Assignment #2 Part 1 (Total points: 60)
(Course: CS 301)

For regular students, the deadline is **October 22**, Tuesday in class.

For special needs students, the deadline is **October 29**, Tuesday in class.

No late assignments will be accepted.

Special note: Any answer that is not sufficiently clear even after a reasonably careful reading will not be considered a correct answer, and only what is written in the answer will be used to verify accuracy. No hand waiving, vague descriptions or sufficiently ambiguous statements that can be interpreted in multiple ways will be considered as a correct answer, nor will the student be allowed to add any explanations to his/her answer after it has been submitted.

Problem 1 **(**30 **points):** Let L be a regular language over some alphabet Σ. Prove that the following language is also regular:

$$L_{\text{strange}} = \left\{ b_2\, b_1\, b_4\, b_3 \ldots b_{2n}\, b_{2n-1} \,\middle|\, b_1\, b_2 \ldots b_{2n-1}\, b_{2n} \in L \right\}$$

For example, if the language L over $\Sigma = \{0, 1, 2\}$ contains 01 21 11 then L_{strange} contains 10 12 11.

Optional hint: Since L is a regular language, there is a DFA M for L. Using M, think about constructing a new DFA M_{strange} for L_{strange} by processing input symbols in pairs.

Problem 2 **(**20 **points):** Let L be the following language over the alphabet $\Sigma = \{a, b\}$:

$$L = \left\{ a^m b^n \,|m \geq 1,\, n \geq 1,\, m \geq n^2 \right\}$$

For example, $0^{2^2}1^2 = 0^4 1^2 = 000011$ and $0^{3^2+2}1^3 = 0^{11}1^3 = 00000000000111$ is in L but $0^3 1^2 = 00011$ is not in L.

Using pumping lemma prove that L is *not* regular.

Optional hint: The pumping lemma states that if a string $z \in L$ is written as uvw where $|uv| \leq p$ and $|v| \geq 1$ where p is the constant in the pumping lemma, then $uv^i w$ must belong to L for all $i \geq 0$. **Note that** $i = 0$ **is allowed**.

Problem 3 **(**10 **points):** Write down a context-free grammar (CFG) for the following language L over alphabet $\Sigma = \{0, 1, \#\}$:

$$L = \left\{ 0^{n+3} \# 1^n \,\middle|\, n \geq 0 \right\}$$