2023/04/25

task 2

导入基本需要的计算以及绘图模块：

```python
dec = 6 # 设置每一步计算保留小数点后位数（精度，可以自己调整）
import numpy as np
np.set_printoptions(formatter={'float': ('{: 0.' + str(dec) + 'f}').format})
import matplotlib.pyplot as plt
import matplotlib as mpl
mpl.rcParams['text.usetex'] = True
import sympy as sp
```

1 习题2.11

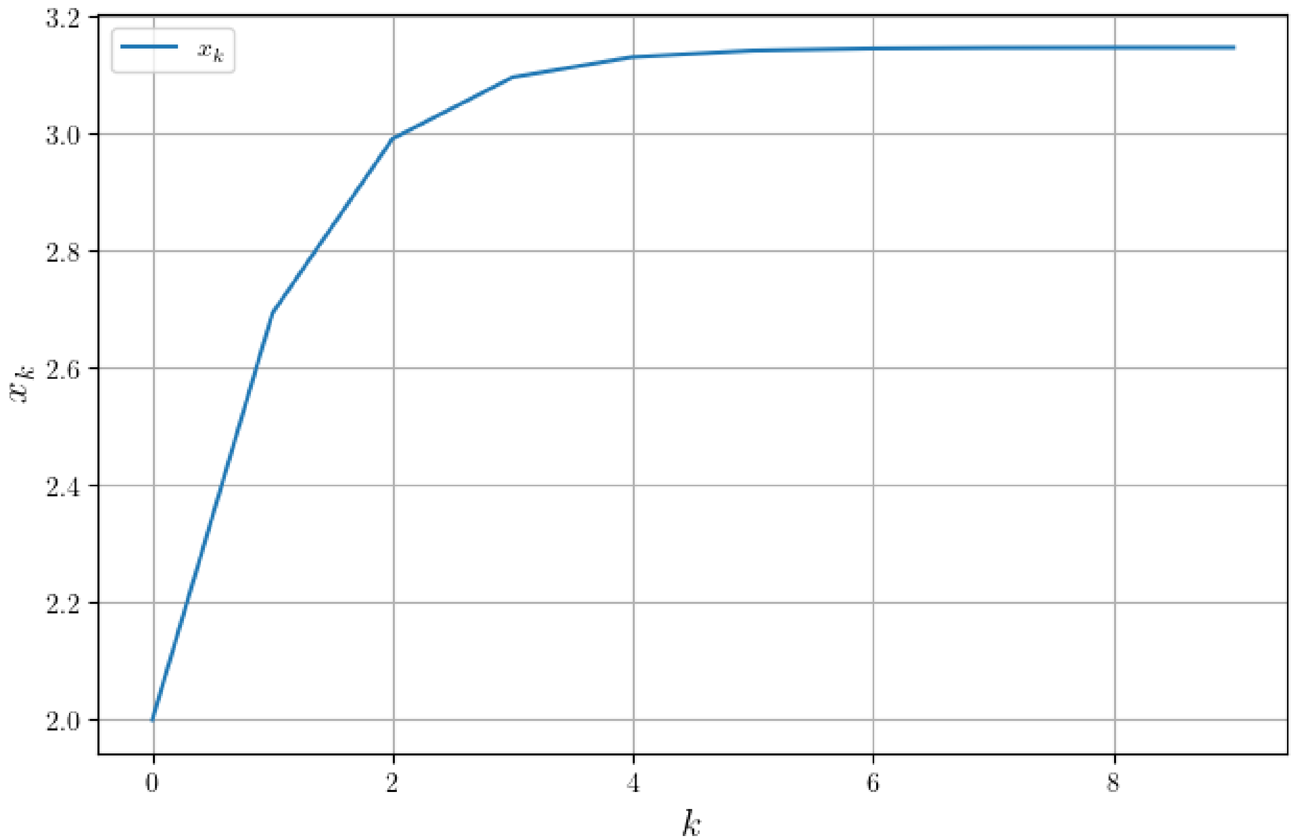
迭代格式一：

$$x_{k+1} = \ln x_k + 2$$

```python
def iterationFunc(x):
    return np.round(np.log(x) + 2, dec)
    pass

step = 10
xk = np.zeros(step)
x0 = 2
xk[0] = x0
for i in range(step-1):
    xk[i+1] = iterationFunc(xk[i])
    pass

error = xk[1:] - xk[:-1]

print("xk= ", xk)
print("error= ", error)
```

```
xk=  [ 2.000000  2.693147  2.990710  3.095511  3.129953  3.141018  3.144547
  3.145670  3.146027  3.146140]
error=  [ 0.693147  0.297563  0.104801  0.034442  0.011065  0.003529  0.001123
  0.000357  0.000113]
```

```python
plt.figure(figsize=(8, 5), facecolor="white")
plt.plot(np.arange(0, step), xk, label=r"$x_k$")
plt.legend()
plt.xlabel("$k$", fontsize=15)
plt.ylabel("$x_k$", fontsize=15)
plt.grid(True)
plt.show()
```



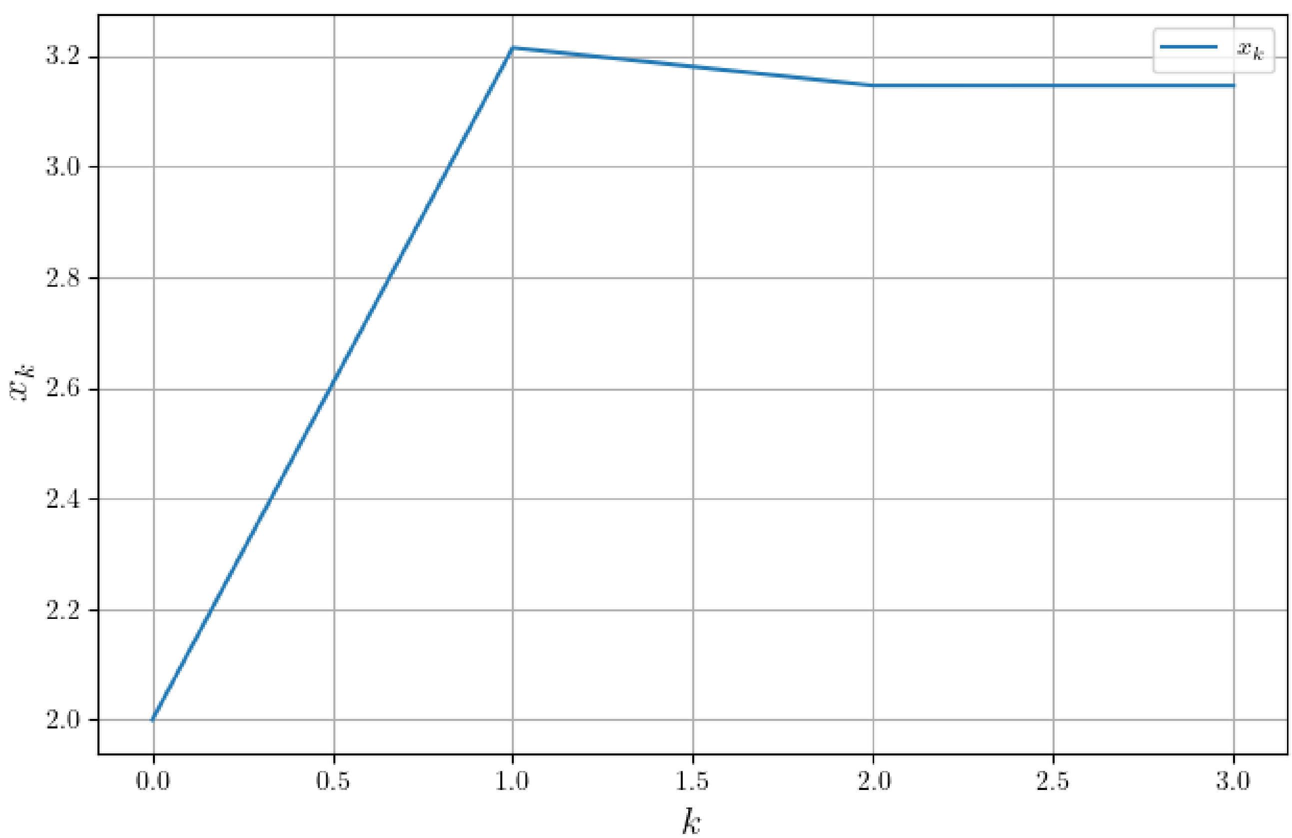
调整一下迭代格式，并只计算4步如下：

$$\begin{cases} y_k = \ln x_k + 2 \\ z_k = \ln y_k + 2 \\ x_{k+1} = x_k - \dfrac{(y_k - z_k)^2}{z_k + x_k - 2y_k} \end{cases}$$

```
def iterationFuncStephenson(xi):
    yi = iterationFunc(xi)
    zi = iterationFunc(yi)
    x_next = xi - (yi-xi)**2 / (zi+xi-2*yi)
    return np.around(x_next, dec)
    pass

step = 4
xk = np.zeros(step)
x0 = 2
xk[0] = x0
for i in range(step-1):
    xk[i+1] = iterationFuncStephenson(xk[i])
    pass

error = xk[1:] - xk[:-1]

print("xk= ", xk)
print("error= ", error)
```

```
xk=  [ 2.000000  3.214540  3.146300  3.146193]
error=  [ 1.214540 -0.068240 -0.000107]
```

```python
plt.figure(figsize=(8, 5), facecolor="white")
plt.plot(np.arange(0, step), xk, label=r"$x_k$")
plt.legend()
plt.xlabel("$k$", fontsize=15)
plt.ylabel("$x_k$", fontsize=15)
plt.grid(True)
plt.show()
```



效果拔群。

2 习题2.13

```python
x_k, x_k_next, x_real, a, p, q, r = sp.symbols('x_k, x_{k+1}, x^*, a, p, q, r')
expr = p*x_k + q*a/x_k**2 + r*a**2/x_k**5
x_real = a**sp.Rational(1, 3)
expr
iter = sp.Eq(x_k_next, expr)
print("迭代格式: ")
iter
```

```
迭代格式:
```

$$x_{k+1} = \frac{a^2 r}{x_k^5} + \frac{aq}{x_k^2} + p x_k$$

```
print("真实解: ")
x_real
```

```
真实解:
```

$$\sqrt[3]{a}$$

```
expr1 = expr.diff(x_k)
print("一阶导: ")
expr1
```

```
一阶导:
```

$$-\frac{5a^2 r}{x_k^6} - \frac{2aq}{x_k^3} + p$$

```
expr2 = expr1.diff(x_k)
print("二阶导: ")
expr2
```

```
二阶导:
```

$$\frac{30a^2 r}{x_k^7} + \frac{6aq}{x_k^4}$$

由此建立三个等式如下:

```
eq1 = sp.Eq(expr.subs(x_k, x_real), x_real)
eq1
```

$\sqrt[3]{a}p + \sqrt[3]{a}q + \sqrt[3]{a}r = \sqrt[3]{a}$

```
eq2 = sp.Eq(expr1.subs(x_k, x_real), 0)
eq2
```

$p - 2q - 5r = 0$

```
eq3 = sp.Eq(expr2.subs(x_k, x_real), 0)
eq3
```

$\frac{6q}{\sqrt[3]{a}} + \frac{30r}{\sqrt[3]{a}} = 0$

```
res = sp.solve(
    [eq1, eq2, eq3],
    [p, q, r]
)
print("p, q, r: ")
res
```

```
p, q, r:
```

```
{p: 5/9, q: 5/9, r: -1/9}
```

查看一下三阶导数，代入上述的 p, q, r 和 $x = a^{\frac{1}{3}}$ ：

```
expr3 = expr2.diff(x_k).subs(x_k, x_real).subs(res)
expr3
```

$\frac{10}{a^{\frac{2}{3}}}$

显然上式是不满足等于零的，因此这个迭代格式三阶收敛。

3 习题2.14

显然这个序列满足如下迭代格式：

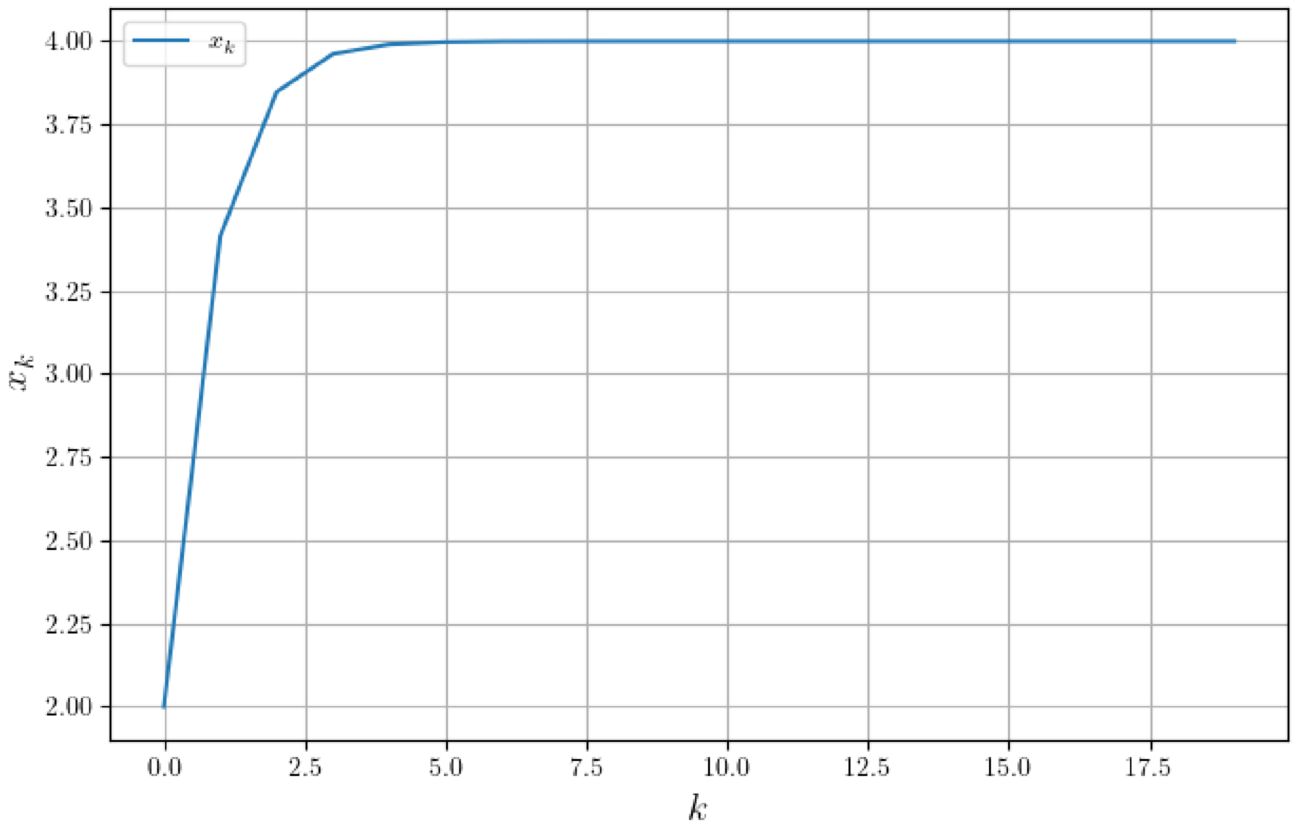
$$x_{k+1} = 2 + \sqrt{x_k}$$

因此稍加计算即可：

```
def iterationFunc(xi):
    return np.around(
        2+np.sqrt(xi),
        dec
    )
    pass

x0 = 2
step = 20
xk = np.zeros(step)
xk[0] = x0
for i in range(step-1):
    xk[i+1] = iterationFunc(xk[i])
    pass
print("xk=\n", xk)
```

```
xk=
 [ 2.000000  3.414214  3.847759  3.961571  3.990370  3.997591  3.999398
  3.999849  3.999962  3.999990  3.999997  3.999999  4.000000  4.000000
  4.000000  4.000000  4.000000  4.000000  4.000000  4.000000]
```

```
plt.figure(figsize=(8, 5), facecolor="white")
plt.plot(np.arange(0, step), xk, label=r"$x_k$")
plt.legend()
plt.xlabel("$k$", fontsize=15)
plt.ylabel("$x_k$", fontsize=15)
plt.grid(True)
plt.show()
```



4 习题2.16

事实上这是求解方程的解：

$$f(x) = x^2 - \frac{1}{a} = 0$$

也就是方程：

$$g(x) = a - \frac{1}{x^2} = 0$$

ref

```
x = sp.symbols("x", positive=True)
a = sp.symbols("a", positive=True)
expr = a - 1/x**2
expr1 = expr.diff(x)
print("原函数: ")
expr
```

```
原函数:
```

$a - \frac{1}{x^2}$

```python
print("一阶导: ")
expr1
```

```
一阶导:
```

$\frac{2}{x^3}$

```python
expr_iter = (x - expr/expr1).expand()
print("迭代格式: ")
expr_iter
```

```
迭代格式:
```

$-\frac{ax^3}{2} + \frac{3x}{2}$

5 习题2.17

```python
expr = 2*x**2 + 2*x - sp.exp(2*x) + 1
expr1 = expr.diff(x)
iter_expr = x - expr / expr1
iterationFunc = sp.lambdify(x, iter_expr, "numpy")
print("表达式: ")
expr
```

```
表达式:
```

$2x^2 + 2x - e^{2x} + 1$

```python
print("一阶导: ")
expr1
```

```
一阶导:
```

$4x - 2e^{2x} + 2$

```
print("迭代格式: ")
iter_expr
```

```
迭代格式:
```

$$x - \frac{2x^2 + 2x - e^{2x} + 1}{4x - 2e^{2x} + 2}$$

```
x0 = 0.5
step = 40
xk = np.zeros(step)
xk[0] = x0
for i in range(step-1):
    xk[i+1] = iterationFunc(xk[i])
    pass
print("xk=\n", xk)
```

```
xk=
 [ 0.500000  0.348053  0.239056  0.162643  0.109929  0.073967  0.049618
  0.033216  0.022206  0.014831  0.009900  0.006605  0.004406  0.002938
  0.001959  0.001306  0.000871  0.000581  0.000387  0.000258  0.000172
  0.000115  0.000076  0.000051  0.000034  0.000023  0.000015  0.000010
  0.000007  0.000005  0.000002  0.000002  0.000002  0.000002  0.000002
  0.000002  0.000002  0.000002  0.000002  0.000002]
```

我这里取了40步进行计算，效果如上，一直熟练不到 0 ，接下来我用修正牛顿进行操作：

```
m = 3
iter_expr = x - expr / expr1 * m
iterationFunc = sp.lambdify(x, iter_expr, "numpy")
print("修正牛顿迭代公式: ")
iter_expr
```

```
修正牛顿迭代公式:
```

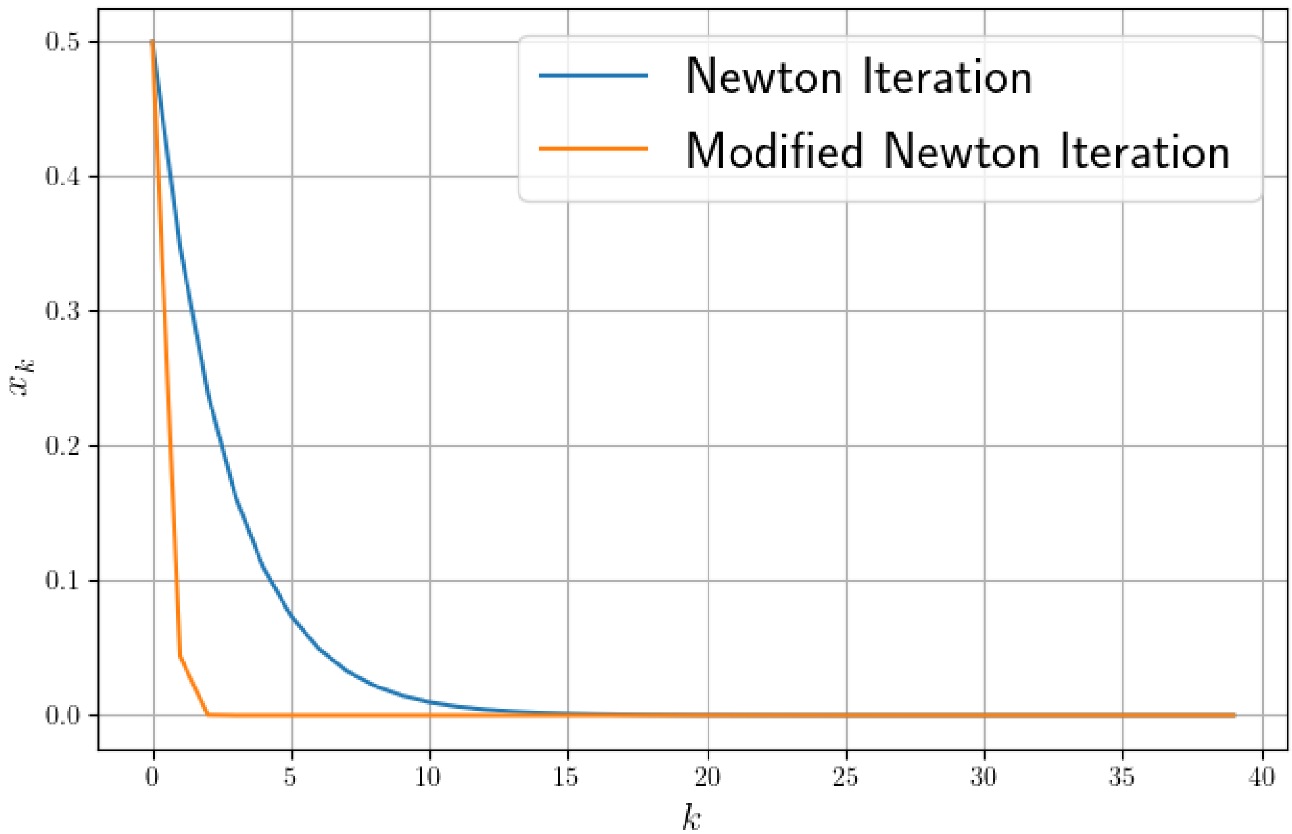
$$x - \frac{3 \cdot \left(2x^2 + 2x - e^{2x} + 1\right)}{4x - 2e^{2x} + 2}$$

```python
x0 = 0.5
step = 40
xkk = np.zeros(step)
xkk[0] = x0
for i in range(step-1):
    xkk[i+1] = iterationFunc(xkk[i])
    pass
print("xkk=\n", xkk)
```

```
xkk=
 [ 0.500000  0.044158  0.000327  0.000000  0.000000  0.000000  0.000000
  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
  0.000000  0.000000  0.000000  0.000000  0.000000]
```

只能说效果拔群↑:

```python
plt.figure(figsize=(8, 5), facecolor="white")
plt.plot(np.arange(0, step), xk, label="Newton Iteration")
plt.plot(np.arange(0, step), xkk, label="Modified Newton Iteration")
plt.legend(fontsize=20)
plt.xlabel("$k$", fontsize=15)
plt.ylabel("$x_k$", fontsize=15)
plt.grid(True)
plt.show()
```



6 习题2.19

```
expr = (x**3 - 3*x**2 + 3*x - 1) * (x+3)
expr = expr.factor()
print("原表达式：")
expr
```

```
原表达式:
```

$$\left(x-1\right)^{3}\left(x+3\right)$$

```
print("一阶导函数：")
expr1 = expr.diff(x)
expr1 = expr1.factor()
expr1
```

```
一阶导函数:
```

$$4(x-1)^2\,(x+2)$$

```python
m = 3
x_k1 = sp.symbols("x_{k+1}")
```

```python
print("x = 1 局部牛顿迭代收敛表达式：")
sp.Eq(x_k1, x_k - expr.subs(x, x_k) / expr1.subs(x, x_k)*m)
```

```
x = 1 局部牛顿迭代收敛表达式:
```

$$x_{k+1} = x_k - \frac{3\,(x_k-1)\,(x_k+3)}{4\,(x_k+2)}$$

```python
print("x = -3 局部牛顿迭代收敛表达式：")
sp.Eq(x_k1, x_k - expr.subs(x, x_k) / expr1.subs(x, x_k))
```

```
x = -3 局部牛顿迭代收敛表达式:
```

$$x_{k+1} = x_k - \frac{(x_k-1)\,(x_k+3)}{4\,(x_k+2)}$$