CS 301

Solutions of Assignment #1 Part 4

Problem 1:

We will prove our result by showing $(\mathbf{r}^*\mathbf{s}^*)^* \subseteq (\mathbf{r} \cup \mathbf{s})^*$ and $(\mathbf{r} \cup \mathbf{s})^* \subseteq (\mathbf{r}^*\mathbf{s}^*)^*$.

Proof of $(\mathbf{r}^*\mathbf{s}^*)^* \subseteq (\mathbf{r} \cup \mathbf{s})^*$

This follows since $(\mathbf{r} \cup \mathbf{s})^*$ contains *every* possible regular expression involving **r** and **s**.

Proof of $(\mathbf{r} \cup \mathbf{s})^* \subseteq (\mathbf{r}^*\mathbf{s}^*)^*$

Consider a string $\alpha \in (\mathbf{r} \cup \mathbf{s})^*$. Note that α can be written as $\mathbf{r}^{a_1}\mathbf{s}^{b_1}\mathbf{r}^{a_2}\mathbf{s}^{b_2}\ldots\mathbf{r}^{a_k}\mathbf{s}^{b_k}$ for some integers $a_1, a_2, \ldots, a_k, b_1, b_2, \ldots, b_k \geq 0$. Now, for any $1 \leq j \leq k$, it hold that $\mathbf{r}^{a_j}\mathbf{s}^{b_j} \in \mathbf{r}^*\mathbf{s}^*$. Thus, it follows that $\alpha \in (\mathbf{r}^*\mathbf{s}^*)^k \subset (\mathbf{r}^*\mathbf{s}^*)^*$.