homework3 - task2

作业要求:


```python
dec = 6 # 设置每一步计算保留小数点后位数 (精度, 可以自己调整)
import numpy as np
np.set_printoptions(formatter={'float': ('{: 0.' + str(dec) + 'f}').format})
import matplotlib.pyplot as plt
import matplotlib as mpl
mpl.rcParams['text.usetex'] = True
import sympy as sp
```

1 习题3.8

1.1 第一问

```
1  A = sp.Matrix([
2      [2, 1, 2],
3      [4, 3, 1],
4      [6, 1, 5]
5  ])
6  b = sp.Matrix([6, 11, 13])
7  A
```

$$\begin{bmatrix} 2 & 1 & 2 \\ 4 & 3 & 1 \\ 6 & 1 & 5 \end{bmatrix}$$

```
1  L, U = A.LUdecomposition()[0: 2]
2  print("L=")
3  L
```

```
1  L=
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -2 & 1 \end{bmatrix}$$

```
1  print("U=")
2  U
```

```
1  U=
```

$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & -7 \end{bmatrix}$$

```
1  x = A.solve(b)
2  print("Ax=b->x=")
3  x
```

```
1  Ax=b->x=
```

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

```
1  det = U.det()
2  print("det(U)=", det)
3  det
```

```
1  det(U)= -14
```

-14

1.2 第二问

```
1  A = sp.Matrix([
2      [2, 2, 3],
3      [4, 7, 7],
4      [-2, 4, 5]
5  ])
6  b = sp.Matrix([3, 1, -7])
7  A
```

$$\begin{bmatrix} 2 & 2 & 3 \\ 4 & 7 & 7 \\ -2 & 4 & 5 \end{bmatrix}$$

```
1  L, U = A.LUdecomposition()[0: 2]
2  print("L=")
3  L
```

```
1  L=
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$$

```
1  print("U=")
2  U
```

```
1  U=
```

$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 6 \end{bmatrix}$$

```
1  x = A.solve(b)
2  print("Ax=b->x=")
3  x
```

```
1  Ax=b->x=
```

$$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

```
1  det = U.det()
2  print("det(U)=", det)
3  det
```

```
1  det(U)= 36
```

36

2   习题3.9

```
1  A = sp.Matrix([
2      [6, 2, 1, -1],
3      [2, 4, 1, 0],
4      [1, 1, 4, -1],
5      [-1, 0, -1, 3]
6  ])
7  b = sp.Matrix([6, -1, 5, -5])
8  A
```

$$\begin{bmatrix} 6 & 2 & 1 & -1 \\ 2 & 4 & 1 & 0 \\ 1 & 1 & 4 & -1 \\ -1 & 0 & -1 & 3 \end{bmatrix}$$

```
1  x = A.solve(b)
2  x.T
```

$$[1 \quad -1 \quad 1 \quad -1]$$

3　习题3.10

此题中不知道是什么情况，有点奇怪，似乎第一行应该是：

$$x_1 + 2x_2 + x_3 = 3$$

而不是

$$2x_1 + x_2 + x_3 = 3$$

这里我都算一下。

```
1  A = sp.Matrix([
2      [1, 2, 1],
3      [1, 3, 2],
4      [1, 2, -3]
5  ])
6  b = sp.Matrix([3, 3, 11])
7  x = A.solve(b)
8  print("Ax=b->x=")
9  x.T
```

```
1  Ax=b->x=
```

$$[1 \quad 2 \quad -2]$$

```
1  A = sp.Matrix([
2      [2, 1, 1],
3      [1, 3, 2],
4      [1, 2, -3]
5  ])
6  b = sp.Matrix([3, 3, 11])
7  x = A.solve(b)
8  print("Ax=b->x=")
9  x.T
```

```
1  Ax=b->x=
```

$$\begin{bmatrix} \frac{35}{22} & \frac{39}{22} & -\frac{43}{22} \end{bmatrix}$$

4 习题3.11

参考 C 程序可以见文件夹 ThomasMethod ，这里就直接算了。

```
1  A = sp.Matrix([
2      [2, 1, 0, 0],
3      [1, 3, 1, 0],
4      [0, 1, 1, 1],
5      [0, 0, 2, 1]
6  ])
7  b = sp.Matrix([1, 2, 2, 0])
8  x = A.solve(b)
9  print("Ax=b->x=")
10 x.T
```

```
1  Ax=b->x=
```

$$\begin{bmatrix} 0 & 1 & -1 & 2 \end{bmatrix}$$

5 习题3.14(2)

```
1  A = sp.Matrix([
2      [2, -1, 0],
3      [-1, 2, -1],
4      [0, -1, 2]
5  ])
6  f = sp.Matrix([0, 1, 0])
7  x = A.solve(f)
8  print("Ax=b->x=")
9  x.T
```

```
1  Ax=b->x=
```

$$\begin{bmatrix} \frac{1}{2} & 1 & \frac{1}{2} \end{bmatrix}$$

6   习题3.15

```
1  x = sp.Matrix([2, -3, 4])
2  print("一范数: ", x.norm(1))
3  print("二范数: ", x.norm(2))
4  print("无穷范数: ", x.norm(sp.oo))
```

```
1  一范数:  9
2  二范数:  sqrt(29)
3  无穷范数:  4
```

```
1  A = sp.Matrix([
2      [1, 0, 0],
3      [0, 2, 4],
4      [0, -2, 4]
5  ])
6  print("一范数: ", A.norm(1))
7  print("二范数: ", A.norm(2))
8  print("无穷范数: ", A.norm(sp.oo))
```

```
1  一范数:  8
2  二范数:  4*sqrt(2)
3  无穷范数:  6
```

7   习题3.16

前者是的，因为显然其具有：

- $f(\lambda \vec{x}) = \lambda f(x)$ ；

- $f(x) \geq 0$，当且仅当 $\vec{x} = \vec{0}$ 时取等。

后者不是，显然给出一个：

$$\vec{x} = [-3, 1, 0]$$

时会让 $f(\vec{x}) = 0$ ，非范数。

8 补充题目1

```
1  A = sp.Matrix([
2      [2, -1, 3],
3      [4, 2, 5],
4      [1, 2, 0]
5  ])
6  b = sp.Matrix([1, 4, 7])
7  x_real = A.solve(b)
8  x_real.T
```

$$\begin{bmatrix} 9 & -1 & -6 \end{bmatrix}$$

9 补充题目2

```
1  A = sp.Matrix([
2      [11, -3, -2],
3      [-23, 11, 1],
4      [1, 2, 2]
5  ])
6  b = sp.Matrix([3, 0, -1])
7  x_real = A.solve(b)
8  x_real.T
```

$$\begin{bmatrix} \frac{41}{193} & \frac{106}{193} & -\frac{223}{193} \end{bmatrix}$$

10 补充题目3

```
1   A = sp.Matrix([
2       [2, 1, 0, 0, 0],
3       [5/14, 2, 9/14, 0, 0],
4       [0, 3/5, 2, 2/5, 0],
5       [0, 0, 3/7, 2, 4/7],
6       [0, 0, 0, 1, 2]
7   ])
8   b = sp.Matrix([-5.5200, -4.3144, -3.2664, -2.4287, -2.1150])
9   x_real = A.solve(b)
10  x_real
```

$$\begin{bmatrix} -2.02856318234611 \\ -1.46287363530778 \\ -1.03314692218351 \\ -0.80595493612079 \\ -0.654522531939605 \end{bmatrix}$$

11 习题3.22

11.1 第一问

```
1  A = sp.Matrix([
2      [10, -2, -2],
3      [-2, 10, -1],
4      [-1, -2, 3]
5  ])
6  b = sp.Matrix([1, 1/2, 1])
7  x = A.solve(b)
8  x.T
```

$$[0.23109243697479 \quad 0.147058823529412 \quad 0.508403361344538]$$

11.2 第二问

```
1  A = sp.Matrix([
2      [10, -1, 2, 0],
3      [-1, 11, -1, 3],
4      [2, -1, 10, -1],
5      [0, 3, -1, 8]
6  ])
7  b = sp.Matrix([6, 25, -11, 15])
8  x = A.solve(b)
9  x.T
```

$$[1 \quad 2 \quad -1 \quad 1]$$

12 习题3.23

```
1  A = sp.Matrix([
2      [1, 2, -2],
3      [1, 1, 1],
4      [2, 2, 1]
5  ])
6  b = sp.Matrix([1, 3, 5])
7  x_real = A.solve(b)
8  x_real.T
```

$$[1 \quad 1 \quad 1]$$

13 习题3.24

```
1  A = sp.Matrix([
2      [4, 3, 0],
3      [3, 4, -1],
4      [0, -1, 4]
5  ])
6  b = sp.Matrix([24, 30, -24])
7  x_real = A.solve(b)
8  x_real.T
```

$$[3 \quad 4 \quad -5]$$

14 习题3.25

```
1  A = sp.Matrix([
2      [2, -1, 0, 0],
3      [-1, 2, -1, 0],
4      [0, -1, 2, -1],
5      [0, 0, -1, 2]
6  ])
7  b = sp.Matrix([1, 0, 1, 0])
8  x_real = A.solve(b)
9  x_real.T
```

$$\left[\frac{6}{5} \quad \frac{7}{5} \quad \frac{8}{5} \quad \frac{4}{5}\right]$$