
PUTNAM 2024 A

Math Competitions

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Problem 1. Find all $n \in \mathbb{Z}^+$ such that there exist solutions to

$$2a^n + 3b^n = 4c^n,$$

where $a, b, c \in \mathbb{Z}^+$.

Problem 2. Find all polynomials $p(x)$ such that there exist polynomials $q(x)$ that satisfy the equation:

$$p(p(x)) - x = (p(x) - x)^2 q(x).$$

Problem 3. Let S be the set of bijections

$$T : \{1, 2, 3\} \times \{1, 2, \dots, 2024\} \rightarrow \{1, 2, \dots, 6072\} \quad (1)$$

such that $T(1, j) < T(2, j) < T(3, j)$ for all $j \in \{1, 2, \dots, 2024\}$ and $T(i, j) < T(i, j+1)$ for all $i \in \{1, 2, 3\}$ and $j \in \{1, 2, \dots, 2023\}$. Do there exist a and c in $\{1, 2, 3\}$ and b and d in $\{1, 2, \dots, 2024\}$ such that the fraction of elements in T in S for which $T(a, b) < T(c, d)$ is at least $1/3$ and at most $2/3$?

Problem 4. Find all primes $p > 5$ for which there exists an integer a and an integer r satisfying $1 \leq r \leq p-1$ with the following property: the sequence $1, a, a^2, \dots, a^{p-5}$ can be arranged to form a sequence b_0, b_1, \dots, b_{p-5} such that $b_n - b_{n-1} - r$ is divisible by p for $1 \leq n \leq p-5$.

Problem 5. Let Ω be a circle of radius 9 centered at the origin. Let Δ be a disk of radius 1 with center $(r, 0)$, $r \in [0, 8]$. For two points P, Q on Ω , find the r that minimizes the probability for PQ to be in Δ .

Problem 6. For some sufficiently small x , write

$$\frac{1 - 3x - \sqrt{(3x-1)^2 - 8x}}{4} = \sum_{k=0}^{\infty} c_k x^k.$$

For some $n \in \mathbb{Z}^n$, let A be the $n \times n$ matrix with i, j entry c_{i+j-1} for $i, j \in \{1, \dots, n\}$. Find the determinant of A .