

解析解

求解约束反力

```
In[1]:= Clear["Global`*"];
```

清除

```
d1 = 0.03;
```

```
d2 = 0.02;
```

```
case = {
```

```
Es → 200 * 109,
```

```
q → 200,
```

```
F → 1000,
```

```
M → 2000,
```

```
L → 0.12,
```

```
J1 →  $\frac{\pi}{64} d1^4$ ,
```

```
J2 →  $\frac{\pi}{64} d2^4$ 
```

```
};
```

$$wrq = \frac{q L^3}{6 Es J1} L + \frac{q L^4}{8 Es J1};$$

$$wrF = \frac{F L^2}{2 Es J1} L + \frac{F L^3}{3 Es J1};$$

$$wrM = \frac{M L}{Es J1} L + \frac{M L^2}{2 Es J1} + \frac{M L^2}{2 Es J2};$$

$$wrN = - \left(\left(\frac{FN L^2}{2 Es J1} + \frac{FN L L}{Es J1} \right) L + \frac{FN L^3}{3 Es J1} + \frac{FN L L^2}{2 Es J1} + \frac{FN L^3}{3 Es J2} \right);$$

```
FNres = Solve[wrq + wrF + wrM + wrN == 0, FN][[1]][[1]] // FullSimplify;
```

解方程

完全简化

```
FN = FN /. FNres // FullSimplify;
```

完全简化

```
FN // TraditionalForm
```

传统格式

```
Out[11]/TraditionalForm=
```

$$\frac{20 F J2 L + 12 J1 M + 7 J2 L^2 q + 36 J2 M}{8 J1 L + 56 J2 L}$$


```
In[20]:= Fx[x] // TraditionalForm
```

传统格式

```
Mx[x] // TraditionalForm
```

传统格式

```
{Fx[0] == F0,
 Mx[0] == M0,
 Fx[2 L] + FN == 0,
 Mx[2 L] + M == 0}
```

```
Out[20]//TraditionalForm=
```

$$\begin{cases} \frac{4 F L (2 J_1 + 9 J_2) + L^2 q (8 J_1 + 49 J_2) - 12 M (J_1 + 3 J_2)}{8 L (J_1 + 7 J_2)} - q x & 0 \leq x < L \\ -\frac{20 F J_2 L + 12 J_1 M + 7 J_2 L^2 q + 36 J_2 M}{8 J_1 L + 56 J_2 L} & L \leq x \leq 2 L \end{cases}$$

```
Out[21]//TraditionalForm=
```

$$\begin{cases} \frac{1}{8 L (J_1 + 7 J_2)} (4 F L (2 J_1 (x - L) - 4 J_2 L + 9 J_2 x) + L^2 q x (8 J_1 + 49 J_2) - \\ 2 L^3 q (2 J_1 + 7 J_2) + 16 L M (J_1 + J_2) - 4 L q x^2 (J_1 + 7 J_2) - 12 M x (J_1 + 3 J_2)) & 0 \leq x < L \\ \frac{20 F J_2 L (2 L - x) + 16 L M (J_1 + J_2) - 12 M x (J_1 + 3 J_2) - 7 J_2 L^2 q x + 14 J_2 L^3 q}{8 L (J_1 + 7 J_2)} & L \leq x \leq 2 L \end{cases}$$

```
Out[22]= {True, True, True, True}
```

求解 θ, w 分布

```
In[23]:=  $\theta q[x_] := Assuming[L > 0,$ 
```

假定

```
Piecewise[{
```

分段函数

$$\begin{aligned} & \left\{ -\frac{q x^3}{6 E s J_1} - \frac{q (L - x) x^2}{2 E s J_1} - \frac{\frac{1}{2} q (L - x)^2 x}{E s J_1}, 0 \leq x < L \right\}, \\ & \left\{ -\frac{q L^3}{6 E s J_1}, L \leq x \leq 2 L \right\} \\ & \}, 0] // FullSimplify \end{aligned}$$

完全简化

```
];
```

```
 $\theta F[x_] := Assuming[L > 0,$ 
```

假定

```
Piecewise[{
```

分段函数

$$\begin{aligned} & \left\{ -\frac{F x^2}{2 E s J_1} - \frac{F (L - x) x}{E s J_1}, 0 \leq x < L \right\}, \\ & \left\{ -\frac{F L^2}{2 E s J_1}, L \leq x \leq 2 L \right\} \\ & \}, 0] // FullSimplify \end{aligned}$$

完全简化

```
];
```

```
 $\theta M[x_] := Assuming[L > 0,$ 
```

假定

```

Piecewise[ $\left\{ \begin{aligned} & -\frac{M x}{E_s J_1}, \quad 0 \leq x < L \end{aligned} \right\},$ 
分段函数
 $\left\{ -\frac{M L}{E_s J_1} - \frac{M (x - L)}{E_s J_2}, \quad L \leq x \leq 2 L \right\}$ 
 $\}, 0]$  // FullSimplify
完全简化

];

 $\theta N[x_] := Assuming[L > 0,$ 
假定
Piecewise[ $\left\{ \begin{aligned} & \frac{F_N x^2}{2 E_s J_1} + \frac{F_N (2 L - x) x}{E_s J_1}, \quad 0 \leq x < L \end{aligned} \right\},$ 
分段函数
 $\left\{ \frac{F_N L^2}{2 E_s J_1} + \frac{F_N L^2}{E_s J_1} + \frac{F_N (x - L)^2}{2 E_s J_2} + \frac{F_N (2 L - x) (x - L)}{E_s J_2}, \quad L \leq x \leq 2 L \right\}$ 
 $\}, 0]$  // FullSimplify
完全简化

];

 $\theta x[x_] := Assuming[L > 0, (\theta q[x] + \theta F[x] + \theta M[x] + \theta N[x])] // FullSimplify;$ 
假定 完全简化

In[28]:=  $wq[x_] := Assuming[L > 0,$ 
假定
Piecewise[ $\left\{ -\frac{q x^4}{8 E_s J_1} - \frac{q (L - x) x^3}{3 E_s J_1} - \frac{\frac{1}{2} q (L - x)^2 x^2}{2 E_s J_1}, \quad 0 \leq x < L \right\},$ 
分段函数
 $\left\{ -\frac{q L^4}{8 E_s J_1} - \frac{q L^3}{6 E_s J_1} (x - L), \quad L \leq x \leq 2 L \right\}$ 
 $\}, 0]$  // FullSimplify
完全简化

];

 $wF[x_] := Assuming[L > 0,$ 
假定
Piecewise[ $\left\{ -\frac{F x^3}{3 E_s J_1} - \frac{F (L - x) x^2}{2 E_s J_1}, \quad 0 \leq x < L \right\},$ 
分段函数
 $\left\{ -\frac{F L^3}{3 E_s J_1} - \frac{F L^2}{2 E_s J_1} (x - L), \quad L \leq x \leq 2 L \right\}$ 

```

```

    }, 0] // FullSimplify
    [完全简化

];

wM[x_] := Assuming[L > 0,
    [假定

    Piecewise[{
    [分段函数

        { $-\frac{M x^2}{2 E_s J_1}, 0 \leq x < L$ },
        { $-\frac{M L^2}{2 E_s J_1} - \frac{M L}{E_s J_1} (x - L) - \frac{M (x - L)^2}{2 E_s J_2}, L \leq x \leq 2 L$ }
    }, 0] // FullSimplify
    [完全简化

];

wN[x_] := Assuming[L > 0,
    [假定

    Piecewise[{
    [分段函数

        { $\frac{F_N x^3}{3 E_s J_1} + \frac{F_N (2 L - x) x^2}{2 E_s J_1}, 0 \leq x < L$ },
        { $\left(\frac{F_N L^2}{2 E_s J_1} + \frac{F_N L^2}{E_s J_1}\right) (x - L) +$ 
 $\frac{F_N L^3}{3 E_s J_1} + \frac{F_N L L^2}{2 E_s J_1} + \frac{F_N (x - L)^3}{3 E_s J_2} + \frac{F_N (2 L - x) (x - L)^2}{2 E_s J_2}, L \leq x \leq 2 L$ }
    }, 0] // FullSimplify
    [完全简化

];

wx[x_] := Assuming[L > 0, (wq[x] + wF[x] + wM[x] + wN[x])] // FullSimplify;
    [假定 [完全简化

```

验证与打印

```

In[33]:= EJ[x_] := Assuming[L > 0,
    Piecewise[{
        {Es J1, 0 ≤ x < L},
        {Es J2, L ≤ x ≤ 2 L}
    }, 1] // FullSimplify
];
(∂x wx[x] - ∅x[x]) // FullSimplify
(∂x ∅x[x] -  $\frac{Mx[x]}{EJ[x]}$ ) // FullSimplify

```

Out[34]= 0

Out[35]= 0

```

In[36]:= ∅x[x] // TraditionalForm
wx[x] // TraditionalForm

```

Out[36]//TraditionalForm=

$$\left\{ \begin{array}{ll} x \left(\frac{3(4L-x)(20FJ_2L+12M(J_1+3J_2)+7J_2L^2q)}{L(J_1+7J_2)} + 24F(x-2L) - 8q(3L^2-3Lx+x^2) - 48M \right) & x \geq 0 \wedge L > x \\ - \frac{\frac{48EsJ_1}{3(3L^2(J_1-J_2)-4J_1Lx+J_1x^2)(20FJ_2L+12M(J_1+3J_2)+7J_2L^2q)} + 24FL^2 + \frac{48M(J_1(x-L)+J_2L)}{J_2} + 8L^3q}{48EsJ_1} & L \leq x \wedge 2L \geq x \end{array} \right.$$

Out[37]//TraditionalForm=

$$\left\{ \begin{array}{ll} \frac{1}{48EsJ_1L(J_1+7J_2)} & x \geq 0 \wedge L > x \\ x^2(4FL(x(2J_1+9J_2)-6L(J_1+2J_2))+L^2qx(8J_1+49J_2)-6L^3q(2J_1+7J_2)+48LM(J_1+J_2)-2Lqx^2(J_1+7J_2)-12Mx(J_1+3J_2)) & \\ \frac{1}{48EsJ_1J_2L(J_1+7J_2)}(2L-x)(J_1x^2(20FJ_2L+12M(J_1+3J_2)+7J_2L^2q)-4J_1Lx(20FJ_2L+6J_1M+7J_2L^2q-6J_2M)+4L^2(FJ_2L(11J_1-3J_2)+3M(J_1-J_2)^2)+J_2L^4q(15J_1-7J_2)) & L \leq x \wedge 2L \geq x \end{array} \right.$$

带入本题数据

```
In[38]:= excase[x_] := ex[x] /. case // Expand;
```

展开

```
wxcase[x_] := wx[x] /. case // Expand;
```

展开

```
excase[x] // TraditionalForm
```

传统格式

```
wxcase[x] // TraditionalForm
```

传统格式

Out[40]//TraditionalForm=

$$\begin{cases} -0.00419174 x^3 - 0.999404 x^2 + 0.243843 x & x \geq 0 \wedge 0.12 > x \\ -5.38543 x^2 + 1.31177 x - 0.0649994 & 0.12 \leq x \wedge 0.24 \geq x \end{cases}$$

Out[41]//TraditionalForm=

$$\begin{cases} -0.00104793 x^4 - 0.333135 x^3 + 0.121922 x^2 & x \geq 0 \wedge 0.12 > x \\ -1.79514 x^3 + 0.655884 x^2 - 0.0649994 x + 0.00263701 & 0.12 \leq x \wedge 0.24 \geq x \end{cases}$$