

Problem 1A: Equations Explanation

Say that you have a gene  $P$  which produces a protein  $p$ . The starting material balance equations for the (1) mRNA made from gene  $P$  and then (2) translated to make protein  $p$  are:

$$(1) \dot{m} = r_X u - (\mu + \theta_m)m + \lambda$$

$$(2) \dot{p} = r_L w - (\mu + \theta_p)p$$

Where  $\dot{m}$  and  $\dot{p}$  are the rate of concentration change in mRNA and protein, respectively, and  $r_X$  and  $r_L$  are the regulated specific rates of transcription and translation. The  $u$  and  $w$  terms describe the control logic of the cell for transcription and translation. The terms  $\theta_m$  and  $\theta_p$  govern the rate of lumped non-specific degradation mechanisms for transcription and translation, and the  $\lambda$  term is the unregulated rate of transcription.

Of particular concern in this case is the dilution rate,  $\mu$ , that is due to cell growth. We are considering a cell-free system that is contained in an abstract volume  $B$ . Therefore the dilution term is zero. The resulting material balance equations are:

$$(3) \dot{m} = r_X u - \theta_m m + \lambda$$

$$(4) \dot{p} = r_L w - \theta_p p$$