Hermite插值

In[1]:=
```
Clear["Global`*"];
```
清除
```
rule = x2 → (x1 + h);
rule2 = {x → x1 + t, x2 → (x1 + h)};
matx = {
    {1, x1, x1^2, x1^3},
    {0, 1, 2 x1, 3 x1^2},
    {1, x2, x2^2, x2^3},
    {0, 1, 2 x2, 3 x2^2}
   };
ϕ = Dot[{1, x, x^2, x^3}, Inverse[matx]] // FullSimplify;
```
点积 逆 完全简化
```
ϕ // TraditionalForm
```
传统格式
```
ϕ /. rule2 // FullSimplify // TraditionalForm
```
完全简化 传统格式
```
∂x ϕ /. rule2 // FullSimplify // TraditionalForm
```
完全简化 传统格式

Out[6]//TraditionalForm=

$$\left\{-\frac{(x-\text{x2})^2\,(2\,x-3\,\text{x1}+\text{x2})}{(\text{x1}-\text{x2})^3},\ \frac{(x-\text{x1})\,(x-\text{x2})^2}{(\text{x1}-\text{x2})^2},\ \frac{(x-\text{x1})^2\,(2\,x+\text{x1}-3\,\text{x2})}{(\text{x1}-\text{x2})^3},\ \frac{(x-\text{x1})^2\,(x-\text{x2})}{(\text{x1}-\text{x2})^2}\right\}$$

Out[7]//TraditionalForm=

$$\left\{\frac{(h-t)^2\,(h+2\,t)}{h^3},\ \frac{t\,(h-t)^2}{h^2},\ \frac{t^2\,(3\,h-2\,t)}{h^3},\ \frac{t^2\,(t-h)}{h^2}\right\}$$

Out[8]//TraditionalForm=

$$\left\{\frac{6\,t\,(t-h)}{h^3},\ \frac{(h-3\,t)\,(h-t)}{h^2},\ \frac{6\,t\,(h-t)}{h^3},\ \frac{t\,(3\,t-2\,h)}{h^2}\right\}$$

In[9]:=
```
ϕ2 = ∂x,x ϕ // FullSimplify;
```
完全简化
```
Kpart = Integrate[KroneckerProduct[ϕ2, ϕ2], {x, x1, x2}] // FullSimplify;
```
积分 克罗内克积 完全简化
```
Kpart /. rule // FullSimplify // TraditionalForm
```
完全简化 传统格式

Out[11]//TraditionalForm=

$$\begin{pmatrix} \frac{12}{h^3} & \frac{6}{h^2} & -\frac{12}{h^3} & \frac{6}{h^2} \\ \frac{6}{h^2} & \frac{4}{h} & -\frac{6}{h^2} & \frac{2}{h} \\ -\frac{12}{h^3} & -\frac{6}{h^2} & \frac{12}{h^3} & -\frac{6}{h^2} \\ \frac{6}{h^2} & \frac{2}{h} & -\frac{6}{h^2} & \frac{4}{h} \end{pmatrix}$$

In[12]:=
```
bpart = Integrate[ϕ, {x, x1, x2}] // FullSimplify;
bpart /. rule // FullSimplify // TraditionalForm
```

Out[13]//TraditionalForm=

$$\left\{\frac{h}{2}, \frac{h^2}{12}, \frac{h}{2}, -\frac{h^2}{12}\right\}$$

In[14]:=
```
Integrate[∂x ϕ, {x, x1, x2}] /. rule // FullSimplify // TraditionalForm
```

Out[14]//TraditionalForm=

$$\{-1, 0, 1, 0\}$$

In[15]:=
```
Integrate[∂x,x ϕ, {x, x1, x2}] /. rule // FullSimplify // TraditionalForm
```

Out[15]//TraditionalForm=

$$\{0, -1, 0, 1\}$$

In[16]:=
```
(∂x ϕ) /. {x → x1} /. rule // FullSimplify // TraditionalForm
```

Out[16]//TraditionalForm=

$$\{0, 1, 0, 0\}$$

In[17]:=
```
(∂x ϕ) /. {x → x1 + h} /. rule // FullSimplify // TraditionalForm
```

Out[17]//TraditionalForm=

$$\{0, 0, 0, 1\}$$