

CS1632: Static Analysis, Part 3

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Symbolic Model Checking

- Model checking can be categorized into:
 1. Enumerative model checking
 - What we learned in the last chapter
 - Hard to escape state explosion
 2. Symbolic model checking
 - What we will learn in this chapter
 - Model checking using *symbolic execution*
 - Can fundamentally solve the state explosion problem

Symbolic Execution

- **Symbolic value**: Math symbol that stands for an *input value*
 - A, B, \dots, X, Y, Z
- **Symbolic expression**: An expression using *symbolic values*
 - $A + 1, A * B, \dots$
- **Symbolic execution**: Assigning *symbolic expressions* instead of actual values to variables during execution
 - Instead of assigning values like: $x = 10, y = 20$
 - Assign symbolic expressions like: $x = A + 1, y = A * B$
- Example: let's say $x == A+1$ and $y == A+2$ at a given point
 - We can prove that `assert(x < y)` always passes. **Why?**
 $\text{assert}(x < y) \equiv \text{assert}(A+1 < A+2) \equiv \text{assert}(0 < 1)$
for all possible input values A

Notation We Will Use

- Program variables: lower case

- `int x, y, z;`

- Symbolic values: UPPER CASE

- `A, B, ..., X, Y, Z`

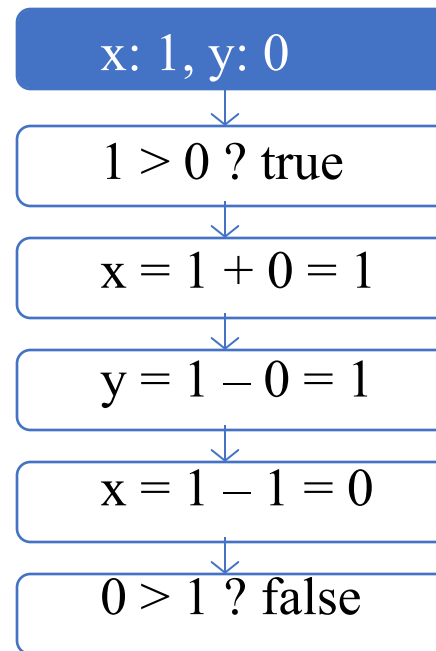
- Represent input values that are given to the code
 - Can be values from user input
 - Can be values from command line arguments
 - Can be values passed into method parameters

Example: Enumerative Model Checking

Code that swaps 2 integers

```
int x, y; // parameters
if (x > y) {
    x = x + y;
    y = x - y;
    x = x - y;
    if (x > y)
        assert false;
}
```

Execution Path for x=1, y=0



- Must do this for all values of x and y.
- But is that how a human would do it?

Symbolic Model Checking

- Trace through a program like a human being would
- In a symbolic execution:
 - Inputs are *symbolic values* instead of concrete data values

- Example:

```
int x, y; // parameters
```

```
x = x + y;
```

```
y = x - y;
```

```
x = x - y;
```

[Code]

x: 1, y: 0

x = 1 + 0 = 1

y = 1 - 0 = 1

x = 1 - 1 = 0

[Concrete]

x: A, y: B

x = A + B

y = A + B - B = A

x = A + B - A = B

[Symbolic]

- Symbolic execution proves that the swap works for all A and B!

Symbolic Model Checking

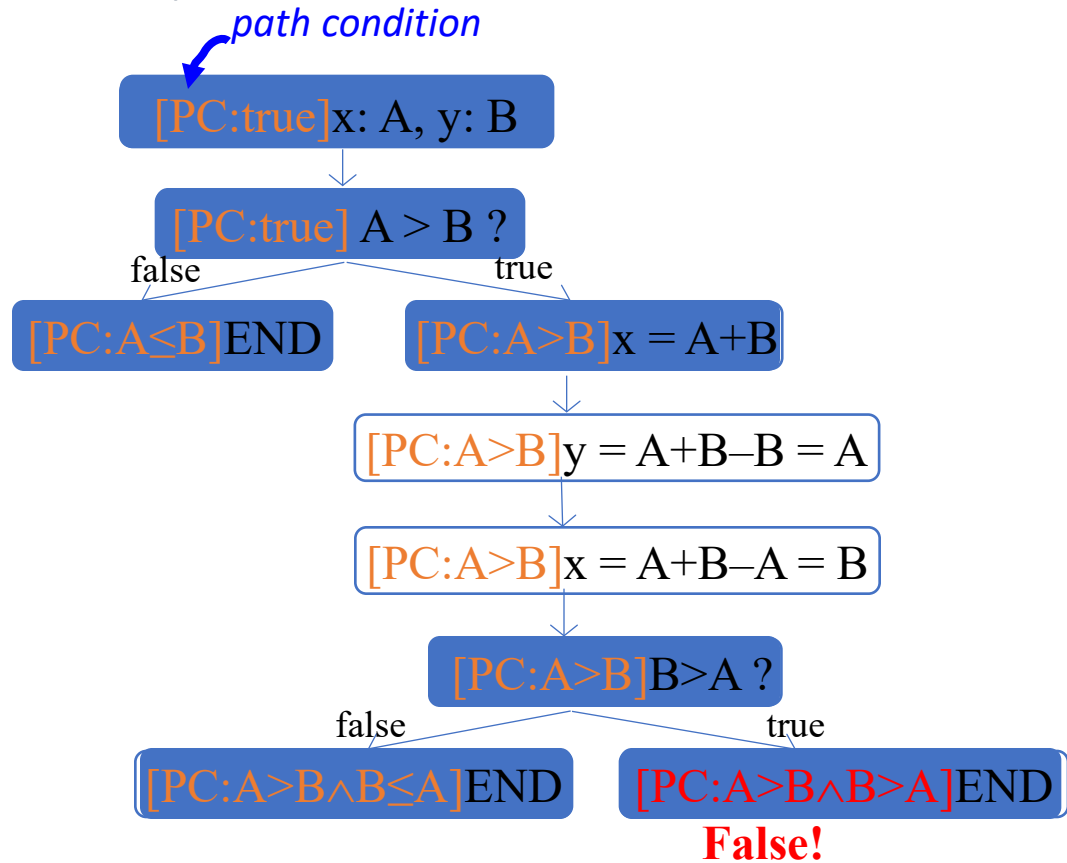
- What if there is path divergence?
 - if statement
 - for loop
 - while loop
- For each path, build a **Path Condition (PC)**
 - Condition on symbolic values (the As and the Bs)

Example: Symbolic Execution

Code that swaps 2 integers:

```
int x, y;  
  
if (x > y) {  
    x = x + y;  
    y = x - y;  
    x = x - y;  
    if (x > y)  
        assert false;  
}
```

Symbolic Execution Tree:



Is the Path Condition Feasible?

- Each path condition is checked using a constraint solver



- If path is infeasible, does not continue down that path
 - Hence, **assert false** is never reached

Symbolic Model Checking Uses

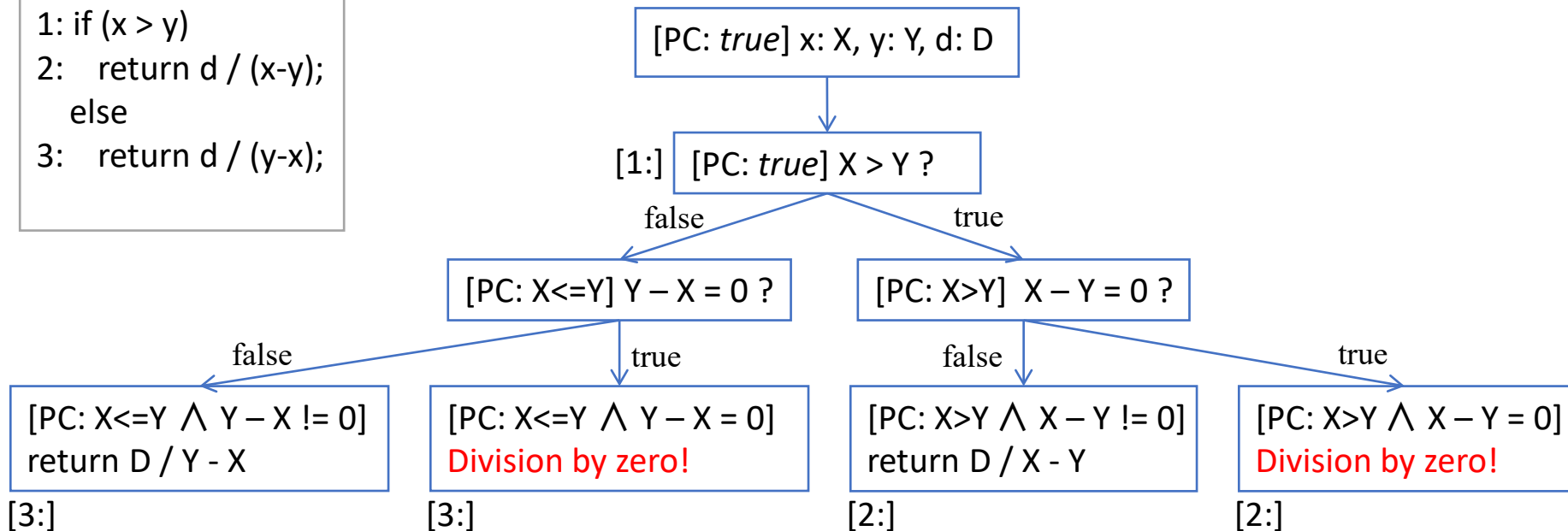
- Prove a program correct
 - Much less state explosion than enumerative checking
- Generate test vectors that exercise each path
 - For the purposes of increasing test coverage of code
 - The output sets of the constraint solver are the test vectors
- Generate program invariants
 - Invariants enhance programmer's understanding of code
 - The path conditions themselves *are* the invariants

Generating Test Cases out of Path Conditions

Method $m(x, y, d)$:

```
1: if (x > y)
2:   return d / (x-y);
   else
3:   return d / (y-x);
```

Symbolic execution tree:



Solve path conditions → test inputs

Auto-generated JUnit Tests

- Constraint solver returns values satisfying each Path Condition
- ➡ Can achieve full code coverage by exercising each path!

```
@Test public void t1() {
    m(10, 20, 0);
}
```

Pass ✓ PC: $X \leq Y \wedge Y - X \neq 0 \Leftrightarrow X < Y$
 $(X=10, Y=20, D=0) \in \text{PC}$

```
@Test public void t2() {      Fail X PC:  $X \leq Y \wedge Y - X = 0 \Leftrightarrow X = Y$   
    m(10, 10, 0);            $(X=10, Y=10, D=0) \in PC$   
}
```

```
@Test public void t3() {      Pass ✓ PC:  $X > Y \wedge X - Y \neq 0 \Leftrightarrow X > Y$   
    m(20, 10, 0);           (X=20, Y=10, D=0)  $\in$  PC  
}
```

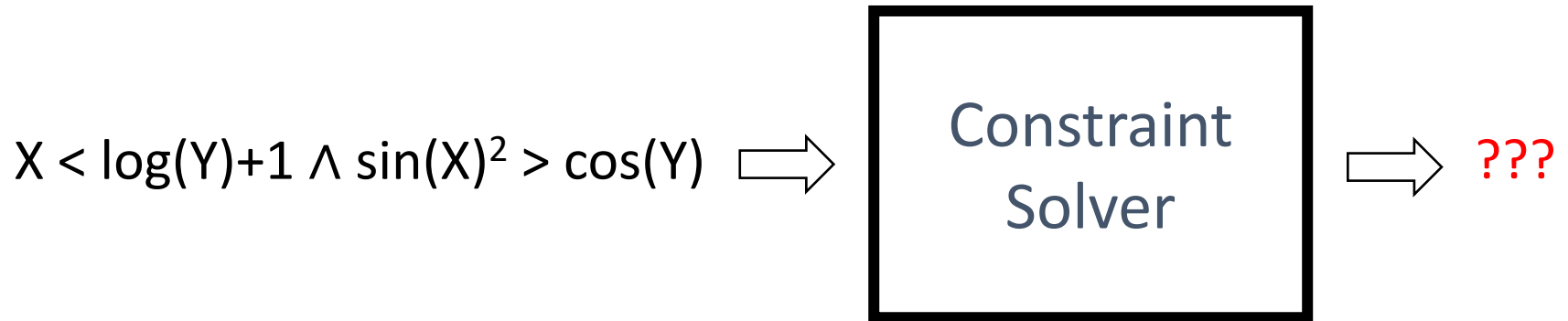
Skip  PC: $X > Y \wedge X - Y = 0 \Leftrightarrow \{ \}$

Symbolic Model Checking Challenges

- Symbolic model checking does have challenges
 - ... Or everyone would be using symbolic model checking
- Some examples are:
 - Complex math constraints
 - Complex data structures

Challenges: Complex Math Constraints

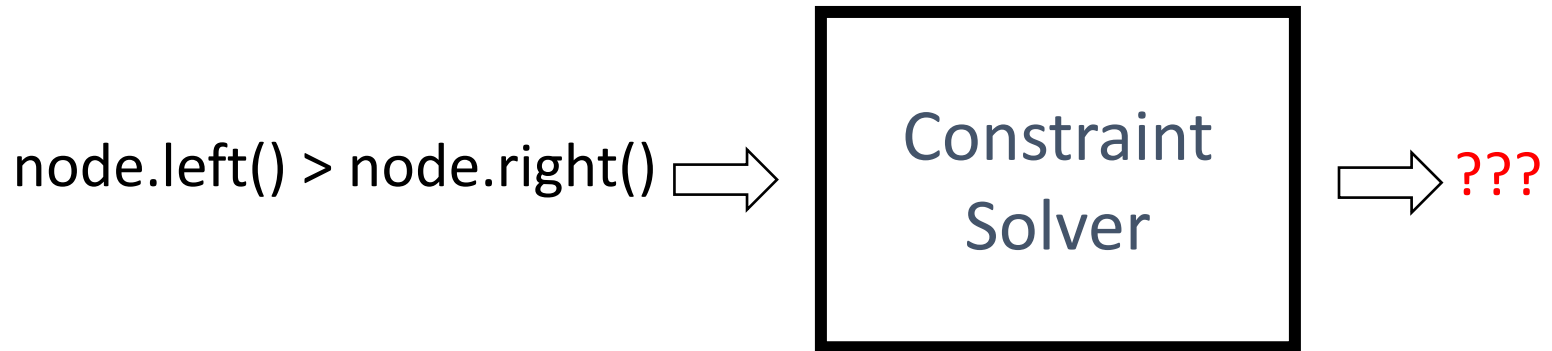
- Constraint solvers are not particularly good at math



- Constraint solving is NP-Complete in general
 - Research resulted in some subproblems being proved polynomial
 - State-of-the-art solvers have trouble when constraints are non-linear

Challenges: Complex Data Structures

- Say “node” is a tree node with two children, left() and right()



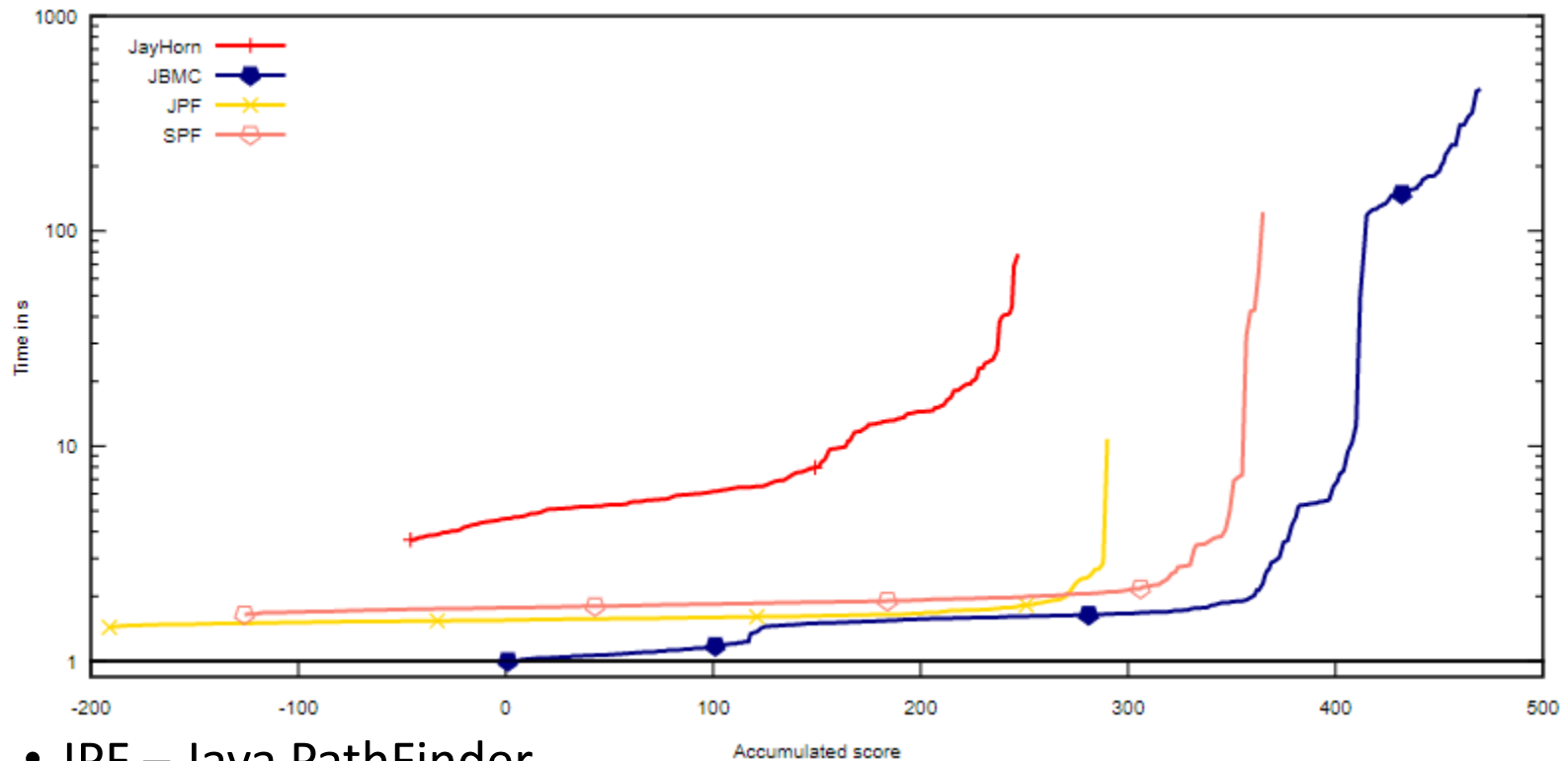
- In order to solve above constraint, solver must know:
 - What the tree data structure looks like
 - Where in the tree “node” is
- Solvers understand some data structures, but not many

The Best of Both Worlds

- Symbolic Model Checking (Symbolic Execution)
 - + Much less state explosion
 - Hard time dealing with loops, math, data structures
- Enumerative Model Checking (Concrete Execution)
 - Serious state explosion
 - + No problems with complex math or data structures
- The best of both worlds: Concolic Execution
 - Concolic = Concrete + Symbolic
 - Symbolic Java Path Finder does exactly this!
 - Chooses between the two depending on the method

Model Checking is Getting Better Every Year

<https://sv-comp.sosy-lab.org/2019/results/results-verified/>



- JPF – Java PathFinder
- SPF – Symbolic Java PathFinder (JPF with symbolic execution)
- JBMC – Java Bounded Model Checker (2018 newcomer)

References

- Ranjit Jhala and Rupak Majumdar. 2009. “Software model checking”. ACM Computing Surveys: <https://people.mpi-sws.org/~rupak/Papers/SoftwareModelChecking.pdf>
- Cristian Cadar and Koushik Sen. 2013. “Symbolic execution for software testing: three decades later”. Communications of the ACM: <https://people.eecs.berkeley.edu/~ksen/papers/cacm13.pdf>
- 8th Competition on Software Verification (SV-COMP), 2025: <https://sv-comp.sosy-lab.org/2025/results/results-verified/>