Assignment #1 part 1, Total points: 55
(Course: CS 401)

These are the first two problems for Assignment 1. **The remaining problems of Assignment 1 will be given out later.**

For regular students, the deadline is **September 29, Monday, in class**.

For special needs students, the deadline is **October 6, Monday, in class**.

Please submit a hard copy of your solution (handwritten or typed) in person in class. No late assignments will be accepted.

Special note: Any answer that is not sufficiently clear even after a reasonably careful reading will not be considered a correct answer, and only what is written in the answer will be used to verify accuracy. No vague descriptions or sufficiently ambiguous statements that can be interpreted in multiple ways will be considered as a correct answer, nor will the student be allowed to add any explanations to his/her answer after it has been submitted.

Problem 1 (20 points): Prof. Smart thinks he is smarter than all the students in this CS 401 class. He has made the following claim to show how smart he is.

> **Claim made by Prof. Smart**: Consider the stable matching problem as taught in class. Suppose that we have only two men, say m_1 and m_2, and two women, say w_1 and w_2, with their corresponding preference lists. Suppose also that the matching $m_1 - w_1$ and $m_2 - w_2$ is a stable matching. Prof. Smart claims that in that case the matching $m_1 - w_2$ and $m_2 - w_1$ can **never** be a stable matching.

Your task is to decide if Prof. Smart is indeed so smart. For this purpose, do the following.

- Either prove the claim made by Prof. Smart is indeed correct. Such a proof should work **no matter** what the preferences of the men and women are, as long as $m_1 - w_1$ and $m_2 - w_2$ is a stable matching.
- Or, prove the claim made by Prof. Smart is wrong by giving a counter-example. The counter-example should provide the preferences lists of every man and woman, and show that for these preference lists **both** $m_1 - w_1$, $m_2 - w_2$ and $m_1 - w_2$, $m_2 - w_1$ are indeed stable matchings.

Problem 2 (35 points): Let $G = (V, E)$ be a **directed** graph and let s and t be two nodes of G. Let n and m be the number of nodes and edges of G, respectively. In the class we say how to decide if there is a path **from** s **to** t, namely, we start a (directed) BFS starting from s and check if t appears among the list of nodes that are visited during BFS. The purpose of the assignment is to decide if such a path exists under some **additional constraints**. Let u and v be two other nodes of G that are **not** s or t.

***(i)* [15 points]** Decide if G has a path from s and t that **avoids** using **both** the nodes u and v.

***(ii)* [20 points]** Decide if G has a path from s and t that **uses both** the nodes u and v.

Both of your algorithms should run in $O(m + n)$ time. You may assume that the graph is given in its adjacency list representation. If you are using BFS, there is no need to give codes for it; simply saying "do a BFS starting at such-and-such node" will suffice.