AIC-4301C Artificial Intelligence
Part 2

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2022 - 2023

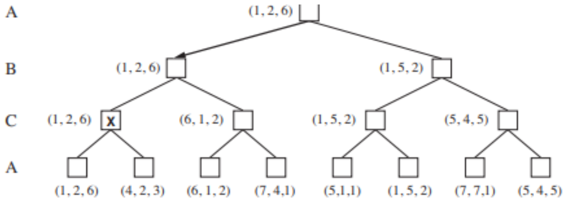
Plan

Mixed Layer Games

- Many games don't follow the exact pattern of alternation maximizer/minimizer (minimax) and maximizer/chance (expectimax) nodes
- In Pacman, after Pacman moves, there are usually multiple ghosts that take turns making moves, not a single ghost.
- Can be done by having a maximizer layer followed by consecutive ghost/minimizer layers before the second Pacman/maximizer layer.

General Games

- Different agents may have actions in a game that are not competing with another.
- Such games can be set up with trees characterized by multi-agent utilities.
- Each node is labeled with values from the viewpoint of each player.
- Multiplayer games usually involve alliances, whether formal or informal, among the players.



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Markov Decision Processes

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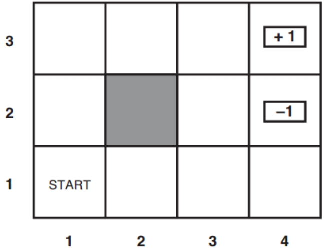
- We changed our model to account for **adversaries**.
- Now, we change our model to account for the **dynamic of the environment**:
  - the environment may subject the agent's actions to being nondeterministic.
  - there are multiple possible successor states that can result from an action taken in some state.
  - problems where environment have a degree of uncertainty = **nondeterministic search problems**.
  - can be solved with models known as **Markov decision processes** = **MDPs**.

Markov Decision Processes



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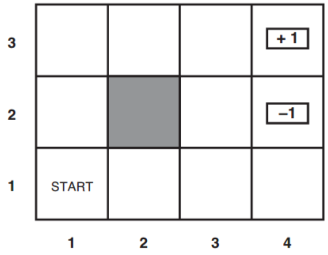

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- **Actions**: UP, DOWN, LEFT, RIGHT.

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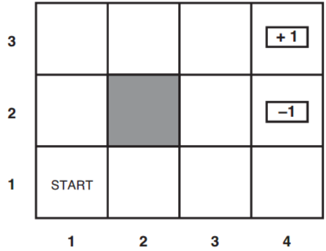

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- **Actions**: UP, DOWN, LEFT, RIGHT.
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  - the "intended" successor occurs with probability 0.8.
  - the agent moves at right angles to the intended direction with probability 0.2.
  - collision with a wall results in no movement.

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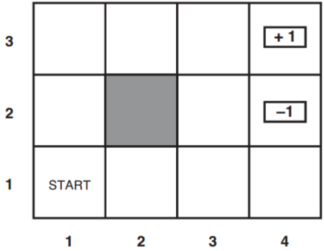

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- **Transition system**:
  - the "intended" successor occurs with probability 0.8.
  - the agent moves at right angles to the intended direction with probability 0.2.
  - collision with a wall results in no movement.
- **Goal**: two goal states have **reward** $+1$ and -1, respectively, and all other states have a reward of -0.04.

Markov Decision Processes

An MDP is defined by:

- S: a set of states
- ACTION: set of actions.
- s_0: initial state.
- T(s, a, s'): a transition function with **P(s'/ s, a)** the conditional probability that the action a from s leads to s'.
- R(s, a, s'): a **reward** function, sometimes defined only for states, R(s).
- A terminal state (not always defined).

MDP Example

A robot car wants to travel far and quickly:

- Three states: Cool, Warm, Overheated.
- Initial state: Cool.
- Two actions: Slow, Fast.
- Going fast gets reward $+2$: $R(s, Fast, s') = +2$
- Going slow gets reward $+1$: $R(s, Slow, s') = +1$

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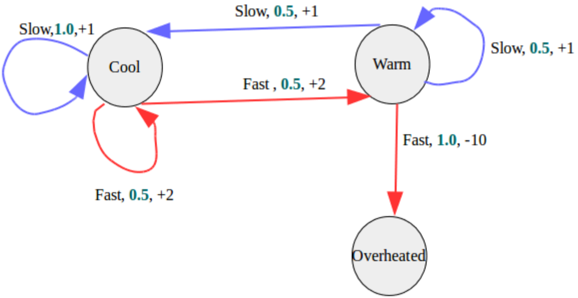
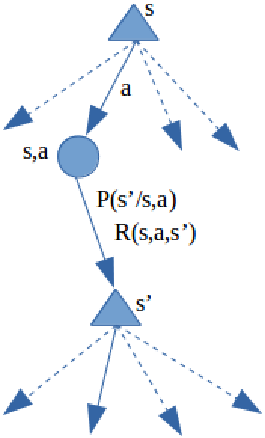
Transition Function:



MDP Search Tree

- **s** is a state.
- **(s, a)** is a q-state = take the action a from state s.



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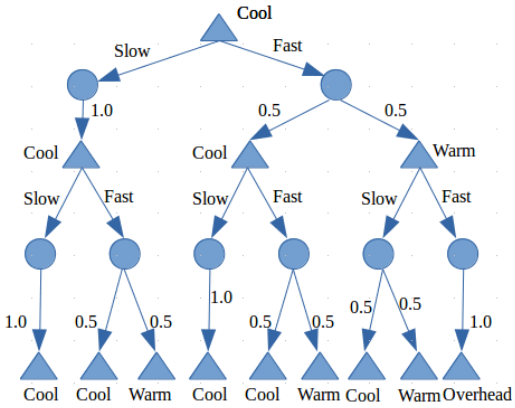

MDP Sequences

- An agent goes through different MDP states over time with discrete timesteps.
- $s_t \in S$ the state in which an agent is at timestep t.
- $a_t \in ACTION(s_t)$ the action which an agent takes at timestep t.
- s_0: initial state.
- The execution of an agent through a MDP is modeled as:

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \dots$$

- We call $[s_0, a_0, s_1, a_1, s_2, a_2 \dots]$ a sequence.

Markov ?

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- Formally:
  - S_t denotes the random variable representing agent's state at time t.
  - A_t denotes the random variable representing the action the agent takes at time t.
  - $P(S_{t+1} = s_{t+1} / S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, \ldots, S_0 = s_0)$

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- Expectimax didn't compute entire policies, it computes the action to take for a single state only.

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$$\forall k > 0, U([s_0, a_0, s_1, a_1, \ldots, s_{N+k}]) = U([s_0, a_0, s_1, a_1, \ldots, s_N])$$

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- Under stationarity there are just two ways to assign utilities to sequences: Additive utility and Discounted utility.

Additive utility

$$U([s_0, a_0, s_1, a_1, s_2, \ldots]) = R(s_0, a_0, s_1) + R(s_1, a_1, s_2) + R(s_2, a_2, s_3) + \ldots$$

Discounted utility

$U([s_0, a_0, s_1, a_1, s_2, \ldots]) = R(s_0, a_0, s_1) + \gamma R(s_1, a_1, s_2) + \gamma^2 R(s_2, a_2, s_3) + \ldots$
where $|\gamma| \leq 1$ is the discount factor.

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- Discounting appears to be a good model of human preferences over time.

Discounted utility

$$U([s_0, a_0, s_1, \ldots]) \leq R_{max}/(1-\gamma) \text{ } if \text{ } \gamma < 1$$

where R_{max} is the maximum possible reward reachable at any given timestep in the MDP.

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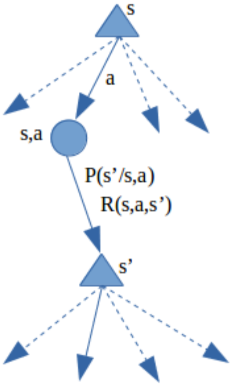
$$\begin{array}{rcl} U([s_0, a_0, s_1, \ldots]) & = & R(s_0, a_0, s_1) + \gamma R(s_1, a_1, s_2) + \gamma^2 R(s_2, a_2, s_3) + \ldots \\ & = & \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t, s_{t+1}) \\ & \leq & \sum_{t=0}^{\infty} \gamma^t R_{max} \\ & \leq & R_{max}(1-\gamma^{\infty})/(1-\gamma) \\ & \leq & R_{max}/(1-\gamma) \; if \; \gamma < 1 \end{array}$$

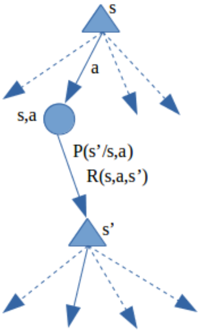
Optimality

- $V^*(s)$: expected utility starting in s and acting optimally.
- $Q^*(s,a)$: expected utility having taken action a from state s and (thereafter) acting optimally.
- $\pi^*(s)$: optimal action from state s.



Optimality Recursive Equations

- $V^*(s) = max_a Q^*(s, a)$
- $Q^*(s, a) = \sum_{s'} T(s, a, s')[R(s, a, s') + \gamma V^*(s')]$



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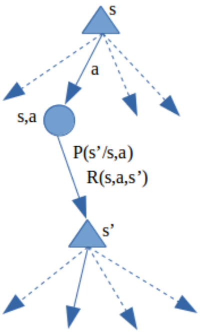

The Bellman Equation:

$$V^*(s) = max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

Value Iteration (An algorithm to solve MDPs)

A dynamic programming algorithm that uses an iteratively longer time limit to compute time-limited values until convergence ($\forall s, V_{k+1}(s) = V_k(s)$).

1. $\forall s \in S, V_0(s) = 0$
2. Repeat the following update rule until convergence:
   $\forall s \in S, V_{k+1}(s) = max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V_k(s')]$

Value Iteration Example

$V_1(cool) = max\{1 \times [1 + 0.5 \times 0], 0.5 \times [2 + 0.5 \times 0] + 0.5 \times [2 + 0.5 \times 0]\} =$
$max\{1, 2\} = 2$
$V_1(warm) = max\{0.5 \times [1 + 0.5 \times 0] + 0.5 \times [1 + 0.5 \times 0], 1 \times [-10 + 0.5 \times 0]\} =$
$max\{1, -10\} = 1$
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$V_2(cool) = max\{1 \times [1 + 0.5 \times 2], 0.5 \times [2 + 0.5 \times 2] + 0.5 \times [2 + 0.5 \times 1]\} =$
$max\{2, 2.75\} = 2.75$
$V_2(warm) = max\{0.5 \times [1 + 0.5 \times 2] + 0.5 \times [1 + 0.5 \times 1], 1 \times [-10 + 0.5 \times 0]\} =$
$max\{1.75, -10\} = 1.75$
$V_2(overheated) = max\{\} = 0$

Policy Extraction

- In every state take the action a which yields the maximum expected utility.
- a is the action which takes us to the q-state with maximum q-value: $\forall s \in S$,

$$\pi^*(s) = argmax_a Q^*(s,a) = argmax_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

we must recompute all necessary q-values with the Bellman equation before applying argmax.

Complexity

- At each iteration, we must update the values of all $|S|$ states, each of which requires iteration over all $|A|$ actions as we compute the q-value for each action.
- The computation of each of these q-values, in turn, requires iteration over each of the $|S|$ states again
- Time complexity $= O(|S|^2|A|)$.

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- Repeat steps until policy converges.

Policy Iteration Algorithm

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$$V^{\pi_i}(s) = \cancel{max_a} \sum_{s'} T(s, \cancel{a}\ \pi_i(s), s')[R(s, \cancel{a}\ \pi_i(s), s') + \gamma V^{\pi_i}(s')$$

$V^{\pi_i}(s)$ can be computed by:

- Method 1: solving the generated system of $|S|$ equations system.
- Method 2: using the following update rule until convergence
  $V^{\pi_i}_{k+1}(s) = \sum_{s'} T(s, \pi_i(s), s')[R(s, \pi_i(s), s') + \gamma V^{\pi_i}_k(s')]$
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        just like in value iteration, this method is typically slower in practice
   2. Policy improvement of π_i to generate a better policy π_{i+1}.
      Uses policy extraction on the values of states generated by policy evaluation.

      $$\pi_{i+1}(s) = argmax_a \sum_{s'} T(s, a, s')[R(s, a, s') + \gamma V^{\pi_i}(s')]$$

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2. Policy evaluation π_0:

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3. Policy improvement of π_0:

$$\begin{cases} \pi_1(cool) & = argmax\{slow : 1 \times [1 + 0.5 \times 2], \\ & fast : 0.5 \times [2 + 0.5 \times 2] + 0.5 \times [2 + 0.5 \times 2]\} = fast \\ \pi_1(warm) & = argmax\{slow : 0.5 \times [1 + 0.5 \times 2] + 0.5 \times [1 + 0.5 \times 2], \\ & fast : 1 \times [-10 + 0.5 \times 0]\} = slow \end{cases}$$

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$$\begin{cases} V^{\pi_0}(cool) & = 1 \times [1 + 0.5 \times V^{\pi_0}(cool)] \\ V^{\pi_0}(warm) & = 0.5 \times [1 + 0.5 \times V^{\pi_0}(cool)] + \\ & 0.5 \times [1 + 0.5 \times V^{\pi_0}(warm)] \end{cases}$$

$$V^{\pi_0}(cool) = 2, V^{\pi_0}(warm) = 2, V^{\pi_0}(overheated) = 0$$

3. Policy improvement of π_0:

$$\begin{cases} \pi_1(cool) & = argmax\{slow : 1 \times [1 + 0.5 \times 2], \\ & fast : 0.5 \times [2 + 0.5 \times 2] + 0.5 \times [2 + 0.5 \times 2]\} = fast \\ \pi_1(warm) & = argmax\{slow : 0.5 \times [1 + 0.5 \times 2] + 0.5 \times [1 + 0.5 \times 2], \\ & fast : 1 \times [-10 + 0.5 \times 0]\} = slow \end{cases}$$

$$\pi_1(cool) = fast, \pi_1(warm) = slow, \pi_1(overheated) = -$$

Policy Iteration Example

1. Initial policy π_0:

$$\pi_0(cool) = slow, \pi_0(warm) = slow, \pi_0(overheated) = -$$

2. Policy evaluation π_0:

$$\begin{cases} V^{\pi_0}(cool) & = 1 \times [1 + 0.5 \times V^{\pi_0}(cool)] \\ V^{\pi_0}(warm) & = 0.5 \times [1 + 0.5 \times V^{\pi_0}(cool)] + \\ & 0.5 \times [1 + 0.5 \times V^{\pi_0}(warm)] \end{cases}$$

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4. Policy evaluation π_1:

...

Plan

1. Adversarial Search
2. Markov Decision Processes
3. Reinforcement Learning

Machine Learning

- Supervised Learning:
  - Learning with labeled instances
  - Ex. : Decision Trees, Neural Networks, SVMS, ...
- Unsupervised Learning:
  - Learning without labels
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- Reinforcement Learning:
  - Learning with rewards
  - App. : robots, autonomous vehicles, ...

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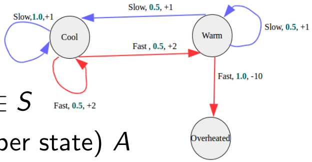
- In reinforcement learning we assume no knowledge of the environment and the reward function.

Reinforcement Learning

A Markov decision process (MDP):



- A set of states $s \in S$
- A set of actions (per state) A
- A model $T(s, a, s')$
- A reward function $R(s, a, s')$
- Looking for a policy $\pi(s)$

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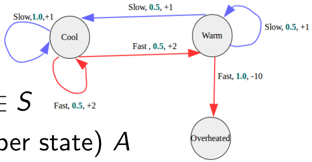

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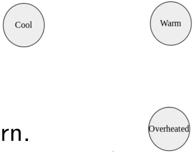

- A MDP.
- Don't know T or R.
- Must try actions to learn.

Exploration

- Exploration step: At each time step an agent starts in a state s, then takes an action a and ends up in a successor state s' reaching some reward r until arriving at a terminal state.

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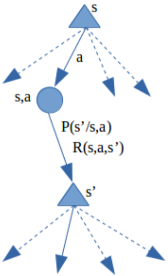
Agents go through many episodes during exploration in order to collect sufficient data needed for learning.

Model-based/ Model-free Learning

- Model-based learning: estimate the transition and reward functions with the samples reached during exploration before using these estimates to solve the MDP normally with value or policy iteration.
- Model-free learning: estimate the utilities ($V(s)$) or q-utilities ($Q(s, a)$) without ever constructing a model of the rewards and transitions in the MDP.

Model-based Learning

Agent generates an approximation $\hat{T}(s, a, s')$ of $T(s, a, s')$, by counting during the exploration the number of times it arrives in each state s' after entering each q-state (s, a).



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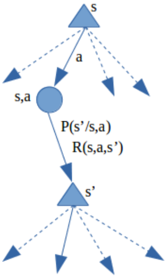

Step 1: Learn empirical MDP model

- Exploration
- Count outcomes s' for each s, a
- Normalize to give an estimate of $\hat{T}(s, a, s')$
- Discover each $\hat{R}(s, a, s')$ when we explore (s, a, s')

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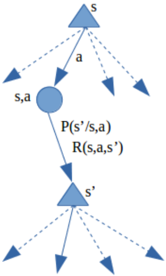

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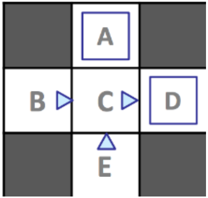
Step 2: Solve the learned MDP

Model-based Learning Example (with $\gamma = 1$)



Exploration	Learned MDP
Episode 1	$\hat{T}(s, a, s')$
B, east, C, -1	$\hat{T}(B, east, C) = 1.00$
C, east, D, -1	$\hat{T}(C, east, D) = 0.75$
D, exit, x, +10	$\hat{T}(C, east, A) = 0.25$
Episode 2	...
B, east, C, -1	$\hat{R}(s, a, s')$
C, east, D, -1	$\hat{R}(B, east, C) = -1$
D, exit, x, +10	$\hat{R}(C, east, D) = -1$
Episode 3	$\hat{R}(D, exit, x) = +10$
E, north, C, -1	...
C, east, D, -1	
D, exit, x, +10	
Episode 4	
E, north, C, -1	
C, east, A, -1	
A, exit, x, -10	

Model-free Learning

- Passive Reinforcement Learning: the agent is given a policy to follow and learns the utilities of states under that policy during exploration.
- Active Reinforcement Learning: the agent must also learn the policy.

Passive Reinforcement Learning

- Input: a fixed policy $\pi(s)$.
- You don't know the transitions $T(s, a, s')$.
- You don't know the rewards $R(s, a, s')$.
- Goal: learn the state utilities.

In this case:

- No choice about what actions to take (given policy).
- Just execute the policy and learn from experience.

Passive Reinforcement Learning

1. Direct Evaluation
2. Temporal Difference Learning

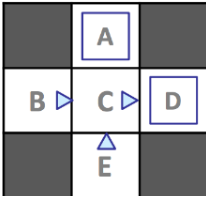
Direct Evaluation

Idea: Average observed sample utilities

- Act according to π
- Every time the agent visits a state, write down the sum of discounted utilities.
- Average those samples.

Direct Evaluation Example (with $\gamma = 1$)

Input Policy π



Exploration
Episode 1
B, east, C, -1
C, east, D, -1
D, exit, x, +10
Episode 2
B, east, C, -1
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Episode 4
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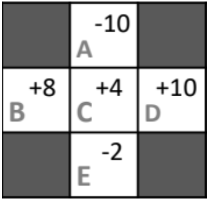
A, exit, x, -10

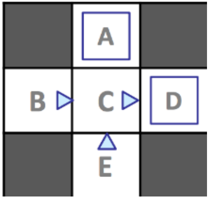
Output state utilities $V(s)$

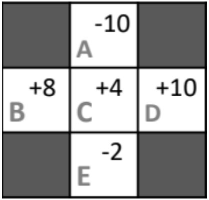
s	Total Utilities	Times Visited	$V^\pi(s)$
A	-10	1	-10
B	16	2	8
C	16	4	4
D	30	3	10
E	-4	2	-2



Direct Evaluation Problems

- It wastes information about state connections
- Each state must be learned separately, so it takes a long time to learn





If B and E both go to C under this policy, how can their values be different?

Temporal Difference Learning (TD learning)

Idea: learning from every experience rather than simply keeping track of total rewards and number of times states are visited and learning at the end as direct evaluation does.

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Reminder: policy evaluation uses the system of equations generated by a fixed policy π and the Bellman equation to determine the utilities of states under that policy.

$$V^{\pi}(s) = \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V^{\pi}(s')]$$

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$$V^\pi(s) = \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^\pi(s')]$$

- Uses exponential moving average: $\bar{x}_n = (\alpha - 1)\bar{x}_{n-1} + \alpha x_n$
  - $\alpha, 0 \leq \alpha \leq 1$ is a parameter known as the learning rate.
  - Makes recent samples more important.
  - Forgets about the past (distant past values were wrong anyway).

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Update $V^{\pi}(s)$ each time we experience a transition (s, a, s', r) during exploration using exponential moving.

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- compute a sample using (s, a, s', r)

$$\textit{sample} = R(s, \pi(s), s') + \gamma V^\pi(s')$$

- Update $V^\pi(s)$ using exponential moving average:
$V^\pi(s) = (1 - \alpha)V^\pi(s) + \alpha \ \textit{sample}$

Temporal Difference Learning Algorithm

1. Start $V^\pi(s) = 0, \forall s \in S$
2. Compute a sample:

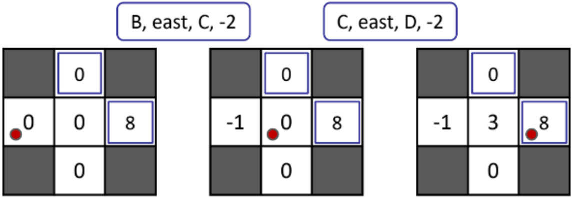
$$sample = R(s, \pi(s), s') + \gamma V^\pi(s')$$

3. Update $V^\pi(s)$: $V^\pi(s) = (1-\alpha)V^\pi(s) + \alpha\ sample$

It is typical to start with learning rate of $\alpha = 1$ and slowly shrinking it towards 0.

Temporal Difference Learning Example

$$\alpha = 0.5, \gamma = 1$$



$$V^\pi(s) = (1-\alpha)V^\pi(s) + \alpha\,(R(s,\pi(s),s') + \gamma V^\pi(s'))$$

Problems with Passive Reinforcement Learning

1. Learn the utilities value of all states under a given policy.
2. Finding an optimal policy for our agent requires knowledge of the q-utilities of states.
3. Computing q-utilities from the state utilities requires a transition function and reward function as dictated by the Bellman equation.

$$Q^*(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

4. Passive Reinforcement Learning is used in tandem with some model-based learning to acquire estimates of T and R in order to effectively update the policy followed by the learning agent.

Active Reinforcement Learning: Q-learning

Idea of Q-learning: learning the q-utilities directly, as a result, Q-learning is entirely model-free.

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$$Q^*(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

$$V^*(s') = \max_{a'} Q^*(s',a')$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma \max_{a'} Q^*(s',a')]$$

Q-learning Algorithm

Q-learning uses the following update known as q-value iteration:

$$Q_{k+1}(s,a) = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')]$$

1. Compute a sample:

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

2. Update $Q(s,a)$: $Q(s,a) = (1-\alpha)Q(s,a) + \alpha\ sample$

Q-learning

Q-learning Algorithm

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- As long as we spend enough time in exploration and decrease the learning rate α at an appropriate pace, Q-learning learns the optimal Q-values for every Q-state
- TD learning and direct evaluation learn the values of states under a policy by following the policy before determining policy optimality via other techniques
- Q-learning can learn the optimal policy directly by taking suboptimal or random actions.

Exploration: ϵ-greedy

1. Simplest scheme for exploration: Random actions (ϵ-greedy)
    1. Select n, a random uniform number in $[0, 1]$
    2. If $n < \epsilon$, act randomly (with probability ϵ)
    3. If $n \geq \epsilon$, act on current policy (with probability $1 - \epsilon$)

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   3. If $n \geq \epsilon$, act on current policy (with probability $1 - \epsilon$)
2. Problems with random actions?
   1. For a large ϵ: even after learning the optimal policy, the agent will still behave mostly randomly
   2. For a small ϵ: the agent will explore infrequently, leading Q-learning to learn the optimal policy very slowly.
   3. One solution: lower ϵ over time
   4. Another solution: exploration functions

Exploration: Exploration functions

- Manually tuning ϵ is avoided by exploration functions
- Use a modified Q-value iteration update to give some preference to visiting less-visited states.

$$\textit{Regular } Q\textit{-Update}: \; Q(s,a) = (1-\alpha)Q(s,a) + \alpha\left[R(s,a,s') + \gamma \max_{a'} Q(s',a')\right]$$

$$\textit{Modified } Q\textit{-Update}: \; Q(s,a) = (1-\alpha)Q(s,a) + \alpha\left[R(s,a,s') + \gamma \max_{a'} f(s',a')\right]$$

f is an exploration function, with a common choice of f is

$$f(s,a) = Q(s,a) + \frac{k}{N(s,a)}$$

with k predetermined value and $N(s,a)$ is the number of times Q-state (s,a) has been visited

Exploration: Exploration functions

- A state s select the action that has the highest $f(s, a)$ from each state
- Agents never make a probabilistic decision between exploration and exploitation
- Exploration is encoded by the exploration function, since $\frac{k}{N(s,a)}$ give a "bonus" to some infrequently taken action
- This bonus decreases when As time goes states are visited more frequently state and $f(s, a)$ regresses towards $Q(s, a)$, making exploitation more exclusive.

Approximate Q-learning

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Idea of approximate Q-learning:

- Represents each state as feature vector $(f_1(s,a), f_2(s,a), \ldots)$
  For example a feature vector for Pacman may encode: the distance to the closest ghost, the distance to the closest food pellet, the number of ghosts.
- Store a single weight vector to compute approximate Q-values. For example for linear Q-functions

$$Q(s,a) = w_1 \times f_1(s,a) + w_2 \times f_2(s,a) + \ldots + w_n \times f_n(s,a)$$

- Compute Q-values on-demand as needed.

Approximate Q-learning for linear Q-functions

1. Difference:

$$difference = [R(s, a, s') + \gamma \max_{a'} Q(s', a')] - Q(s, a)$$

2. Approximate Q-learning works almost identically to Q-learning, using the following update rule:

$$Approximate\ Q(s, a) : w_i = w_i + \alpha \times difference \times f_i(s, a)$$

   - Rather than storing Q-values for each state, approximate Q-learning only need to store a single weight vector and can compute Q-values on-demand as needed.
   - Significantly more memory efficient

3.

$$Exact\ Q(s, a) : Q(s, a) = Q(s, a) + \alpha \times difference$$