Reverse Engineering of Molecular Networks from a Common Combinatorial Approach

Bhaskar DasGupta
Department of Computer Science
University of Illinois at Chicago
Chicago, IL 60607
Email: dasgupta@cs.uic.edu

Paola Vera-Licona Institut Curie, Paris Cedex 05, France Email: paola.vera-licona@curie.fr

Eduardo Sontag
Mathematics Department
Rutgers University
Piscataway, NJ 08854
Email: sontag@math.rutgers.edu

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1 Introduction

The understanding of molecular cell biology requires insight into the structure and dynamics of networks that are made up of thousands of interacting molecules of DNA, RNA, proteins, metabolites, and other components. One of the central goals of systems biology is the unraveling of the as yet poorly characterized complex web of interactions among these components. This work is made harder by the fact that new species and interactions are continuously discovered in experimental work, necessitating the development of adaptive and fast algorithms for network construction and updating. Thus, the "reverse-engineering" of networks from data has emerged as one of the central concern of systems biology research.

A variety of reverse-engineering methods have been developed, based on tools from statistics, machine learning, and other mathematical domains. In order to effectively use these methods, it is essential to develop an understanding of the fundamental characteristics of these algorithms. With that in mind, this chapter is dedicated to the reverse-engineering of biological systems.

Specifically, we focus our attention on a particular class of methods for reverse-engineering, namely those that rely algorithmically upon the so-called "hitting-set" problem, which is a classical combinatorial and computer science problem, Each of these methods utilizes a different algorithm in order to obtain an exact or an approximate solution of the hitting set problem. We will explore the ultimate impact that the alternative algorithms have on the inference of published *in silico* biological networks.

2 Reverse Engineering of Biological Networks

Systems biology aims at a systems-level understanding of biology, viewing organisms as integrated and interacting networks of genes, proteins, and other molecular species through biochemical reactions that result in particular form and function (phenotype). Under this "system" conceptualization, it is the interactions among components that gives rise to emerging properties.

Systems-level ideas have been a recurrent theme in biology for several decades, as exemplified by Cannon's work on homeostasis [7], Wiener's biological cybernetics [33], and Ludwig von Bertalanffy's foundations of general systems theory [32]. So what has brought systems biology to the mainstream of biological science research in recent years? The answer can be found in large part in enabling technological advances, ranging from high-throughput biotechnology (gene expression arrays, mass spectrometers, etc.) to advances in information technology, that have revolutionized the way that biological knowledge is stored, retrieved and processed.

A systems approach to understanding biology can be described as an iterative process which includes: (1) data collection and integration of all available information (ideally, regarding all the components and their relationships in the organism of interest), (2) system modeling, (3) experimentation at a global level, and (4) generation of new hypotheses (see Fig. 2.1).

The current chapter focuses on the system modeling aspects, and, specifically, on the top-down modeling approach broadly known as the biological "reverse-engineering", which can be very broadly described as follows:

The biological reverse engineering problem is that of analyzing a given system in order to identify, from biological data, the components of the system and their relationships.

In broad terms, there are two very different levels of representation for biological networks. They are described as follows.

(a) Network Topology Representations

Also known as "wiring diagrams" or "static graphs", these are coarse diagrams or maps that represent the connections (physical, chemical, or statistical) among the various molecular components of a network. At this level, no detailed kinetic information is included. A network of molecular interactions can

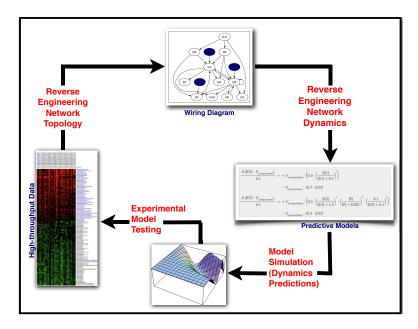


Figure 2.1: Iterative Process in Systems Biology.

be viewed as a graph: cellular components are nodes in a network, and the interactions (binding, activation, inhibition, etc.) between these components are the edges that connect the nodes. A reconstruction of network topology allows one to understand properties that might remain hidden without the model or with a less relevant model.

These type of models can be enriched by adding information on nodes or edges. For instance, '+' or '-' labels on edges may be used in order to indicate positive or negative regulatory influences. The existence of an edge might be specified as being conditional on the object being studied (for instance a cell) being in a specific global state, or on a particular gene that regulates that particular interaction being expressed above a given threshold. These latter types of additional information, however, refer implicitly to notions of state and temporal evolution, and thus lead naturally towards qualitative dynamical models.

Different reverse-engineering methods for topology identification differ on the types of graphs considered. For example, in the work in [3, 9, 11, 24, 26, 28, 34, 35], edges represent statistical correlation between variables. In [10, 13, 15, 17], edges represent causal relationships among nodes.

(b) Network Dynamical Models

Dynamical models represent the time-varying behavior of the different molec-

ular components in the network, and thus provide a more accurate representation of biological function.

Models can be used to simulate the biological system under study. Different choices of values for parameters correspond either to unknown system characteristics or to environmental conditions. The comparison of simulated dynamics with experimental measurements helps refine the model and provide insight on qualitative properties of behavior, such as the identification of steady states or limit cycles, multi-stable (e.g., switch-like) behavior, the characterization of the role of various parts of the network in terms of signal processing (such as amplifiers, differentiators and integrators, logic gates), and the assessment of robustness to environmental changes or genetic perturbations.

Examples of this type of inference include those leading to various types of Boolean networks [2, 20–22] or systems of differential equations [12, 16, 30], as well as multi-state discrete models [19].

Depending upon the type of network analyzed, data availability and quality, network size, and so forth, the different reverse engineering methods offer different advantages and disadvantages relative to each other. In Section 3.1, we will explore some of the common approaches to their systematic evaluation and comparison.

2.1 Evaluation of the Performance of Reverse Engineering Methods

The reverse-engineering problem is by its very nature highly "ill-posed", in the sense that solutions will be far from unique. This lack of uniqueness stems from the many sources of uncertainty: measurement error, lack of knowledge of all the molecular species that are involved in the behavior being analyzed ("hidden variables"), stochasticity of molecular processes, and so forth. In that sense, reverse-engineering methods can at best provide approximate solutions for the network that one wishes to reconstruct, making it very difficult to evaluate their performance through a theoretical study. Instead, their performance is usually assessed empirically, in the following two ways:

Experimental testing of predictions: after a model has been inferred, the newly found interactions or predictions can be tested experimentally for network topology and network dynamics inference, respectively.

Benchmarking testing: this type of performance evaluation consists on measuring how "close" the method of our interest is from recovering a known network, referred to as the "gold standard" for the problem. In the case of dynamical models, one evaluates the ability of the method of interest to reproduce observations that were not taken into account in the "training" phase involved in the construction of the model. On the another hand, for methods that only reconstruct the network topology (wiring diagram), a varierty of standard metrics may be applied.

Metrics for Network Topology Benchmarking

Suppose that Γ is the graph representing the network topology of a chosen "gold standard" network. Let Γ_i be the graph representing the inferred network topology. Each one of the interactions in Γ_i can be classified into one of the these four classes, when comparing to the gold standard:

- (a) Correct interactions inferred (true positives, TP)
- (b) Incorrect interactions inferred (false positives, FP)
- (c) Correct non-interactions inferred (true negatives, TN)
- (d) Incorrect non-interactions inferred (false negatives FN)

From this classification of the interactions, we compute the following metrics:

- The Recall or True Positive Rate TPR = TP/(TP + FN)
- The False Positive Rate FPR = FP/(FP + TN).
- The Accuracy ACC = (TP + TN)/TotI where TotI is the total number of possible interactions in a network.
- The Precision or Positive Predictive Value PPV = TP/(TP + FP).

As mentioned earlier, the reverse-engineering problem is underconstrained. Every algorithm will have one or more free parameters that helps select a "best" possible prediction. Hence, a more objective evaluation of performance has to somehow involve a range of parameter values. One way to evaluate performance across ranges of parameters is the **receiver operating characteristic (ROC)** method, based on the plot of FPR vs. TPR values. The resulting **ROC plot** depicts relative trade-offs between true positive predictions and false positive prediction across different parameter values (See Fig. 2.2). A closely related approach is the **Recall-Precision plot**, obtained by plotting TPR vs. PPV values.

3 Classical Combinatorial Algorithms: A Case Study

We have briefly discussed some basic aspects of reverse-engineering of biological systems. Next, as a case of study, we focus our attention on some reverse-engineering algorithms that rely upon the solution of the so-called "Hitting Set Problem". The Hitting Set Problem is a classical problem in combinatorics and computer science. It is defined as follows:

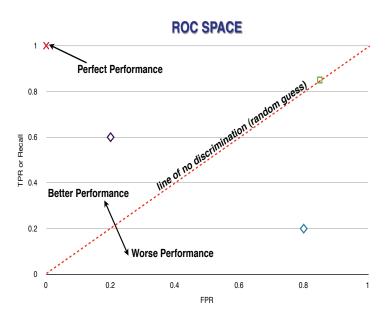


Figure 2.2: Receiver operating characteristic -ROC-space. Defined by FPR vs. TPR values in a two dimensional coordinate system: a perfect reverse engineering method will ideally have score (FPR, TPR) = (0,1) whereas the worst possible network will have coordinates (FPR, TPR) = (1,0) and scores below the identity line (diagonal) indicate methods that perform no better than a random guess.

Problem 1 (HITTING SET Problem) Given a collection \mathcal{H} of subsets of $E = \{1, ..., n\}$, find the smallest set $L \subseteq E$ such that $L \cap \mathcal{X} \neq \emptyset$ for all $\mathcal{X} \in \mathcal{H}$.

The Hitting Set problem is NP-hard, as can be shown via transformation from its dual, the (Minimum) Set Cover problem [14].

We next introduce some reverse engineering methods based on the hitting set approach.

• Ideker *et al.* [15].

This paper introduces two methods to infer the topology of a gene regulatory network from gene expression measurements. The first "network inference" step consists of the estimation of a set of Boolean networks consistent with an observed set of steady-state gene expression profiles, each generated from a different perturbation to the genetic network studied. Next, an "optimization step" involves the use of an entropy-based approach to select an additional perturbation experiment in order to perform a model selection from the set of predicted Boolean networks. In order to compute the sparsest network that interpolates the data, Ideker et al. rely upon the "Minimum Set Cover" problem. An approximate solution for the Hitting Set problem is obtained by means of a branch and bound technique [25]. Assessment is performed "in Numero": the proposed method is evaluated on simulated networks with varying number of genes and numbers of interactions per gene.

• Jarrah *et al.* [13]

This paper introduces a method for the inference of the network topology from gene expression data, from which one extracts state transition measurements of wild-type and perturbation data. The goal of this reverseengineering algorithm is to output one or more most likely network topologies for a collection x_1, \ldots, x_n of molecular species (genes, proteins, etc), which we will refer to as variables. The state of a molecular species can represent its levels of activation. That is, each variable x_i takes values in the set $X = \{0, 1, 2, \ldots\}$ and the interactions among species indicate causal relationships among molecular species. The inference algorithm takes as input one or more time courses of observational data. The output is a most likely network structure for the interactions among x_1, \ldots, x_n that is consistent with the observational data: The notion of consistency with observational data makes the assumption that the regulatory network for x_1, \ldots, x_n can be viewed as a dynamical system that is described by a function $f: X_n \to X_n$, which transforms an input state (s_1, \ldots, s_n) , $s_i \in X$, of the network into an output state (t_1, \ldots, t_n) at the next time step. A directed edge $x_i \to x_j$ in the graph of the network topology of this dynamical system f indicates that the value of x_i under application of f depends on the value of x_i . Hence a directed graph is consistent with a given time course s_1, \ldots, s_r of states in X_n , if it is the network topology of a function $f: X_n \to X_n$ that reproduces the time course, that is, $f(s_i) = s_{i+1}$ for all i.

One possible drawback of reverse engineering approaches lies in the fact that they construct the "sparest" possible network consistent with the given data. However, real biological networks are known to be not minimal [31]. Although accurate measures of deviation from sparsity are difficult to estimate, nonetheless it seems reasonable to allow additional edges in the network in a "controlled" manner that is consistent with the given data. As already commented in [15], it is possible to add redundancies to the reverse engineering construction. The basic hitting set approach provides only a minimal set of connections, whereas real biological networks are known to contain redundancies (e.g., see [31]). To account for this, one can modify the hitting set approach to add redundancies systematically by allowing additional parameters to control the extra connections. Theoretically, in terms of the algorithm this corresponds to a standard generalization of the set-cover problem, known as the set-multicover problem, which is well-studied in the literature, and for which approximation algorithms are known [4].

The search for the topologies that interpolate the input data involves directly the Hitting Set problem, which is solved analytically with the use of a computational algebra tools.

The algorithms presented in [5,17] also make use of Hitting Set algorithms, but we will restrict our attention to the comparison of the two methods described above.

3.1 Benchmarking RE Combinatorial-Based Methods

3.1.1 In Silico Gene Regulatory Networks

We use data from two different regulatory networks. These contain some features that are common in real regulatory networks, such as time delays and the need for a measurement data presented into discrete states (0, 1, 2, ...).

In Silico Network 1: Gene Regulatory Network with External Perturbations. This network was originally introduced in [6]. It was generated using the software package given in [23], the interactions between genes in this regulatory network are phenomenological, and represent the net effect of transcription, translation, and post-translation modifications on the regulation of the genes in the network. The model is implemented as a system of ODEs in Copasi [18].

This network, shown in Fig. 3.3, consists of 13 species: ten genes plus three different environmental perturbations. The perturbations affect the transcription rate of the gene on which they act directly (through inhibition or activation)

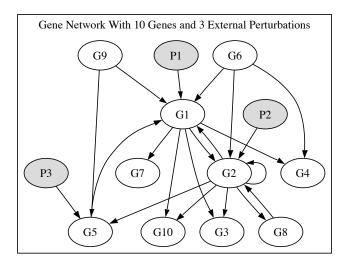


Figure 3.3: Network 1: 10 genes and 3 environmental perturbations. In this network, the 3 environmental perturbations P1, P2 and P3 directly affect the expression rate of genes G1, G2 and G5, respectively.

and their effect is propagated throughout the network by the interactions between the genes.

Network 2: Segment Polarity Genes Network in *D. melanogaster*. The network of segment polarity genes is responsible for pattern formation in the *Drosophila melanogaster* embryo. Albert and Othmer [1] proposed and analyzed a Boolean model based on the binary ON/OFF representation of mRNA and protein levels of five segment polarity genes. This model was constructed based on the known topology and it was validated using published gene and expression data. We generated time courses from this model, from which we will attempt to reverse-engineer the network in order to benchmark the performance of the reverse-engineering algorithms being evaluated.

The network of the segment polarity genes represents the last step in the hierarchical cascade of gene families initiating the segmented body of the fruit fly. The genes of this network include engrailed (en), wingless (wg), hedgehog (hh), patched (ptc), cubitus interruptus (ci) and sloppy paired (slp), coding for the corresponding proteins, which are represented by capital letters (EN, WG, HH, PTC, CI and SLP). Two additional proteins, resulting from transformations of the protein CI, also play important roles: CI may be converted into a transcriptional activator, CIA, or may be cleaved to form a transcriptional repressor CIR. The expression of the segment polarity genes occurs in stripes that encircle the embryo. These key features of these patterns can be

represented in one dimension by a line of 12 interconnected cells, grouped into 3 parasegment primordia, in which the genes are expressed every fourth cell. In Albert and Othmer [1], parasegments are assumed to be identical, and thus only one parasegment of four cells is considered. Therefore, in the model, the variables are the expression levels of the segment polarity genes and proteins (listed above) in each of the four cells, and the network can be seen as a $15 \times 4 = 60$ node network. Using the wild-type pattern from [1], we consider one wild-type time series of length 23.

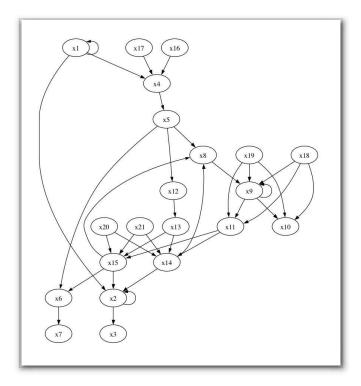


Figure 3.4: Segment Polarity Genes Network on the D. melanogaster. This network consists of the interaction of 60 molecular species: genes and proteins.

3.1.2 Results of Comparison

In this section we compare the results obtained after running Jarrah et al.'s and Ideker et al.'s methods on each of the above networks. Computations were made

	TP	FP	TN	FN	TPR	FPR	ACC	PPV
Network1								
Jarrah Exact Sol D7	12	34	113	10	.5454	.231	.7396	.2608
Jarrah Exact Sol Q5	9	49	98	13	.4090	.3334	.6331	.1551
Jarrah Exat Sol I5	7	46	101	15	.3181	.313	.6390	.1320
Karp Greedy Approx R1	9	49	98	13	.4090	.666	.016	.084
Karp Greedy Approx R2	11	63	84	11	.5	.571	.020	.115
Karp LP Approx. R1	7	46	101	15	.318	.687	.014	.064
Karp LP Approx. R2	9	59	88	13	.409	.598	.018	.092
Network 2								
Jarrah Exact Sol	_	_	_	_	_	_	_	_
Karp Greedy Approx R1	4	3321	91	124	.031	.026	.923	.042
Karp Greedy Approx R2	15	3254	218	113	.117	.062	.908	.064
Karp LP Approx. R1	3	3279	93	125	.023	.026	.939	.031
Karp LP Approx. R2	9	3285	187	119	.070	054	.915	.045

Table 3.1: Comparison of RE methods

on Mac OS X, Processor 2GHz Intel Core 2 Duo.

As we mentioned in Section 3, for Jarrah et al.'s method, the input data must be discrete. Hence in order to apply this reverse-engineering method to network 1 we discretize the input data, considering then different discretizations as our running parameter to test Jarrah et al.'s method in the ROC space. We specifically use three discretization methods: a graph-theoretic based approached "D" (see [8]), as well as quantile "Q" (discretization method on which each variable state receives an equal number of data values) and interval "I" discretization (discretization method on which we select thresholds for the different discrete values).

For Ideker et al.'s method we have considered both Greedy and Linear Programming approximations to the Hitting set problem as well as redundancy values (how many extra edges one allows) of R=1 or 2.

We have displayed some our results on Table 3.1. We observe that for network 1, Jarrah et. al.'s method obtains better results than Ideker et. al.'s method when considering these values in the ROC space, although both fare very poorly. On the another hand, we observe that Ideker et. al.'s method achieves a performance no better than random guessing on this network. In contrast, for network 2, Jarrah et al.'s method could not obtain any results after running their method for over 12 hours, but Ideker et al.'s method was able to compute results for such network in less than 1 minute. Also Ideker et al.'s method improved slightly its results when the redundancy number is increased; this might indicate the shortcoming of inferring sparser networks when they are of larger size containing redundancies.

3.2 Software Availability

The implementation of Jarrah et. al.'s algorithm [13] is available online through the web interface provided at http://polymath.vbi.vt.edu/polynome/. The implementation of Ideker et al.'s algorithm [15] is available online through the web interface provided at http://sts.bioengr.uic.edu/causal/.

4 Concluding Remarks

In this chapter, we first provided a brief discussion of the biological reverseengineering problem, which is a central problem in systems biology. As a case study, we then focused on two methods that rely upon the solution of the "Hitting Set problem", but which differ in their approach to solve this problem, thus leading to different performance.

In terms of network inference power, we hypothesize that, for the smaller network, the poor quality of the results when using Jarrah's approach might be ascribed to the type of data used: in [13] it is claimed that the method performs better if perturbation data is added. The algorithm has the ability of considering both wild-type and mutant data to infer the network, and probably results would improve if using such additional data. In the case of Ideker et. al.'s method, in both networks we think that it is possible that the low quality of results could be due to lack of ability of using more than one time series at a time, as well as the fact that the implementation of the method does not include self loops (self-loops are edges connecting a node to itself which may, for example, represent degradation terms in biochemical systems). We believe that this feature is fundamental for a good performance of the algorithm.

When comparing the computational efficiency of the approaches, one should keep in mind that there will always be a difference between exact solutions and approximate solutions based upon greedy algorithms or linear programming relaxations. However, since the size of the networks was fairly small, it is possible that the reason for which Jarrah's method did not find a solution within a reasonable time might lie in encoding issues rather than intrinsic computational complexity of the problem.

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