Assignment #1, Part 4 (final part)

(Course: CS 301, 1 problem, Total points: 20)

For regular students, the deadline is October 8, 2024, in class. For special needs students, the deadline is October 15, 2024, in class. No late assignments will normally be accepted.

Special note: Any answer that is not sufficiently clear even after a reasonably careful reading will not be considered a correct answer, and only what is written in the answer will be used to verify accuracy. No hand waiving, vague descriptions or sufficiently ambiguous statements that can be interpreted in multiple ways will be considered as a correct answer, nor will the student be allowed to add any explanations to his/her answer after it has been submitted.

Problem 1 (20 points): Show that for any two regular expressions **r** and **s** the following is true:

$$(\mathbf{r}^* \mathbf{s}^*)^* = (\mathbf{r} \cup \mathbf{s})^*$$

For example, taking $\mathbf{r} = 10^*11(0 \cup 1)$ and $\mathbf{s} = (1 \cup \varepsilon)11^*(0 \cup 1)$, the above equality implies that

$$\Big(\big(\underbrace{10^*11(0 \cup 1)}_{\mathbf{r}} \big)^* \big(\underbrace{(1 \cup \varepsilon)11^*(0 \cup 1)}_{\mathbf{s}} \big)^* \Big)^* = \Big(\big(\underbrace{10^*11(0 \cup 1)}_{\mathbf{r}} \big) \cup \big(\underbrace{(1 \cup \varepsilon)11^*(0 \cup 1)}_{\mathbf{s}} \big) \Big)^*$$

Note that proof by example and proof by vague-o-logy (i.e., vague descriptions) are not valid proof methods.

Hint: Show that $(\mathbf{r}^* \mathbf{s}^*)^* \subseteq (\mathbf{r} + \mathbf{s})^*$ and $(\mathbf{r}^* \mathbf{s}^*)^* \subseteq (\mathbf{r} + \mathbf{s})^*$. In other words, show that:

- If a string is in $(\mathbf{r}^*\mathbf{s}^*)^*$ then the string is also in $(\mathbf{r} + \mathbf{s})^*$.
- If a string is in $(\mathbf{r} + \mathbf{s})^*$ then the string is also in $(\mathbf{r}^* \mathbf{s}^*)^*$.