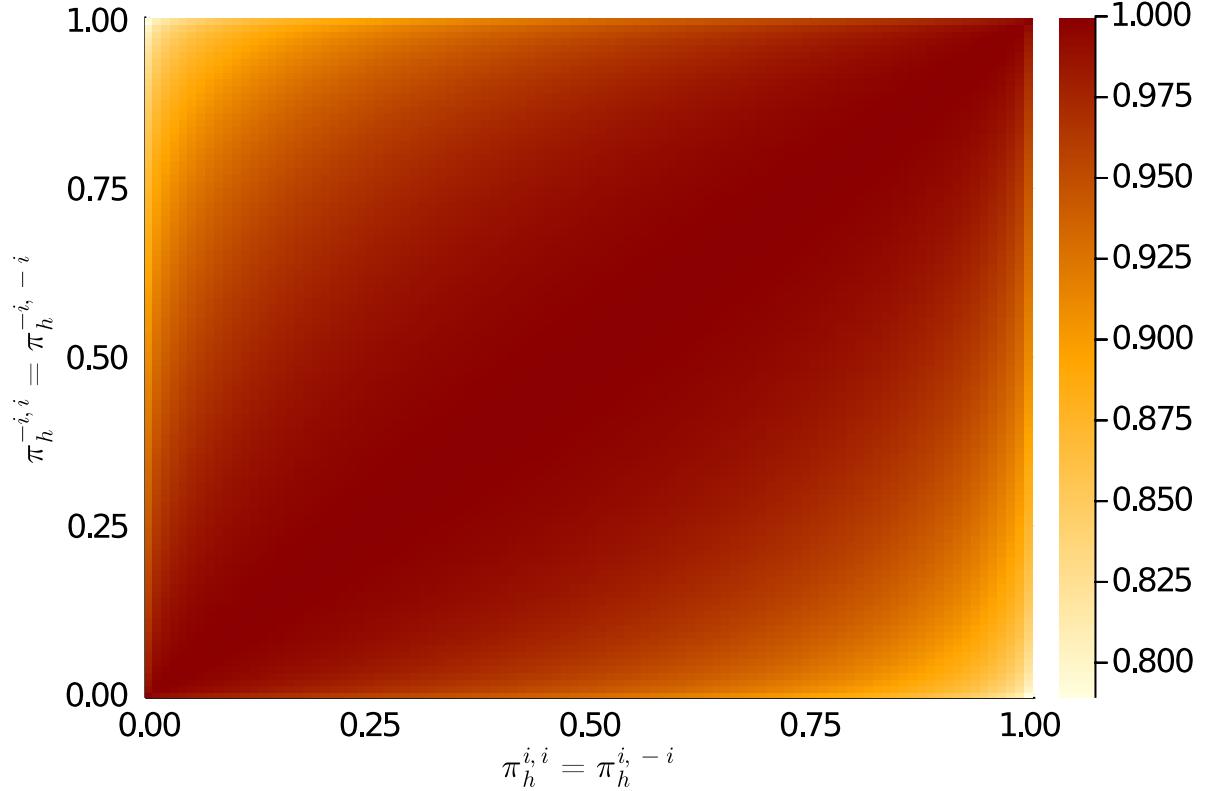
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## A Figures in Manuscript



**Figure 10: Ratio between Market Allocation Diffusion with a multi-sector and a single-sector framework.** Multi-sector to single-sector diffusion ratio under the Market Allocation. Here, we assume each sector use-sector to have identical degrees of openness with respect to each supply-sector ( $\pi_{hi} = \pi_h^{i,-i}$ ,  $\pi_h^{-i,i} = \pi_h^{-i,-i}$ ) but they may differ across sectors. By definition, the aggregate trade shares a convex combination of the sectoral trade share. In the upper left sector  $i$  sources almost all of its inputs abroad, while sector  $-i$  buys most of its inputs domestically and losses induced by a multi-sector framework reach a maximum. Over the diagonal, the trade shares of the sectors are identical, and differences reach zero.

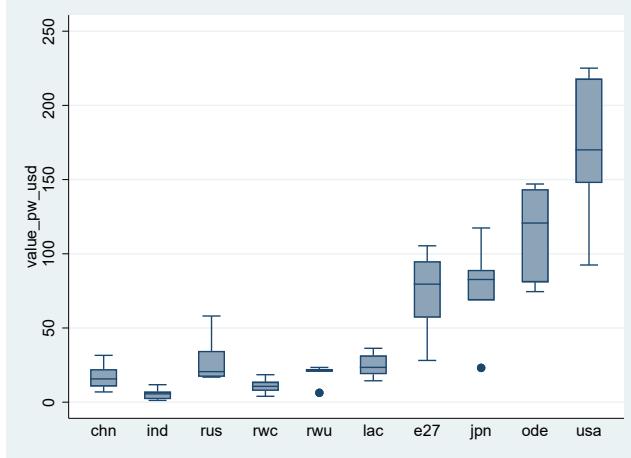


Figure 11: Distribution of the calibrated  $\lambda_{si,0}$  parameters across industries  $i$  of each region  $s$  in our model. We assume that this parameter is proportional to PPP-adjusted labor productivity in each sector-country. After the initial period, the location parameter of the sector-country-specific Fréchet distribution  $\lambda_{si,t}$  evolves endogenously according to the model.

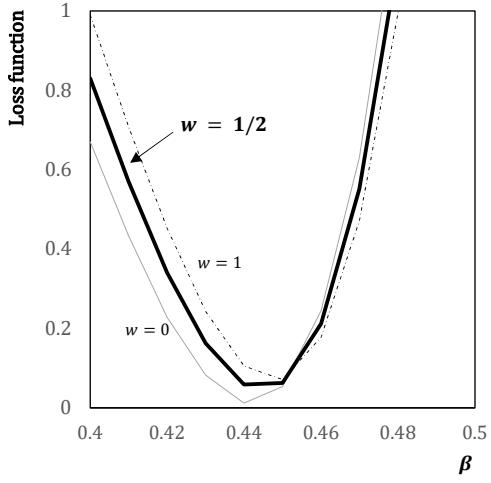


Figure 12: Plots the loss function for values of  $\beta \in [0.4, 0.5]$ . The solid gray line shows the loss function with the parameterization of  $w^{GDPpc} = 0$ , which is minimized at  $\beta = 0.44$ . The dotted gray line shows the loss function with the parameterization of  $w^{GDPpc} = 1$ , which is minimized at  $\beta = 0.45$ . The thicker black line shows the loss function with the parameterization of  $w^{GDPpc} = 1/2$ , which is minimized at  $\beta = 0.44$ .

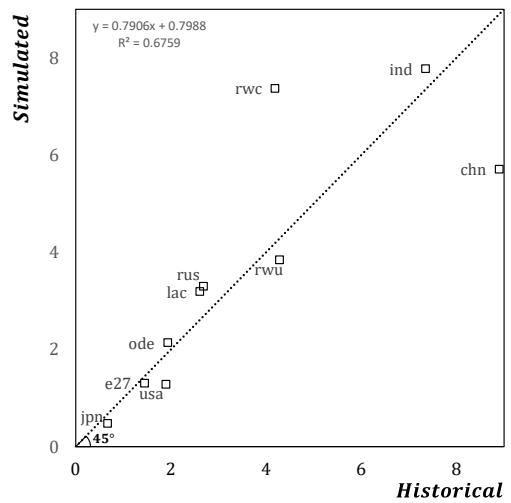


Figure 13: Historical and simulated (for  $\beta = 0.44$ ) GDP growth rates (average 2004-2019)

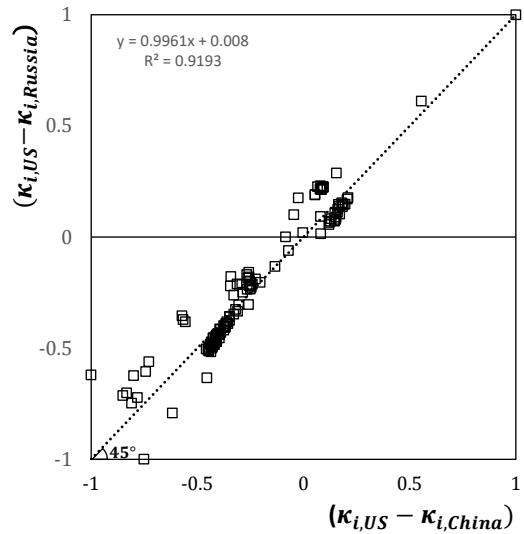


Figure 14: Differential Foreign Policy Similarity Index. Values are normalized such that 1 represents maximum relative similarity with the U.S. and  $-1$  represents maximum relative similarity with China or Russia. See the caption of Figure 9 for further details on the indices.

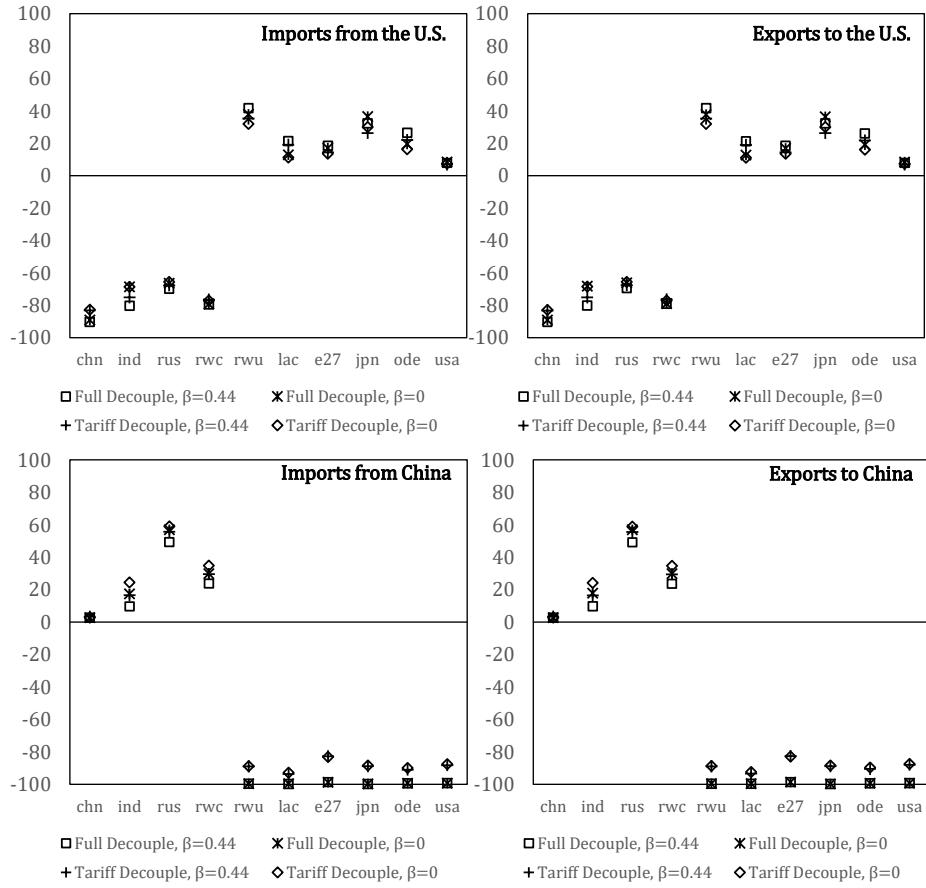


Figure 15: Cumulative Percentage Change in Trade Flows with China and the United States, respectively, after the policy change, by 2040. *Full Decouple* increases iceberg trade costs  $\tau_{sd,t}^i$  by 160 percentage points. *Tariff decouple* increases bilateral tariffs  $tm_{sd,t}^i$ , across groups, by 32 percentage points.  $\beta$  is a parameter that controls the diffusion of ideas according to equation 12, assumed to be homogeneous across sectors. Country codes: **chn**, China; **ind**, India; **rus**, Russia; **rwc**, Rest of Eastern bloc; **rwu**, Rest of Western bloc; **lac**, Latin America; **e27**, European Union; **ode**, Other Developed; **usa**, United States. Tables with the values for these charts can be found in the Appendix.

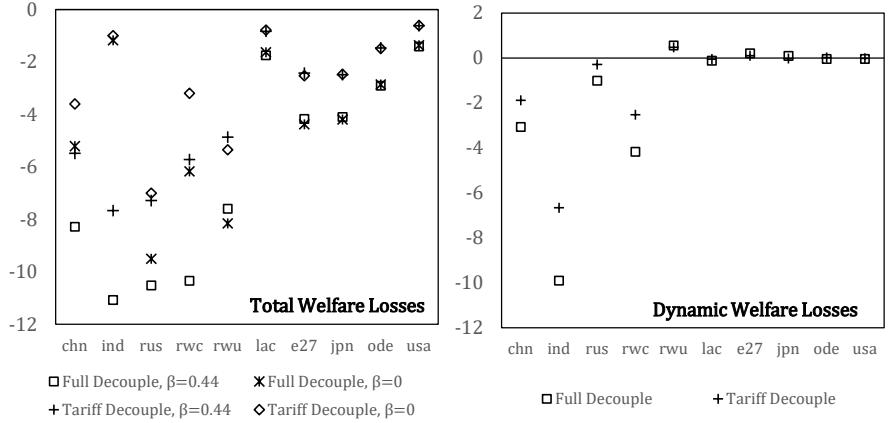


Figure 16: Cumulative Percentage Change in Real Income, after the policy change, by 2040. *Full Decouple* increases iceberg trade costs  $\tau_{sd,t}^i$  by 160 percentage points. *Tariff decouple* increases bilateral tariffs  $tm_{sd,t}^i$ , across groups, by 32 percentage points.  $\beta$  is a parameter that controls the diffusion of ideas according to equation 12, assumed to be homogeneous across sectors. Country codes: **chn**, China; **ind**, India; **rus**, Russia; **rwc**, Rest of Eastern bloc; **rwu**, Rest of Western bloc; **lac**, Latin America; **e27**, European Union; **ode**, Other Developed; **usa**, United States. Tables with the values for these charts can be found in the Appendix.

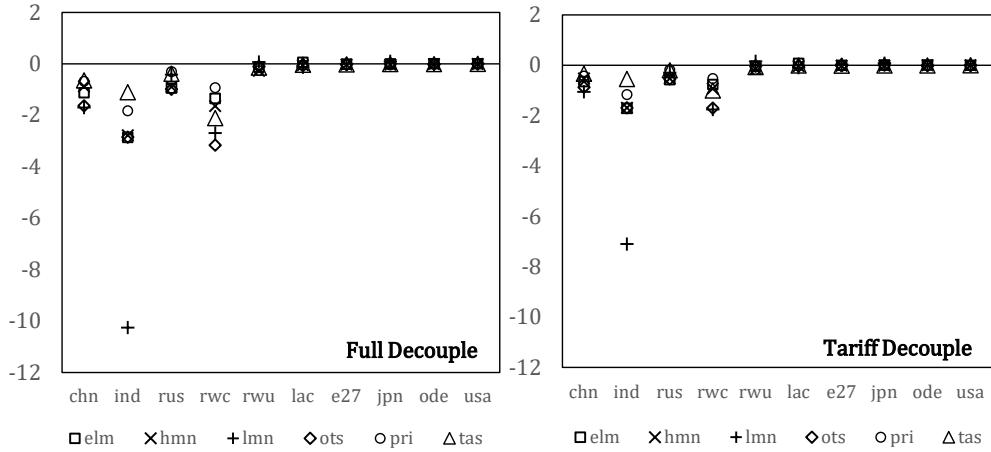


Figure 17: Cumulative Percentage Change in the Fréchet Distribution location parameter  $\lambda_{di,t}$ , after policy change, by 2040. *Full Decouple* increases iceberg trade costs  $\tau_{sd,t}^i$  by 160 percentage points. *Tariff decouple* increases bilateral tariffs  $tm_{sd,t}^i$ , across groups, by 32 percentage points.  $\beta$  is a parameter that controls the diffusion of ideas according to equation 12, assumed to be homogeneous across sectors. Country codes: **chn**, China; **ind**, India; **rus**, Russia; **rwc**, Rest of Eastern bloc; **rwu**, Rest of Western bloc; **lac**, Latin America; **e27**, European Union; **ode**, Other Developed; **usa**, United States. Sector codes: **elm**, Electronic Equipment; **hmn**, Heavy manufacturing; **lmn**, Light manufacturing; **ots**, Other Services; **pri**, Primary Sector; **tas**, Business services. Tables with the values for these charts can be found in the Appendix.

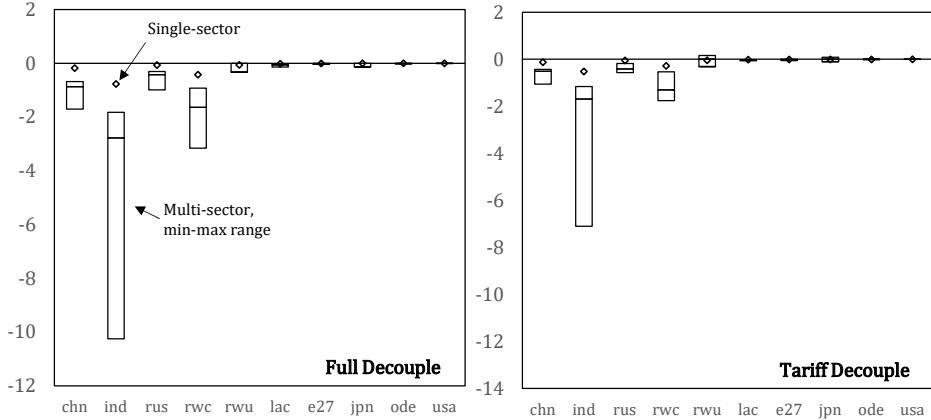


Figure 18: Multi-sector vs. Single-sector: Cumulative Percentage Change in the Fréchet Distribution location parameter  $\lambda_{di,t}$ , after policy change, by 2040. *Full Decouple* increases iceberg trade costs  $\tau_{sd,t}^i$  by 160 percentage points. *Tariff decouple* increases bilateral tariffs  $tm_{sd,t}^i$ , across groups, by 32 percentage points. Country codes: **chn**, China; **ind**, India; **rus**, Russia; **rwc**, Rest of Eastern bloc; **rwu**, Rest of Western bloc; **lac**, Latin America; **e27**, European Union; **ode**, Other Developed; **usa**, United States. Sector codes: **elm**, Electronic Equipment; **hmn**, Heavy manufacturing; **lmm**, Light manufacturing; **ots**, Other Services; **pri**, Primary Sector; **tas**, Business services. Tables with the values for these charts can be found in the Appendix.

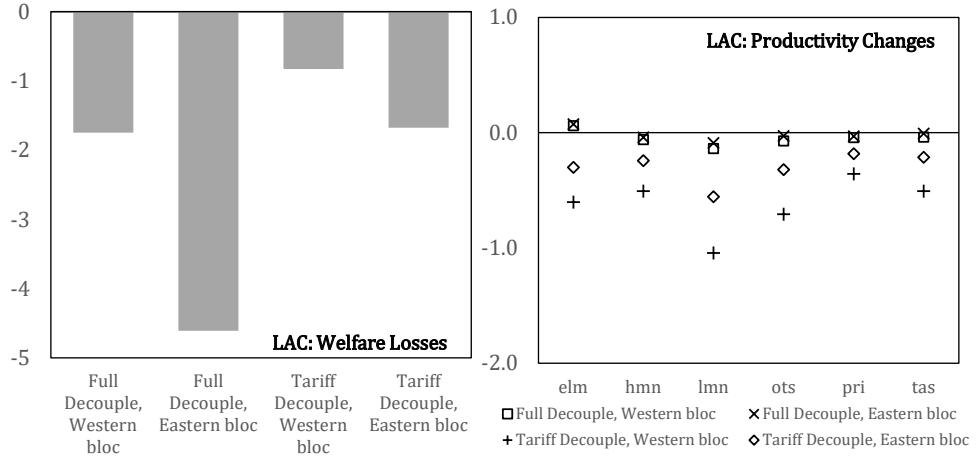


Figure 19: Left Panel: Cumulative Percentage Change in Real Income in LAC Region, by scenario. Right Panel: Cumulative Percentage Change of the Fréchet Distribution scale parameter  $\lambda_{di,t}$  in LAC Region, by scenario. *Full Decouple* increases iceberg trade costs  $\tau_{sd,t}^i$  by 160 percentage points. *Tariff decouple* increases bilateral tariffs  $tm_{sd,t}^i$ , across groups, by 32 percentage points. In all cases, we set the parameter that controls the diffusion of ideas to  $\beta = 0.44$ . Sector codes: **elm**, Electronic Equipment; **hmnn**, Heavy manufacturing; **lmnn**, Light manufacturing; **ots**, Other Services; **pri**, Primary Sector; **tas**, Business services. Tables with the values for these charts can be found in the Appendix.

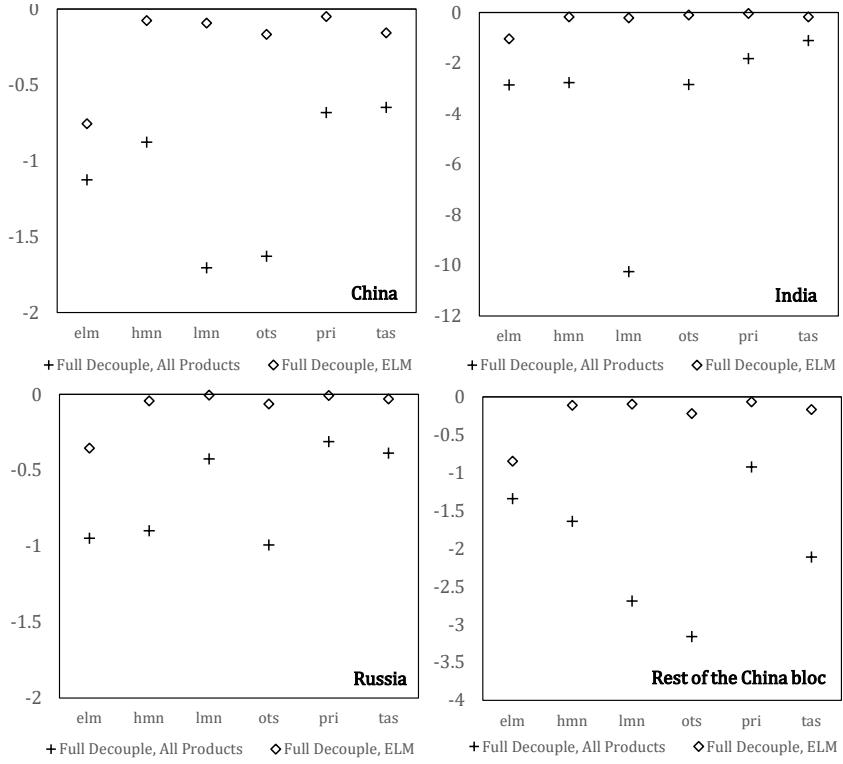


Figure 20: Cumulative Percentage Change of the Fréchet Distribution scale parameter  $\lambda_{di,t}$ , by scenario. *Full Decouple* increases iceberg trade costs  $\tau_{sd,t}^i$  by 160 percentage points in either all sectors or only in the Electronic Equipment (`elm`) sector. In both cases, we set the parameter that controls the diffusion of ideas to  $\beta = 0.44$ . Country codes: `chn`, China; `ind`, India; `rus`, Russia; `rwc`, Rest of Eastern bloc; `rwu`, Rest of Western bloc; `lac`, Latin America; `e27`, European Union; `ode`, Other Developed; `usa`, United States. Tables with the values for these charts can be found in the Appendix.

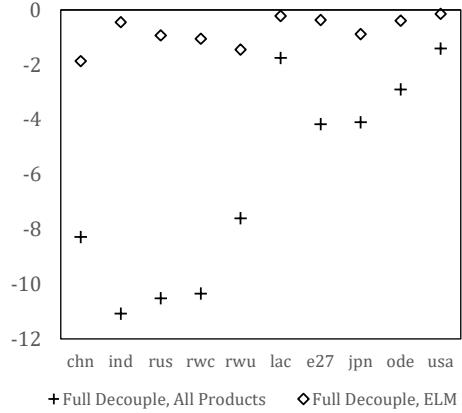
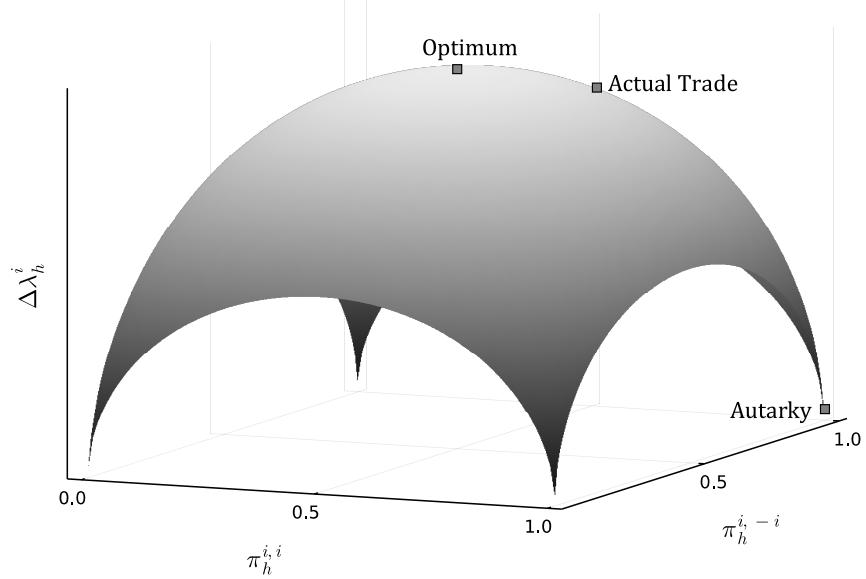


Figure 21: Cumulative Percentage Change in Welfare (Real Income), by scenario. *Full Decouple* increases iceberg trade costs  $\tau_{sd,t}^i$  by 160 percentage points in either all sectors or only in the Electronic Equipment (**elm**) sector. In both cases, we set the parameter that controls the diffusion of ideas to  $\beta = 0.44$ . Country codes: **chn**, China; **ind**, India; **rus**, Russia; **rwc**, Rest of Eastern bloc; **rwu**, Rest of Western bloc; **lac**, Latin America; **e27**, European Union; **ode**, Other Developed; **usa**, United States. Tables with the values for these charts can be found in the Appendix.

## B Additional Figures



**Figure 22: Idea diffusion function in a two-by-two economy.** The graph shows  $\Delta\lambda^i \propto \kappa_i[(\pi_{hi})^{1-\beta}(\lambda_{hi})^\beta + (1 - \pi_{hi})^{1-\beta}(\lambda_f^i)^\beta] + (1 - \kappa_{di})[(\pi_h^{i,-i})^{1-\beta}(\lambda_h^{-i})^\beta + (1 - \pi_h^{i,-i})^{1-\beta}(\lambda_f^{-i})^\beta]$  as a function of domestic trade share in sectors  $i, -i$ :  $\pi_h^{i,i}, \pi_h^{-i}$ . If countries and sectors are identical and  $\kappa_i = 1/2$ , Optimal, Actual Trade, and Autarky allocations are represented in this figure. The marginal contributions of each sector to total diffusion are as shown in the left panel of Figure 1

## C Additional tables calibration exercise

Table A1: Growth Rate of Real GDP and Real GDP per Capita, respectively, Historically and in Simulations, using different values of  $\beta$

$\beta$	Mean	St.Dev.	max	min
GDP				
Historical	3.60	2.66	8.90	0.67
0.40	3.09	2.02	6.26	0.41
0.41	3.21	2.13	6.58	0.43
0.42	3.34	2.26	6.94	0.44
0.43	3.48	2.40	7.34	0.46
0.44	3.64	2.56	7.78	0.48
0.45	3.82	2.73	8.26	0.50
0.46	4.02	2.92	8.79	0.53
0.47	4.24	3.13	9.36	0.56
0.48	4.48	3.36	9.97	0.59
0.49	4.75	3.60	10.62	0.63
0.50	5.04	3.87	11.32	0.68
GDP per capita				
Historical	2.70	2.51	8.36	0.75
0.40	2.20	1.65	4.91	0.47
0.41	2.32	1.76	5.23	0.48
0.42	2.44	1.89	5.59	0.50
0.43	2.59	2.03	5.98	0.51
0.44	2.75	2.19	6.41	0.53
0.45	2.92	2.36	6.89	0.54
0.46	3.12	2.55	7.41	0.57
0.47	3.34	2.76	7.97	0.59
0.48	3.58	2.98	8.57	0.62
0.49	3.84	3.22	9.22	0.65
0.50	4.13	3.48	9.91	0.69

Table A2: The squared difference between the sum of the historical and simulated mean and standard deviation of GDP, GDP per capita and their sum

$\beta$	GDP	GDP pc	Sum
0.40	0.67	1.00	1.67
0.41	0.43	0.72	1.15
0.42	0.23	0.46	0.69
0.43	0.08	0.25	0.33
0.44	0.01	0.11	0.12
0.45	0.06	0.07	0.13
0.46	0.25	0.17	0.42
0.47	0.64	0.46	1.10
0.48	1.28	0.98	2.26
0.49	2.22	1.79	4.01
0.50	3.54	2.96	6.50

Table A3: Growth Rate of Real GDP and Real GDP per Capita, respectively, Historically and in Simulations, using different values of  $\beta$  between 0 and 0.6

$\beta$	Mean	St.Dev.	max	min
GDP				
Historical	3.60	2.66	8.90	0.67
0	2.17	1.10	3.79	0.32
0.5	2.19	1.12	3.84	0.32
0.10	2.20	1.13	3.87	0.32
0.15	2.23	1.16	3.93	0.32
0.20	2.27	1.20	4.02	0.33
0.25	2.34	1.26	4.19	0.33
0.30	2.46	1.39	4.49	0.35
0.35	2.68	1.61	5.08	0.37
0.40	3.09	2.02	6.26	0.41
0.45	3.82	2.73	8.26	0.50
0.50	5.04	3.87	11.32	0.68
0.55	6.89	5.42	15.39	1.01
0.60	9.50	7.32	20.53	1.66
GDP per capita				
Historical	2.70	2.51	8.36	0.75
0	1.29	0.72	2.27	0.40
0.5	1.31	0.75	2.33	0.40
0.10	1.32	0.76	2.36	0.40
0.15	1.35	0.78	2.41	0.40
0.20	1.39	0.82	2.53	0.40
0.25	1.46	0.89	2.74	0.41
0.30	1.58	1.01	3.10	0.42
0.35	1.80	1.24	3.75	0.44
0.40	2.20	1.65	4.91	0.47
0.45	2.92	2.36	6.89	0.54
0.50	4.13	3.48	9.91	0.69
0.55	5.96	5.02	13.92	0.98
0.60	8.54	6.88	18.97	1.55

Table A4: Growth Rate of Real GDP and Real GDP per Capita, respectively, Historically and in Simulations, using different values of  $\beta$  between 0 and 0.6 with an Autonomous Technology Growth Rate of  $\alpha = 2.36$

$\beta$	Mean	St.Dev.	max	min
GDP				
Historical	3.60	2.66	8.90	0.67
0	2.17	1.10	3.79	0.32
0.5	2.19	1.12	3.84	0.32
0.10	2.21	1.14	3.88	0.32
0.15	2.23	1.16	3.94	0.32
0.20	2.28	1.21	4.04	0.33
0.25	2.35	1.28	4.23	0.33
0.30	2.49	1.41	4.56	0.35
0.35	2.73	1.65	5.22	0.37
0.40	3.17	2.09	6.48	0.42
0.45	3.95	2.85	8.60	0.52
0.50	5.23	4.03	11.76	0.71
0.55	7.16	5.62	15.92	1.08
0.60	9.84	7.52	21.17	1.77
GDP per capita				
Historical	2.70	2.51	8.36	0.75
0	1.29	0.72	2.27	0.40
0.5	1.31	0.75	2.33	0.40
0.10	1.33	0.77	2.36	0.40
0.15	1.35	0.79	2.43	0.40
0.20	1.40	0.83	2.56	0.40
0.25	1.47	0.91	2.79	0.41
0.30	1.61	1.04	3.18	0.42
0.35	1.85	1.28	3.89	0.44
0.40	2.28	1.72	5.13	0.48
0.45	3.05	2.48	7.22	0.56
0.50	4.32	3.64	10.35	0.72
0.55	6.22	5.21	14.45	1.04
0.60	8.87	7.08	19.56	1.66

## D Estimation of $\lambda_{si,t}$ and $\beta_i$ (from gravity and diffusion moments)

This appendix collects implementation notes for estimating the time-varying sectoral Fréchet location parameters  $\lambda_{si,t}$  and diffusion elasticities  $\beta_i$ . We keep the paper's index conventions: exporters/sources  $s, n \in \mathcal{D}$ , importers/destinations  $d \in \mathcal{D}$ , sectors  $i, j \in \mathcal{I}$ , and time  $t \in \mathcal{T}$ .

### D.1 First stage: estimating gravity fixed effects with normalizations and constraints

Levchenko and Zhang (2016) estimate the following gravity equation in normalized share form (notation adapted):

$$\ln\left(\frac{X_{sdi,t}}{X_{ddi,t}}\right) = \underbrace{\ln\left(\lambda_{si,t} (c_{si,t})^{-\theta_i}\right)}_{\text{Exporter fixed effect}} - \theta_i ex_{si,t} - \underbrace{\ln\left(\lambda_{di,t} (c_{di,t})^{-\theta_i}\right)}_{\text{Importer fixed effect}} \\ - \underbrace{\theta_i d_{\ell,t} - \theta_i b_{\{sd\}t} - \theta_i CU_{\{sd\}t} - \theta_i RTA_{\{sd\}t}}_{\text{Bilateral observables}} - \underbrace{\theta_i \nu_{sdi,t}}_{\text{Error term}}. \quad (\text{A-1})$$

All bilateral observables are symmetric in this specification. As recommended in Waugh (2010), any asymmetry in bilateral trade costs is attributed to the exporter side. In practice, this just implies an estimated gravity model that allows the importer and exporter fixed effects to vary and the use of the “uncontaminated” importer term in subsequent analysis. Note, though, that the term  $\theta_i ex_{si,t}$  is the only difference between the terms encompassed by the exporter and importer fixed effects here; they each just reflect a given country's technology and unit costs (either in its capacity as exporter or importer).

$\theta_i ex_{si,t}$  can be interpreted as the average level of trade costs faced by exporters in country  $s$ . In reality, some asymmetric costs may additionally appear along the importer side or even vary fully bilaterally (e.g. discriminatory barriers which are not reciprocated). Asymmetric pairwise fixed effects are now the gravity literature's preferred approach to fully flexibly (cross-sectionally, at least) controlling out trade costs (Baier and Bergstrand, 2007; Head and Mayer, 2014). This lends itself to the following amended estimating equation:

$$\ln\left(\frac{X_{sdi,t}}{X_{ddi,t}}\right) = \underbrace{\ln\left(\lambda_{si,t} (c_{si,t})^{-\theta_i}\right)}_{\text{Exporter fixed effect}} - \underbrace{\ln\left(\lambda_{di,t} (c_{di,t})^{-\theta_i}\right)}_{\text{Importer fixed effect}} + \underbrace{\left(\ln(\tau_{sdi})^{-\theta_i}\right)}_{\text{Pair fixed effect}} \\ - \underbrace{\theta_i CU_{\{sd\}t} - \theta_i RTA_{\{sd\}t} - \theta_i Glob_{sdi,t}}_{\text{Bilateral observables}} - \underbrace{\theta_i \nu_{sdi,t}}_{\text{Error term}}. \quad (\text{A-2})$$

Where, importantly, the pairwise fixed effect is designed to pick up all time-invariant trade costs. Additional time-varying bilateral observables can then also be included to control for trade cost changes that occur within-sample<sup>28</sup>.

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<sup>28</sup>The pair fixed effect can also be augmented by a time interval interaction over any period less frequent than  $t$

There are some important implications for estimation of (A-2). First, the exporter-year and importer-year fixed effects should be constrained to share the mirror vector of estimated values:  $\ln(\lambda_{si,t} (c_{si,t})^{-\theta_i}) \equiv \ln(\lambda_{di,t} (c_{di,t})^{-\theta_i}), \forall s = d$ . This is not trivial to implement in standard high dimensional fixed effects packages but can be achieved by explicitly designing the model matrix.

Separately, we also need to drop an appropriate set of fixed effects levels as reference categories. Unlike in many gravity applications, we need to be able to directly interpret the fixed effect estimates. The normalizations and reference levels applied to them are therefore important. In Levchenko and Zhang (2016), in the absence of any pairwise fixed effects, it is sufficient to drop one country's importer fixed effect (analogously one per year in a panel setting) and use it as the numeraire for subsequent analysis. Once we add pair fixed effects, dropping these for domestic trade observations (i.e. setting  $\ln(\tau_{ddi}) = 0$ ) is standard and is already an underlying structural assumption in many gravity settings.

However, this still leaves a rank deficiency of  $N - 1$ , where  $N$  is the number of countries in the data. Consider the simplified version of the model<sup>29</sup>  $\eta_{sdi,t} = \xi_{si,t} - \xi_{di,t} + \gamma_{sd}$ . For a given  $c \neq$  reference country across all years, we can perturb  $\xi_{ci,t} \leftarrow \xi_{ci,t} + \delta$  on the exporter side but also  $\gamma_{cd} \leftarrow \gamma_{cd} - \delta$  and  $\gamma_{sc} \leftarrow \gamma_{sc} + \delta$ . Then, for any observation  $(s, d, t)$ :

- If  $s = c$ : the  $+\delta$  from  $\xi_{ci,t}$  cancels with the  $-\delta$  in  $\gamma_{cd}$ ;
- If  $d = c$ : the  $-\delta$  from  $-\xi_{ci,t}$  cancels with the  $+\delta$  in  $\gamma_{sc}$ ;
- Otherwise: no term shifts.

Thus in all cases, this yields the same predictor. In other words, there remain many different combinations of  $(\xi_{si,t}, \gamma_{sd})$  which would yield the same  $\eta_{sdi,t}$ , so there is no unique solution to our fixed effects system to pin down from the data.

This issue has a fairly intuitive interpretation here: the estimated fixed effects face an ambiguity as to where to attribute the average level of trade costs faced by a given country over the sample. They can be considered as part of the relevant country-year terms, as a shifter of the relevant pairwise terms, or spread in some proportion across both.

Given that we want to interpret the country-year terms as functions of technology and unit costs only, we need to anchor  $\gamma_{sd}$  in such a way that ensures it absorbs the average level of trade costs (and equivalently prevents any potential to directly compensate for perturbations in the country fixed effects). We achieve the required  $N - 1$  normalizations by additionally setting  $\gamma_{s,\text{ref}} = 0, \forall s$ , where ref represents the reference country whose country-year fixed effects we are already dropping. This normalization pins down the full system of fixed effects to a unique set, but ensures the average level of trade costs can remain in the pairwise term. This contrasts with an alternative normalization, often implemented by default in high dimensional fixed effects software, which is to enforce a sum-to-zero constraint on the pair fixed effect estimates. This would force each country's average bilateral barrier to zero, pushing that average into the country-year terms and thereby mixing productivity/unit costs with trade costs in a way that would not be appropriate for further auxiliary analysis.

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<sup>29</sup>This issue also applies in the case where the fixed effects aren't constrained to mirror each other

## D.2 Second stage: estimating technology levels $\lambda$

From (A-2), the exporter-year fixed effect in sector  $i$  collects the (log) technology term and the unit-cost term raised to the trade elasticity:

$$\text{FE}_{si,t}^i \equiv \ln(\lambda_{si,t}(c_{si,t})^{-\theta_i}) - \ln(\lambda_{\text{ref},i,t}(c_{\text{ref},i,t})^{-\theta_i}).$$

With Cobb–Douglas production and (time- and country-) flexible cost shares we have:

$$c_{si,t} = \frac{1}{\lambda_{si,t}} w_{s,t}^{\psi_{si,t}\gamma_{si,t}} r_{s,t}^{(1-\psi_{si,t})\gamma_{si,t}} \prod_{j \in \mathcal{I}} P_{sj,t}^{(1-\gamma_{si,t})\gamma_{sij,t}},$$

so that, relative to a reference country ref in the same year:

$$\ln \frac{\lambda_{si,t}}{\lambda_{\text{ref},i,t}} = \text{FE}_{si,t}^i + \theta_i \left[ \psi_{si,t} \gamma_{si,t} \ln \frac{w_{s,t}}{w_{\text{ref},t}} + (1 - \psi_{si,t}) \gamma_{si,t} \ln \frac{r_{s,t}}{r_{\text{ref},t}} + \sum_j (1 - \gamma_{si,t}) \gamma_{sij,t} \ln \frac{P_{sj,t}}{P_{\text{ref},j,t}} \right]. \quad (\text{A-3})$$

We assemble data on the cost components and shares as follows. Labour cost shares in value added  $\{\psi_{si,t}\}$  are inferred from OECD Trade in Employment (TiM) factor-share data. Value-added and intermediate input shares  $\{\gamma_{si,t}, \gamma_{sij,t}\}$  are computed from the OECD ICIO tables by sector and year and scaled by gross output. Wages  $w_{s,t}$  (compensation per worker) are constructed from TiM compensation and employment. The rental rate  $r_{s,t}$  is built from data in the Penn World Tables on real interest and depreciation rates.

For the intermediate-input price term  $\ln(P_{sj,t}/P_{\text{ref},j,t})$ , we follow [Levchenko and Zhang \(2016\)](#). Let  $\pi_{ssj,t}$  denote the domestic expenditure share in sector  $j$  for country  $s$ , and  $\pi_{\text{ref},\text{ref},j,t}$  the corresponding share in the reference country.

With  $\text{FE}_{\text{ref},i,t}^i = 0$  by construction we have

$$\exp(\text{FE}_{si,t}^i) = \frac{\lambda_{si,t}}{\lambda_{\text{ref},i,t}} \left( \frac{c_{si,t}}{c_{\text{ref},i,t}} \right)^{-\theta_i} \equiv S_{si,t}^i.$$

Thus  $S_{si,t}^i$  is observed directly from the gravity first stage.

Relative sectoral prices can then be obtained from the adjusted domestic-share “double ratio”:

$$\frac{P_{sj,t}}{P_{\text{ref},j,t}} = \left( \frac{\pi_{ssj,t}/\pi_{\text{ref},\text{ref},j,t}}{S_{sj,t}^i} \right)^{1/\theta_j}, \quad (\text{A-4})$$

which corresponds to equation (A.5) in [Levchenko and Zhang \(2016\)](#).

We still need a set of values for  $\lambda_{\text{ref},i,t}$  to anchor the technology levels. Using a standard production function residual approach and KLEMS data provides an observed sectoral TFP index for the reference country,  $\Lambda_{it}^{\text{obs}}$ . However, this conflates the latent technology scale with access to foreign suppliers via the price index; we do not observe the full distribution of productivity draws (see, e.g., [Finicelli, Pagano and Sbracia \(2013\)](#); [Costinot, Donaldson and Komunjer \(2012\)](#)). Under EK, the reference-country price index satisfies

$$P_{\text{ref},it}^{-\theta_i} = \sum_s \lambda_{si,t} (c_{si,t})^{-\theta_i} \tau_{s,\text{ref},i}^{-\theta_i} = \sum_s \exp(\text{FE}_{si,t}^i) \exp(\text{pair}_{s,\text{ref}}^i) \equiv \phi_{it},$$

where  $\text{pair}_{s,\text{ref}}^i$  captures time-invariant bilateral costs (normalized so that domestic pairs are zero). Finicelli et al. show that the latent technology scale for the reference country is obtained by removing this market-access term:

$$\lambda_{\text{ref},i,t} = \left( \frac{\Lambda_{it}^{\text{obs}\theta_i}}{\phi_{it}} \right)^{1/\theta_i}, \quad \phi_{it} = \sum_s \exp(\text{FE}_{si,t}^i) \exp(\text{pair}_{s,\text{ref}}^i).$$

Combining the reference technology with  $S_{si,t}^i$  and the cost ratio from (A-3), we obtain

$$\lambda_{si,t} = \left( \frac{\Lambda_{it}^{\text{obs}\theta_i}}{\phi_{it}} \right)^{1/\theta_i} \exp(\text{FE}_{si,t}^i) (c_{si,t}/c_{\text{ref},i,t})^{\theta_i}.$$

### D.3 Third stage: estimating diffusion elasticities $\beta$

Let  $\Delta\lambda_{di,t} \equiv \lambda_{di,t} - \lambda_{di,t-1}$ . The law of motion implied by the model can be written as

$$\Delta\lambda_{di,t} = \alpha_{i,t} Q_{di,t-1}(\beta, \theta) + u_{di,t}, \quad (\text{A-5})$$

where

$$Q_{di,t-1}(\beta, \theta) = \sum_j \tilde{\gamma}_{dij,t-1} \Gamma\left(1 - \frac{\beta_i \theta_i}{\theta_j}\right) \sum_s \pi_{sdj,t-1} \left( \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} \right)^{\frac{\beta_i \theta_i}{\theta_j}}.$$

Here  $\tilde{\gamma}_{dij,t-1}$  are input-cost shares for sector  $i$  in destination  $d$ , and  $\pi_{sdj,t-1}$  are bilateral expenditure shares on good  $j$ .

We compute  $\tilde{\gamma}_{dij,t}$  from the OECD ICIO as above, with bilateral shares  $\pi_{sdj,t}$  coming from summing sector- $j$  shipments from  $s$  to  $d$  across uses and normalizing by the destination-sector total. When countries are dropped in the estimation of  $\lambda$ , we renormalize  $\pi_{sdj,t-1}$  over the set of exporters  $s$  with observed  $\lambda_{sj,t-1}$  so that  $\sum_s \pi_{sdj,t-1} = 1$  within each  $(d, j, t)$  used in (A-5).

We estimate  $\beta_i$  by nonlinear least squares with  $\alpha_{i,t}$  profiled out. For any candidate vector  $\beta$  we compute  $Q_{di,t-1}(\beta, \theta)$  via (A-5) and set

$$\hat{\alpha}_{i,t}(\beta) \equiv \frac{\sum_d Q_{di,t-1}(\beta, \theta) \Delta\lambda_{di,t}}{\sum_d Q_{di,t-1}(\beta, \theta)^2}.$$

The objective is the sum of squared residuals,

$$\min_{\beta} \sum_{i,t} \sum_d \left[ \Delta\lambda_{di,t} - \hat{\alpha}_{i,t}(\beta) Q_{di,t-1}(\beta, \theta) \right]^2,$$

treating  $\{\theta_j\}$  as given.

## E Equilibrium

### E.1 Equilibrium in levels

$$c_{di,t} = \Theta_{di} \left( w_{d,t}^{\psi_{di}} r_{d,t}^{1-\psi_{di}} \right)^{\gamma_{di}} \prod_{j \in \mathcal{I}} P_{dj,t}^{(1-\gamma_{di})\gamma_{dij}} \quad (\text{A-6})$$

$$P_{di,t} = \Gamma_i \left( \sum_{n \in \mathcal{D}} \lambda_{ni,t} (\tau_{ndi} c_{ni,t})^{-\theta_i} \right)^{-\frac{1}{\theta_i}} \quad (\text{A-7})$$

$$\pi_{sdi,t} = \frac{\lambda_{si,t} (\tau_{sdi} c_{si,t})^{-\theta_i}}{\sum_{n \in \mathcal{D}} \lambda_{ni,t} (\tau_{ndi} c_{ni,t})^{-\theta_i}} \quad (\text{A-8})$$

$$w_{d,t} L_{d,t} = \sum_{i \in \mathcal{I}} \frac{\theta_i}{1 + \theta_i} \gamma_{di} \psi_{di} X_{di,t} \quad (\text{A-9})$$

$$r_{d,t} K_{d,t} = \sum_{i \in \mathcal{I}} \frac{\theta_i}{1 + \theta_i} \gamma_{di} (1 - \psi_{di}) X_{di,t} \quad (\text{A-10})$$

$$T_{d,t} = \sum_{s \in \mathcal{D}} \sum_{i \in \mathcal{I}} \frac{t_{sdi}}{\tau_{sdi}} \pi_{sdi,t} X_{si,t} \quad (\text{A-11})$$

$$\Pi_{si,t} = \frac{1}{1 + \theta_i} \sum_{d \in \mathcal{D}} \pi_{sdi,t} X_{di,t} \quad (\text{A-12})$$

$$Y_{d,t} = w_{d,t} L_{d,t} + r_{d,t} K_{d,t} + \sum_{i \in \mathcal{I}} \Pi_{di,t} + T_{d,t} \quad (\text{A-13})$$

$$X_{si,t}^F = \sum_{d \in \mathcal{D}} \frac{\pi_{sdi,t}}{\tau_{sdi}} \kappa_{di} (1 - s_d) Y_{d,t} \quad (\text{A-14})$$

$$X_{si,t}^I = \sum_{d \in \mathcal{D}} \frac{\pi_{sdi,t}}{\tau_{sdi}} \chi_{di} (s_d - b_{d,t}) Y_{d,t} \quad (\text{A-15})$$

$$X_{si,t}^M = \sum_{d \in \mathcal{D}} \frac{\pi_{sdi,t}}{\tau_{sdi}} \sum_{j \in \mathcal{I}} \frac{\theta_j}{1 + \theta_j} (1 - \gamma_{dij}) \gamma_{dij} X_{dj,t} \quad (\text{A-16})$$

$$X_{si,t} = X_{si,t}^F + X_{si,t}^I + X_{si,t}^M \quad (\text{A-17})$$

$$b_{d,t} Y_{d,t} = \sum_{i \in \mathcal{I}} \left( \sum_{s \in \mathcal{D} \setminus \{d\}} \pi_{sdi,t} X_{si,t} - \sum_{n \in \mathcal{D} \setminus \{d\}} \pi_{dni,t} X_{ni,t} \right) \quad (\text{A-18})$$

## E.2 Hat algebra

According relative changes to equilibrium outcomes, given relative changes in select parameters such as those for tariffs, subsidies and trade costs, are worked out with common hat algebra and implemented in the solution algorithm. For each variable  $x$ , we denote  $\hat{x}_t \equiv x_t/x_{t-1}$ , are the equilibrium value of such a variable before and after, respectively, a change in trade costs.

$$\hat{c}_{di,t} = \left( \hat{w}_{d,t}^{\psi_{di}} \hat{r}_{d,t}^{1-\psi_{di}} \right)^{\gamma_{di}} \prod_{j \in \mathcal{I}} \hat{P}_{dj,t}^{(1-\gamma_{di})\gamma_{dij}} \quad (\text{A-19})$$

$$\hat{P}_{di,t} = \left( \sum_{n \in \mathcal{D}} \pi_{ndi,t} \hat{\lambda}_{ni,t} (\hat{\tau}_{ndi} \hat{c}_{ni,t})^{-\theta_i} \right)^{-\frac{1}{\theta_i}} \quad (\text{A-20})$$

$$\hat{\pi}_{sdi,t} = \frac{\hat{\lambda}_{si,t} (\hat{\tau}_{sdi} \hat{c}_{si,t})^{-\theta_i}}{\sum_{n \in \mathcal{D}} \pi_{ndi,t} \hat{\lambda}_{ni,t} (\hat{\tau}_{ndi} \hat{c}_{ni,t})^{-\theta_i}} \quad (\text{A-21})$$

$$\hat{w}_{d,t} \hat{L}_{d,t} = \sum_{i \in \mathcal{I}} \frac{\frac{\theta_i}{1+\theta_i} \gamma_{di} \psi_{di} X_{di,t}}{w_{d,t} L_{d,t}} \hat{X}_{di,t} \quad (\text{A-22})$$

$$\hat{r}_{d,t} \hat{K}_{d,t} = \sum_{i \in \mathcal{I}} \frac{\frac{\theta_i}{1+\theta_i} \gamma_{di} (1 - \psi_{di}) X_{di,t}}{r_{d,t} K_{d,t}} \hat{X}_{di,t} \quad (\text{A-23})$$

$$\hat{T}_{d,t} = \sum_{s \in \mathcal{D}} \sum_{i \in \mathcal{I}} \frac{\frac{t_{sdi}}{\tau_{sdi}} \pi_{sdi,t} X_{si,t}}{T_{d,t}} \frac{\hat{t}_{sdi}}{\hat{\tau}_{sdi}} \hat{\pi}_{sdi,t} \hat{X}_{si,t} \quad (\text{A-24})$$

$$\hat{\Pi}_{si,t} = \sum_{d \in \mathcal{D}} \frac{\pi_{sdi,t} X_{di,t}}{\Pi_{si,t}} \hat{\pi}_{sdi,t} \hat{X}_{di,t} \quad (\text{A-25})$$

$$\hat{Y}_{d,t} = \frac{w_{d,t} L_{d,t}}{Y_{d,t}} \hat{w}_{d,t} \hat{L}_{d,t} + \frac{r_{d,t} K_{d,t}}{Y_{d,t}} \hat{r}_{d,t} \hat{K}_{d,t} + \sum_{i \in \mathcal{I}} \frac{\Pi_{di,t}}{Y_{d,t}} \hat{\Pi}_{di,t} + \frac{T_{d,t}}{Y_{d,t}} \hat{T}_{d,t} \quad (\text{A-26})$$

$$\hat{X}_{si,t}^F = \sum_{d \in \mathcal{D}} \frac{\frac{\pi_{sdi,t}}{\tau_{sdi}} \kappa_{di} (1 - s_d) Y_{d,t}}{X_{si,t}^F} \frac{\hat{\pi}_{sdi,t}}{\hat{\tau}_{sdi}} \hat{Y}_{d,t} \quad (\text{A-27})$$

$$X_{si,t}^I = \frac{\sum_{d \in \mathcal{D}} \frac{\pi_{sdi,t}}{\tau_{sdi}} \chi_{di} (s_d - b_{d,t}) Y_{d,t}}{X_{si,t}^I} \frac{\hat{\pi}_{sdi,t}}{\hat{\tau}_{sdi}} (\widehat{s_d - b_{d,t}}) \hat{Y}_{d,t} \quad (\text{A-28})$$

$$\hat{X}_{si,t}^M = \sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{I}} \frac{\frac{\pi_{sdi,t}}{\tau_{sdi}} \frac{\theta_j}{1+\theta_j} (1 - \gamma_{dj}) \gamma_{dji} X_{dj,t}}{X_{si,t}^M} \frac{\hat{\pi}_{sdi,t}}{\hat{\tau}_{sdi}} \hat{X}_{dj,t} \quad (\text{A-29})$$

$$\hat{X}_{si,t} = \frac{X_{si,t}^F}{X_{si,t}} \hat{X}_{si,t}^F + \frac{X_{si,t}^I}{X_{si,t}} X_{si,t}^I + \frac{X_{si,t}^M}{X_{si,t}} X_{si,t}^M \quad (\text{A-30})$$

$$\hat{b}_{d,t} \hat{Y}_{d,t} = \sum_{i \in \mathcal{I}} \left( \sum_{s \in \mathcal{D} \setminus \{d\}} \frac{\pi_{sdi,t} X_{si,t}}{b_{d,t} Y_{d,t}} \hat{\pi}_{sdi,t} \hat{X}_{si,t} - \sum_{n \in \mathcal{D} \setminus \{d\}} \frac{\pi_{dni,t} X_{ni,t}}{b_{d,t} Y_{d,t}} \hat{\pi}_{dni,t} \hat{X}_{ni,t} \right) \quad (\text{A-31})$$

## F Construction of Initial Sectoral Productivity Parameters $\lambda_{s,0}^i$

In our model, the location parameter of Fréchet distribution of a given industry-country  $\lambda_{d,t}^i$  evolves endogenously according to a law of motion, as described by equation (12). To calibrate the model, we need initial values  $(\lambda_{d,0}^i)_{d \in \mathcal{D}, i \in \mathcal{I}}$ . This appendix describes the steps required to estimate the initial sectoral Fréchet location parameters  $\{\lambda_{si,0}\}$  using a structural gravity approach consistent with the model equilibrium described in Appendix E. We follow the steps of Levchenko and Zhang (2016) adapted to our mode. Throughout, indices are as follows:  $s, n \in \mathcal{S}$  denote source (exporter) countries,  $d \in \mathcal{D}$  denotes destination (importer) countries,  $i \in \mathcal{I}$  denotes traded sectors, and  $t = 0$  denotes the base year.

In the baseline model, sectoral trade shares are given by

$$\pi_{sdi,t} = \frac{X_{sdi,t}}{X_{di,t}} = \frac{\lambda_{si,t} (\tau_{sdi} c_{si,t})^{-\theta_i}}{\sum_{n \in \mathcal{S}} \lambda_{ni,t} (\tau_{ndi} c_{ni,t})^{-\theta_i}}, \quad (\text{A-32})$$

where  $\lambda_{si,t}$  is the Fréchet location parameter governing productivity draws in sector  $i$  and country  $s$ ;  $\theta_i > 0$  is the sector-specific trade elasticity;  $\tau_{sdi} \geq 1$  is an iceberg trade cost;  $c_{si,t}$  is the unit cost of the sector- $i$  input bundle in country  $s$ ;  $X_{sdi,t}$  is total expenditure by destination  $d$  on sector- $i$  goods produced in country  $s$ ; and

$$X_{di,t} \equiv \sum_s X_{sdi,t}$$

denote total absorption in sector  $i$  in destination  $d$ . As known, then  $\pi_{sdi,t} = X_{sdi,t}/X_{di,t}$ . The objective of this appendix is to explain how  $\lambda_{si,0}$  can be identified and estimated from observed trade flows  $\{X_{sdi,0}\}$ .

**Basic mapping of model to the data.** Fix a destination  $d$  and sector  $i$  and period  $t = 0$ . Consider the ratio of imports from  $s$  relative to domestic expenditure:

$$\frac{X_{sdi,0}}{X_{ddi,0}} = \frac{\lambda_{si,0} (\tau_{sdi} c_{si,0})^{-\theta_i}}{\lambda_{di,0} (c_{di,0})^{-\theta_i}}, \quad (\text{A-33})$$

where we use  $\tau_{ddi} = 1$  by normalization. Taking logs yields

$$\ln \left( \frac{X_{sdi,0}}{X_{ddi,0}} \right) = \ln(\lambda_{si,0} c_{si,0}^{-\theta_i}) - \ln(\lambda_{di,0} c_{di,0}^{-\theta_i}) - \theta_i \ln \tau_{sdi}. \quad (\text{A-34})$$

We must now make an assumption about the parameterization of trade costs  $\tau_{sdi}$ . Following standard practice in the gravity literature, we assume:

$$\ln \tau_{sdi} = \delta_{\text{dist}}^i(s, d) + \delta_{\text{border}}^i(s, d) + \delta_{\text{rta}}^i(s, d) + \delta_{\text{cu}}^i(s, d) + ex_{si} + \nu_{sdi}, \quad (\text{A-35})$$

where:  $\delta_{\text{dist}}^i(s, d)$  are distance-bin indicators;  $\delta_{\text{border}}^i(s, d)$  captures common borders;  $\delta_{\text{rta}}^i(s, d)$  captures regional trade agreements;  $\delta_{\text{cu}}^i(s, d)$  captures currency unions;  $ex_{si}$  is an exporter-sector fixed effect (exporter-specific trade friction);  $\nu_{sdi}$  is an idiosyncratic residual.

The exporter component  $ex_{si}$  allows exporter-wide frictions to be separated from technology. Substituting (A-35) into (A-34) gives

$$\ln\left(\frac{X_{sdi,0}}{X_{ddi,0}}\right) = \underbrace{\ln(\lambda_{si,0} c_{si,0}^{-\theta_i}) - \theta_i ex_{si}}_{\text{Exporter fixed effect}} - \underbrace{\ln(\lambda_{di,0} c_{di,0}^{-\theta_i})}_{\text{Importer fixed effect}} \\ - \theta_i \left[ \delta_{\text{dist}}^i(s, d) + \delta_{\text{border}}^i(s, d) + \delta_{\text{rta}}^i(s, d) + \delta_{\text{cu}}^i(s, d) \right] - \theta_i \nu_{sdi}. \quad (\text{A-36})$$

Equation (A-36) is estimated separately for each sector  $i$  using OLS with exporter-sector and importer-sector fixed effects.

From the importer fixed effect, the regression identifies (up to normalization)

$$S_{di} \equiv \frac{\lambda_{di,0} c_{di,0}^{-\theta_i}}{\lambda_{Ri,0} c_{Ri,0}^{-\theta_i}} \quad (\text{A-37})$$

relative to a reference country  $R$ , for which we assume  $\lambda_{Ri,0} c_{Ri,0}^{-\theta_i} = 1$ . Importantly, up to normalization, we can directly identify  $S_{di}$  from the coefficients of the gravity regression.

**Identifying relative price levels** From (A-32), the domestic expenditure share satisfies

$$\frac{X_{ddi,0}}{X_{di,0}} = \pi_{ddi,0} = \frac{\lambda_{di,0} (\tau_{ddi} c_{di,0})^{-\theta_i}}{\sum_{n \in S} \lambda_{ni,0} (\tau_{ndi} c_{ni,0})^{-\theta_i}} = \lambda_{di,0} \left( \frac{\Gamma_i c_{di,0}}{P_{di,0}} \right)^{-\theta_i}, \quad (\text{A-38})$$

where  $P_{d=i,0}$  is the sector- $i$  price index. Dividing this by the equivalent expression in the reference country  $R$ :

$$\frac{\pi_{ddi,0}}{\pi_{RRi,0}} = \frac{\lambda_{di,0}}{\lambda_{Ri,0}} \left( \frac{c_{di,0}}{c_{Ri,0}} \right)^{-\theta_i} \left( \frac{P_{Ri,0}}{P_{di,0}} \right)^{-\theta_i} = S_{di} \left( \frac{P_{Ri,0}}{P_{di,0}} \right)^{-\theta_i}$$

and therefore:

$$\frac{P_{di,0}}{P_{ri,0}} = \left[ \frac{\pi_{ddi,0}}{\pi_{RRi,0}} \frac{1}{S_{di}} \right]^{1/\theta_i}. \quad (\text{A-39})$$

Thus, sectoral price indices can be recovered from domestic expenditure shares and the gravity-implied  $S_{di}$ .

**Recovering unit costs** Given the model's unit cost function (Appendix E), relative unit costs satisfy

$$\frac{c_{di,0}}{c_{ri,0}} = \left( \frac{w_d}{w_R} \right)^{\alpha_i \gamma_i} \left( \frac{r_d}{r_R} \right)^{(1-\alpha_i) \gamma_i} \prod_j \left( \frac{P_{dj,0}}{P_{rj,0}} \right)^{(1-\gamma_i) \gamma_{ki}}, \quad (\text{A-40})$$

where  $\alpha_i$ ,  $\gamma_i$ , and  $\gamma_{ki}$  are sectoral factor and input shares. Combining (A-37) and (A-40) yields

$$\frac{\lambda_{di,0}}{\lambda_{ri,0}} = S_{di} \left( \frac{c_{di,0}}{c_{Ri,0}} \right)^{\theta_i}. \quad (\text{A-41})$$

Equation (A-41) delivers the calibrated initial sectoral productivity parameters  $\{\lambda_{si,0}\}$  up to a sector-specific normalization.

## G Mathematical Derivations

### G.1 Proof of Proposition 1

In this section, we largely follow the steps of the mathematical appendix to [Buera and Oberfield \(2020\)](#) to the particularities of our model.

$$F_{d,t+\Delta}^i(z) = \underbrace{F_{di,t}(z)}_{Pr\{\text{productivity} < z \text{ at } t\}} \times \underbrace{\left(1 - \int_t^{t+\Delta} \int \alpha_{i,\tau} z^{-\theta_i} (z')^{\beta_i \theta_i} dG_{di,\tau}(z') d\tau\right)}_{Pr\{\text{no better draws in } (t, t+\Delta)\}}$$

Rearranging and using the definition of the derivative:

$$\frac{d}{dt} \ln F_{si,t}(z) = \lim_{\Delta \rightarrow 0} \frac{F_{s,t+\Delta}^i(z) - F_{si,t}(z)}{F_{si,t}(z)} = - \int \alpha_t z^{-\theta} (z')^{\beta \theta} dG_{di,t}(z')$$

Define  $\lambda_{si,t} = \int_{-\infty}^t \alpha_{i,\tau} \int (z')^{\beta \theta} dG_{s,\tau}^i(z') d\tau$  and integrate both sides wrt to time:

$$\begin{aligned} \int_0^t \frac{d}{d\tau} \ln F_{s,\tau}^i(z) d\tau &= -z^{-\theta} \int_0^t \int \alpha_{i,\tau} (z')^{\beta \theta} dG_{di,\tau}(z') d\tau \\ \ln \left( \frac{F_{s,\tau}^i(z)}{F_{s,0}^i(z)} \right) &= -z^{-\theta} (\lambda_{si,t} - \lambda_{si,0}) \\ F_{si,t}(z) &= F_{si,0}(z) \exp\{-z^{-\theta} (\lambda_{si,t} - \lambda_{si,0})\} \end{aligned}$$

Assuming that the initial distribution is Fréchet  $F_{si,0}(z) = \exp\{-\lambda_{si,0} z^{-\theta}\}$  guarantees that the distribution will be Fréchet at any point in time:

$$F_{si,t}(z) = \exp\{-\lambda_{si,t} z^{-\theta}\} \tag{A-42}$$

As seen above, we have defined:

$$\lambda_{si,t} = \int_{-\infty}^t \alpha_{i,\tau} \int (z')^{\beta \theta_i} dG_{s,\tau}^i(z') d\tau$$

Differentiating this definition with respect to time and applying Leibniz's Lemma yields:

$$\dot{\lambda}_{di,t} = \alpha_{i,t} \int (z')^{\beta_i \theta_i} dG_{di,t}(z')$$

We use these results and work with a discrete approximation of the law of motion for productivity:

$$\Delta \lambda_{d,t}^i = \alpha_{i,t} \int (z')^{\beta_i \theta_i} dG_{di,t}(z') \tag{A-43}$$

The source distribution  $G_{di,t}(z') \equiv \sum_{j \in \mathcal{I}} \gamma_{dij} \sum_{s \in \mathcal{D}} H_{sjdi,t-1}(z')$ , where  $\gamma_{dij}$  is the expenditure share of sector  $j$  in gross output when producing good  $i$  in region  $d$ ; and  $H_{sjdi,t-1}(z')$  is the fraction of commodities for which the lowest cost supplier in period  $t-1$  is a firm located in  $s$  with productivity weakly less than  $z'$ .

We focus our attention on the integral  $\int z^{\beta_i \theta_i} dH_{sdi,t}(z)$ . Let  $F_{si,t}(z_2, z_2) = \exp\{-\lambda_{si,t} z_2^{-\theta_i}\}$  and  $F_{si,t}(z_1, z_2) = (1 + \lambda_{si,t}[z_2^{-\theta_i} - z_1^{-\theta_i}]) \exp\{-\lambda_{si,t} z_2^{-\theta_i}\}$  are, respectively, the probability that a productivity draw is below  $z_2$ , and that the maximum productivity is  $z_1$  and the second highest productivity is  $z_2$ <sup>30</sup>. Let for each  $n$ ,  $A_{ndj,t} \equiv \frac{\tau_{ndj} c_{nj,t}}{\tau_{sdj} c_{sj,t}}$ , such that  $s$  will have a lower cost than  $d$  iff  $A_{ndj,t} z_{nj,t}(\omega) < z_{sj,t}(\omega)$ . Region  $s$  with highest productivity producers  $z_1, z_2$  will supply the commodity  $i \in \mathcal{I}$  in region  $d$  with the following probability:

$$\begin{aligned} \mathcal{F}_{sdi,t-1}(z_1, z_2) &= \int_0^{z_2} \prod_{n \neq s} F_{nj,t-1}(A_{ndj,t-1}y, A_{ndj,t-1}y) dF_{sj,t-1}(y, y) \\ &+ \int_{z_2}^{z_1} \prod_{n \neq s} F_{nj,t-1}(A_{ndj,t-1}z_2, A_{ndj,t-1}z_2) \frac{\partial}{\partial y} F_{sj,t-1}(y, z_2) dy \end{aligned}$$

The first term in the right hand side denotes the probability that the lowest cost producer at destination  $d$  is from  $s$  and has productivity lower than  $z_2$ , while the second term accounts for the probability that the lowest cost producer at destination  $d$  is from  $s$  and has productivity in the range  $[z_2, z_1]$ . In particular,  $\prod_{n \neq s} F_{nj,t-1}(A_{ndj,t-1}z_2, A_{ndj,t-1}z_2)$  denotes the probability of the lowest cost from all other sources  $n \neq s$  have a cost higher than a producer from  $s$  with productivity  $z_2$  and  $\frac{\partial}{\partial y} F_{sj,t-1}(y, z_2)$  (the density along  $y$  of the joint distribution) is probability of the lowest cost producer having productivity over  $y > z_2$  conditional on the second-lowest cost producer having productivity  $z_2$ . We will evaluate each integral separately. First, take the first term:

$$\begin{aligned} &\int_0^{z_2} \prod_{n \neq s} F_{nj,t-1}(A_{ndj,t-1}y, A_{ndj,t-1}y) dF_{sj,t-1}(y, y) \\ &= \int_0^{z_2} \exp \left\{ - \sum_{n \neq s} \lambda_{nj,t-1} (A_{ndj,t-1}y)^{-\theta_j} \right\} \theta \lambda_{sj,t-1} y^{-\theta_j-1} \exp\{-\lambda_{sj,t-1} y^{-\theta_j}\} dy \\ &= \lambda_{sj,t-1} \int_0^{z_2} \theta y^{-\theta_j-1} \exp \left\{ - \sum_n \lambda_{nj,t-1} (A_{ndj,t-1})^{-\theta_j} y^{-\theta_j} \right\} dy \\ &= \lambda_{sj,t-1} \frac{1}{\sum_n \lambda_{nj,t-1} (A_{ndj,t-1})^{-\theta_j}} \exp \left\{ - \sum_n \lambda_{nj,t-1} (A_{ndj,t-1})^{-\theta_j} y^{-\theta_j} \right\} \Big|_{y=0}^{y=z_2} \\ &= \frac{\lambda_{sj,t-1} (\tau_{sdj} c_{sj,t})^{-\theta_j}}{\sum_n \lambda_{nj,t-1} (\tau_{ndj} c_{nj,t})^{-\theta_j}} \exp \left\{ - \frac{\sum_n \lambda_{nj,t-1} (\tau_{ndj} c_{nj,t})^{-\theta_j}}{(\tau_{sdj} c_{sj,t})^{-\theta_j}} y^{-\theta_j} \right\} \Big|_{y=0}^{y=z_2} \\ &= \pi_{sdj,t-1} \exp \left\{ - \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} z_2^{-\theta_j} \right\} \end{aligned}$$

Now consider the second term.

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<sup>30</sup>To see the latter, note that:

$$\begin{aligned} Prob(z_1 \leq Z_1, z_2 \leq Z_2) &= F_{si,t}(Z_2) + \int_0^{Z_2} \int_{Z_2}^{Z_1} F_{si,t}(y) F_{si,t}(y') dy' dy \\ &= F_{si,t}(Z_2) + F_{si,t}(Z_2)(F_{si,t}(Z_1) - F_{si,t}(Z_2)) \\ &= (1 + \lambda_{si,t}[Z_2^{-\theta} - Z_1^{-\theta}]) \exp\{-\lambda_{si,t} Z_2^{-\theta}\} \end{aligned}$$

$$\begin{aligned}
& \int_{z_2}^{z_1} \prod_{n \neq s} F_{nj,t-1}(A_{ndj,t} z_2, A_{ndj,t} z_2) \frac{\partial}{\partial y} F_{sj,t-1}(y, z_2) dy \\
&= \int_{z_2}^{z_1} \exp \left\{ - \sum_{n \neq s} \lambda_{nj,t-1} \left( A_{ndj,t-1}^i z_2 \right)^{-\theta_j} \right\} \theta_j \lambda_{sj,t-1} y^{-\theta_j-1} \exp \{ -\lambda_{sj,t-1} z_2^{-\theta_j} \} dy \\
&= \exp \left\{ - \sum_n \lambda_{nj,t-1} \left( A_{ndj,t-1} z_2 \right)^{-\theta_j} \right\} \lambda_{sj,t-1} \int_{z_2}^{z_1} \theta_j y^{-\theta_j-1} dy \\
&= \exp \left\{ - \sum_n \lambda_{nj,t-1} \left( A_{ndj,t-1} z_2 \right)^{-\theta_j} \right\} \lambda_{sj,t-1} \int_{z_2}^{z_1} \theta_j y^{-\theta_j-1} dy \\
&= \exp \left\{ - \frac{\sum_n \lambda_{nj,t-1} (\tau_{ndj} c_{nj,t})^{-\theta_j}}{(\tau_{sdj} c_{sj,t})^{-\theta_j}} z_2^{-\theta_j} \right\} \lambda_{sj,t-1} y^{-\theta_j} \Big|_{y=z_1}^{y=z_2} \\
&= \exp \left\{ - \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} z_2^{-\theta_j} \right\} \lambda_{sj,t-1} (z_2^{-\theta_j} - z_1^{-\theta_j})
\end{aligned}$$

Therefore:

$$\mathcal{F}_{sjdi,t-1}(z_1, z_2) = \exp \left\{ - \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} z_2^{-\theta_j} \right\} \left( \pi_{sdj,t-1} + \lambda_{sj,t-1} (z_2^{-\theta_j} - z_1^{-\theta_j}) \right) \quad (\text{A-44})$$

Note that:

$$\int z^{\beta \theta_i} dH_{sjdi,t}(z) = \int_0^\infty \int_{z_2}^\infty z_1^{\beta \theta_i} \frac{\partial^2 \mathcal{F}_{sjdi,t-1}(z_1, z_2)}{\partial z_1 \partial z_2} dz_1 dz_2 \quad (\text{A-45})$$

and that we can calculate the joint density explicitly:

$$\begin{aligned}
\frac{\partial^2 \mathcal{F}_{sjdi,t-1}(z_1, z_2)}{\partial z_1 \partial z_2} &= \frac{\partial}{\partial z_2} \exp \left\{ - \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} z_2^{-\theta_j} \right\} \theta_j \lambda_{sj,t-1} z_1^{-\theta_j-1} \\
&= \frac{1}{\pi_{sdj,t-1}} \exp \left\{ - \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} z_2^{-\theta_j} \right\} (\theta_j \lambda_{sj,t-1} z_1^{-\theta_j-1}) (\theta_j \lambda_{nj,t-1} z_2^{-\theta_j-1})
\end{aligned}$$

Plugging this into (A-45):

$$\begin{aligned}
& \int_0^\infty \int_{z_2}^\infty z_1^{\beta_i \theta_i} \frac{1}{\pi_{sdj,t-1}} \exp \left\{ - \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} z_2^{-\theta_j} \right\} (\theta_j \lambda_{sj,t-1} z_1^{-\theta_j-1}) (\theta_j \lambda_{nj,t-1} z_2^{-\theta_j-1}) dz_1 dz_2 \\
&= \int_0^\infty \frac{1}{\pi_{sdj,t-1}} \exp \left\{ - \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} z_2^{-\theta_j} \right\} (\theta_j \lambda_{sj,t-1} z_2^{-\theta_j-1}) \lambda_{sj,t-1} \int_{z_2}^\infty (\theta_j z_1^{\beta_i \theta_i - \theta_j - 1}) dz_1 dz_2 \\
&= \int_0^\infty \frac{1}{\pi_{sdj,t-1}} \exp \left\{ - \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} z_2^{-\theta_j} \right\} (\theta_j \lambda_{sj,t-1} z_2^{-\theta_j-1}) \lambda_{sj,t-1} \left[ \frac{\theta_j}{\beta_i \theta_i - \theta_j} z_1^{\beta_i \theta_i - \theta_j} \right]_{z_1=z_2}^{z_1=\infty} dz_2
\end{aligned} \quad (\text{A-46})$$

Now, note that this integral only converges as  $z_1 \rightarrow \infty$  if  $\beta_i \theta_i < \theta_j$  or  $\beta_i \theta_i / \theta_j < 1$ . This condition is analogous to what [Deng and Zhang \(2023\)](#) show in the context of

Ricardian trade with perfect competition. Here we show that the same restriction holds when dealing with Ricardian trade with Bertrand competition.

Consider two cases. First, assume the trade elasticity is the same in each sector  $\theta_i = \theta_j = \theta$ . In that case, the restriction holds by default, and we can use a change of variables to show the resulting law of motion. Let  $\eta \equiv \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} z_2^{-\theta}$ , which implies that  $d\eta = -\theta \frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} z_2^{-\theta-1} dz$ . Replacing above:

$$\begin{aligned} &= (\lambda_{sj,t-1})^{\beta_i} (\pi_{sdj,t-1})^{1-\beta_i} \frac{1}{1-\beta_i} \int_0^\infty \exp\{-\eta\} \eta^{(1-\beta_i)} d\eta \\ &= (\lambda_{sj,t-1})^{\beta_i} (\pi_{sdj,t-1})^{1-\beta_i} \frac{1}{1-\beta_i} \Gamma(2-\beta_i) \\ &= \Gamma(1-\beta_i) (\lambda_{sj,t-1})^{\beta_i} (\pi_{sdj,t-1})^{1-\beta_i} \quad (\because \Gamma(y+1) = y\Gamma(y)) \end{aligned}$$

Therefore, replacing into the law of motion for the location parameter of the Fréchet distribution with homogeneous  $\theta_i = \theta_j$  satisfying  $\beta_i < 1$ :

$$\begin{aligned} \Delta \lambda_{di,t} &= \alpha_{i,t} \int z^{\beta_i \theta_i} dG_{di,t}(z) \\ &= \alpha_{i,t} \sum_{j \in \mathcal{I}} \gamma_{dij} \sum_{s \in \mathcal{D}} \int z^{\beta_i \theta_i} dH_{sdi,t-1}(z) \\ &= \alpha_{i,t} \Gamma(1-\beta_i) \sum_{j \in \mathcal{I}} \gamma_{dij} \sum_{s \in \mathcal{D}} (\lambda_{sj,t-1})^{\beta_i} (\pi_{sdj,t-1})^{1-\beta_i} \\ \Delta \lambda_{di,t} &= \alpha_{i,t} \Gamma(1-\beta_i) \sum_{j \in \mathcal{I}} \gamma_{dij} \sum_{s \in \mathcal{D}} (\lambda_{sj,t-1})^{\beta_i} (\pi_{sdj,t-1})^{1-\beta_i} \end{aligned} \tag{A-47}$$

Now, assume the trade elasticity varies across sectors and the restriction  $\beta_i < \theta_j/\theta_i$  holds for each sector pair. Let  $\eta \equiv \frac{\lambda_{sj,t-1}^i}{\pi_{sd,t-1}^i} z_2^{-\theta_j}$ , which implies that  $d\eta = -\theta_j \frac{\lambda_{sj,t-1}^i}{\pi_{sd,t-1}^i} z_2^{-\theta_j-1} dz$ . Replacing in equation (A-46):

$$\begin{aligned} &= \int_0^\infty \frac{1}{\pi_{sdj,t-1}} \exp\left\{-\frac{\lambda_{sj,t-1}}{\pi_{sdj,t-1}} z_2^{-\theta_j}\right\} (\theta_j \lambda_{sj,t-1} z_2^{-\theta_j-1}) \lambda_{sj,t-1} \frac{\theta_j}{\beta_i \theta_i - \theta_j} z_2^{\beta_i \theta_i - \theta_j} dz_2 \\ &= \int_0^\infty \exp\{-\eta\} \lambda_{sj,t-1} \frac{\theta_j}{\theta_j - \beta_i \theta_i} \left(\frac{\pi_{sdj,t-1}}{\lambda_{sj,t-1}} \eta\right)^{\frac{\theta_j - \beta_i \theta_i}{\theta_j}} d\eta \\ &= (\lambda_{sj,t-1})^{\beta_i \theta_i / \theta_j} (\pi_{sdj,t-1})^{1-\beta_i \theta_i / \theta_j} \frac{\theta_j}{\theta_j - \beta_i \theta_i} \int_0^\infty \exp\{-\eta\} \eta^{\frac{\theta_j - \beta_i \theta_i}{\theta_j}} d\eta \\ &= (\lambda_{sj,t-1})^{\beta_i \theta_i / \theta_j} (\pi_{sdj,t-1})^{1-\beta_i \theta_i / \theta_j} \frac{\theta_j}{\theta_j - \beta_i \theta_i} \Gamma\left(1 + \frac{\theta_j - \beta_i \theta_i}{\theta_j}\right) \\ &= \Gamma\left(\frac{\theta_j - \beta_i \theta_i}{\theta_j}\right) (\lambda_{sj,t-1})^{\beta_i \theta_i / \theta_j} (\pi_{sdj,t-1})^{1-\beta_i \theta_i / \theta_j} \end{aligned}$$

Therefore, replacing into the law of motion for the location parameter of the Fréchet distribution for heterogeneous  $\theta_i \neq \theta_j$  but satisfying  $\beta_i \theta_i / \theta_j < 1$ :

$$\begin{aligned}
\dot{\lambda}_{di,t} &= \alpha_{i,t} \int z^{\beta_i \theta_i} dG_{di,t}(z) \\
&= \alpha_{i,t} \sum_{j \in \mathcal{I}} \gamma_{dij} \sum_{s \in \mathcal{D}} \int z^{\beta_i \theta_i} dH_{sjdi,t-1}(z) \\
&= \alpha_{i,t} \sum_{j \in \mathcal{I}} \Gamma \left( \frac{\theta_j - \beta_i \theta_i}{\theta_j} \right) (\lambda_{sj,t-1})^{\beta_i \theta_i / \theta_j} (\pi_{sdj,t-1})^{1 - \beta_i \theta_i / \theta_j} \\
&= \alpha_{i,t} \sum_{j \in \mathcal{I}} \Gamma (1 - \beta_{ij}) (\lambda_{sj,t-1})^{\beta_{ij}} (\pi_{sdj,t-1})^{1 - \beta_{ij}}
\end{aligned}$$

which is the same expression as in equation (12).

## G.2 Pairwise Diffusion-Elasticities

Starting from law of motion of the knowledge frontier in destination country  $d$  and industry  $i$ :

$$\dot{\lambda}_{di} = \alpha_i \Gamma(1 - \beta_{ij}) \sum_j \tilde{\gamma}_{dij} \sum_s \pi_{sdj}^{1-\beta_{ij}} \lambda_{sj}^{\beta_{ij}} = \alpha_i \Gamma(1 - \beta_{ij}) S_{di} \quad (\text{A-48})$$

where:  $S_{di} \equiv \sum_j \tilde{\gamma}_{dij} \sum_s \pi_{sdj}^{1-\beta_{ij}} \lambda_{sj}^{\beta_{ij}}$ . Taking logs and differentiating with respect to  $\log \lambda_{sj}$  yields:

$$\varepsilon_{di,sj} \equiv \frac{\partial \log \dot{\lambda}_{di}}{\partial \log \lambda_{sj}} = \frac{\lambda_{sj}}{S_{di}} \frac{\partial S_{di}}{\partial \lambda_{sj}} = \beta_{ij} \frac{\tilde{\gamma}_{dij} \pi_{sdj}^{1-\beta_{ij}} \lambda_{sj}^{\beta_{ij}}}{S_{di}} = \beta_{ij} \omega_{di,sj} \quad (\text{A-49})$$

where  $\omega_{di,sj}$  is the diffusion-flow weight satisfying:

$$\omega_{di,sj} \equiv \frac{\tilde{\gamma}_{dij} \pi_{sdj}^{1-\beta_{ij}} \lambda_{sj}^{\beta_{ij}}}{\sum_{j'} \tilde{\gamma}_{dij'} \sum_{s'} \pi_{s'dj'}^{1-\beta_{ij'}} \lambda_{s'j'}^{\beta_{ij'}}}, \quad \text{with } \sum_{s,j} \omega_{di,sj} = 1,$$

This aggregates naturally to a weighted average diffusion elasticity for each  $(di)$  cluster:

$$\varepsilon_{di} = \sum_{j \in \mathcal{I}} \beta_{ij} \sum_{s \in \mathcal{N}} \omega_{di,sj} = \sum_{j \in \mathcal{I}} \beta_{ij} \omega_{di,j} \quad (\text{A-50})$$

where  $\omega_{di,j} \equiv \sum_{s \in \mathcal{N}} \omega_{di,sj}$  denotes the total weight of industry  $j$  in the diffusion response of cluster  $(di)$ .

## G.3 Diffusion Multiplier along BGP

**Proposition 3** (Knowledge-Diffusion Multiplier for Source-TFP (Stock) Shocks). *Let  $\lambda \in \mathbb{R}^{RI} = \{\lambda_{di}\}$  be the stacked vector of productivities. Assume that global economy is along its balanced growth path, such that  $\dot{\lambda}_{di}/\lambda_{di} = g$  for all  $di$ .*

*Index receivers by  $(d, i)$  and sources by  $(s, j)$ , and stack  $x = \log \lambda \in \mathbb{R}^{NI}$  in the order of  $(d, i)$ . Let  $W \in \mathbb{R}^{NI \times NI}$  collect bilateral idea-inflow shares,  $W_{(di),(sj)} = \omega_{di,sj}$ , with  $\sum_{(s,j)} W_{(di),(sj)} = 1$  for all  $(d, i)$ . Let  $B \in \mathbb{R}^{NI \times NI}$  collect bilateral diffusion elasticities  $\beta_{ij}$ , i.e.  $B_{(di),(sj)} = \beta_{ij}$ . Assume the local stationary-growth restriction  $g_{di} = \dot{\lambda}_{di}/\lambda_{di}$  is held fixed for comparative statics and  $\rho(BW) < 1$ .*

Consider \*\*exogenous TFP shocks\*\*  $z$  that scale source stocks multiplicatively:

$$\lambda_{sj}^{exog} = z_{sj} \bar{\lambda}_{sj} \quad \Rightarrow \quad d \log \lambda^{exog} = d \log z.$$

Let  $P \in \mathbb{R}^{NI \times NI}$  be a selection (injection) matrix that maps  $d \log z$  into the coordinates of  $x$  that are shocked (e.g.,  $P = I$  if all  $(s, j)$  can be shocked; otherwise  $P$  picks a subset).

Then the \*\*reduced-form comparative statics\*\* of equilibrium knowledge stocks with respect to source-TFP shocks is

$$d \log \lambda = (I - BW)^{-1} P d \log z, \quad (\text{A-51})$$

so entrywise

$$\frac{\partial \log \lambda_{di}}{\partial \log z_{s'j'}} = \left[ (I - BW)^{-1} P \right]_{(di), (s'j')}.$$

Moreover, for  $\rho(BW) < 1$ ,

$$(I - BW)^{-1} = I + (BW) + (BW)^2 + \dots,$$

so the effect of a source-TFP shock equals the \*\*direct first-round impact\*\* plus all higher-order diffusion rounds through the network.

*Proof.* Let  $x = \log \lambda$ . The log of the inflow aggregator satisfies  $d \log S = (BW) dx$  because  $\partial \log S_{di} / \partial \log \lambda_{sj} = \beta_{ij} \omega_{di,sj}$ . Under the stationary-growth restriction  $d \log S = dx$ , we have  $dx = (BW) dx$ . Introduce an \*\*exogenous\*\* perturbation to the state:  $dx = P d \log z + x^{\text{endo}}$ , where  $P d \log z$  injects the TFP shock on the shocked coordinates and  $x^{\text{endo}}$  is the induced adjustment required by the stationary condition. Substituting,

$$P d \log z + x^{\text{endo}} = (BW) (P d \log z + x^{\text{endo}}) \Rightarrow (I - BW) x^{\text{endo}} = (BW) P d \log z.$$

Since  $\rho(BW) < 1$ ,  $(I - BW)$  is invertible and

$$x^{\text{endo}} = (I - BW)^{-1} (BW) P d \log z, \quad dx = P d \log z + x^{\text{endo}} = (I - BW)^{-1} P d \log z,$$

which is (A-51). The Neumann series follows from  $\rho(BW) < 1$ .  $\square$

## G.4 Proof of Proposition 2

**Diffusion Maximum** trade shares that maximize diffusion solve the program:

$$\begin{aligned} & \max_{\{\pi_{sdj,t-1}\}_{j \in \mathcal{I}, s \in \mathcal{D}}} (1 - \gamma_{di}) \sum_{j \in \mathcal{I}} \gamma_{dij} \sum_{s \in \mathcal{D}} (\pi_{sdj,t-1})^{1-\beta} (\lambda_{sj,t-1})^\beta \\ & \text{s.t. } \forall (i, j) \in \mathcal{I} \times \mathcal{I} \quad \sum_{s \in \mathcal{D}} \pi_{sdj,t-1} = 1 \end{aligned} \quad (\text{A-52})$$

whose solutions satisfy:<sup>31</sup>:

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<sup>31</sup>Let  $\varphi$  be the Lagrange multiplier. Then, for each  $(s, i, j)$  first order conditions satisfy:

$$\begin{aligned} (1 - \beta) \gamma_{dij} (\pi_{sdj,t-1})^\beta (\lambda_{sj,t-1})^\beta &= \varphi \\ (\pi_{sdj,t-1})^{\text{Diffusion Optimum}} &= \varphi^{-\frac{1}{\beta}} [(1 - \beta) \gamma_{dij}]^{\frac{1}{\beta}} \lambda_{sj,t-1} \end{aligned}$$

using the constraint:

$$\sum_{s \in \mathcal{D}} (\varphi^{-\frac{1}{\beta}} [(1 - \beta) \gamma_{dij}]^{\frac{1}{\beta}} \lambda_{sj,t-1}) = 1 \iff \varphi^{-\frac{1}{\beta}} = [(1 - \beta) \gamma_{dij}]^{-\frac{1}{\beta}} \left( \sum_{s \in \mathcal{D}} \lambda_{sj,t-1} \right)^{-1}$$

$$(\pi_{sdj,t-1})^{\text{Diffusion Optimum}} = \frac{\lambda_{sj,t-1}}{\sum_{k \in \mathcal{D}} \lambda_{kj,t-1}} \quad (\text{A-53})$$

Replacing (A-53) into the objective function results in the following diffusion function:

$$\begin{aligned} (\Delta\lambda_{di,t})^{\text{Diffusion Optimal}} &\propto (1 - \gamma_{di}) \sum_{j \in \mathcal{I}} \gamma_{dij} \sum_{s \in \mathcal{D}} \left( \frac{\lambda_{sj,t-1}}{\sum_{k \in \mathcal{D}} \lambda_{kj,t-1}} \right)^{1-\beta} (\lambda_{sj,t-1})^\beta \\ &\propto (1 - \gamma_{di}) \sum_{j \in \mathcal{I}} \gamma_{dij} \sum_{s \in \mathcal{D}} \lambda_{sj,t-1} \left( \sum_{k \in \mathcal{D}} \lambda_{kj,t-1} \right)^{-(1-\beta)} \\ &\propto (1 - \gamma_{di}) \sum_{j \in \mathcal{I}} \gamma_{dij} \left( \sum_{s \in \mathcal{D}} \lambda_{sj,t-1} \right)^\beta \end{aligned}$$

The single-sector analog is of the diffusion optimal function, holding exogenous parameters constant, is:

$$(\Delta\lambda_{d,t})^{\text{Diffusion Optimal}} = \alpha_t \Gamma(1 - \beta)(1 - \gamma_d) \left( \sum_{s \in \mathcal{D}} \lambda_{s,t-1} \right)^\beta \quad (\text{A-54})$$

where  $(1 - \gamma_d) \equiv \sum_i \kappa_{di}(1 - \gamma_{di})$  is the aggregate intermediate cost share and . Let  $h(x) = x^\beta$  for  $\beta \in [0, 1]$ , a strictly concave function. Assume that  $\lambda_{s,t-1} \geq \sum_j \gamma_{dij} \lambda_{sj,t-1}$ , i.e. in each source country  $s$  aggregate productivity are at least as large as the cost-weighted average of its sectoral productivities in every destination country  $d$ . Then:

$$\begin{aligned} (\Delta\lambda_{d,t})^{\text{Diffusion Optimal}} &= \alpha_t \Gamma(1 - \beta)(1 - \gamma_d) \times h \left( \sum_{s \in \mathcal{D}} \lambda_{s,t-1} \right) \\ &= \alpha_t \Gamma(1 - \beta) \sum_{i \in \mathcal{I}} \kappa_{di}(1 - \gamma_{di}) \times h \left( \sum_{s \in \mathcal{D}} \lambda_{s,t-1} \right) \\ &\geq \alpha_t \Gamma(1 - \beta) \sum_{i \in \mathcal{I}} \kappa_{di}(1 - \gamma_{di}) \times h \left( \sum_{j \in \mathcal{I}} \gamma_{dij} \lambda_{sj,t-1} \right) \\ &> \alpha_t \Gamma(1 - \beta) \sum_{i \in \mathcal{I}} \kappa_{di}(1 - \gamma_{di}) \sum_{j \in \mathcal{I}} \gamma_{dij} \times h \left( \sum_{s \in \mathcal{D}} \lambda_{sj,t-1} \right) \\ &= \sum_{i \in \mathcal{I}} \kappa_{di} (\Delta\lambda_{di,t})^{\text{Diffusion Optimal}} \end{aligned}$$

where the first inequality comes from the assumption about aggregate productivities and that  $h(\cdot)$  is an increasing function; and the second inequality comes from applying Jensen's Inequality in the context of strictly concave functions.

**Market Allocation** Let  $g(\pi, \lambda) \equiv \pi^{1-\beta} \lambda^\beta$ . Note that  $g(\pi, \lambda)$  is strictly concave in both of its arguments. Denote  $\pi_{sd}, \lambda_s$  the value for aggregate trade shares and aggregate productivity, respectively, in a single sector mode. For ease of exposition, we omit time subscripts below whenever possible. Due to the concavity of  $g(\cdot, \cdot)$ :

$$g(\pi_{sdj}, \lambda_{sj}) \leq g(\pi_{sd}, \lambda_s) + g_\pi(\pi_{sd}, \lambda_s)(\pi_{sdj} - \pi_{sd}) + g_\lambda(\pi_{sd}, \lambda_s)(\lambda_{sj} - \lambda_s) \quad (\text{A-55})$$

By definition,  $\pi_{sd} = \sum_i \kappa_{di}(1 - \gamma_{di}) \sum_j \gamma_{dij} \pi_{sdj}$ , where  $\pi_{sd}$  are intermediate trade shares.<sup>32</sup> Multiplying both sides of (A-55) by  $\kappa_{di}(1 - \gamma_{di})\gamma_{dij}$  and summing over  $i$  and  $j$  results in:

$$\begin{aligned} \sum_{i \in \mathcal{I}} \kappa_{di}(1 - \gamma_{di}) \sum_{j \in \mathcal{I}} \gamma_{dij} g(\pi_{sdj}, \lambda_{sj}) &\leq \sum_{i \in \mathcal{I}} \kappa_{di}(1 - \gamma_{di}) g(\pi_{sd}, \lambda_s) \\ &+ g_\pi(\pi_{sd}, \lambda_s) \left( \sum_{i \in \mathcal{I}} \kappa_{di} \sum_{j \in \mathcal{I}} \tilde{\gamma}_{dij} \pi_{sdj} - \pi_{sd} \right) \\ &+ g_\lambda(\pi_{sd}, \lambda_s) \left( \sum_{i \in \mathcal{I}} \kappa_{di} \sum_{j \in \mathcal{I}} \tilde{\gamma}_{dij} \lambda_{sj} - \lambda_s \right) \\ &= \sum_{i \in \mathcal{I}} \kappa_{di}(1 - \gamma_{di}) g(\pi_{sd}, \lambda_s) \\ &+ g_\lambda(\pi_{sd}, \lambda_s) \left( \sum_{i \in \mathcal{I}} \kappa_{di} \sum_{j \in \mathcal{I}} \tilde{\gamma}_{dij} \lambda_{sj} - \lambda_s \right) \end{aligned}$$

Recall the definition of the law of motion for productivity in a multi-sector framework (12), using a common arrival rate  $\alpha_{i,t} = \alpha_t$  and a common  $\beta_i = \beta$  for every  $i$  is:

$$\Delta \lambda_{di} = \alpha_t \Gamma(1 - \beta) \sum_{j \in \mathcal{I}} (1 - \gamma_{di}) \gamma_{dij} \sum_{s \in \mathcal{D}} (\pi_{sdj})^{1-\beta} (\lambda_{sj})^\beta = \alpha_{i,t} \Gamma(1 - \beta) \sum_{j \in \mathcal{I}} \tilde{\gamma}_{dij} g(\pi_{sdj}, \lambda_{sj})$$

In a single sector framework the law of motion of productivity is:  $\Delta \lambda_d = \alpha_t \Gamma(1 - \beta) (1 - \gamma_d) \sum_{s \in \mathcal{D}} g(\pi_{sd}, \lambda_s)$ . Using these definitions, multiplying both sides of the inequality above by  $\alpha_t \Gamma(1 - \beta)$  and summing over source countries results in:

$$\sum_{i \in \mathcal{I}} \kappa_{di} \Delta \lambda_{di} \leq \Delta \lambda_d + \alpha_t \Gamma(1 - \beta) \sum_{i \in \mathcal{I}} \kappa_{di} \sum_{s \in \mathcal{D}} g_\lambda(\pi_{sd}, \lambda_s) \left( \sum_{j \in \mathcal{I}} \gamma_{dij} \lambda_{sj,t-1} - \lambda_{s,t-1} \right) \quad (\text{A-56})$$

Since  $g_\lambda(\pi_{sd}, \lambda_s) > 0$  and by assumption,  $\sum_{j \in \mathcal{I}} \gamma_{dij} \lambda_{sj,t-1} \geq \lambda_{s,t-1}$ , the second term on the right side of the inequality is positive. Therefore:

$$\left( \sum_{i \in \mathcal{I}} \kappa_{di} \Delta \lambda_{di,t} \right)^{\text{Market Allocation}} \leq (\Delta \lambda_{d,t})^{\text{Market Allocation}} \quad (\text{A-57})$$

which completes the proof.

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<sup>32</sup>To see that, note:  $\pi_{sd} = \frac{e_{sd}}{e_d} = \frac{\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}} e_{sd}^{i,j}}{e_d} = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}} \frac{e_d^i}{e_d} \frac{e_d^{i,j}}{e_d^i} \frac{e_{sd}^{i,j}}{e_d^{i,j}} = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}} \kappa_{di} \cdot \gamma_{dij} \cdot \pi_{sdj}$ .

## G.5 Summary Statistics

Note that the aggregate diffusion function is:

$$\Delta\lambda_{d,t} = \alpha_t \Gamma(1 - \beta) \sum_{d \in \mathcal{D}} (\pi_{sd})^{1-\beta} (\lambda_s)^\beta$$

Hence:

$$\frac{\partial \Delta\lambda_{d,t}}{\partial \tilde{\tau}_{s'd}} = \alpha_t \Gamma(1 - \beta)(1 - \beta) \sum_{d \in \mathcal{D}} (\pi_{sd})^{1-\beta} (\lambda_s)^\beta \frac{1}{\pi_{sd}} \frac{\partial \pi_{sd}}{\partial \tilde{\tau}_{s'd}}$$

Now calculate  $\frac{\partial \pi_{sd}}{\partial \tilde{\tau}_{s'd}}$ :

$$\frac{\partial \pi_{sd}}{\partial \tilde{\tau}_{s'd}} = \begin{cases} -\theta \frac{\pi_{s'd}}{\tilde{\tau}_{s'd}} [1 - \pi_{s'd}], & \text{if } s = s' \\ +\theta \frac{\pi_{sd}}{\tilde{\tau}_{s'd}} \pi_{s'd}, & \text{if } s \neq s' \end{cases}$$

Hence:

$$\frac{\partial \Delta\lambda_{d,t}}{\partial \tilde{\tau}_{s'd}} = -\theta(1-\beta) \frac{1}{\tilde{\tau}_{s'd}} \alpha_t \Gamma(1-\beta) \left[ (1 - \pi_{s'd}) (\pi_{s'd})^{1-\beta} (\lambda_{s'})^\beta - (\pi_{s'd}) \sum_{k \in \mathcal{D} \setminus \{s'\}} (\pi_{kd})^{1-\beta} (\lambda_k)^\beta \right]$$

Finally, define  $\omega_{sd} \equiv \frac{\alpha_t \Gamma(1-\beta) (\pi_{sd})^{1-\beta} (\lambda_s)^\beta}{\Delta\lambda_d}$ , the pairwise diffusion term. Then:

$$\begin{aligned} \frac{\partial \Delta\lambda_{d,t}}{\partial \tilde{\tau}_{s'd}} \frac{\tilde{\tau}_{s'd}}{\Delta\lambda_{d,t}} &= -\theta(1 - \beta) \left[ (1 - \pi_{s'd}) \omega_{s'd} - (\pi_{s'd}) \sum_{k \in \mathcal{D} \setminus \{s'\}} \omega_{kd} \right] \\ \frac{\partial \Delta\lambda_{d,t}}{\partial \tilde{\tau}_{s'd}} \frac{\tilde{\tau}_{s'd}}{\Delta\lambda_{d,t}} &= -\theta(1 - \beta) [(1 - \pi_{s'd}) \omega_{s'd} - \pi_{s'd} (1 - \omega_{s'd})] \\ \frac{\partial \Delta\lambda_{d,t}}{\partial \tilde{\tau}_{s'd}} \frac{\tilde{\tau}_{s'd}}{\Delta\lambda_{d,t}} &= -\theta(1 - \beta) [\omega_{s'd} - \pi_{s'd}] \end{aligned}$$

## H Other Mathematical Derivations

### H.1 Trade shares

In this model, since there are infinitely many varieties in the unit interval, the expenditure share of destination region  $d \in \mathcal{D}$  on goods coming from source country  $s \in \mathcal{D}$  converge to their expected values. Let  $\pi_{sd,t}^i$  denote the share of expenditures of consumers in region  $d \in \mathcal{D}$  on commodity  $i \in \mathcal{I}^m$  coming from region  $s \in \mathcal{D}$  and, let for each  $n$ ,  $(A_{n,t}^i)^{-1} \equiv \frac{\tilde{x}_{sd,t}^i}{\tilde{x}_{nd,t}^i}$ . This share will satisfy:

$$\begin{aligned}
\pi_{sd,t}^i &= Pr\left(\frac{\tilde{x}_{sd,t}^i}{z_{s,t}^i(\omega)} < \min_{(n \neq s)} \left\{ \frac{\tilde{x}_{nd,t}^i}{z_{n,t}^i(\omega)} \right\}\right) \\
&= \int_0^\infty Pr(z_{s,t}^i(\omega) = z) Pr(z_{n,t}^i(\omega) < z A_n^i) dz \\
&= \int_0^\infty F_{si,t}(z) \Pi_{(n \neq s)} F_{n,t}(z A_n^i) dz \\
&= \int_0^\infty \theta \lambda_{si,t} z^{-(1+\theta)} e^{-(\sum_{n \in \mathcal{D}} \lambda_{n,t}^i (A_n^i)^{-\theta}) z^{-\theta}} dz \\
&= \frac{\lambda_{si,t} (\tilde{x}_{sd,t}^i)^{-\theta}}{\sum_{n \in \mathcal{D}} \lambda_{n,t}^i (\tilde{x}_{nd,t}^i)^{-\theta}} \\
&= \frac{\lambda_{si,t} (\tilde{x}_{sd,t}^i)^{-\theta}}{\Phi_{d,t}^i}
\end{aligned} \tag{A-58}$$

Similarly, since countries use the same aggregate final goods as intermediate inputs, cost shares in intermediates for each supplying sector  $j$  and region  $s$  used in the production of good  $i$  in region  $d$  satisfies:

$$\pi_{sdj,t} = \frac{\lambda_{s,t}^j (\tilde{x}_{sd,t}^j)^{-\theta}}{\Phi_{d,t}^j} \tag{A-59}$$

which are the same as expressed in (18).

### H.2 Price levels

Recall that the prices of commodities and intermediate goods can be expressed, respectively, as:

$$p_{d,t}^i = \left[ \int_{[0,1]} p_{d,t}^i(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

Let  $\Omega_{sd,t}^i$  and  $\Omega_{sd,t}^{i,j}$  denote the subsets of  $\Omega = [0, 1]$  for which the region  $s \in \mathcal{D}$  is a supplier in destination region  $d \in \mathcal{D}$ . We can then rewrite price levels above as:

$$p_{d,t}^i = \left[ \sum_{s \in \mathcal{D}} \int_{\Omega_{sd,t}^i} p_{d,t}^i(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

Similarly, we restate  $\mathcal{F}_{sd,t}^i(z_1, z_2)$  and the analogous measure  $\mathcal{F}_{sd,t}^{i,j}(z_1, z_2)$ :

$$\mathcal{F}_{sd,t}^i(z_1, z_2) = \exp \left\{ -\frac{\lambda_{si,t}}{\pi_{sd,t}^i} z_2^{-\theta} \right\} \left( \pi_{sd,t}^i + \lambda_{si,t} (z_2^{-\theta} - z_1^{-\theta}) \right) \quad (\text{A-60})$$

which denote the fraction of varieties that  $d$  purchases from  $s$  with productivity up to  $z_1$  and whose second best producer is not more efficient than than  $z_2$ . Recall that, from the Bertrand competition assumption, we can write, for each variety  $\omega$ :

$$p_{d,t}^i(\omega) = \min \left\{ \frac{\sigma}{\sigma-1} \frac{\tilde{x}_{sd,t}^i}{z_{1s,t}^i(\omega)}, \frac{\tilde{x}_{sd,t}^i}{z_{2s,t}^i(\omega)} \right\}$$

So we can rewrite the equation  $\int_{\Omega_{sd,t}^i} p_{d,t}^i(\omega)^{1-\sigma} d\omega$  in the following fashion:

$$\begin{aligned} & \int_{\Omega_{sd,t}^i} p_{d,t}^i(\omega)^{1-\sigma} d\omega \\ = & \int_0^\infty \int_{z_2}^\infty (p_{d,t}^i)^{1-\sigma} \frac{\partial^2 \mathcal{F}_{sd,t}^i(z_1, z_2)}{\partial z_1 \partial z_2} dz_1 dz_2 \\ = & \int_0^\infty \int_{z_2}^\infty \min \left\{ \frac{\sigma}{\sigma-1} \frac{\tilde{x}_{sd,t}^i}{z_1}, \frac{\tilde{x}_{sd,t}^i}{z_2} \right\}^{1-\sigma} \frac{1}{\pi_{sd,t}^i} \exp \left\{ -\frac{\lambda_{si,t}}{\pi_{sd,t}^i} z_2^{-\theta} \right\} (\theta \lambda_{si,t} z_1^{-\theta-1}) (\theta \lambda_{si,t} z_2^{-\theta-1}) dz_1 dz_2 \end{aligned}$$

With a change of variables, denote  $\Psi_1 \equiv \frac{\lambda_{si,t}}{\pi_{sd,t}^i} z_1^{-\theta}$  and  $\Psi_2 \equiv \frac{\lambda_{si,t}}{\pi_{sd,t}^i} z_2^{-\theta}$  and  $d\Psi_1 = -\frac{\theta \lambda_{si,t} z_1^{-\theta-1}}{\pi_{sd,t-1}^i} dz_1$ ,  $d\Psi_2 = -\frac{\theta \lambda_{si,t} z_2^{-\theta-1}}{\pi_{sd,t-1}^i} dz_2$ , which allows us to rewrite the equation above as:

$$\begin{aligned}
& \int_{\Omega_{sd,t}^i} p_{d,t}^i(\omega)^{1-\sigma} d\omega \\
&= \pi_{sd,t}^i \int_0^\infty \int_0^{\Psi_2} \min \left\{ \frac{\sigma}{\sigma-1} \frac{\tilde{x}_{sd,t}^i}{z_1}, \frac{\tilde{x}_{sd,t}^i}{z_2} \right\}^{1-\sigma} \exp \left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \\
&= \pi_{sd,t}^i \left( \frac{\lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}_{sd,t}^i)^{1-\sigma} \int_0^\infty \int_0^{\Psi_2} \min \left\{ \left( \frac{\sigma}{\sigma-1} \right)^\theta \Psi_1, \Psi_2 \right\}^{\frac{1-\sigma}{\theta}} \exp \left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \\
&= \pi_{sd,t}^i \left( \frac{\lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}_{sd,t}^i)^{1-\sigma} \left[ \int_0^\infty \int_{\left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \Psi_2}^{\Psi_2} \Psi_2^{\frac{1-\sigma}{\theta}} \exp \left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \right. \\
&\quad \left. + \int_0^\infty \int_0^{\left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \Psi_2} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \Psi_1^{\frac{1-\sigma}{\theta}} \exp \left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \right] \\
&= \pi_{sd,t}^i \left( \frac{\lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}_{sd,t}^i)^{1-\sigma} \left[ 1 - \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} + \frac{\theta}{1-\sigma+\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \right] \cdot \int_0^\infty \Psi_2^{\frac{1-\sigma}{\theta}+1} \exp \left\{ -\Psi_2 \right\} d\Psi_2 \\
&= \pi_{sd,t}^i \left( \frac{\lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}_{sd,t}^i)^{1-\sigma} \left[ 1 - \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} + \frac{\theta}{1-\sigma+\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \right] \Gamma \left( \frac{1-\sigma}{\theta} + 2 \right) \\
&= \pi_{sd,t}^i \left( \frac{\lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}_{sd,t}^i)^{1-\sigma} \left[ 1 - \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} + \frac{\theta}{1-\sigma+\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \right] \frac{1-\sigma+\theta}{\theta} \Gamma \left( \frac{1-\sigma+\theta}{\theta} \right) \\
&= \left[ 1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \right] \cdot \Gamma \left( \frac{1-\sigma+\theta}{\theta} \right) \cdot \pi_{sd,t}^i \left( \frac{\lambda_{si,t} (\tilde{x}_{sd,t}^i)^{-\theta}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} \\
&= \left[ 1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \right] \cdot \Gamma \left( \frac{1-\sigma+\theta}{\theta} \right) \cdot \pi_{sd,t}^i \left( \frac{\lambda_{si,t} (\tilde{x}_{sd,t}^i)^{-\theta}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} \\
&= \left[ 1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \right] \cdot \Gamma \left( \frac{1-\sigma+\theta}{\theta} \right) \cdot \pi_{sd,t}^i \left( \sum_{n \in \mathcal{D}} \lambda_{n,t}^i (\tilde{x}_{nd,t}^i)^{-\theta} \right)^{-\frac{1-\sigma}{\theta}}
\end{aligned}$$

Therefore:

$$\begin{aligned}
p_{d,t}^i &= \left[ \sum_{s \in \mathcal{D}} \int_{\Omega_{sd,t}^i} p_{d,t}^i(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \\
p_{d,t}^i &= \left[ 1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \right]^{\frac{1}{1-\sigma}} \cdot \Gamma \left( \frac{1-\sigma+\theta}{\theta} \right)^{\frac{1}{1-\sigma}} \cdot \left( \sum_{n \in \mathcal{D}} \lambda_{n,t}^i (\tilde{x}_{nd,t}^i)^{-\theta} \right)^{-\frac{1}{\theta}} \cdot \left[ \sum_{s \in \mathcal{D}} \pi_{sd,t}^i \right]^{\frac{1}{1-\sigma}} \\
p_{d,t}^i &= \left[ 1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \right]^{\frac{1}{1-\sigma}} \cdot \Gamma \left( \frac{1-\sigma+\theta}{\theta} \right)^{\frac{1}{1-\sigma}} \cdot \left( \sum_{n \in \mathcal{D}} \lambda_{n,t}^i (\tilde{x}_{nd,t}^i)^{-\theta} \right)^{-\frac{1}{\theta}}
\end{aligned} \tag{A-61}$$

Which is the same as (17) after allowing the elasticities to be sector-specific.

### H.3 Marginal costs and profits

Let  $q_{d,t}^i$  denote the CES aggregate quantity of varieties:

$$q_{d,t}^i \equiv \left[ \int_{[0,1]} q_{d,t}^i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}. \tag{A-62}$$

From equation (A-62) we can derive standard CES demand functions as:

$$q_{d,t}^i(\omega) = \left( \frac{p_{d,t}^i(\omega)}{p_{d,t}^i} \right)^{-\sigma} \frac{e_{d,t}^i}{p_{d,t}^i} \quad (\text{A-63})$$

$$c_{d,t}^{i,j}(\omega) = \left( \frac{p_{d,t}^{m,j}(\omega)}{p_{d,t}^{m,j}} \right)^{-\sigma} \frac{e_{d,t}^{i,j}}{p c_{d,t}^{m,j}} \quad (\text{A-64})$$

where  $p_{d,t}^i$  satisfies equations (17);  $e_{d,t}^i$  denotes expenditure on commodity  $i$  of macro-sector  $m$  in country  $d$ ; and  $e_{d,t}^{i,j}$  denotes expenditure on intermediate input  $j$  used in the production of commodity  $i$  of macro-sector  $m$  in country  $d$ .

As in previous subsections of the Appendix, we will derive the expression for the marginal cost and mark-up for the production of variety  $q_{d,t}^i(\omega)$  and state a corresponding expression for  $c_{d,t}^{i,j}(\omega)$ . The marginal cost of producing variety  $\omega$  sourced in country  $s$  and consumed in country  $s$  is:

$$\frac{\tilde{x}_{d,t}^i}{z_1(\omega)} q_{d,t}^i(\omega)$$

and total cost of varieties sourced in country  $s$  and consumed in country  $s$  can be expressed as:

$$\int_{\Omega_{sd,t}^i} \frac{\tilde{x}_{d,t}^i}{z_1(\omega)} q_{d,t}^i(\omega) d\omega = \int_{\Omega_{sd,t}^i} \frac{\tilde{x}_{d,t}^i}{z_1(\omega)} \left( \frac{p_{d,t}^i(\omega)}{p_{d,t}^i} \right)^{-\sigma} \frac{e_{d,t}^i}{p_{d,t}^i} d\omega$$

As in the previous section of the Appendix, we let  $\Omega_{sd,t}^i$  and  $\Omega_{sd,t}^{i,j}$  denote the subsets of  $\Omega = [0, 1]$  for which the region  $s \in \mathcal{D}$  is a supplier in destination region  $d \in \mathcal{D}$ . We can then rewrite the integral above as:

$$\begin{aligned} & \int_{\Omega_{sd,t}^i} \frac{\tilde{x}_{d,t}^i}{z_1(\omega)} \left( \frac{p_{d,t}^i(\omega)}{p_{d,t}^i} \right)^{-\sigma} \frac{e_{d,t}^i}{p_{d,t}^i} d\omega \\ &= \tilde{x}_{d,t}^i \frac{e_{d,t}^i}{(p_{d,t}^i)^{1-\sigma}} \int_0^\infty \int_{z_2}^\infty (z_1)^{-1} (p_{d,t}^i)^{-\sigma} \frac{\partial^2 \mathcal{F}_{sd,t}^i(z_1, z_2)}{\partial z_1 \partial z_2} dz_1 dz_2 \\ &= \tilde{x}_{d,t}^i \frac{e_{d,t}^i}{(p_{d,t}^i)^{1-\sigma}} \int_0^\infty \int_{z_2}^\infty \frac{1}{z_1} \min \left\{ \frac{\sigma}{\sigma-1} \frac{\tilde{x}_{sd,t}^i}{z_1}, \frac{\tilde{x}_{sd,t}^i}{z_2} \right\}^{-\sigma} \frac{1}{\pi_{sd,t}^i} \exp \left\{ -\frac{\lambda_{si,t}}{\pi_{sd,t}^i} z_2^{-\theta} \right\} (\theta \lambda_{si,t} z_1^{-\theta-1}) (\theta \lambda_{si,t} z_2^{-\theta-1}) dz_1 dz_2 \end{aligned}$$

Once again, use a change of variables, denote  $\Psi_1 \equiv \frac{\lambda_{si,t}}{\pi_{sd,t}^i} z_1^{-\theta}$  and  $\Psi_2 \equiv \frac{\lambda_{si,t}}{\pi_{sd,t}^i} z_2^{-\theta}$  and  $d\Psi_1 = -\frac{\theta \lambda_{si,t} z_1^{-\theta-1}}{\pi_{sd,t}^i} dz_1$ ,  $d\Psi_2 = -\frac{\theta \lambda_{si,t} z_2^{-\theta-1}}{\pi_{sd,t}^i} dz_2$ , which allows U.S. to rewrite the equation above as:

$$\begin{aligned}
& \int_{\Omega_{sd,t}^i} \frac{\tilde{x}_{d,t}^i}{z_1(\omega)} \left( \frac{p_{d,t}^i(\omega)}{p_{d,t}^i} \right)^{-\sigma} \frac{e_{d,t}^i}{p_{d,t}^i} d\omega \\
&= \pi_{sd,t}^i \frac{e_{d,t}^i}{(p_{d,t}^i)^{1-\sigma}} \left( \frac{\lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}_{sd,t}^i)^{1-\sigma} \int_0^\infty \int_0^{\Psi_2} \Psi_1^{\frac{1}{\theta}} \min \left\{ \left( \frac{\sigma}{\sigma-1} \right)^\theta \Psi_1, \Psi_2 \right\}^{-\frac{\sigma}{\theta}} d\Psi_1 d\Psi_2 \\
&= \pi_{sd,t}^i \frac{e_{d,t}^i}{(p_{d,t}^i)^{1-\sigma}} \left( \frac{\lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}_{sd,t}^i)^{1-\sigma} \left[ \int_0^\infty \int_{\left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \Psi_2}^{\Psi_2} \Psi_1^{\frac{1}{\theta}} \Psi_2^{\frac{-\sigma}{\theta}} \exp \left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \right. \\
&\quad \left. + \int_0^\infty \int_0^{\left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \Psi_2} \left( \frac{\sigma}{\sigma-1} \right)^{-\sigma} \Psi_1^{\frac{1-\sigma}{\theta}} \exp \left\{ -\Psi_2 \right\} d\Psi_1 d\Psi_2 \right] \\
&= \pi_{sd,t}^i \frac{e_{d,t}^i}{(p_{d,t}^i)^{1-\sigma}} \left( \frac{\lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}_{sd,t}^i)^{1-\sigma} \left[ \int_0^\infty \frac{\theta}{1+\theta} \left[ 1 - \left( \frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Psi_2^{\frac{1-\sigma+\theta}{\theta}} \exp \left\{ -\Psi_2 \right\} d\Psi_2 \right. \\
&\quad \left. + \int_0^\infty \frac{\theta}{1-\sigma+\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-1-\theta} \Psi_1^{\frac{1-\sigma+\theta}{\theta}} \exp \left\{ -\Psi_2 \right\} d\Psi_2 \right] \\
&= \pi_{sd,t}^i \frac{e_{d,t}^i}{(p_{d,t}^i)^{1-\sigma}} \left( \frac{\lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}_{sd,t}^i)^{1-\sigma} \left[ \frac{\theta}{1+\theta} \left[ 1 - \left( \frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] + \frac{\theta}{1-\sigma+\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Gamma \left( \frac{1-\sigma}{\theta} + 2 \right) \\
&= \pi_{sd,t}^i \frac{e_{d,t}^i}{(p_{d,t}^i)^{1-\sigma}} \left( \frac{\lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}_{sd,t}^i)^{1-\sigma} \left[ \frac{\theta}{1+\theta} \left[ 1 - \left( \frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] + \frac{\theta}{1-\sigma+\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \frac{1-\sigma+\theta}{\theta} \Gamma \left( \frac{1-\sigma}{\theta} + 1 \right) \\
&= \left[ 1 - \frac{\sigma}{1+\theta} + \frac{\sigma}{1+\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Gamma \left( \frac{1-\sigma+\theta}{\theta} \right) \pi_{sd,t}^i \frac{e_{d,t}^i}{(p_{d,t}^i)^{1-\sigma}} \left( \frac{\lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1-\sigma}{\theta}} (\tilde{x}_{sd,t}^i)^{1-\sigma} \\
&= \left[ 1 - \frac{\sigma}{1+\theta} + \frac{\sigma}{1+\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Gamma \left( \frac{1-\sigma+\theta}{\theta} \right) \pi_{sd,t}^i \frac{e_{d,t}^i}{(p_{d,t}^i)^{1-\sigma}} \left( \frac{(\tilde{x}_{sd,t}^i)^{-\theta} \lambda_{si,t}}{\pi_{sd,t}^i} \right)^{\frac{\sigma}{\theta}} \left( \frac{(\tilde{x}_{sd,t}^i)^{-\theta} \lambda_{si,t}}{\pi_{sd,t}^i} \right)^{-\frac{1}{\theta}} \\
&= \left[ 1 - \frac{\sigma}{1+\theta} + \frac{\sigma}{1+\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Gamma \left( \frac{1-\sigma+\theta}{\theta} \right) \pi_{sd,t}^i \frac{e_{d,t}^i}{(p_{d,t}^i)^{1-\sigma}} \left( \sum_{n \in \mathcal{D}} (\tilde{x}_{nd,t}^i)^{-\theta} \lambda_{n,t}^i \right)^{-\frac{1-\sigma}{\theta}}
\end{aligned}$$

Using the expression for  $(p_{d,t}^i)^{1-\sigma}$ :

$$\begin{aligned}
&= \frac{\left[ 1 - \frac{\sigma}{1+\theta} + \frac{\sigma}{1+\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right] \Gamma \left( \frac{1-\sigma+\theta}{\theta} \right) \pi_{sd,t}^i e_{d,t}^i \left( \sum_{n \in \mathcal{D}} (\tilde{x}_{nd,t}^i)^{-\theta} \lambda_{n,t}^i \right)^{-\frac{1-\sigma}{\theta}}}{\left[ 1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \right] \Gamma \left( \frac{1-\sigma+\theta}{\theta} \right) \left( \sum_{n \in \mathcal{D}} \lambda_{n,t}^i (\tilde{x}_{nd,t}^i)^{-\theta} \right)^{-\frac{1-\sigma}{\theta}}} \\
&= \frac{\left[ 1 - \frac{\sigma}{1+\theta} + \frac{\sigma}{1+\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-1-\theta} \right]}{\left[ 1 - \frac{\sigma-1}{\theta} + \frac{\sigma-1}{\theta} \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \right]} \pi_{sd,t}^i e_{d,t}^i \\
&= \frac{\theta}{1+\theta} \frac{1+\theta-\sigma+\sigma^{-\theta}(\sigma-1)^{1+\theta}}{1+\theta-\sigma+\sigma^{-\theta}(\sigma-1)^{1+\theta}} \pi_{sd,t}^i e_{d,t}^i
\end{aligned}$$

Therefore, total cost equals:

$$C_{s,t}^i = \sum_{d \in \mathcal{D}} \int_{\Omega_{sd,t}^i} \frac{\tilde{x}_{d,t}^i}{z_1(\omega)} \left( \frac{p_{d,t}^i(\omega)}{p_{d,t}^i} \right)^{-\sigma} \frac{e_{d,t}^i}{p_{d,t}^i} d\omega = \frac{\theta}{1+\theta} \sum_{d \in \mathcal{D}} \pi_{sd,t}^i e_{d,t}^i \quad (\text{A-65})$$

Profits can be expressed compactly as total revenue minus total cost:

$$\Pi_{s,t}^i = \sum_{d \in \mathcal{D}} \pi_{sd,t}^i e_{d,t}^i - \frac{\theta}{1+\theta} \sum_{d \in \mathcal{D}} \pi_{sd,t}^i e_{d,t}^i = \frac{1}{1+\theta} \sum_{d \in \mathcal{D}} \pi_{sd,t}^i e_{d,t}^i \quad (\text{A-66})$$

Analogously, total costs and profits of intermediary producers are, respectively:

$$c_{s,t}^i = \frac{\theta}{1+\theta} \sum_{d \in \mathcal{D}} \pi_{sdj,t} e_{d,t}^{i,j}, \quad \Pi_{s,t}^i = \frac{1}{1+\theta} \sum_{d \in \mathcal{D}} \pi_{sdj,t} e_{d,t}^{i,j} \quad (\text{A-67})$$

These are allow analogous to the expression in the paper after allowing the elasticities to be sector-specific.