**1.** Deduction and Programming: consider the boundary value problem for the function in two dimensional domain D enclosed by the surface S, governed by the Poisson’s equation and the boundary conditions as follows.

|  |  |
| --- | --- |
| for | (1) |
| for | (2) |
| for | (3) |

where  and  are given values, and ***n*** is the normal vector to the surface .

(1) Show that eqs. (1) to (3) can be obtained by minimizing the total potential energy U defined by:

|  |  |
| --- | --- |
|  | (4) |

with the boundary conditions given by eqn (2).

(2) Let  be the solution for the following weak form:

|  |  |
| --- | --- |
|  | (5) |

where *w* is an arbitrary weight function. Show that if  and  on ,  in eqn (5) coincides with the solution *u* in eqs. (1) to (3).

(3) Programming: As shown by Fig. 1, let the domain *D* be the triangular region, in which *u* satisfies eqn (1) subjected to no body force () and the following boundary conditions:

|  |  |
| --- | --- |
| on | (6) |
| on | (7) |
| on | (8) |

Use finite element method to determine the distribution of u within domain *D* and show the validity of your result together with FEM codes.

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| Fig. 1 Problem 1 |

**2.** Programming: Use the minimum possible number of Bernoulli-Euler beam elements, unless stated otherwise, to analyze at the beam structures shown in Fig. 2. Show your results together with FEM codes.

(Note: dia. means diameter.)

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| Fig. 2 Problem 2 |