© Journal of Peace Research, vol. 26, no. 1, 1989, pp. 91-99

The Probability of Nuclear War*

RUDOLF AVENHAUS
Universität der Bundeswehr,
München

STEVEN J. BRAMS
Department of Political Science,
New York University

JOHN FICHTNER
Siemens,
München

D. MARC KILGOUR
Department of Mathematics,
Wilfrid Laurier University

A theoretical analysis of the probability of nuclear war is developed that assumes a starting probability and an annual reduction factor. Whatever the starting probability is, a constant reduction factor leads to an eventual probability that is less than 1, whereas the eventual probability goes to 1 if there is no reduction or if the reduction proportion decreases at a constant rate. Numerical calculations and graphical results illustrate trade-offs between the starting probabilities and the reduction factors, demonstrating especially the significance of the latter. In addition, upper and lower limits for, and approximations of, the eventual probabilities – along with measures of the rate of convergence – are derived. The applicability of the analysis to lowering the probability of nuclear war is discussed, with particular attention paid to real-life factors that seem to affect this probability.

1. *Introduction*

At the height of the Cuban missile crisis in October 1962, John F. Kennedy estimated the chances of a major war between the United States and the Soviet Union to be somewhere between one-third and one-half (Sorensen, 1965, p. 705), though other participants in the crisis thought the probability of nuclear war was considerably lower (Lebow, 1987, p. 15). There is no indication that President Kennedy had any firm basis for making such an estimate, much less a model for deriving this prediction, but his intuition about the danger of the superpower confrontation escalating to nuclear conflict was probably as good as anybody else's.

Other world leaders have frequently made predictions about the probability of nuclear war, as have military analysts, but these predictions have generally not been rooted in any systematic approach. Indeed, it is hard to see what kind of technique could be used to forecast reliably a unique event that involves two or more nuclear powers. The only prior wartime use of nuclear weapons was by the United States against Japan in 1945, and in that case only the United States had nuclear weapons. Moreover, many other circumstances have changed drastically since 1945, making extrapolations from this first wartime use dubious.

Most analysts view a 'bolt from the blue' – a massive and unexpected first strike of one superpower against the other – as exceedingly unlikely (e.g. Bracken, 1983, p. 73). Instead, the main danger is seen as a steady erosion of trust, which in an extreme crisis may precipitate the first use of nuclear weapons, particularly if the initiator faces a desperate situation and believes that only nuclear weapons might provide an escape from certain defeat. Alternatively, there is always some small but positive probability that nuclear weapons will be used accidentally because of failures in command, communication, control, and intelligence (C^3I).[1]

Whatever the reason for the outbreak of nuclear war, we assume that there is some positive probability that it will occur in, say, the next year. If, for example, this probability is 1% per year into the indefinite future, then the probability is greater than

* Steven J. Brams gratefully acknowledges the financial support of the National Science Foundation under Grant No. SES85-20154. D. Marc Kilgour gratefully acknowledges the financial support of the Natural Sciences and Engineering Research Council of Canada under Grant No. A8974. The authors would like to thank K. Bielesch, Universität der Bundeswehr München, for performing the numerical calculations.

0.5 that nuclear war will occur within 69 years. This value is the minimum integer n satisfying the inequality

$$0.01 + (1 - 0.01)(0.01) + (1 - 0.01)^2(0.01) + \ldots + (1 - 0.01)^{n-1}(0.01) > 0.5,$$

in which the terms on the left side are the (constant) conditional probabilities of nuclear war in years 1, 2, . . ., n, given that war did not occur in any preceding year, times the probability of no earlier war. It is not difficult to show that as the number of years goes to infinity, the probability of eventual nuclear war approaches 1, no matter how small (but positive and constant) the probability of war per year may be.[2]

More conveniently for purposes of calculation, the above inequality is equivalent to

$$1 - (0.99)^n > 0.5,$$

where the left side can be interpreted as 1 minus the probability that nuclear war will not occur at some time within n years. Simplifying and taking logarithms gives

$$n > \frac{\log 0.5}{\log 0.99} = 68.97,$$

confirming that the smallest integer solution is $n = 69$.

This is not a happy picture as one peers into the future. Indeed, it may be a future we want to shrink from, as Fred Charles Iklé somberly pointed out: 'We all turn away . . . from the thought that nuclear war may be as inescapable as death, and may end our lives and our society within this generation or the next' (Iklé, 1973, p. 267; see also Quester 1986; Nye 1987).

More hopefully, suppose that the probability of nuclear war in each subsequent year can be decreased by, say, 20% per year. Then Richard Garwin (1985, p. 39) observed that the cumulative probability of nuclear war – over an eternity – would be dramatically reduced to less than 5%. This result follows from a simple calculation, where the terms in the sum on the left side of the equation below are upper bounds for the unconditional probabilities of nuclear war in years 1, 2, . . . :

$$0.01 + (0.8)(0.01) + (0.8)^2(0.01) + \ldots = 0.01\left(\frac{1}{1 - 0.8}\right) = 0.05.$$

The evaluation of the expression follows from the identity

$$1 + x + x^2 + x^3 + \ldots = \sum_{i=0}^{\infty} x^i = \frac{1}{1 - x},$$

where $|x| < 1$, for the sum of an infinite geometric series. (The justification of this method of calculating an upper bound will be examined in more detail later.)

In this paper we shall generalize Garwin's observation, assuming both a constant reduction factor (as he did) and a non-constant reduction factor. To give a quantitative feel of how the probability of nuclear war changes with changes in both the starting probability of nuclear war and the reduction factor after the first year, we have done computer calculations in which these two parameters are varied. After presenting our analytic and computational results, we shall conclude with some observations about their significance for the prevention of nuclear war.

2. *The Model*

Assume that the probability that a war will occur in the i^{th} year, provided there was no war in any preceding year, is p_i, where $0 \leqq p_i \leqq 1, i = 1, 2, \ldots$. Then the probability that a war will occur at some time within n years is

$$W_n = 1 - \prod_{i=1}^{n}(1 - p_i), n = 1, 2, \ldots,$$

and the probability that a war will occur eventually is $W = \lim_{n\to\infty} W_n$.

Next, consider how various sequences p_i, $i = 1, 2, \ldots$, can affect the values of W_n, $n = 1, 2, \ldots$, and ultimately of W. To begin with, assume, as above, that the (conditional) probability of war in any year is constant: $p_i = p$, $i = 1, 2, \ldots$, where $0 < p < 1$. Then

$$W_n = 1 - (1 - p)^n, n = 1, 2, \ldots.$$

Because $(1 - p)^n \to 0$ as $n \to \infty$, it follows that W, the probability that a war will occur at some time, equals 1. In other words, eventual war is certain.

There are many other sequences, p_1, p_2, . . . , besides constant sequences, that lead to the probability of war's becoming, eventually, one. To determine these, it is useful to recall that the infinite product $\prod_{i=1}^{\infty} (1 - p_i)$ converges (to a finite non-zero value) if and only if the sum $\sum_{i=1}^{\infty} p_i$ converges (Apostol, 1974, p. 207).

Now because each factor $(1 - p_i)$ lies in the open interval (0, 1), either the product converges to a number in that interval or it diverges to zero. The latter case, which implies that the probability of eventual war is 1, is therefore equivalent to the divergence of the sum $\sum_{i=1}^{\infty} p_i$ – that is, the sum becomes infinite. In particular, this infinite sum must diverge, so that the eventual probability of war must equal one, unless the sequence $p_1, p_2, \ldots$ converges to zero. These considerations prove

Theorem 1

If $W_n = 1 - \prod_{i=1}^{n}(1 - p_i)$, where $0 \leqq p_i \leqq 1$, and $W = \lim_{n\to\infty} W_n$, then $W = 1$ if and only if $\sum_{i=1}^{\infty} p_i$ diverges. In particular, if $W < 1$, then $\lim_{i\to\infty} p_i = 0$.

Theorem 1 characterizes those sequences $p_1, p_2, \ldots$ of conditional probabilities of war that imply the certainty of eventual war, as signalled by the divergence of the infinite sum. For instance, if the conditional probability of war in year i does not approach zero in the limit (as i goes to infinity), then the probability that war occurs at some time is one. The constant sequence, $p_i = p > 0$, fits into this category, because p_i does not approach zero.

After analyzing in detail examples of the kind suggested by Garwin – in which the probability of eventual war is non-zero but less than one – we will exhibit another class of sequences which can be shown, using Theorem 1, to imply that eventual war is certain.

Garwin's example involves a sequence in which the conditional probability that a war will occur in a given year is decreased each year by a constant *reduction factor*, r, where $0 \leqq r < 1$. This factor is the ratio of the (conditional) probability of war in year $i + 1$ to the corresponding probability in year i – in Garwin's example $r = 0.8$, indicating a 20% reduction each year.

Let the probability of war in year 1 be $p_1 = s$, where $0 \leqq s \leqq 1$. Henceforth we will call s the *starting probability*. Then the (conditional) probability that a war will occur in year, i, $i = 2, 3, \ldots$, is $p_i = sr^{i-1}$, and the eventual probability of war is

$$(1) \quad W(s, r) = 1 - \prod_{i=1}^{\infty}(1 - sr^{i-1}).$$

It does not seem possible to give an explicit or closed form for this probability; however, we show in the Appendix

Theorem 2

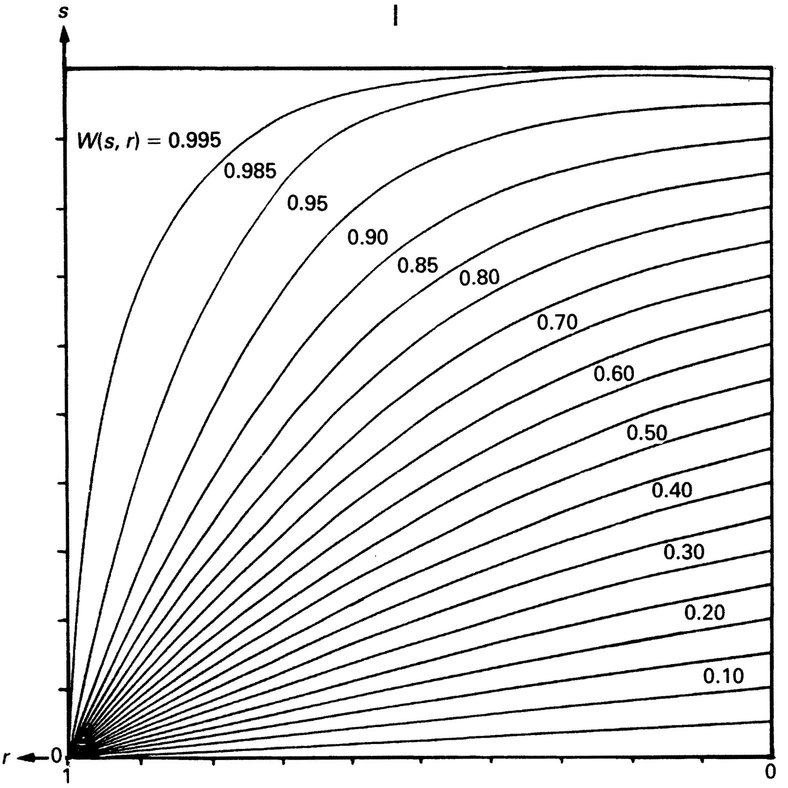
If $W(s, r)$ is defined by (1), where $0 < s \leqq 1$ and $0 < r < 1$, then

$$(2) \quad 1 - \exp\left(-\frac{s}{1 - r}\right) < W(s, r) < \min\left(1, \frac{s}{1 - r}\right).$$

The single most important conclusion of Theorem 2 is that $W(s, r) < 1$ since the strict upper bound can never exceed 1. Hence, if the conditional probability of war is reduced by a *constant* reduction factor r each year, then, no matter how small this annual percentage reduction is, eventual war is *not* a certainty. This finding is consistent with Theorem 1, because $\sum_{i=1}^{\infty} p_i = \sum_{i=1}^{\infty} sr^{i-1}$ is an infinite geometric series and therefore convergent. Of course, $p_i = sr^{i-1} \to 0$ as $i \to \infty$.

We have carried out numerical calculations of actual values of $W(s, r)$ and of the upper and lower bounds given by (2). We observed (see below) that, whenever $s/(1 - r)$ is small, the two bounds in (2) are very close (which can be demonstrated analytically using a Taylor series expansion). This justifies the approximation

Fig. 1. Lines of constant $W(s, r)$ for varying values of s and r.



$$(3) \quad W(s, r) \simeq \frac{s}{1 - r},$$

provided $s/(1 - r)$, and therefore s, is small. Approximation (3) indicates that if the reduction factor is held constant, then the eventual probability of war is proportional to the starting probability, providing the starting probability is small.

Two other features of the values of $W(s, r)$ are noteworthy. When the reduction factor, r, is zero, it is easy to evaluate (1) directly to show that $W(s, 0) = s$. In other words, given the reduction factor is zero (so the probability of war in subsequent years is zero), the eventual probability of war equals the starting probability. Also, $W(s, r) > W(s, 0)$ if $r > 0$; that is, the eventual probability of war is minimized when the reduction factor is zero, providing the starting probability is held fixed. This implies, of course, that $W(s, r) \geqq s$.

Fig. 1 shows the curves of constant value of $W(s, r)$, illustrating the trade-off between starting probability and reduction that leads to the same probability of eventual war. In general, an increase in the starting probability s can be counterbalanced by a decrease in the reduction factor r (i.e. an increase in the reduction fraction $1 - r$) to keep $W(s, r)$ constant. For example, the rate of substitution of s for r can be seen from Fig. 1 to be about 10% when $W(s, r) \simeq 0.1$. This means that an increase in s of $\triangle s$ must be accompanied by a decrease in r of about $10\triangle s$ in order that $W(s, r)$ be unaffected, making s a much more potent factor than r. When $W(s, r) \simeq 0.5$, the rate of substitution of s for r decreases from about 70% when s is small to about 25% when s is large. When $W(s, r) \simeq 0.9$, the corresponding rate decreases from about 250% when s is small to about 10% when s is large.

In general, the ratio of an increase in the starting probability s to the compensating decrease in the reduction factor r is approximately constant [and equal to $W(s, r)$] when the eventual probability of war, $W(s, r)$, is small. This trade-off is highly sensitive to s, however, when $W(s, r)$ is large, with the importance of r increasing as s falls. Nonetheless, greater constant reductions are less effective than an immediately lower s when s is quite small, which is probably the case today.

The nearly constant rate of substitution of s for r when $W(s, r)$ is small reflects approximation (3). Another consequence of (3) is evident in Fig. 1; the curves of constant value of $W(s, r)$ are nearly straight lines when the value of $W(s, r)$ is small.

We remark that the method in the Appendix can be adapted to improve the lower bound in (2), but no such improvement seems possible for the upper bound. However, an alternative derivation of $W(s, r) < s/(1 - r)$ is of some interest.

This upper bound could have been obtained by using *Bonferroni's Inequality*, which says that the probability of a union of events is not greater than the sum of the probabilities of the events (Rohatgi, 1976, p. 27). For $i > 1$, the unconditional probability of war in the i^{th} year is equal to the conditional probability (that is, $p_i = sr^{i-1}$) times the probability of no earlier war. Since the latter probability cannot exceed one, the unconditional probability of war in the i^{th} year cannot exceed $p_i = sr^{i-1}$. Thus, an upper bound for the eventual probability of war is

$$\sum_{i=1}^{\infty} sr^{i-1} = \frac{s}{1-r},$$

as we illustrated earlier in the case of Garwin's example.

It is interesting to note that this expression would give an *exact* value for the eventual probability of war if we had defined $p_i = sr^{i-1}$ not as the conditional probability of war (given that no war occurred previously) but rather as the unconditional probability that the first war occurs in the i^{th} year. In this case, we would have disjoint events – that the first war occurs in year i, $i = 1, 2, \ldots$ – and we could simply sum the probabilities p_i, $i = 1, 2, \ldots$, of these events' occurring. We believe, however, that it is intuitively more plausible to think in terms of the conditional probabilities used here – reflecting the fact that a nuclear war did not occur in prior years, which are *not* disjoint events for each conditional probability – than the unconditional probabilities that a nuclear war will occur for the first time in years 1, 2, ... Indeed, estimating the probability that a war will first break out in, say, year 10 (versus any previous year) seems to us a well-nigh impossible task, whereas estimating the probability that a war will occur in year 10, *given* that there was no war in years 1–9 (whose probability can be computed), seems a somewhat more reasonable task, especially since the tenth-year conditional probability is related to the ninth, eighth, etc.

We now return to Garwin's example and report briefly on some additional calculations concerning not only W but also the rate of convergence of W_n to W. For four different sets of values of the arguments (s, r), Table I shows six representative values of W_n as well as our estimate of the limit W (from summing several hundred terms in the series), and the upper and lower bounds for W given by Theorem 2.

The trade-offs shown in Table I illustrate the effects of the starting probabilities s and (constant) reduction factors r. Although Garwin's example (1% starting, decreased

Table I. Probabilities of War and Rates of Convergence.

	W_1	W_5	W_{10}	W_{20}	W_{50}	W_{100}	W	Bounds from (2)	$T_{0.5}$	$T_{0.9}$
$s = 0.01, r = 0.80$	0.01	0.0332	0.0438	0.0484	0.0489	0.0489	0.0489	$0.0488 < W < 0.05$	4	11
$s = 0.01, r = 0.95$	0.01	0.0444	0.0774	0.1208	0.1690	0.1807	0.1817	$0.1813 < W < 0.20$	13	44
$s = 0.05, r = 0.80$	0.05	0.1574	0.2028	0.2217	0.2240	0.2240	0.2240	$0.2212 < W < 0.25$	3	10
$s = 0.05, r = 0.95$	0.05	0.2067	0.3362	0.4795	0.6078	0.6348	0.6369	$0.6321 < W < 1.00$	10	36

by 20% per year) augurs well for the future – culminating in an eventual probability of less than 5% – an eventual probability of less than one out of five (18%) can still be achieved if the reduction (r) is 95% rather than 80%.

More dismal, the eventual probabilities in these two cases (80% and 95% reduction factors) go to about 22% and 64% if the starting probability is 5% rather than 1%. As suggested at the beginning of this paper, we know of no way of estimating either the starting probabilities or the reduction factors, but the most 'optimistic' (1% starting, decreased by 20% per year) and 'pessimistic' (5% starting, decreased by 5% per year) estimates given here seem to be plausible bounds that might well bracket the true values.

Using the estimate of W, it is possible to estimate $T_{0.5}$, the least integer n satisfying $W_n \geqq 0.5W$, and $T_{0.9}$, defined analogously. These values are also shown, for the four sequences, in Table I. To interpret $T_{0.5}$ and $T_{0.9}$, note that, if a war is to occur eventually, there is at least a 50% chance it will occur within $T_{0.5}$ years, and at least a 90% chance within $T_{0.9}$ years. Given that a war occurs eventually, $T_{0.5}$ and $T_{0.9}$ are percentiles of the distribution of the time of occurrence. Specifically, $T_{0.5}$ can be thought of as the 'half-life' of a war's eventually occurring for a given s and r.

A curious aspect of the interplay of these factors is illustrated by the half-life, $T_{0.5}$, for reaching the eventual probability. If the reduction factor is substantial (80%), avoiding nuclear war in the first 3–4 years cuts the eventual probability by more than half, whether one assumes 1% or 5% starting probability. By contrast, the much slower reduction rate implied by a reduction factor of 95% requires 10 and 13 years in the cases of 1% and 5% starting probabilities, respectively, before the eventual probabilities are cut by more than half. Ninety-percent reductions in the eventual probabilities, given by $T_{0.9}$, require roughly three times as many years as the 50% reductions in each case. (Recall that the eventual probability of nuclear war is strictly less than one, so 'half-life' here does not refer to a 50% chance of war but rather to the initial time period which includes at least half of the overall probability of war.)

We turn now to the analysis of the implications of a different class of sequences of conditional probabilities $p_1, p_2, \ldots$. Recall that in the sequences suggested by Garwin, the initial probability $p_1 = s$ was reduced by the proportion $(1 - r)$ in each subsequent year. Instead of sequences which decrease at a constant rate, we now consider sequences which decrease at a decreasing rate.

For s, $\bar{r}$, and q satisfying $0 < s, \bar{r}, q < 1$, define

$$(4) \quad p_1 = s, p_i = s \prod_{j=1}^{i=1} (1 - \bar{r}q^{j-1}), i = 2, 3, \ldots.$$

Because $p_n - p_{n+1} = \bar{r}q^{n-1}p_n$, (4) implies that the conditional probability of nuclear war is reduced by the proportion $\bar{r}q^{n-1}$ from the n^{th} year to the $(n + 1)^{\text{st}}$ year. In other words, the proportion of reduction is decreasing from one year to the next. The requirement that $q < 1$ means that this year-to-year reduction proportion approaches 0 – in other words, successive conditional probabilities become more nearly equal as time passes. Setting $q = 1$ (no decrease in the year-to-year reduction proportion – it is constant) would reproduce examples like Garwin's, with reduction factor $r = 1 - \bar{r}$. (We have changed notation from r to $\bar{r}$ to indicate that $\bar{r}$ does not represent a reduction factor as used earlier, but $1 - \bar{r}$ does.)

Theorem 2 states that if the conditional probability of war decreases at a constant rate, then there is a positive probability that war will not occur at any time. In contrast, if the rate of decrease of the conditional probability itself decreases at a constant rate – no matter how slowly – then eventual war is certain. In formal terms:

Theorem 3

For the sequence $p_1, p_2, \ldots$ defined by (4), if $W_n = 1 - \prod_{i=1}^{n}(1 - p_i)$, then $W = \lim_{n\to\infty} W_n = 1$.

Thus, sequences of the class defined by (4) correspond to the certainty of eventual war. The proof of Theorem 3 is given in the Appendix.

3. *Conclusions*

One major step in averting nuclear war is to make sure that its eventual probability does not go to 1. We will always be successful in this task if, no matter what the probability of nuclear conflict in the next year is, we can succeed in reducing it by at least a constant factor every year thereafter. Of course, if we can reduce the probability of war not just at a constant rate but at an increasing rate (not considered here), then the eventual probability is diminished even farther below 1 than a constant rate of decrease would imply.

More ominously, nuclear war becomes a certainty eventually, whatever the starting probability, if the annual reduction is not constant but itself decreases at a constant rate. That is, if the probability of nuclear war in each subsequent year, even though reduced, is reduced slowly enough, then war will occur for certain in the proverbial long run. (War may occur if the reduction factor is constant, but it is not a *certainty*.) Thus, the economist's usual assumption of marginally decreasing returns, if applicable to the year-to-year reduction proportion, is not a good omen: when the probability of war decreases at a decreasing rate, war eventually becomes certain.

Our feeling is that it is unlikely, short of either the abolition of nuclear weapons or perfect Star Wars defenses – whose likelihood seems negligible – that the annual probability of nuclear war will eventually go to zero. We think that, probably beginning after the Cuban missile crisis, the probability of nuclear war, at least between the superpowers, has decreased, perhaps markedly. Speculatively, it may now be not 5% or 1% per year, but considerably less.[3]

Thus, even if eventual nuclear war is a certainty, it may be a long time in coming. In fact, though the dangers of nuclear war probably wax and wane, the world is likely much safer than it was a generation ago when the superpowers had not yet acquired significant reconnaissance and second-strike retaliatory capabilities, and nuclear deterrence was decidedly less stable than today.

Nonetheless, the memory of the Cuban missile crisis has receded, and it may take another searing experience near the nuclear precipice to rejuvenate efforts to avoid nuclear confrontations and slow down the nuclear arms race. The core meltdown at Chernobyl in April 1986 reawakened fears of a nuclear disaster and may have given new momentum to the development of additional safeguards against the accidental or deliberate use of nuclear weapons – as well as the handling of fissionable material – just as the Cuban missile crisis led to the hot line between the superpowers a generation earlier.

Our calculations show how both the starting probability and the reduction factor, especially the starting probability when it is low, act together to diminish the probability of nuclear war. A crisis or disaster – if survived – probably lowers both, but ultimately its sobering effects may prove ephemeral as memories fade.

The key, we think, is to try to set in motion – especially after a crisis or disaster when citizens and governments are aroused and apprehensive – processes that lead in stages to continued reductions in the probability of war. If these agreed-upon reductions are costly for the superpowers to back out of or renege on, and hence stay in place reasonably well, then the cumulative probability of nuclear war, even over long (if not infinite) stretches, may be kept significantly below 1.

Moreover, each year that we survive leads to a new lease on life extending farther and farther ahead if the probability of war continues to decline. To be sure, in an infinite future we will all perish if the possibility of nuclear war is never eliminated altogether, but lengthening that future by steady and ineluctable reductions in the probability of nuclear war – while still allowing for some untoward developments along the way – could help immensely.

APPENDIX

This Appendix contains proofs of Theorem 2 and Theorem 3. To prove Theorem 2, note first that the assertion that $W(s, r) < 1$ is equivalent to

$$(5)\quad -ln[1 - W(s, r)] = \sum_{i=0}^{\infty}\left[-ln(1 - sr^i)\right] < \infty,$$

which is equivalent to

$$\sum_{i=0}^{\infty} \sum_{j=1}^{\infty} \frac{1}{j} \cdot (sr^i)^j < \infty.$$

Since this sum has only positive terms, we get

$$(6) \quad -ln[1 - W(s, r)] = \sum_{j=1}^{\infty} \frac{1}{j} \cdot s^j \cdot \sum_{i=0}^{\infty} (r^j)^i = \sum_{j=1}^{\infty} \frac{1}{j} \cdot \frac{s^j}{1 - r^j}.$$

According to the *Criterion of d'Alembert* (or *ratio test*) (Apostol, 1974, p. 199), if $U_n \geqq 0$ for $n = 1, 2, \ldots$, then the sum

$$\sum^{\infty} U_n$$

converges provided that, for some N and ρ,

$$\frac{U_{n+1}}{U_n} \leqq \rho < 1 \text{ for all } n > N.$$

In our case we have

$$\frac{U_{n+1}}{U_n} = s \cdot \frac{n}{n+1} \cdot \frac{1 - r^n}{1 - r^{n+1}} < s \leqq 1,$$

which proves the first assertion of Theorem 2.

From (6) we get, by taking into account the first term only,

$$-ln[1 - W(s, r)] > \frac{s}{1 - r},$$

which is equivalent to the left-hand inequality of (2). Because

$$(1 - r)^j < 1 - r^j \text{ for } j \leqq 2 \text{ and } 0 < r < 1,$$

which can be proven easily by induction, (6) yields

$$-ln[1 - W(s, r)] < \Sigma \frac{1}{j} \cdot \left(\frac{s}{1 - r}\right)^j = -ln\left(1 - \frac{s}{1 - r}\right),$$

which is equivalent to

$$W(s, r) < \frac{s}{1 - r}.$$

This completes the proof of Theorem 2.

For Theorem 3, note first that, for p_i as defined by (4), $\lim_{i \to \infty} p_i$ must exist because p_i is a positive, decreasing sequence. From (4), it follows that

$$\lim_{i \to \infty} -ln\left[\frac{p_i}{s}\right] = \sum_{j=0}^{\infty} \left[-ln(1 - \bar{r}q^j)\right].$$

Comparison with (5) shows that the argument above applies to this summation, demonstrating that

$$\sum_{j=0}^{\infty} \left[-ln(1 - \bar{r}q^j)\right] < \infty$$

and therefore that $\lim_{i \to \infty} p_i > 0$. But now Theorem 1 implies that $W = \lim_{n \to \infty} W_n = 1$.

NOTES

1. Vulnerabilities in C[3]I are discussed in Blair (1985), Ford (1985), and Bracken (1983). The relationship of 'security reliability' to the probability of nuclear war is analyzed in Cioffi-Revilla (1987).
2. Questions of independence do not arise because we are interested only in the occurrence of the *first* war. Each of the probabilities we take as given, called a 'probability of war per year', is the probability of war in a year *conditional* on there having been no previous war.
3. This tends to be the assessment of specialists. See Nye, Allison & Carnesale (1985, p. 207), who add, 'Not much is proven by this finding. In this realm there are "specialists" but not experts.' Bracken (1985, p. 50) offers a somewhat higher range of estimates of the probability of *accidental* nuclear war (averaging about 10% over the next ten years).

REFERENCES

Apostol, Tom M., 1974. *Mathematical Analysis*, 2nd ed. Reading, Mass: Addison-Wesley.

Blair, Bruce G., 1985. *Strategic Command and Control: Redefining the Nuclear Threat.* Washington, D.C.: Brookings.

Bracken, Paul, 1985. 'Accidental Nuclear War', pp. 25–53 in Graham Allison, Albert Carnesale and Joseph S. Nye, Jr., eds., *Hawks, Doves, and Owls: An Agenda for Avoiding Nuclear War.* New York: Norton.

Bracken, Paul, 1983. *The Command and Control of Nuclear Forces.* New Haven, Conn.: Yale University Press.

Cioffi-Revilla, Claudio, 1987. 'Crises, War and Security Reliability', pp. 49–66 in Claudio Cioffi-Revilla, Richard L. Merritt & Dina A. Zinnes, eds., *Interaction and Communication in Global Politics.* London: SAGE.

Ford, Daniel, 1985. *The Button: The Pentagon's Command and Control System.* New York: Simon & Schuster.

Garwin, Richard, 1985. 'Forum: Is there a Way Out?' *Harper's*, vol. 270, no. 1621 (June), pp. 35–47.

Iklé, Fred Charles, 1973. 'Can Nuclear Deterrence Last Out the Century?' *Foreign Affairs*, vol. 51, no. 2 (January), pp. 267–285.

Lebow, Richard Ned, 1987. *Nuclear Crisis Management: A Dangerous Illusion.* Ithaca, N.Y.: Cornell University Press.

Nye, Joseph S., 1987. 'Nuclear Learning and U.S.–Soviet Security Regimes', *International*

Organization, vol. 41, no. 3 (Summer), pp. 205–222.

Nye, Joseph S., Graham T. Allison & Albert Carnesale, 1985. 'Analytic Conclusions: Hawks, Doves, and Owls', pp. 205–222 in Graham Allison, Albert Carnesale & Joseph S. Nye, Jr., eds., *Hawks, Doves, and Owls: An Agenda for Avoiding Nuclear War*. New York: Norton.

Quester, George H., 1986. *The Future of Nuclear Deterrence*. Lexington, Mass.: Lexington.

Rohatgi, V. K., 1976. *An Introduction to Probability Theory and Mathematical Statistics*. New York: Wiley.

Sorensen, Theodore C., 1965. *Kennedy*. New York: Harper & Row.

RUDOLF AVENHAUS, b. 1938, PhD in Theoretical Physics (University of Karlsruhe, 1967); Habilitation in Statistics (University of Mannheim, 1975); Professor of Statistics and Operations Research at German Armed Forces University Munich (1980–). Most recent book: *Safeguards Systems Analysis* (New York: Plenum, 1986).

STEVEN J. BRAMS, b. 1940, PhD in Political Science (Northwestern University, 1966); Professor, New York University (1969–). Most recent books: *Superpower Games: Applying Game Theory to Superpower Conflict* (Yale University Press, 1985), and, with D. Marc Kilgour, *Game Theory and National Security* (Oxford: Basil Blackwell, 1988).

JOHN FICHTNER, b. 1955, PhD in Mathematics (German Armed Forces University Munich, 1985); Research Assistant (1982–87); Mathematician, Siemens, Munich (1987–). Main interest: mathematical modelling of strategic technical systems.

D. MARC KILGOUR, b. 1946, PhD in Mathematics (University of Toronto, 1973); Professor of Mathematics, Wilfrid Laurier University (1973–); Adjunct Professor of Systems Design Engineering, University of Waterloo (1983–); author (with Steven J. Brams) of *Game Theory and National Security* (Oxford: Basil Blackwell, 1988).