Homework

task1

the equivalent governing equation is:

$$\nabla^2 u + 1 = 0$$

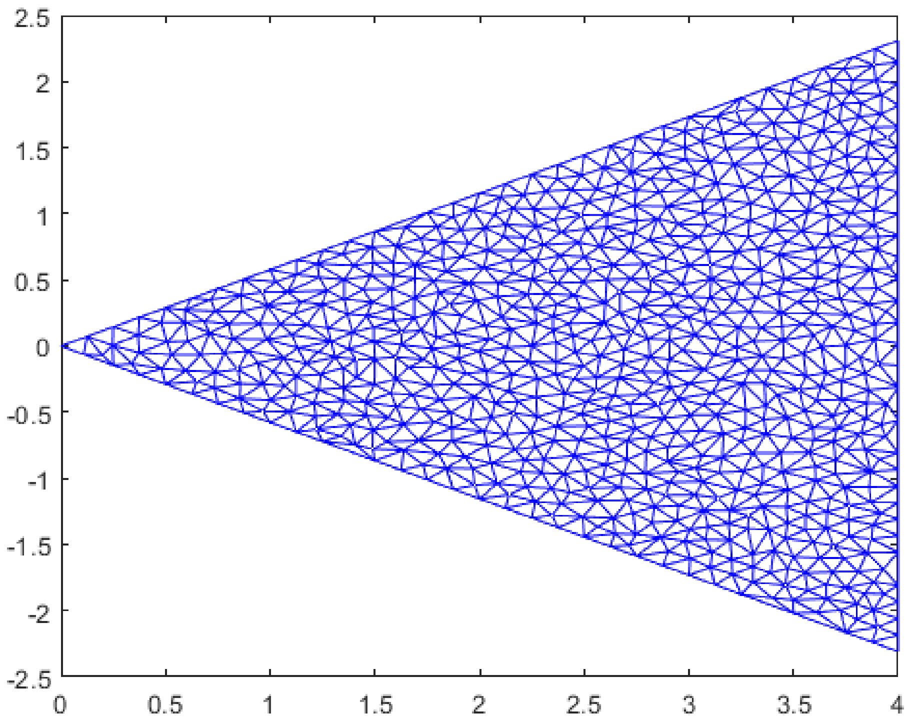
below is the matlab code. This document is generated by matlab's script.

first read the triangulation mesh and draw the reigion, b equals to 4.

```
clear;clc;

b = 4;
alpha = 30*pi/180;
node_elem = textread('hw1.txt', '%d', 2);
node_num = node_elem(1);
elem_num = node_elem(2);
data = readmatrix('hw1.txt');
xy = data(1:node_num, 1:2);
Tri = round(data(node_num+1:node_num+elem_num, 2:4)) + 1;

triplot(Tri, xy(:, 1), xy(:, 2))
```



nodes at the boundary should be excluded, thus:

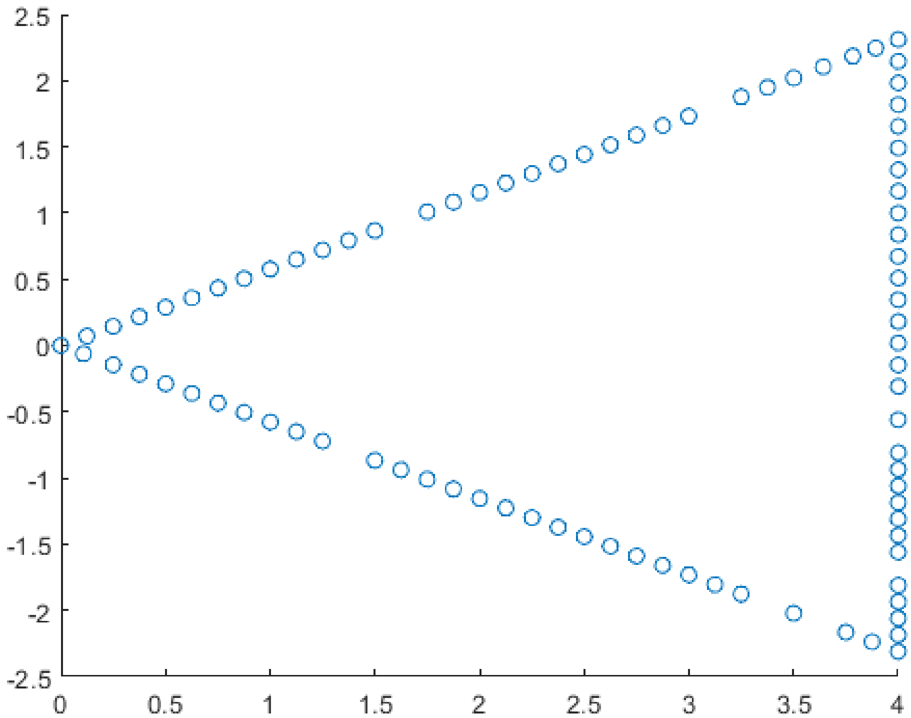
```
node_inside = [];
err = 1e-5;
for i = 1: node_num
    x = xy(i, 1);
    y = xy(i, 2);
    err1 = abs(x - b);
    err2 = abs(y - tan(alpha) * x);
```

```
    err3 = abs(y + tan(alpha) * x);
    if err1 > err && err2 > err && err3 > err
        node_inside = [node_inside, i];
    end
end

node_boundary = setdiff([1:node_num], node_inside);
node_boundary
```

```
node_boundary = 1��88
     1     2     3     4     5     6     8    15    20    24    28    34    44    63    65    66
```

```
scatter(xy(node_boundary, 1), xy(node_boundary, 2))
```



now insert the local stiffness relationship into the total stiffness relationship to get a larger matrix:

```
K = zeros(node_num, node_num);
F = zeros(1, node_num);
u = zeros(1, node_num);
for k = 1: elem_num
    n1 = Tri(k, 1);
    n2 = Tri(k, 2);
    n3 = Tri(k, 3);
    x1 = xy(n1, 1);
    y1 = xy(n1, 2);
    x2 = xy(n2, 1);
    y2 = xy(n2, 2);
    x3 = xy(n3, 1);
    y3 = xy(n3, 2);
    A2 = 2*det([1 x1 y1; 1 x2 y2; 1 x3 y3]);
    k11 = (x2 - x3)^2 + (y2 - y3)^2;
    k12 = (x1 - x3) * (-x2 + x3) + (y1 - y3) * (-y2 + y3);
    k13 = (x1 - x2) * (x2 - x3) + (y1 - y2) * (y2 - y3);
    k22 = (x1 - x3)^2 + (y1 - y3)^2;
    k23 = -(x1^2 + x2 * x3 - x1 * (x2 + x3) + (y1 - y2) * (y1 - y3));
    k33 = (x1 - x2)^2 + (y1 - y2)^2;
```

```
    F1 = (x3 * (y1 - y2) + x1 * (y2 - y3) + x2 * (-y1 + y3)) / 6;
    F2 = (x3 * (y1 - y2) + x1 * (y2 - y3) + x2 * (-y1+y3)) / 6;
    F3 = (x3 * (y1 - y2) + x1 * (y2 - y3) + x2 * (-y1+y3)) / 6;
    K(n1, n1) = K(n1, n1) + k11 / A2;
    K(n1, n2) = K(n1, n2) + k12 / A2;
    K(n1, n3) = K(n1, n3) + k13 / A2;
    K(n2, n1) = K(n2, n1) + k12 / A2;
    K(n2, n2) = K(n2, n2) + k22 / A2;
    K(n2, n3) = K(n2, n3) + k23 / A2;
    K(n3, n1) = K(n3, n1) + k13 / A2;
    K(n3, n2) = K(n3, n2) + k23 / A2;
    K(n3, n3) = K(n3, n3) + k33 / A2;
    F(n1) = F(n1) + F1;
    F(n2) = F(n2) + F2;
    F(n3) = F(n3) + F3;
end

K
```

```
K = 767��767
    0.5893         0         0         0         0         0         0         0         0
         0    0.5893         0         0         0         0         0         0         0
         0         0    0.6195         0         0         0         0         0         0
         0         0         0    1.7773         0         0         0         0         0
         0         0         0         0    2.2396         0         0         0         0
         0         0         0         0         0    1.7700         0         0         0
         0         0         0         0         0         0    3.6042         0         0
         0         0         0         0         0         0         0    1.8412         0
         0         0         0         0         0         0         0         0    3.7771
         0         0         0         0         0         0         0         0         0    4.214
```

```
isdiag(K)
```

```
ans = logical
   0
```

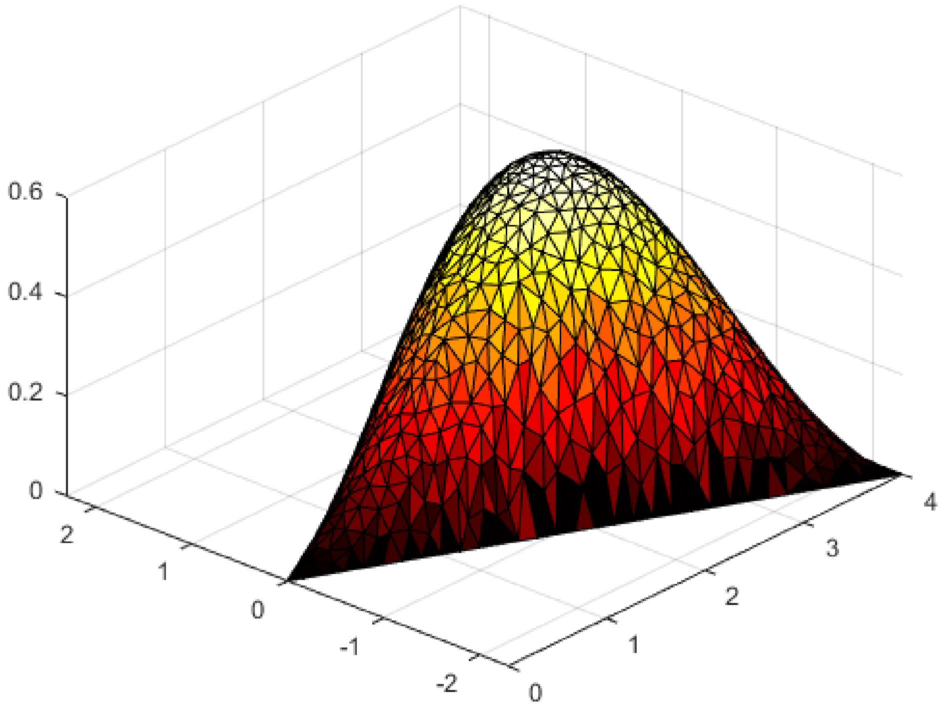
```
F
```

```
F = 1��767
    0.0026    0.0026    0.0030    0.0084    0.0137    0.0072    0.0168    0.0057    0.0149    0.013
```

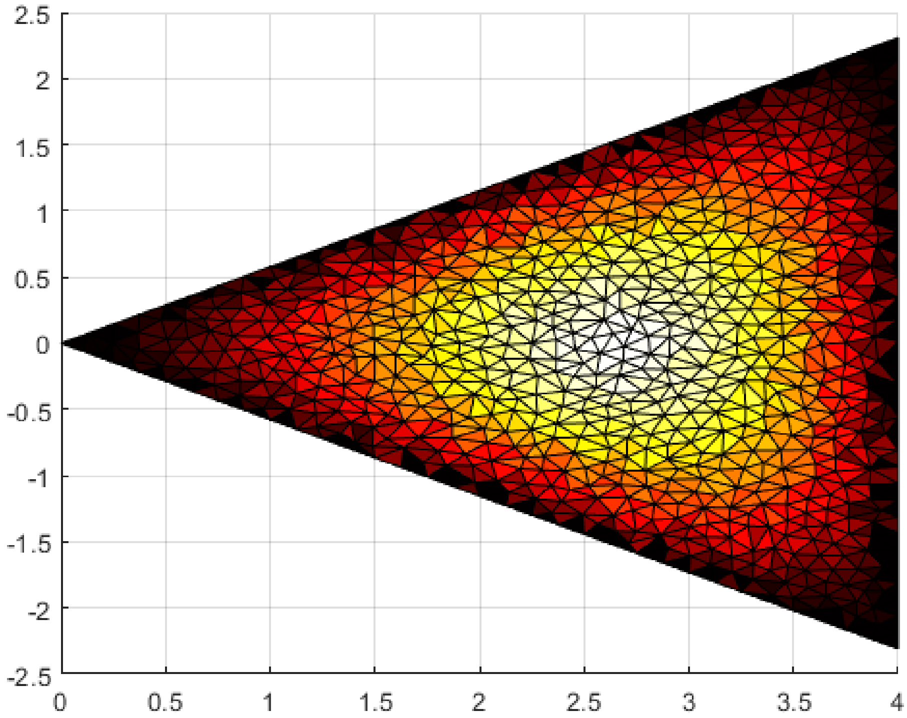
inverse the K to get the result:

```
u(node_inside) = K(node_inside, node_inside) \ F(node_inside).';
trisurf(Tri, xy(:, 1), xy(:, 2), u)
colormap hot;

view([-48.300 40.800])
```




```
view([-0.600 90.000])
```



task2

the Lagrange equation is as below:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \quad i = 1, 2, 3 \ldots$$

where L equals to:

$$L = T - V$$

in which T is the kinetic energy and V is potenial energy. In this case, let $\theta_1 = q_1 \quad \theta_2 = q_2$��

$$\begin{cases} \vec{u}_1 = (l_1\dot{\theta}_1 \cos\theta_1 + \dot{f})\vec{i} + l_1\dot{\theta}_1 \sin\theta_1 \vec{j} \\ \vec{u}_2 = (l_1\dot{\theta}_1 \cos\theta_1 + l_2\dot{\theta}_2 \cos\theta_2 + \dot{f})\vec{i} + (l_1\dot{\theta}_1 \sin\theta_1 + l_2\dot{\theta}_2 \sin\theta_2)\vec{j} \end{cases}$$

kietic energy T:

$$T = \frac{1}{2}m_1||u_1||^2 + \frac{1}{2}m_2||u_2||^2$$

potential energy V:

$$V = m_1 g l_1(1-\cos\theta_1) + m_2 g[l_1(1-\cos\theta_1) + l_2(1-\cos\theta_2)]$$

use Lagrange equation we will have the result:

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}_1} - \frac{\partial L}{\partial\theta_1} \rightarrow$$

$$(m_1+m_2)(\ddot{f}\cos\theta_1 + g\sin\theta_1 + l_1\ddot{\theta}_1) + l_2 m_2\dot{\theta}_2^2 \sin(\theta_1-\theta_2) + l_2 m_2 \ddot{\theta}_2\cos(\theta_1-\theta_2) = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}_2} - \frac{\partial L}{\partial\theta_2} \rightarrow$$

$$\ddot{f}\cos\theta_2 + g\sin\theta_2 - l_1\dot{\theta}_1^2\sin(\theta_1-\theta_2) + l_1\ddot{\theta}\cos(\theta_1-\theta_2) + l_2\ddot{\theta}_2 = 0$$

task3

for this problem, use hermite interpolation to finish the job:

denote that:

$$x_2 = x_1 + h$$
$$x = x_1 + t$$

```
clear;clc;

syms x x1 x2 h t;

mat = [
   1 x1 x1^2 x1^3;
   0 1 2*x1 3*x1^2;
   1 x2 x2^2 x2^3;
   0 1 2*x2 3*x2^2;
];
phi = simplify([1 x x^2 x^3] * inv(mat));
phi_local = subs(subs(phi, x2, x1+h), x, x1+t);
phi_local
```

phi_local =

$$\left(\frac{(h-t)^2\ (h+2\,t)}{h^3}\quad \frac{t\ (h-t)^2}{h^2}\quad \frac{t^2\ (3\,h-2\,t)}{h^3}\quad -\frac{t^2\ (h-t)}{h^2}\right)$$

```
simplify(diff(phi_local, t))
```

ans =

$$\left(-\frac{6\,t\ (h-t)}{h^3}\quad \frac{h^2-4\,h\,t+3\,t^2}{h^2}\quad \frac{6\,t\ (h-t)}{h^3}\quad -\frac{t\ (2\,h-3\,t)}{h^2}\right)$$

thus we have when $0 \le t \le h$:

$$w(x) = \frac{(h-t)^2(h+2t)}{h^3} w_k + \frac{t(h-t)^2}{h^2}\theta_k + \frac{t^2(3h-2t)}{h^3} w_{k+1} - \frac{t^2(h-t)}{h^2}\theta_{k+1}$$

$$\theta(x) = -\frac{6t(h-t)}{h^3} w_k + \frac{(h-t)(h-3t)}{h^2}\theta_k + \frac{6t(h-t)}{h^3} w_{k+1} - \frac{t(2h-3t)}{h^2}\theta_{k+1}$$

below is the caculation parameters:

```
clear;clc;

q = -200;
F = -1000;
M = 2000;
L = 0.12;
d1 = 0.03;
d2 = 0.02;
Es = 200e9;
J1 = pi * d1^4 / 64;
J2 = pi * d2^4 / 64;
N = 101;
x = linspace(0, 2*L, N);
h = 2*L / N;
```

the local stiffness matrix K_{local} and b_{local} are shown as below:

```
K = zeros(2*N, 2*N);
b = zeros(2*N);
stiffness_K = [12/h^3, 6/h^2, -12/h^3, 6/h^2;
   6/h^2, 4/h, -6/h^2, 2/h;
   -12/h^3, -6/h^2, 12/h^3, -6/h^2;
   6/h^2, 2/h, -6/h^2, 4/h];
stiffness_b = h * [1/2, h/12, 1/2, -h/12];
to_solve = [3:2*N-2, 2*N];
stiffness_K
```

```
stiffness_K = 4��4

10^8 ��
    8.9436    0.0106   -8.9436    0.0106
    0.0106    0.0000   -0.0106    0.0000
   -8.9436   -0.0106    8.9436   -0.0106
    0.0106    0.0000   -0.0106    0.0000
```

```
stiffness_b
```

```
stiffness_b = 1��4
    0.0012    0.0000    0.0012   -0.0000
```

put them together:

```
for k = 1: N-1
    if k <= (N-1)/2
        K(2*k-1:2*k+2, 2*k-1:2*k+2) = K(2*k-1:2*k+2, 2*k-1:2*k+2) + Es * J1 * stiffness_K;
        b(2*k-1:2*k+2) = b(2*k-1:2*k+2) + q * stiffness_b;
    else
        K(2*k-1:2*k+2, 2*k-1:2*k+2) = K(2*k-1:2*k+2, 2*k-1:2*k+2) + Es * J2 * stiffness_K;
    end
end
b(N) = b(N) + F;
b(2*N) = b(2*N) - M;
```

solve the $Ku = b$ and get w_i, θ_i

```
bm = b(to_solve);
Km = K(to_solve, to_solve);
solution = zeros(2*N);
solution(to_solve) = Km \ bm.';
w = solution(1:2:2*N);
theta = solution(2:2:2*N);
w
```

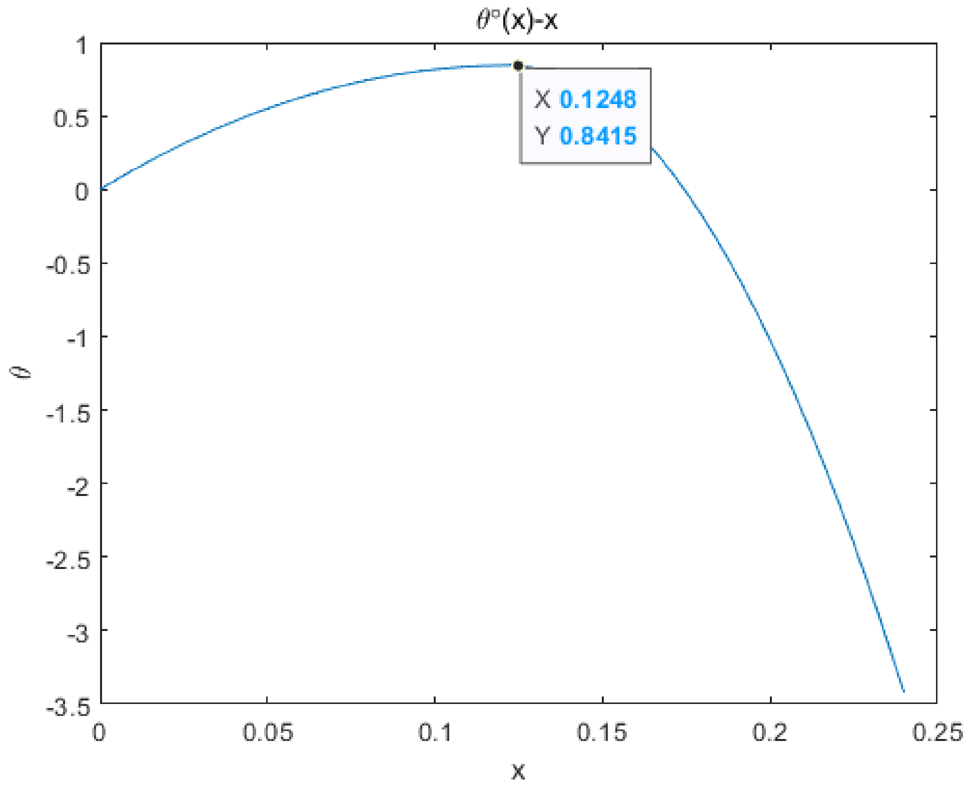
```
w = 1��101
         0    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.000
```

```
theta
```

```
theta = 1��101
         0    0.0006    0.0011    0.0017    0.0022    0.0028    0.0033    0.0038    0.0043    0.004
```

here's the torsion angle $\theta(x) - x$

```
plot(x, theta*180/pi)
xlabel('x')
ylabel('\theta')
title('\theta\circ(x)-x')
ax = gca;
chart = ax.Children(1);
datatip(chart,0.1248,0.8415);
```



below is the deflection:

```
plot(x, w)
xlabel('x')
ylabel('w')
title('w(x)-x')

ax = gca;
```

```
chart = ax.Children(1);
datatip(chart,0.1728,0.001693);
```