Solutions of Assignment #2 Part 2 (Total points: 40)
(Course: CS 301)

For regular students, the deadline is **November 7**, Thursday in class.

For special needs students, the deadline is **November 14**, Thursday in class.

No late assignments will be accepted.

Special note: Any answer that is not sufficiently clear even after a reasonably careful reading will not be considered a correct answer, and only what is written in the answer will be used to verify accuracy. No hand waiving, vague descriptions or sufficiently ambiguous statements that can be interpreted in multiple ways will be considered as a correct answer, nor will the student be allowed to add any explanations to his/her answer after it has been submitted.

Problem 1 **(**20 **points):** Convert the following context-free grammar (CFG) to a pushdown automata (PDA) using the procedure discussed in class (the start symbol for the CFG is S, and the alphabet for the CFG is $\{\,1,\ 2,\ (,\ ),\ +\,\}$).

$$\begin{aligned} S &\rightarrow A \,|\, S + A \\ A &\rightarrow (S) \,|\, 1 \,|\, 2 \end{aligned}$$

Give the PDA either in a diagram form or by listing the 7-tuples

$$\Big(\underset{\text{set of states}}{Q},\ \underset{\substack{\text{input}\\\text{alphabet}}}{\Sigma},\ \underset{\substack{\text{stack}\\\text{alphabet}}}{\Gamma},\ \delta,\ \underset{\substack{\text{initial}\\\text{state}}}{q_0},\ \underset{\substack{\text{bottom}\\\text{stack}\\\text{symbol}}}{\$},\ \underset{\substack{\text{set of}\\\text{final}\\\text{states}}}{F}\Big)$$

Solution:

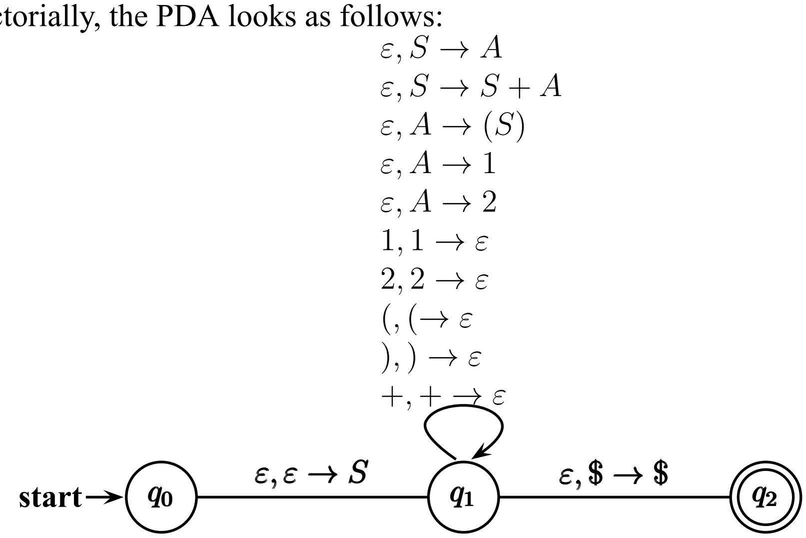
The PDA is $M = \Big(\underset{\text{set of states}}{\{q_0, q_1, q_2\}},\ \underset{\substack{\text{input}\\\text{alphabet}}}{\{\,1,\ 2,\ (,\ ),\ +\,\}},\ \underset{\substack{\text{stack}\\\text{alphabet}}}{\{S, A\}},\ \delta,\ \underset{\substack{\text{initial}\\\text{state}}}{q_0},\ \underset{\substack{\text{bottom}\\\text{stack}\\\text{symbol}}}{\$},\ \underset{\substack{\text{set of}\\\text{final}\\\text{states}}}{\{q_2\}}\Big)$ where

the transition function δ is given by:

current state, input, top of stack (pop), next state, push S

$$
\begin{aligned}
\delta(q_0, \varepsilon, \varepsilon) &= \{(q_1, S)\} \\
\delta(q_1, \varepsilon, S) &= \{(q_1, A),\ (q_1, S + A)\} \\
\delta(q_1, \varepsilon, A) &= \{(q_1, (S)),\ (q_1, 1),\ (q_1, 2)\} \\
\delta(q_1, a, a) &= \{(q_1, \varepsilon)\} \\
\delta(q_1, (, () &= \{(q_1, \varepsilon)\} \\
\delta(q_1,),)) &= \{(q_1, \varepsilon)\} \\
\delta(q_1, 1, 1) &= \{(q_1, \varepsilon)\} \\
\delta(q_1, 2, 2) &= \{(q_1, \varepsilon)\} \\
\delta(q_1, +, +) &= \{(q_1, \varepsilon)\} \\
\delta(q_1, \varepsilon, \$) &= \{(q_2, \$)\}
\end{aligned}
$$

Pictorially, the PDA looks as follows:



Problem 2 **(**20 **points):** Convert the following context-free grammar (CFG) into its equivalent Chomsky Normal Form (CNF) using the method described in class:

$$
\begin{aligned}
S &\to Ba \mid aB \\
A &\to bAA \mid aS \mid a \\
B &\to aBB \mid bS \mid b
\end{aligned}
$$

Solution:

$$
\begin{aligned}
S &\rightarrow BT_a \mid T_a B \\
B &\rightarrow T_a B_1 \mid T_b S \mid b \\
B_1 &\rightarrow BB \\
T_a &\rightarrow a \\
T_b &\rightarrow b
\end{aligned}
$$