Problem 4

a) $K_{0,nonOp} = 10^{-4} = \dfrac{[lacR] \cdot [nonOp]}{[lacR \cdot nonOp]} \approx \dfrac{[lacR] \cdot [1.8E-3]}{[lacR \cdot nonOp]}$

$$\Rightarrow \frac{1}{48} = \frac{[lacR]}{[lacR \cdot nonOp]}$$

$$[lacR] = [lacR]_{total} - [lacR \cdot nonOp]$$

$$\frac{1}{48} = \frac{[lacR]_{total} - [lacR \cdot nonOp]}{[lacR \cdot nonOp]}$$

$$= \frac{[lacR]_{total}}{[lacR \cdot nonOp]} - 1$$

$$\Rightarrow \frac{49}{48} = \frac{[lacR]_{total}}{[lacR \cdot nonOp]}$$

$$\Rightarrow \boxed{\frac{[lacR \cdot nonOp]}{[lacR]_{total}} = \frac{48}{49}}$$

$$\frac{[lacR \cdot nonOp]}{10^{-8}} = \frac{48}{49}$$

$$\Rightarrow [lacR \cdot nonOp] = 10^{-8} \frac{48}{49} = \boxed{9.796 E^{-9} \ M}$$

$$[lacR]_{free} = 10^{-8} - 10^{-8} \cdot \frac{48}{49} = \boxed{2.041 E^{-10} \ M}$$

b) $K_{D,Op} = 10^{-12} = \dfrac{[lacR][Op]}{[lacR \cdot Op]} \rightsquigarrow \dfrac{\frac{1}{49} \cdot 10^{-8}[Op]}{[lacR \cdot Op]}$

$$\Rightarrow 4.9_E^{-3} = \dfrac{[Op]}{[lacR \cdot Op]}$$

$$[Op] = [Op]_{total} - [lacR \cdot Op]$$

$$4.9_E^{-3} = \dfrac{[Op]_{total} - [lacR \cdot Op]}{[lacR \cdot Op]}$$

$$= \dfrac{[Op]_{total}}{[lacR \cdot Op]} - 1$$

$$\Rightarrow \dfrac{[Op]_{total}}{[lacR \cdot Op]} = 4.9_E^{-3} + 1$$

$$\Rightarrow \dfrac{[lacR \cdot Op]}{[Op]_{total}} = (4.9_E^{-3} + 1)^{-1} = \boxed{0.995}$$

c) $K_D = 10^{-9} = \dfrac{\frac{1}{49} 10^{-8} [Op]}{[lacR \cdot Op]}$

$\Rightarrow 4.9 = \dfrac{[Op]}{[lacR \cdot Op]}$

$= \dfrac{[Op]_{total} - [lacR \cdot Op]}{[lacR \cdot Op]}$

$= \dfrac{[Op]_{total}}{[lacR \cdot Op]} - 1$

$\Rightarrow \dfrac{[Op]_{total}}{[lacR \cdot Op]} = 4.9 + 1 = 5.9$

$\Rightarrow \dfrac{[lacR \cdot Op]}{[Op]_{total}} = \dfrac{1}{5.9} = \boxed{0.169}$

d)

Without lactose	With lactose
$K_D = 10^{-12} = \dfrac{10^{-8}[Op]}{[lacR \cdot Op]}$	$K_D = 10^{-9} = \dfrac{10^{-8}[Op]}{[lacR \cdot Op]}$

Without lactose:

$$\Rightarrow 10^{-4} = \frac{[Op]}{[lacR \cdot Op]}$$

$$= \frac{[Op]_{total} - (lacR \cdot Op)}{(lacR \cdot Op)}$$

$$= \frac{[Op]_{total}}{[lacR \cdot Op]} - 1$$

$$\Rightarrow \frac{[lacR \cdot op]}{[Op]_{total}} = (10^{-4} + 1)^{-1}$$

$$= 0.9999$$
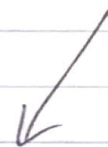
$\downarrow$

Expression
off

With lactose:

$$\Rightarrow 10^{-1} = \frac{[Op]}{[lacR \cdot Op]}$$

$$= \frac{[Op]_{total} - [lacR \cdot Op]}{[lacR \cdot Op]}$$

$$= \frac{[Op]_{total}}{[lacR \cdot Op]} - 1$$

$$\Rightarrow \frac{[lacR \cdot Op]}{[Op]_{total}} = (10^{-1} + 1)^{-1}$$

$$= 0.9091$$

$\downarrow$

Expression
<u>off</u>