导入库

```
import numpy as np
import sympy as sp
from IPython.display import Math
import matplotlib.pyplot as plt
```

用以解决一个类似如下的问题：

$$\begin{bmatrix} b_1 & c_1 & 0 & \cdots & 0 \\ a_2 & b_2 & c_2 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & a_{n-1} & b_{n-1} & c_{n-1} \\ 0 & 0 & \cdots & a_n & b_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_n \end{bmatrix}$$

下面给出一组测试数据并写入`ThomasTestData`：

```
N = 10
a = np.random.randint(20, size=N)
c = np.random.randint(20, size=N)
b = np.random.randint(20, size=N)
d = np.random.randint(20, size=N)
a[0] = 0.0
c[N-1] = 0.0

data = np.array(
    [a, b, c, d],
    dtype=float
)
np.savetxt("ThomasTestData.txt", data)
print("a = ", a)
print("b = ", b)
print("c = ", c)
print("d = ", d)
```

```
a =  [ 0  2  2  1  5 11  0  3  2 16]
b =  [11  0  5 14 17 10 17 19 11  6]
c =  [ 6  1 17  0  7 18  2 11 10  0]
d =  [17  6 15 16  7  2 16  1  4  4]
```

为方便观赏写成矩阵形式：

```
mat = sp.zeros(N, N)
for i in range(N):
    mat[i, i] = b[i]
    pass
for i in range(1, N):
    mat[i, i-1] = a[i]
    pass
for i in range(0, N-1):
    mat[i, i+1] = c[i]
    pass
```

```
d_mat = sp.Matrix([
    d
]).transpose()
x = sp.Matrix(sp.symbols("x_1:"+str(N+1)))
display(
    Math(
        sp.latex(mat)+
        sp.latex(x)+
        "="+
        sp.latex(d_mat)
    )
)
```

$$\begin{bmatrix}11 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 2 & 5 & 17 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 14 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 5 & 17 & 7 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 11 & 10 & 18 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 17 & 2 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 19 & 11 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 11 & 10\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\\ x_7\\ x_8\\ x_9\\ x_{10}\end{bmatrix}=\begin{bmatrix}17\\ 6\\ 15\\ 16\\ 7\\ 2\\ 16\\ 1\\ 4\\ 4\end{bmatrix}$$

Python解

```
sol_python = mat.solve(d_mat)
print("分数解：")
sol_python
```

```
分数解：
```

$$\begin{bmatrix}\frac{689}{236}\\ -\frac{1189}{472}\\ \frac{19}{118}\\ \frac{267}{236}\\ \frac{228280361}{175814454}\\ -\frac{1041316739}{351628908}\\ \frac{15426}{16021}\\ -\frac{2953}{16021}\\ \frac{2350}{16021}\\ \frac{4414}{16021}\end{bmatrix}$$

```
print("python数值解：")
py_res = np.float64(np.array(sol_python.evalf()).T[0])
py_res
```

```
python数值解:
```

```
array([ 2.91949153, -2.5190678 ,  0.16101695,  1.13135593,  1.29841635,
       -2.96140822,  0.96286124, -0.18432058,  0.14668248,  0.27551339])
```

cpp 解

代码在 `ThomasCpp.cpp` 文件中，核心程序如下：

```
std::vector<double> Thomas(std::vector<double> b, std::vector<double> a,
std::vector<double> c, std::vector<double> d){
    int N = b.size();
    std::vector<double> beta(N);
    std::vector<double> y(N);
    std::vector<double> l(N);
    std::vector<double> x(N);
    beta[0] = b[0];
    y[0] = d[0];
    for(int j=1; j<N; j++){
        l[j] = a[j] / beta[j-1];
        beta[j] = b[j] - l[j] * c[j-1];
        y[j] = d[j] - l[j] * y[j-1];
    }
    x[N-1] = y[N-1] / beta[N-1];
    for(int j=N-2; j>-1; j--){
        x[j] = (y[j] - c[j] * x[j+1]) / beta[j];
    }
    return x;
}
```

用 `python` 编译一下并且读取文件：

```
import os
cpp_result_file = "CppResult.txt"
os.system("g++ -o ThomasCpp ThomasCpp.cpp ")
os.system("./ThomasCpp > "+cpp_result_file)
```

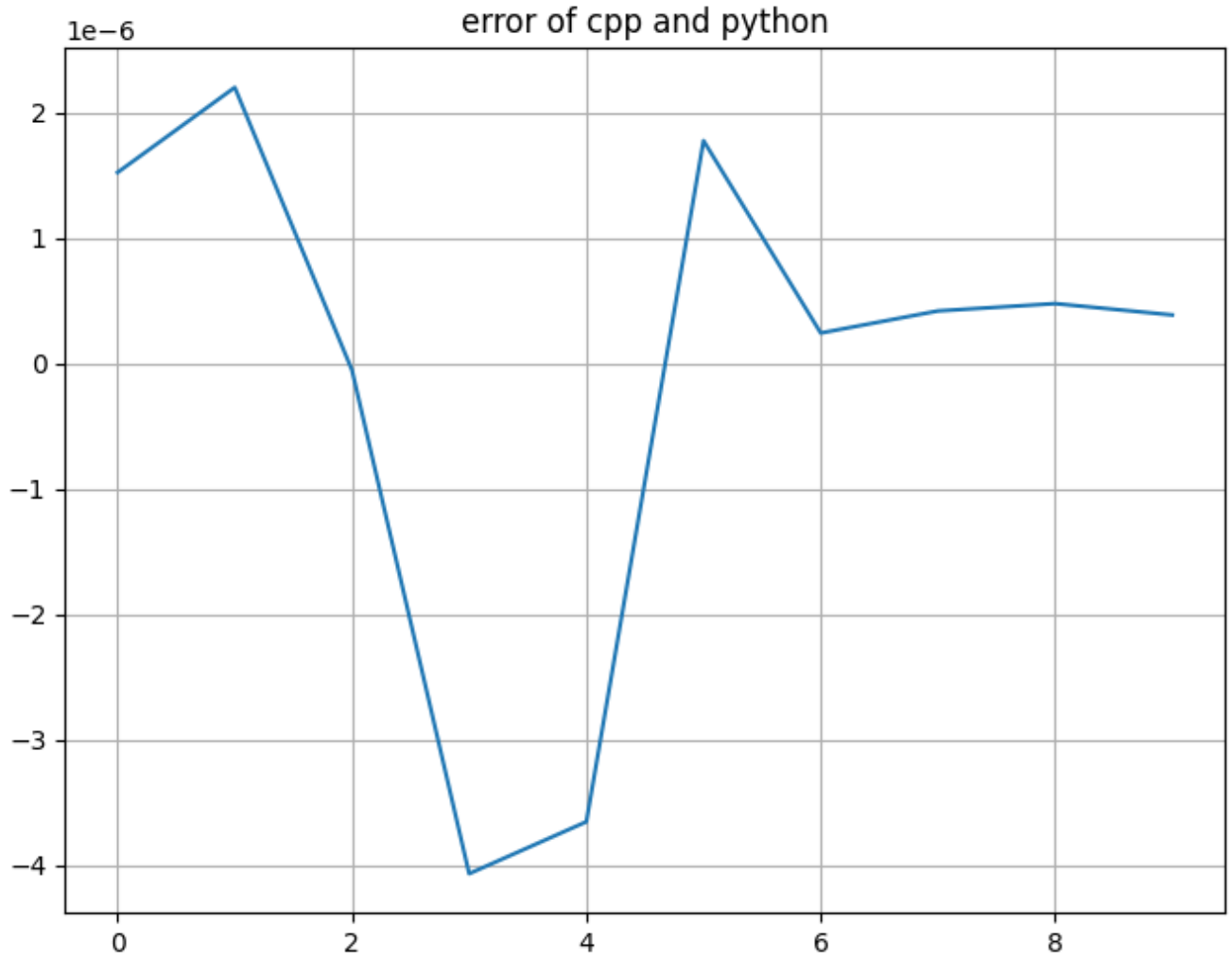
```
0
```

```
cpp_res = np.loadtxt(cpp_result_file).T
print("cpp数值解: ")
print(cpp_res)
```

```
cpp数值解:
[ 2.91949  -2.51907   0.161017  1.13136   1.29842  -2.96141   0.962861
 -0.184321  0.146682  0.275513]
```

比较

```
plt.figure(figsize=(8, 6), facecolor="white")
plt.plot(py_res-cpp_res)
plt.title("error of cpp and python")
plt.grid(True)
plt.show()
```




```
```