mma解

```
In[ ]:= Clear["Global`*"];
        清除
```

1. Suppose we have a functional:

$$J[u(x,y)]=\iint_B\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2-2u\right]dxdy$$

And boundary condition:

$$u|_{\partial B}=0$$

where B is a domain in x-y plane enclosed by:

$$y=\pm\frac{\sqrt{3}}{3}x \quad \text{and} \quad x=b.$$

Let the functional achieve the minimum value.

(1) Write the equivalent governing equation in the domain.

(2) **Programming:** use finite element method to solve the field u within the domain numerically. In the report, please make a brief description of your code, and illustrate the validity of your results. (Please attach the code in another file.)

;

□ 可见，若设变分问题为

$$J[\varphi]=\iiint_v\left\{\frac{\varepsilon}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2\right]-\rho\varphi\right\}dv + \oint_s\varepsilon\left(\frac{1}{2}f_1\varphi^2-f_2\varphi\right)ds = \min \qquad (5\text{-}28)$$

■ 则根据变分方程δJ=0，由其对应的尤拉方程的定解问题[式（5-27a）和(5-27b)]，可知与上述变分问题（5-28）等价的边值问题为

$$\begin{cases}\dfrac{\partial^2\varphi}{\partial x^2}+\dfrac{\partial^2\varphi}{\partial y^2}+\dfrac{\partial^2\varphi}{\partial z^2}=-\dfrac{\rho}{\varepsilon} & (r\in v) \qquad (5\text{-}29a)\\ \dfrac{\partial\varphi}{\partial n}+f_1\varphi=f_2 & (r\in s) \qquad (5\text{-}29b)\end{cases}$$

■ 上式即为泊松方程的第三类边值问题。

;

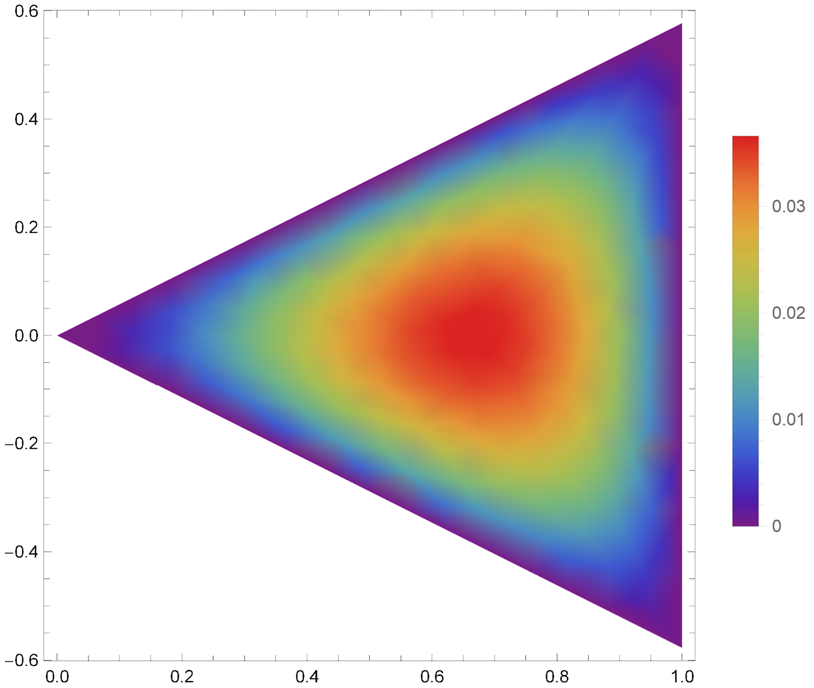
```
α = 30 °;
b = 1;
NodeList = {
    {0, 0},
    {b, -b Tan[α]},
             正切
    {b, b Tan[α]}
            正切
  };
poly = Polygon[NodeList];
       多边形
region = Region[poly];
         几何区域
boundary = (x == b || y == Tan[α] x || y == -Tan[α] x);
                         正切                正切
```

In[ ]:=
```
equation = ∇²{x,y} u[x, y] == -1;
res = NDSolveValue[
      数值解的值
   {equation, DirichletCondition[u[x, y] == 0, boundary]}, u[x, y], {x, y} ∈ region];
                 狄里克雷条件
res
```

Out[ ]= InterpolatingFunction[Domain: {{0., 1.}, {-0.577, 0.577}} Output: scalar][x, y]

In[ ]:=
```
picture =
 DensityPlot[res, {x, y} ∈ region, PlotLegends → Automatic, ColorFunction → "Rainbow"]
 密度图                             绘图的图例        自动            颜色函数
```

Out[ ]=



In[ ]:=
```
res /. {x → 0.5, y → 0}
```

Out[ ]= 0.0312482

In[ ]:=
```
Export["C:\\Users\\bcynuaa\\Desktop\\solution_mma.pdf", picture]
导出      常量
Export["C:\\Users\\bcynuaa\\Desktop\\solution_mma.png", picture]
导出      常量
```

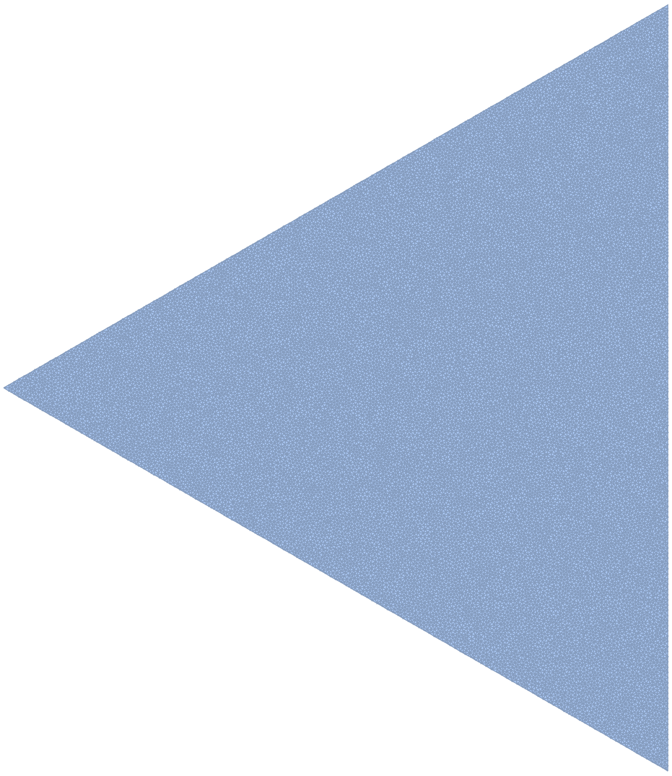
Out[ ]= C:\Users\bcynuaa\Desktop\solution_mma.pdf

Out[ ]= C:\Users\bcynuaa\Desktop\solution_mma.png

Mesh网格

In[ ]:=
```
mesh = TriangulateMesh[poly, MaxCellMeasure → 0.00002]
```
三角剖分网格 最大单元度量

Out[ ]=



In[ ]:=
```
Export["C:\\Users\\bcynuaa\\Desktop\\mesh.ply", mesh];
```
导出 常量

公式推导

```
In[ ]:= Clear["Global`*"];
        清除
        area = Det[{
               行列式
            {1, x1, y1},
            {1, x2, y2},
            {1, x3, y3}
          }] // Simplify;
                化简
        ϕ1 = (area /. {x1 → x, y1 → y}) / A2 // Simplify;
                                               化简
        ϕ2 = (area /. {x2 → x, y2 → y}) / A2 // Simplify;
                                               化简
        ϕ3 = (area /. {x3 → x, y3 → y}) / A2 // Simplify;
                                               化简
        ϕ1
        ϕ2
        ϕ3
        ∇{x,y} ϕ1
        ∇{x,y} ϕ2
        ∇{x,y} ϕ3
```

Out[ ]= $$\frac{x3\ (y-y2)+x\ (y2-y3)+x2\ (-y+y3)}{A2}$$

Out[ ]= $$\frac{x1\ y-x3\ y-x\ y1+x3\ y1+x\ y3-x1\ y3}{A2}$$

Out[ ]= $$\frac{x2\ (y-y1)+x\ (y1-y2)+x1\ (-y+y2)}{A2}$$

Out[ ]= $$\left\{\frac{y2-y3}{A2},\ \frac{-x2+x3}{A2}\right\}$$

Out[ ]= $$\left\{\frac{-y1+y3}{A2},\ \frac{x1-x3}{A2}\right\}$$

Out[ ]= $$\left\{\frac{y1-y2}{A2},\ \frac{-x1+x2}{A2}\right\}$$

In[ ]:=
```
NodeList = {
   {x1, y1},
   {x2, y2},
   {x3, y3}
  };
domain = Region[Polygon[NodeList]]
```
几何区域 多边形

Out[ ]= Region[Embedding dimension: 2
Geometric dimension: 2]

In[ ]:=
```
H[ϕi_, ϕj_] := Integrate[Dot[∇{x,y} ϕi, ∇{x,y} ϕj], {x, y} ∈ domain];
```
积分 点积

```
F[ϕi_] := Integrate[ϕi, {x, y} ∈ domain];
```
积分

In[ ]:=
```
K = {
    {H[ϕ1, ϕ1], H[ϕ1, ϕ2], H[ϕ1, ϕ3]},
    {H[ϕ2, ϕ1], H[ϕ2, ϕ2], H[ϕ2, ϕ3]},
    {H[ϕ3, ϕ1], H[ϕ3, ϕ2], H[ϕ3, ϕ3]}
   } /. A2 → Abs[area] // FullSimplify;
             绝对值          完全简化
b = {F[ϕ1], F[ϕ2], F[ϕ3]} /. A2 → Abs[area] // FullSimplify;
                                   绝对值          完全简化
K
b
```

Out[ ]=

$$\left\{\left\{\frac{(x2-x3)^2+(y2-y3)^2}{2\,\mathrm{Abs}[x2\,y1-x3\,y1-x1\,y2+x3\,y2+x1\,y3-x2\,y3]}\ \text{if condition},\right.\right.$$

$$\frac{(x1-x3)\,(-x2+x3)+(y1-y3)\,(-y2+y3)}{2\,\mathrm{Abs}[x2\,y1-x3\,y1-x1\,y2+x3\,y2+x1\,y3-x2\,y3]}\ \text{if condition},$$

$$\left.\frac{(x1-x2)\,(x2-x3)+(y1-y2)\,(y2-y3)}{2\,\mathrm{Abs}[x2\,y1-x3\,y1-x1\,y2+x3\,y2+x1\,y3-x2\,y3]}\ \text{if condition}\right\},$$

$$\left\{\frac{(x1-x3)\,(-x2+x3)+(y1-y3)\,(-y2+y3)}{2\,\mathrm{Abs}[x2\,y1-x3\,y1-x1\,y2+x3\,y2+x1\,y3-x2\,y3]}\ \text{if condition},\right.$$

$$\frac{(x1-x3)^2+(y1-y3)^2}{2\,\mathrm{Abs}[x2\,y1-x3\,y1-x1\,y2+x3\,y2+x1\,y3-x2\,y3]}\ \text{if condition},$$

$$\left.-\frac{x1^2+x2\,x3-x1\,(x2+x3)+(y1-y2)\,(y1-y3)}{2\,\mathrm{Abs}[x2\,y1-x3\,y1-x1\,y2+x3\,y2+x1\,y3-x2\,y3]}\ \text{if condition}\right\},$$

$$\left\{\frac{(x1-x2)\,(x2-x3)+(y1-y2)\,(y2-y3)}{2\,\mathrm{Abs}[x2\,y1-x3\,y1-x1\,y2+x3\,y2+x1\,y3-x2\,y3]}\ \text{if condition},\right.$$

$$-\frac{x1^2+x2\,x3-x1\,(x2+x3)+(y1-y2)\,(y1-y3)}{2\,\mathrm{Abs}[x2\,y1-x3\,y1-x1\,y2+x3\,y2+x1\,y3-x2\,y3]}\ \text{if condition},$$

$$\left.\left.\frac{(x1-x2)^2+(y1-y2)^2}{2\,\mathrm{Abs}[x2\,y1-x3\,y1-x1\,y2+x3\,y2+x1\,y3-x2\,y3]}\ \text{if condition}\right\}\right\}$$

Out[ ]=

$$\left\{\frac{1}{6}\,(x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3))\ \text{if condition},\right.$$

$$\frac{1}{6}\,(x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3))\ \text{if condition},$$

$$\left.\frac{1}{6}\,(x3\,(y1-y2)+x1\,(y2-y3)+x2\,(-y1+y3))\ \text{if condition}\right\}$$