problem

problem 1

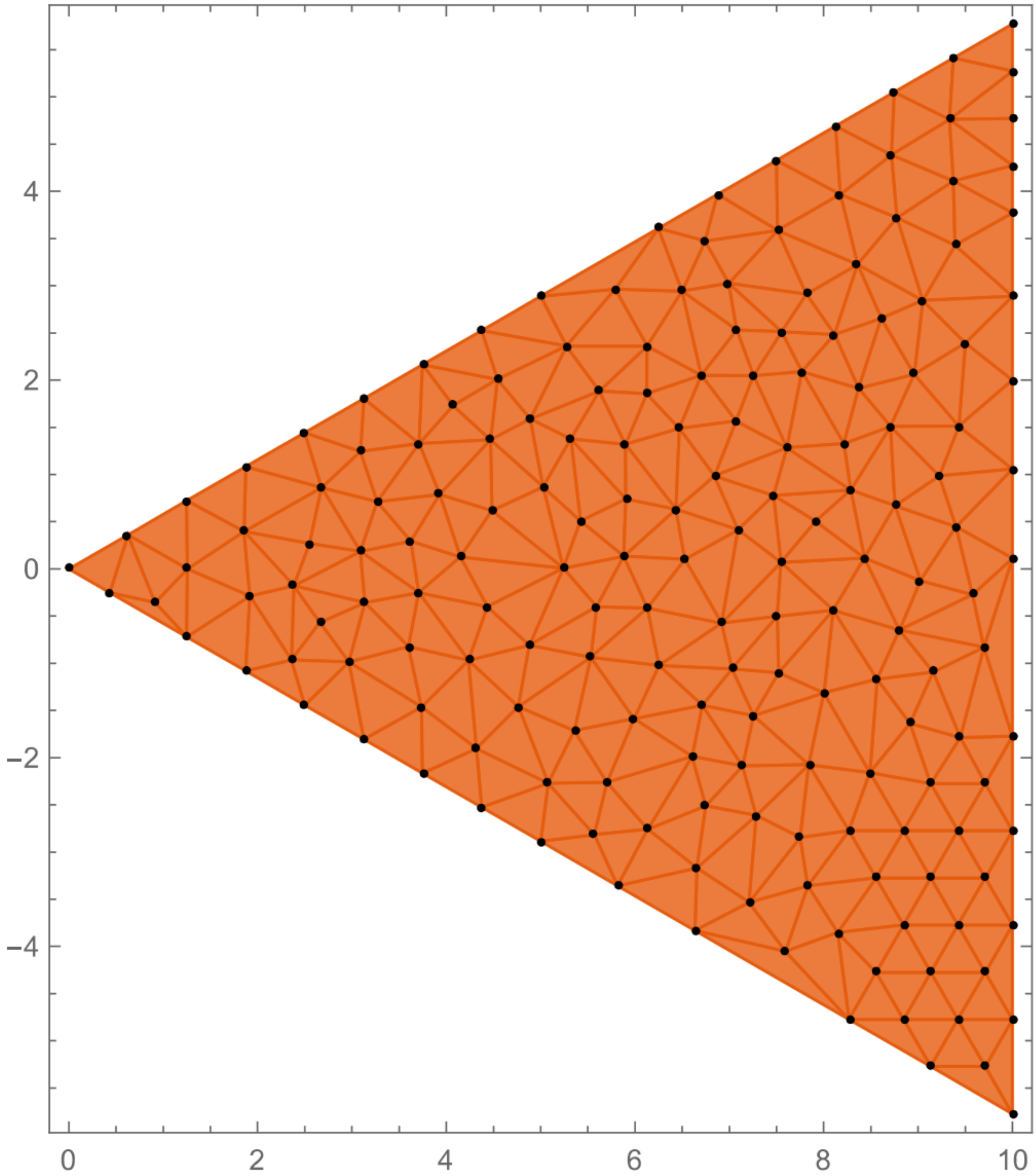
1. Write the equivalent governing equation in the domain

the equivalent governing equation is:

$$-\Delta u = 1$$

2. Programming

get the mesh from triangulation mesh generator:



use matlab to solve problem 1

read mesh information and set b as 10:

```
clear;clc;close all;

NE = textread('mesh.txt', '%d', 2);
N = NE(1);
E = NE(2);
data = readmatrix('mesh.txt');
position = data(1:N, 1:2);
connect = round(data(N+1:N+E, 2:4)) + 1;
b = 10;
alpha = 30*pi/180;
```

the position of node to solve is as below:

```
NI = [];
toler = 1e-5;
for i = 1: N
    x = position(i, 1);
    y = position(i, 2);
    a1 = abs(x - b);
    a2 = abs(y - tan(alpha) * x);
    a3 = abs(y + tan(alpha) * x);
    if a1 > toler && a2 > toler && a3 > toler
        NI = [NI, i];
    end
end
```

the core codes of FEM:

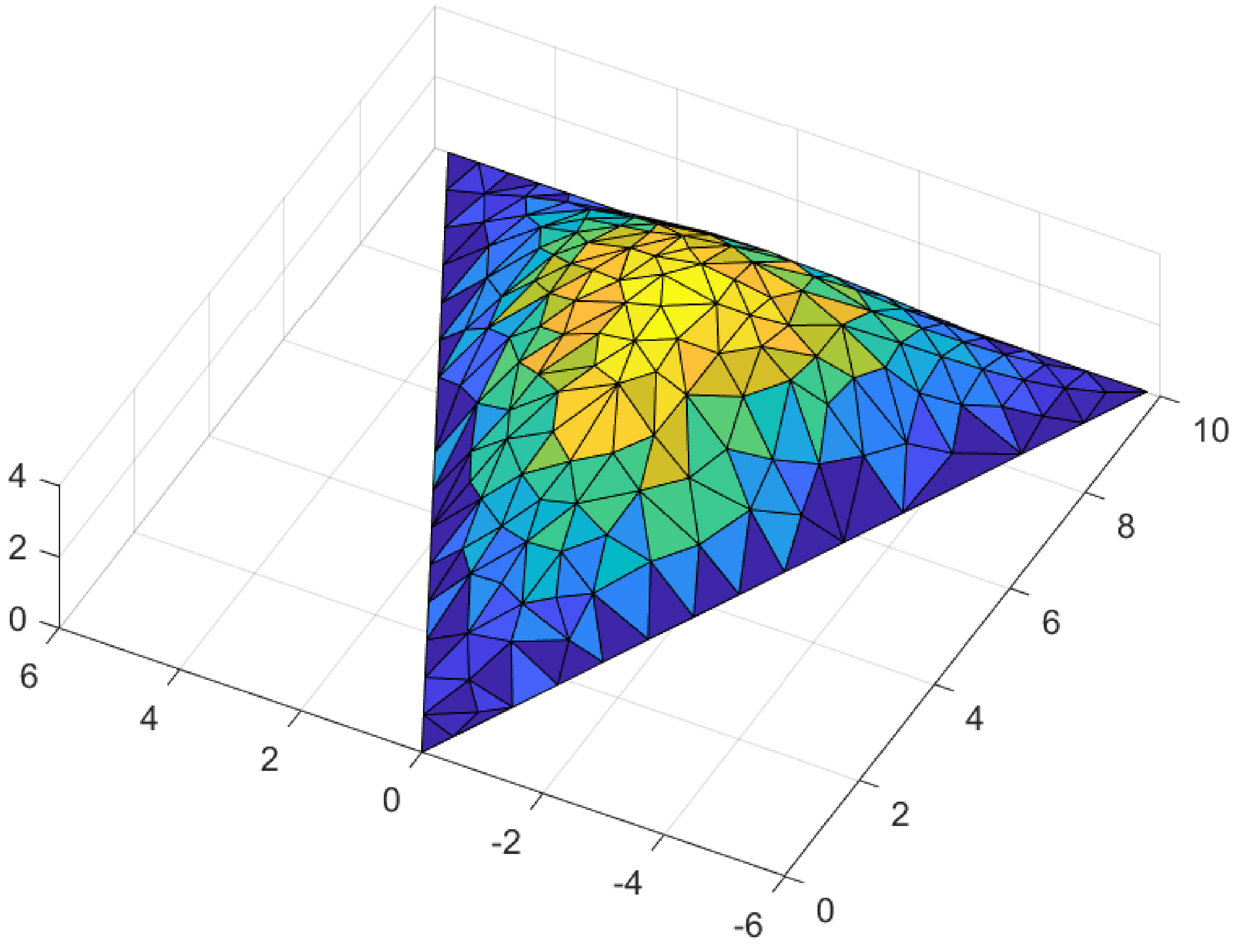
```
K = zeros(N, N);
F = zeros(1, N);
u = zeros(1, N);
for k = 1: E
    node1 = connect(k, 1);
    node2 = connect(k, 2);
    node3 = connect(k, 3);
    x1 = position(node1, 1);
    y1 = position(node1, 2);
    x2 = position(node2, 1);
    y2 = position(node2, 2);
    x3 = position(node3, 1);
    y3 = position(node3, 2);
    A2 = 2*det([1 x1 y1; 1 x2 y2; 1 x3 y3]);
    k11 = (x2 - x3)^2 + (y2 - y3)^2;
    k12 = (x1 - x3) * (-x2 + x3) + (y1 - y3) * (-y2 + y3);
    k13 = (x1 - x2) * (x2 - x3) + (y1 - y2) * (y2 - y3);
    k22 = (x1 - x3)^2 + (y1 - y3)^2;
    k23 = -(x1^2 + x2 * x3 - x1 * (x2 + x3) + (y1 - y2) * (y1 - y3));
    k33 = (x1 - x2)^2 + (y1 - y2)^2;
    F1 = (x3 * (y1 - y2) + x1 * (y2 - y3) + x2 * (-y1 + y3)) / 6;
    F2 = (x3 * (y1 - y2) + x1 * (y2 - y3) + x2 * (-y1+y3)) / 6;
    F3 = (x3 * (y1 - y2) + x1 * (y2 - y3) + x2 * (-y1+y3)) / 6;
    K(node1, node1) = K(node1, node1) + k11 / A2;
    K(node1, node2) = K(node1, node2) + k12 / A2;
    K(node1, node3) = K(node1, node3) + k13 / A2;
    K(node2, node1) = K(node2, node1) + k12 / A2;
    K(node2, node2) = K(node2, node2) + k22 / A2;
    K(node2, node3) = K(node2, node3) + k23 / A2;
    K(node3, node1) = K(node3, node1) + k13 / A2;
    K(node3, node2) = K(node3, node2) + k23 / A2;
    K(node3, node3) = K(node3, node3) + k33 / A2;
    F(node1) = F(node1) + F1;
    F(node2) = F(node2) + F2;
    F(node3) = F(node3) + F3;
```

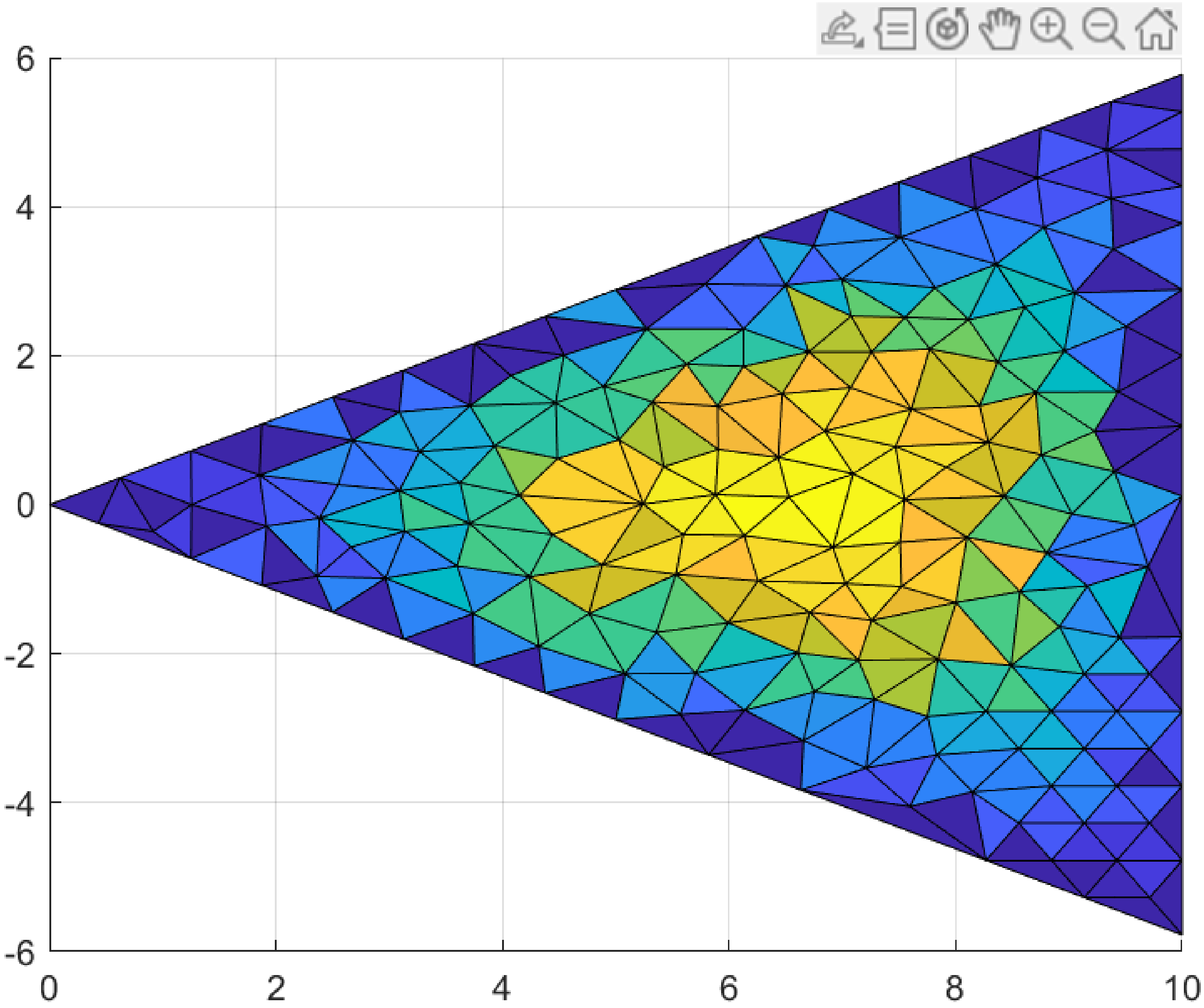
```
end
```

solve by inverse of matrix and draw the 3D result:

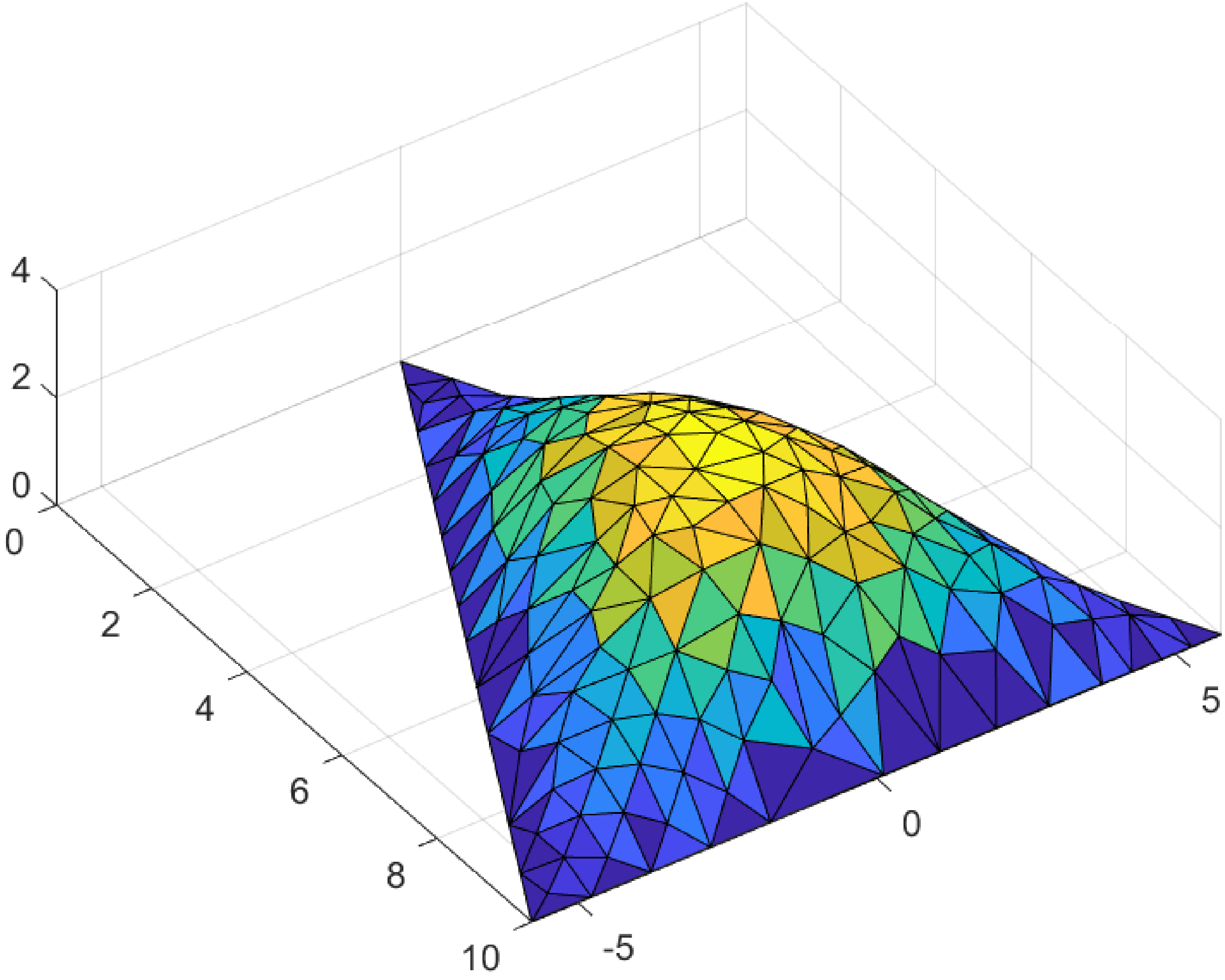
```
u(NI) = K(NI, NI) \ F(NI).';
trisurf(connect, position(:, 1), position(:, 2), u)

view([-62.63 75.27])
exportgraphics(gca, 'res1.png')
view([-0.44 90.00])
exportgraphics(gca, 'res2.png')
view([55.41 66.95])
exportgraphics(gca, 'res3.png')
```

the 3 views of u:





6
4
2
0
-2
-4
-6
0
2
4
6
8
10



problem 2

Lagrange equation:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

for this system, we have

$$\begin{cases} q_1 = \theta_1 \\ q_2 = \theta_2 \end{cases}$$

the velocity:

$$\vec{V}_1 = (l_1\dot{\theta}_1 \cos\theta_1 + \dot{f})\vec{i} + l_1\dot{\theta}_1 \sin\theta_1 \vec{j}$$

$$\vec{V}_2 = (l_1\dot{\theta}_1 \cos\theta_1 + l_2\dot{\theta}_2 \cos\theta_2 + \dot{f})\vec{i} + (l_1\dot{\theta}_1 \sin\theta_1 + l_2\dot{\theta}_2 \sin\theta_2)\vec{j}$$

thus we have kietic energy of the system:

$$T = \frac{1}{2}m_1\vec{V}_1 \cdot \vec{V}_1 + \frac{1}{2}m_2\vec{V}_2 \cdot \vec{V}_2$$

and the potential energy is:

$$V = m_1 g l_1 (1 - \cos\theta_1) + m_2 g [l_1(1 - \cos\theta_1) + l_2(1 - \cos\theta_2)]$$

thus we will have:

$$\begin{cases} (m_1 + m_2)(\ddot{f}\cos\theta_1 + g\sin\theta_1 + l_1\ddot{\theta}_1) + l_2 m_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) + l_2 m_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) = 0 \\ \ddot{f}\cos\theta_2 + g\sin\theta_2 - l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) + l_1\ddot{\theta}\cos(\theta_1 - \theta_2) + l_2\ddot{\theta}_2 = 0 \end{cases}$$

problem 3

governing equation:

$$\frac{d^4 w}{dx^4} = \frac{q}{EJ}$$

local stiffness relationship:

$$EJ \begin{bmatrix} \frac{12}{h^3} & \frac{6}{h^2} & -\frac{12}{h^3} & \frac{6}{h^2} \\ \frac{6}{h^2} & \frac{4}{h} & -\frac{6}{h^2} & \frac{2}{h} \\ -\frac{12}{h^3} & -\frac{6}{h^2} & \frac{12}{h^3} & -\frac{6}{h^2} \\ \frac{6}{h^2} & \frac{2}{h} & -\frac{6}{h^2} & \frac{4}{h} \end{bmatrix} \begin{bmatrix} w_k \\ \theta_k \\ w_{k+1} \\ \theta_{k+1} \end{bmatrix} = q \begin{bmatrix} \frac{h}{2} \\ \frac{h^2}{12} \\ \frac{h}{2} \\ -\frac{h^2}{12} \end{bmatrix} + \begin{bmatrix} F_k \\ -M_k \\ F_{k+1} \\ -M_{k+1} \end{bmatrix}$$

the matlab code and result are as below:

set the calculation parameters

```
clear;clc;close all;

q_ = -200;
F_ = -1000;
M_ = 2000;
L = 0.12;
d1 = 0.03;
d2 = 0.02;
Es = 200e9;
I1 = pi * d1^4 / 64;
I2 = pi * d2^4 / 64;
N = 51;
x = linspace(0, 2*L, N);
h = x(2) - x(1);
```

get the stiffness matrix (core code):

```
K = zeros(2*N, 2*N);
b = zeros(2*N);
localk = [12/h^3, 6/h^2, -12/h^3, 6/h^2;
   6/h^2, 4/h, -6/h^2, 2/h;
   -12/h^3, -6/h^2, 12/h^3, -6/h^2;
   6/h^2, 2/h, -6/h^2, 4/h];
```

```
localb = h * [1/2, h/12, 1/2, -h/12];
not_boundary = [3:2*N-2, 2*N];

for k = 1:N-1
    if k <= (N-1)/2
        K(2*k-1:2*k+2, 2*k-1:2*k+2) = K(2*k-1:2*k+2, 2*k-1:2*k+2) + Es * I1 *
localk;
        b(2*k-1:2*k+2) = b(2*k-1:2*k+2) + q_ * localb;
    else
        K(2*k-1:2*k+2, 2*k-1:2*k+2) = K(2*k-1:2*k+2, 2*k-1:2*k+2) + Es * I2 *
localk;
    end
end
b(N) = b(N) + F_;
b(2*N) = b(2*N) - M_;
```

solve:

```
bm = b(not_boundary);
Km = K(not_boundary, not_boundary);
solution = zeros(2*N);
solution(not_boundary) = Km \ bm.';
w = solution(1:2:2*N);
theta = solution(2:2:2*N);
```

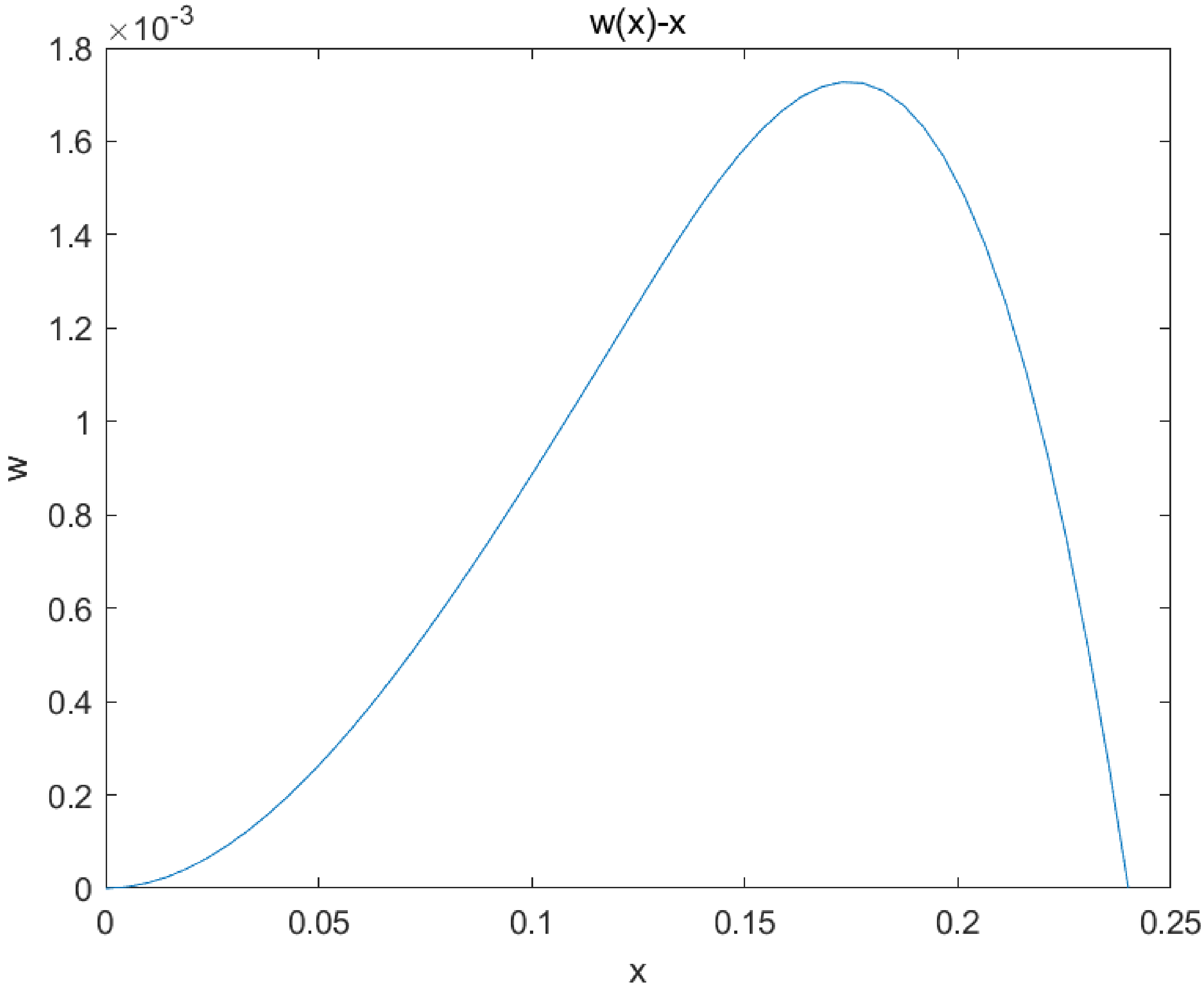
draw the deflection picture:

```
plot(x, w)
xlabel('x')
ylabel('w')
title('w(x)-x')
exportgraphics(gca, 'w.png')
hold off;
```

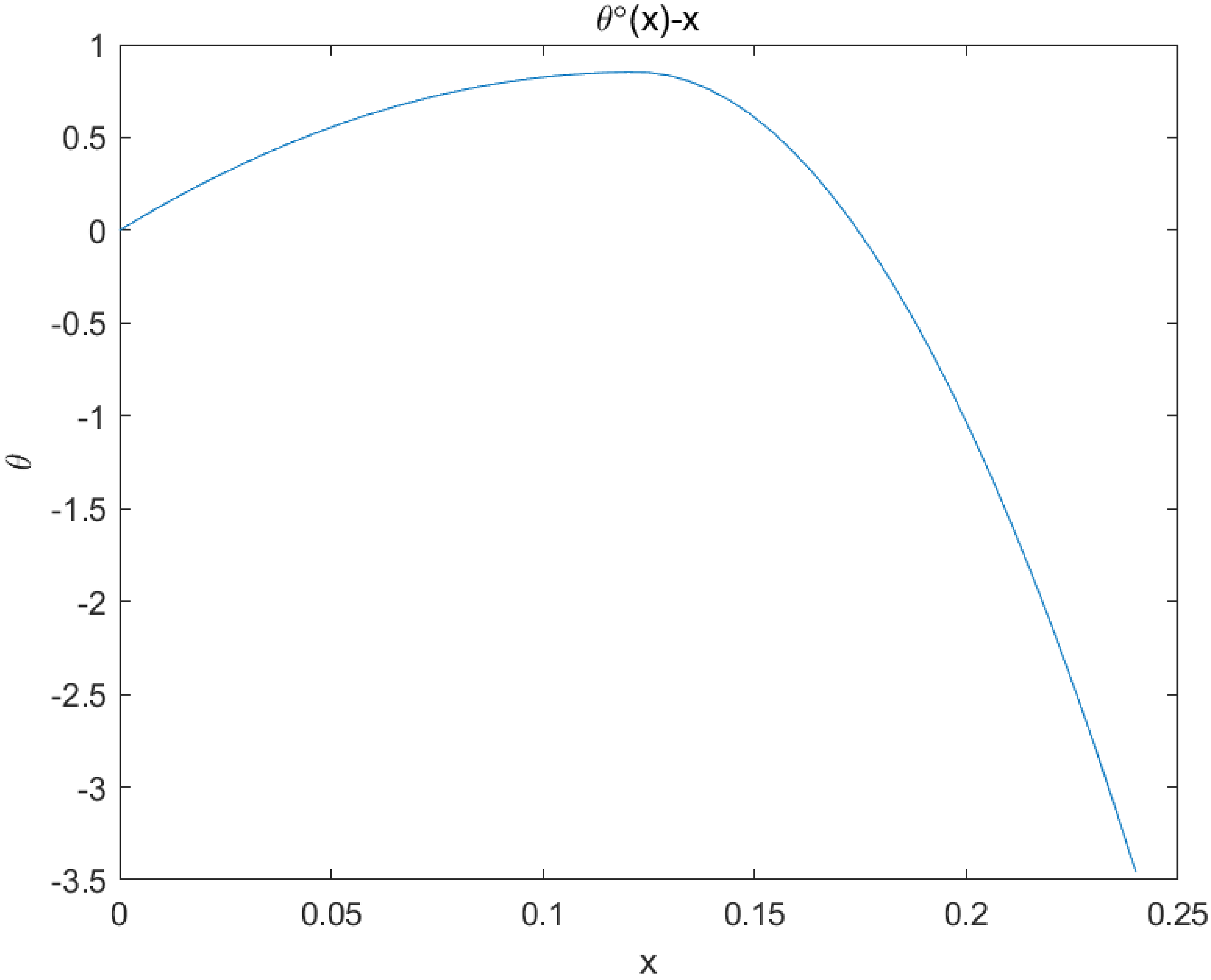
draw the torsion angle deflection:

```
plot(x, theta*180/pi)
xlabel('x')
ylabel('\theta')
title('\theta\circ(x)-x')
exportgraphics(gca, 'theta.png')
```

the solution result is as below, where the deflection of the beam is:



the torsion angle is:


θ°(x)-x
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
θ
0
0.05
0.1
0.15
0.2
0.25
x