Solution of Assignment #1, Part 1
(Course: CS 301)

For regular students, the deadline is **September 17**, Tuesday in class.

For special needs students, the deadline is **September 24**, Tuesday in class.

No late assignments will be accepted.

Special note: Any answer that is not sufficiently clear even after a reasonably careful reading will not be considered a correct answer, and only what is written in the answer will be used to verify accuracy. No vague descriptions or sufficiently ambiguous statements that can be interpreted in multiple ways will be considered as a correct answer, nor will the student be allowed to add any explanations to his/her answer after it has been submitted.

Problem 1 (20 **points**):

Fig. 1 shows a toy for a young child. A marble can be dropped either at X or at Y. Levers α, β and γ cause the marble to fall either to the left or to the right. When a marble hits a lever, it causes the lever to change its direction, so that the next marble to encounter the lever will take the opposite branch.

Model the toy as a deterministic finite automate (DFA) and show the transition diagram of the DFA. Denote a marble dropped at X as input 0 and a marble dropped at Y as input 1. A sequence of inputs is accepted if the last marble comes out at A.

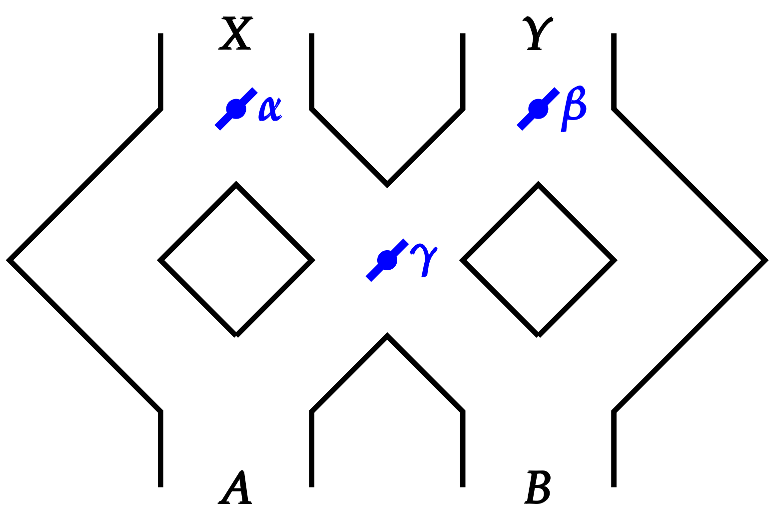

Figure 1: A toy for children's play.

Solution: Let the DFA be symbolically denoted by $(Q, \Sigma, \delta, q_0, F)$ where $\Sigma = \{0, 1\}$.

Note that each lever has *two* possible positions, namely or . We are going to denote and by **0** and **1**, respectively. Notations like $\overset{\alpha}{\mathbf{0}}\overset{\gamma}{\mathbf{1}}\overset{\beta}{\mathbf{1}}$ will indicate that lever α is in position , lever β is in position , and lever γ is in position .

It is tempting to model the system by just $2 \times 2 \times 2 = 8$ states, corresponding to *every* possible position of the levers α, β and γ. However, that may create a problem in deciding which states should be accepting states. For example, if we are currently at $\overset{\alpha\gamma\beta}{\mathbf{100}}$ and the next input is 0 (*i. e.*, the marble is dropped at X) then then we should go to $\overset{\alpha\gamma\beta}{\mathbf{010}}$ and accept since the marbel falls through A, which indicates that we should mark $\overset{\alpha\gamma\beta}{\mathbf{010}}$ as accepting state. However, we can also reach $\overset{\alpha\gamma\beta}{\mathbf{010}}$ from $\overset{\alpha\gamma\beta}{\mathbf{011}}$ on input 1, in which case the marbel falls through B, which indicates that we should *not* mark $\overset{\alpha\gamma\beta}{\mathbf{010}}$ as an accepting state. How can we resolve this conflict?

A simple way to resolve this conflict is to create two copies for each configuration, one for accepting and one for not accepting, and go to the appropriate copy. This gives us $8 \times 2 = 16$ possible states. Symbolically,

$$Q = \left\{ [\overset{\alpha}{p},\overset{\gamma}{q},\overset{\beta}{r},s] \,\middle|\, p \in \{\mathbf{0},\mathbf{1}\}, q \in \{\mathbf{0},\mathbf{1}\}, r \in \{\mathbf{0},\mathbf{1}\}, s \in \{\mathbf{accept},\mathbf{reject}\} \right\}$$

$$q_0 = [\overset{\alpha}{\mathbf{0}},\overset{\gamma}{\mathbf{0}},\overset{\beta}{\mathbf{0}},\mathbf{reject}]$$

The final (accepting) states are all the 8 states that have "**accept**" in it, *i.e.*, symbolically

$$F = \left\{ [\overset{\alpha}{p},\overset{\gamma}{q},\overset{\beta}{r},\mathbf{accept}] \,\middle|\, p \in \{\mathbf{0},\mathbf{1}\}, q \in \{\mathbf{0},\mathbf{1}\}, r \in \{\mathbf{0},\mathbf{1}\} \right\}$$

The transitions are now self-evident, which we provide below in the transition table:

	0	1
$[\overset{\alpha}{\mathbf{0}},\overset{\gamma}{\mathbf{0}},\overset{\beta}{\mathbf{0}},\mathbf{reject}]$	$[\overset{\alpha}{\mathbf{1}},\overset{\gamma}{\mathbf{0}},\overset{\beta}{\mathbf{0}},\mathbf{accept}]$	$[\overset{\alpha}{\mathbf{0}},\overset{\gamma}{\mathbf{1}},\overset{\beta}{\mathbf{1}},\mathbf{accept}]$
$[\overset{\alpha}{\mathbf{0}},\overset{\gamma}{\mathbf{0}},\overset{\beta}{\mathbf{0}},\mathbf{accept}]$	$[\overset{\alpha}{\mathbf{1}},\overset{\gamma}{\mathbf{0}},\overset{\beta}{\mathbf{0}},\mathbf{accept}]$	$[\overset{\alpha}{\mathbf{0}},\overset{\gamma}{\mathbf{1}},\overset{\beta}{\mathbf{1}},\mathbf{accept}]$
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$[\overset{\alpha}{\mathbf{0}},\overset{\gamma}{\mathbf{0}},\overset{\beta}{\mathbf{1}},\mathbf{accept}]$	$[\overset{\alpha}{\mathbf{1}},\overset{\gamma}{\mathbf{0}},\overset{\beta}{\mathbf{1}},\mathbf{accept}]$	$[\overset{\alpha}{\mathbf{0}},\overset{\gamma}{\mathbf{0}},\overset{\beta}{\mathbf{0}},\mathbf{reject}]$
$[\overset{\alpha}{\mathbf{0}},\overset{\gamma}{\mathbf{1}},\overset{\beta}{\mathbf{0}},\mathbf{reject}]$	$[\overset{\alpha}{\mathbf{1}},\overset{\gamma}{\mathbf{1}},\overset{\beta}{\mathbf{0}},\mathbf{accept}]$	$[\overset{\alpha}{\mathbf{0}},\overset{\gamma}{\mathbf{0}},\overset{\beta}{\mathbf{1}},\mathbf{reject}]$
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	0	1
$[\overset{\alpha}{\mathbf{1}}, \overset{\gamma}{\mathbf{1}}, \overset{\beta}{\mathbf{0}}, \textbf{reject}]$	$[\overset{\alpha}{\mathbf{0}}, \overset{\gamma}{\mathbf{0}}, \overset{\beta}{\mathbf{0}}, \textbf{reject}]$	$[\overset{\alpha}{\mathbf{1}}, \overset{\gamma}{\mathbf{0}}, \overset{\beta}{\mathbf{1}}, \textbf{reject}]$
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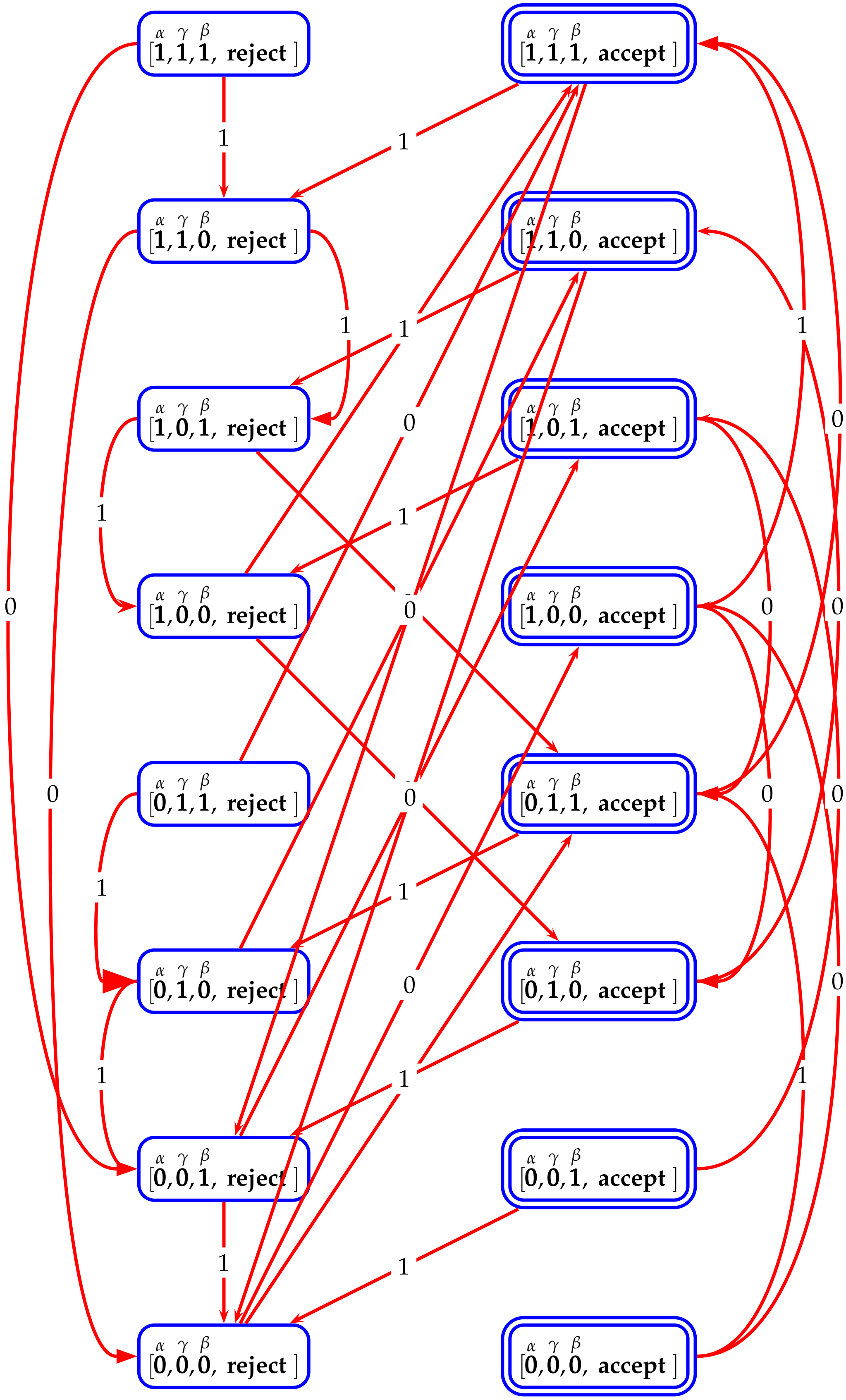
A diagram of the DFA is shown below: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