homework

homework 1

1. Write the equivalent governing equation in the domain

use integrate by part we will have:

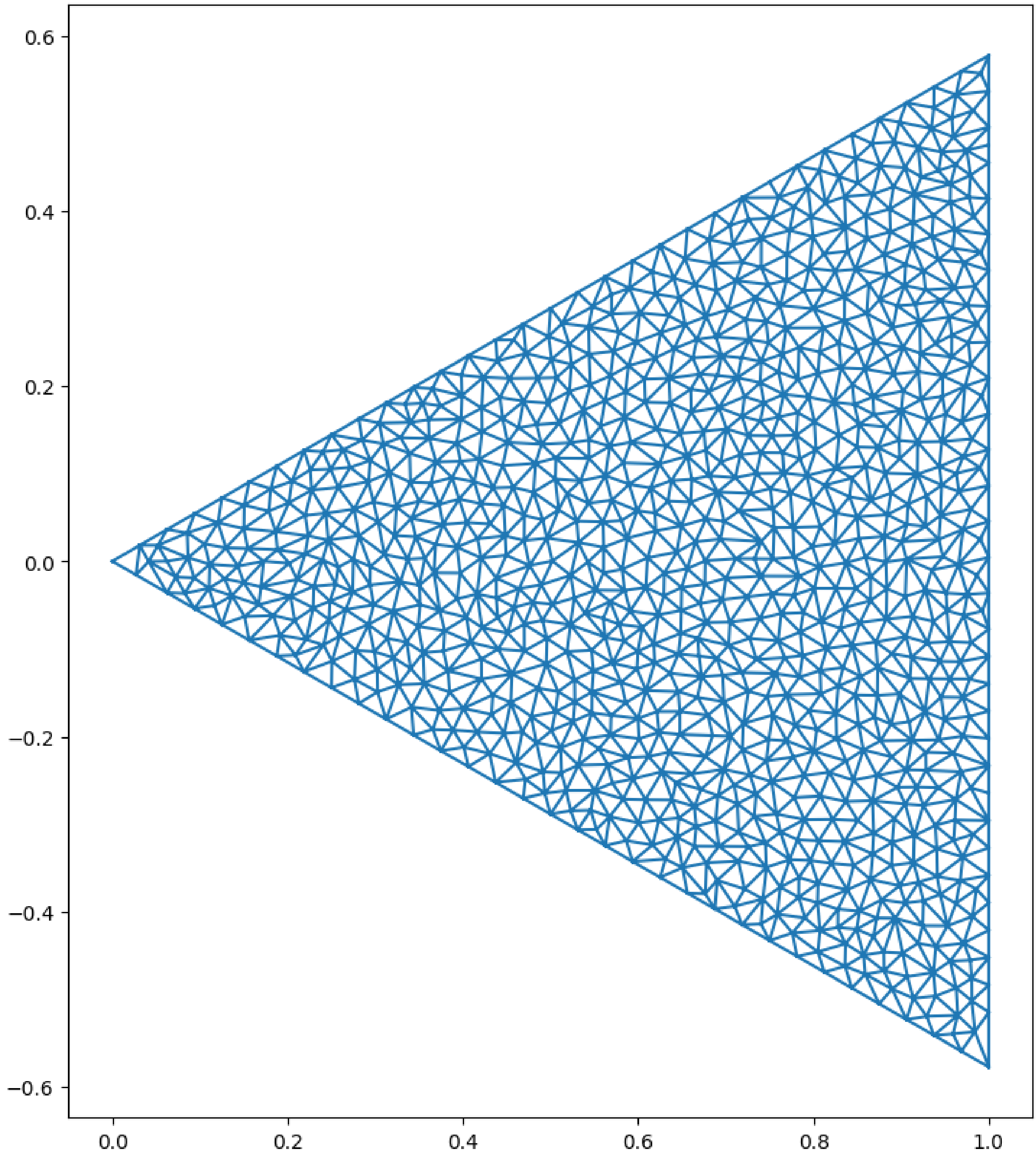
$$Ju = \int_{\Omega} u(-\Delta u - 2) d\Omega$$

thus the equivalent governing equation is:

$$-\Delta u = 1$$

2. Programming

use python to finish the job. The triangle mesh has 963 nodes and 1828 elements shown as below:



here is the description of code:

import the package:

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.tri as tri
```

read information of the mesh:

```
mesh = "mesh.txt"
file = open(mesh, 'r')
tmp = file.readlines()
node_num = int(tmp[0])
elem_num = int(tmp[1])
file.close()
xy = np.loadtxt(mesh, dtype=float, skiprows=2, max_rows=node_num)
[:, 0:2]
elem = np.loadtxt(mesh, dtype=int, skiprows=2+node_num,
max_rows=elem_num)[:, 1:4]
triangle = tri.Triangulation(xy.T[0], xy.T[1], elem)
del tmp
```

draw the picture of mesh:

```
plt.figure(figsize=(10, 10), facecolor="white")
plt.gca().set_aspect(1)
plt.triplot(triangle)
plt.savefig("mesh.png", bbox_inches="tight")
plt.show()
```

mark the nodes inside the domain:

```
b = 1.0
line = np.sqrt(3) / 3
error = 1e-4
not_boundary_ID = []
for k in range(len(xy)):
    x, y = xy[k]
    error1 = np.abs(x-b)
    error2 = np.abs(y - line * x)
    error3 = np.abs(y + line * x)
    if error1 > error and error2 > error and error3 > error:
        not_boundary_ID.append(k)
        pass
    pass
not_boundary_ID = np.array(not_boundary_ID)
```

to get the K and b by finite element method:

```python
K = np.zeros([node_num, node_num])
B = np.zeros(elem_num)
for k in range(elem_num):
    ID1, ID2, ID3 = elem[k]
    x1, y1 = xy[ID1]
    x2, y2 = xy[ID2]
    x3, y3 = xy[ID3]
    A = 2 * np.linalg.det([
        [1, x1, y1],
        [1, x2, y2],
        [1, x3, y3]
    ])
    k11 = (x2 - x3)**2 + (y2 - y3)**2
    k12 = (x1 - x3) * (-x2 + x3) + (y1 - y3) * (-y2 + y3)
    k13 = (x1 - x2) * (x2 - x3) + (y1 - y2) * (y2 - y3)
    k22 = (x1 - x3)**2 + (y1 - y3)**2
    k23 = -(x1**2 + x2 * x3 - x1 * (x2 + x3) + (y1 - y2) * (y1 -
y3))
    k33 = (x1 - x2)**2 + (y1 - y2)**2
    b1 = (x3 * (y1 - y2) + x1 * (y2 - y3) + x2 * (-y1 + y3)) / 6
    b2 = (x3 * (y1 - y2) + x1 * (y2 - y3) + x2 * (-y1+y3)) / 6
    b3 = (x3 * (y1 - y2) + x1 * (y2 - y3) + x2 * (-y1+y3)) / 6

    K[ID1, ID1] += k11 / A;
    K[ID1, ID2] += k12 / A;
    K[ID1, ID3] += k13 / A;

    K[ID2, ID1] += k12 / A;
    K[ID2, ID2] += k22 / A;
    K[ID2, ID3] += k23 / A;

    K[ID3, ID1] += k13 / A;
    K[ID3, ID2] += k23 / A;
    K[ID3, ID3] += k33 / A;

    B[ID1] += b1;
    B[ID2] += b2;
    B[ID3] += b3;
    pass
```

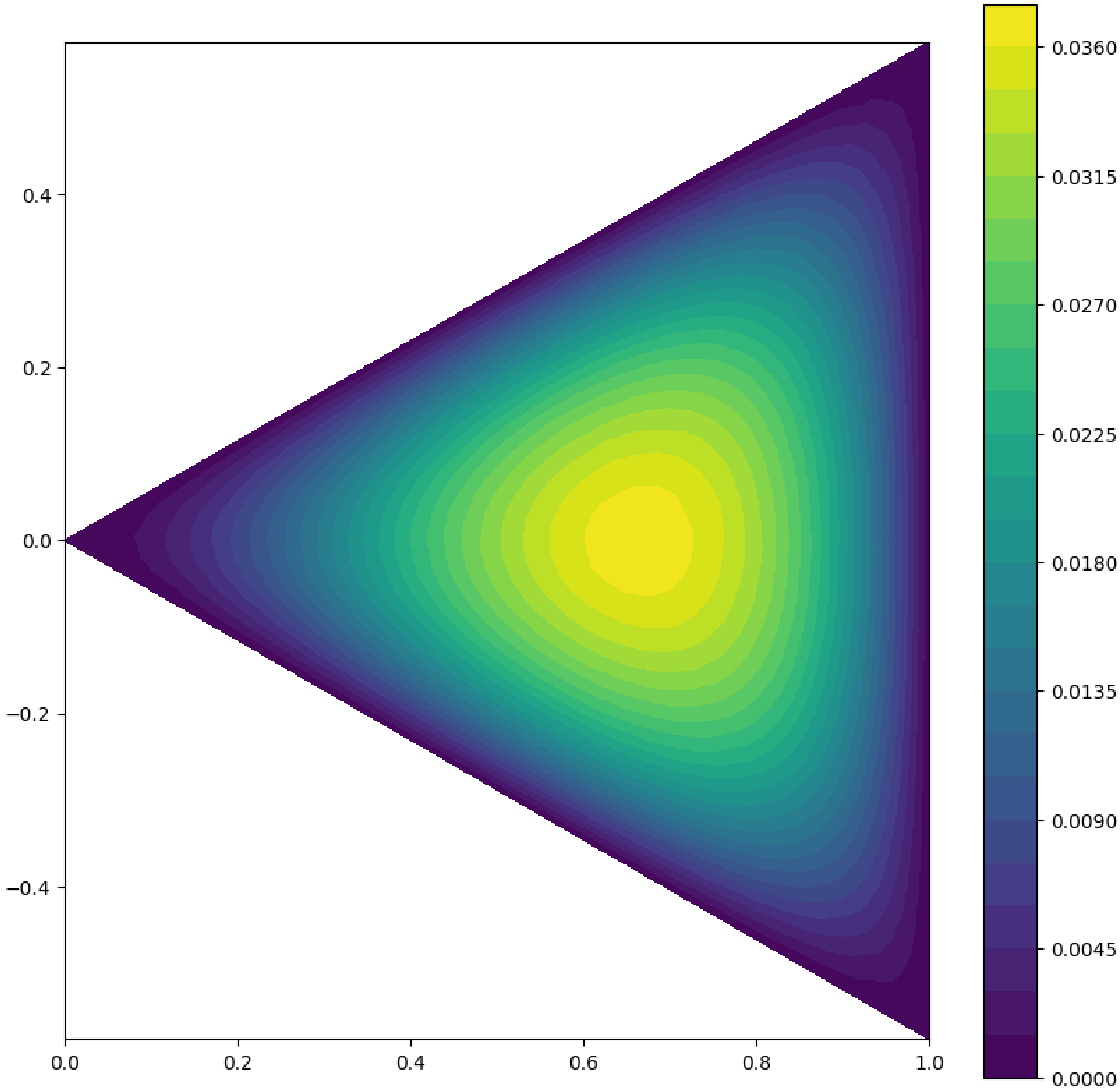
get the solution inside the domain (despite the nodes on the boundary):

```
Km = (K[not_boundary_ID].T[not_boundary_ID]).T
Bm = B[not_boundary_ID]
um = np.linalg.inv(Km) @ Bm
u = np.zeros(node_num)
u[not_boundary_ID] = um
```

draw the picture of result:

```
plt.figure(figsize=(10, 10), facecolor="white")
plt.gca().set_aspect(1)
plt.tricontourf(triangle, u, 30)
plt.colorbar()
plt.savefig("result.png", bbox_inches="tight")
plt.show()
```

the result of u is:



homework 2

by virtue of Lagrange equation we have:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

in this case:

$$\begin{cases} q_1 = \theta_1 \\ q_2 = \theta_2 \end{cases}$$

kietic energy of the system is:

$$T = \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 V_2^2$$

in which:

$$\vec{V_1} = (l_1\dot{\theta}_1 \cos\theta_1 + \dot{f})\vec{i} + l_1\dot{\theta}_1 \sin\theta_1 \vec{j}$$

$$\vec{V_2} = (l_1\dot{\theta}_1 \cos\theta_1 + l_2\dot{\theta}_2 \cos\theta_2 + \dot{f})\vec{i} + (l_1\dot{\theta}_1 \sin\theta_1 + l_2\dot{\theta}_2 \sin\theta_2)\vec{j}$$

the potential energy of the system is:

$$V = m_1 g l_1(1-\cos\theta_1) + m_2 g[l_1(1-\cos\theta_1) + l_2(1-\cos\theta_2)]$$

thus we will have:

$$\begin{cases} (m_1+m_2)(\ddot{f}\cos\theta_1 + g\sin\theta_1 + l_1\ddot{\theta}_1) + l_2 m_2 \dot{\theta}_2^2 \sin(\theta_1-\theta_2) + l_2 m_2 \ddot{\theta}_2 \cos(\theta_1-\theta_2) = 0 \\ \ddot{f}\cos\theta_2 + g\sin\theta_2 - l_1\dot{\theta}_1^2 \sin(\theta_1-\theta_2) + l_1\ddot{\theta}\cos(\theta_1-\theta_2) + l_2\ddot{\theta}_2 = 0 \end{cases}$$

homework 3

governing equation:

$$EJ\frac{d^4 w}{dx^4} = q$$

for one element:

$$EJ\begin{bmatrix} \frac{12}{h^3} & \frac{6}{h^2} & -\frac{12}{h^3} & \frac{6}{h^2} \\ \frac{6}{h^2} & \frac{4}{h} & -\frac{6}{h^2} & \frac{2}{h} \\ -\frac{12}{h^3} & -\frac{6}{h^2} & \frac{12}{h^3} & -\frac{6}{h^2} \\ \frac{6}{h^2} & \frac{2}{h} & -\frac{6}{h^2} & \frac{4}{h} \end{bmatrix} \begin{bmatrix} w_k \\ \theta_k \\ w_{k+1} \\ \theta_{k+1} \end{bmatrix} = q\begin{bmatrix} \frac{h}{2} \\ \frac{h^2}{12} \\ \frac{h}{2} \\ -\frac{h^2}{12} \end{bmatrix}$$

when we have a concentrated force F at node k, F should be added on right hand w_k while concentrated moment of force M be added at right hand of θ_k.

$$EJ\begin{bmatrix} \frac{12}{h^3} & \frac{6}{h^2} & -\frac{12}{h^3} & \frac{6}{h^2} \\ \frac{6}{h^2} & \frac{4}{h} & -\frac{6}{h^2} & \frac{2}{h} \\ -\frac{12}{h^3} & -\frac{6}{h^2} & \frac{12}{h^3} & -\frac{6}{h^2} \\ \frac{6}{h^2} & \frac{2}{h} & -\frac{6}{h^2} & \frac{4}{h} \end{bmatrix} \begin{bmatrix} w_k \\ \theta_k \\ w_{k+1} \\ \theta_{k+1} \end{bmatrix} = q\begin{bmatrix} \frac{h}{2} \\ \frac{h^2}{12} \\ \frac{h}{2} \\ -\frac{h^2}{12} \end{bmatrix} + \begin{bmatrix} F_k \\ -M_k \\ F_{k+1} \\ -M_{k+1} \end{bmatrix}$$

the codes are explained as below:

import the packages:

```
import numpy as np
import matplotlib.pyplot as plt
```

define the parameters:

```
L = 0.12
L2 = 2 * L
Es = 200e9
d1 = 0.03
d2 = 0.02
J1 = d1**4 * np.pi / 64
J2 = d2**4 * np.pi / 64
EJ1 = Es * J1
EJ2 = Es * J2
q = -200
F = -1000
M = 2000

N = 21
x = np.linspace(0, L2, N)
h = x[1] - x[0]

k1 = np.array([
    [12/h**3, 6/h**2, -12/h**3, 6/h**2],
    [6/h**2, 4/h, -6/h**2, 2/h],
    [-12/h**3, -6/h**2, 12/h**3, -6/h**2],
    [6/h**2, 2/h, -6/h**2, 4/h]
])

b1 = q * np.array([
    h/2, h**2/12, h/2, -h**2/12
])
```

get the K and b:

```
K = np.zeros([2*N, 2*N])
b = np.zeros(2*N)

for k in range(N-1):
    if k < (N-1) / 2:
        K[2*k:2*k+4, 2*k:2*k+4] += EJ1 * k1
        b[2*k:2*k+4] += b1
        pass
    else:
```

```
        K[2*k:2*k+4, 2*k:2*k+4] += EJ2 * k1
        pass
    pass

b[N-1] += F
b[-1] += -M

node_to_solve = [i for i in range(2, 2*N-2)]
node_to_solve.append(2*N-1)
node_to_solve = np.array(node_to_solve, dtype=int)
```

solve:

```
Km = (K[node_to_solve].T[node_to_solve]).T
bm = b[node_to_solve]
u = np.zeros(2*N)
u[node_to_solve] = np.linalg.inv(Km) @ bm
w = u[0:-1:2]
theta = u[1::2]
```

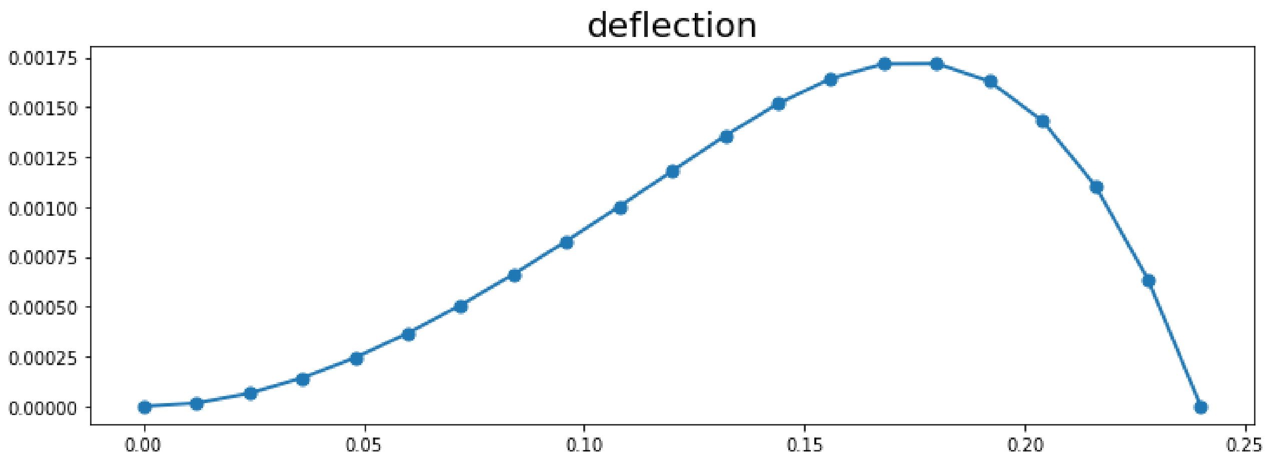
draw the deflection picture:

```
plt.figure(figsize=(12, 4), facecolor="white")
plt.plot(x, w, lw=2)
plt.scatter(x, w, s=50)
plt.title("deflection", fontsize=20)
plt.savefig("deflection.png", bbox_inches="tight")
plt.show()
```

draw the torsion angle deflection:

```
plt.figure(figsize=(12, 4), facecolor="white")
plt.plot(x, theta* 180 / np.pi, lw=2)
plt.scatter(x, theta * 180 / np.pi, s=50)
plt.title("torsion angle", fontsize=20)
plt.savefig("torsion_angle.png", bbox_inches="tight")
plt.show()
```

the solution result is as below, where the deflection of the beam is:



the torsion angle is: