Midterm Revisions

Original Score - 55/100

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March 31, 2019

Question 1a (Received 4 / 10 points)

Suppose we are given 3 points p_0, p_1, and p_2. Compute the equation of the Bezier curve parameterized by t for these three points **recursively** given that the midpoint m_0 lies between p_0 and p_1, m_1 between p_1 and p_2, and m_2 between m_0 and m_1.

With $t\epsilon[0, 1]$,
$m_0 = (1 - t) \cdot p_0 + t \cdot p_1$
$m_1 = (1 - t) \cdot p_1 + t \cdot p_2$
$m_2 = (1 - t) \cdot m_0 + t \cdot m_1$
$m_2 = (1 - t) \cdot [(1 - t) \cdot p_0 + t \cdot p_1] + t \cdot [(1 - t) \cdot p_1 + t \cdot p_2]$
$m_2 = (1 - t)^2 \cdot p_0 + 2t \cdot (1 - t) \cdot p_1 + t^2 \cdot p_2$

Question 1b (Received 5/5 points)

Notice that we have formed a coordinate system based on p_0, p_1, and p_2 where the coefficients (parameterized by t) are the coordinates. What is the name of this coordinate system and what constraint must be placed on t in order for us to stay within the said coordinate system?

I got this question correct.

Question 1c (Received 3/5 points)

Both Chaikin and Bezier curves belong to corner cutting algorithms where points are interpolated around the corners to form a smooth curve. Give the *growth complexity* for these algorithms with respect to n iterations. As $n \to \infty$, we know that Chaikin's curve converges to Bezier's curve. Give the intuition as to why this is so and provide the guarantee that Bezier makes while Chaikin's does not.

With respect to n iterations, both curves have growth complexity of O(2^n). Chaikin's curves take $\frac{1}{4}$ and $\frac{3}{4}$ distances between points, and thus converge to the midpoints generated by Bezier curves.

The guarantee that Bezier provides (that Chaikin does not) is that the end-points will be on the curve.

Question 2a (Received 1/10 points)

Suppose we have a point $c = (c_x, c_y, c_z)$ denoted as the center of a camera and a point $a = (a_x, a_y, a_z)$ denoted as the center of attention for our camera. Generate the coordinate frame for our camera assuming that we have a vector $\vec{y}$ pointed in some "up" direction.

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origin $= c$
$\vec{w} = c - a$
$\vec{u} = \vec{w} \times \vec{y}$
$\vec{v} = \vec{w} \times \vec{u}$

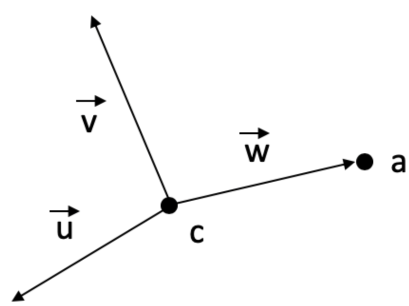

Figure 1: The camera coordinate frame.

Question 2b (Received 5/5 points)

Suppose that we have a point p = *(x, y, z)* that exists in a coordinate frame $F_c =< \vec{u}, \vec{v}, \vec{w}, c >$. Express p with respect to the coordinate frame F_c.

I got this question correct.

Question 2c (Received 0/5 points)

Suppose that $p = (x, y, z)$ exists with respect to the world frame $F_w =< \vec{x}, \vec{y}, \vec{z}, O >$. Write p' as the result of a transformation that places p in the camera coordinate frame $F_c =< \vec{u}, \vec{v}, \vec{w}, c >$.

$p' = C(p)$
**Note*: The camera transform,

$$C = \begin{bmatrix} \vec{u} \cdot [1,0,0] & \vec{u} \cdot [0,1,0] & \vec{u} \cdot [0,0,1] & \vec{u} \cdot \vec{t} \\ \vec{v} \cdot [1,0,0] & \vec{v} \cdot [0,1,0] & \vec{v} \cdot [0,0,1] & \vec{v} \cdot \vec{t} \\ \vec{w} \cdot [1,0,0] & \vec{w} \cdot [0,1,0] & \vec{w} \cdot [0,0,1] & \vec{w} \cdot \vec{t} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 3a (Received 10/10 points)

Given the following transformation matrix, compute its inverse using Gauss-Jordan elimination.

$$\begin{bmatrix} a & 0 & 0 & e \\ 0 & b & 0 & f \\ 0 & 0 & c & g \\ 0 & 0 & 0 & d \end{bmatrix}$$

I got this question correct.

Question 3b (Received 5/5 points)

Give the constraint that matrices A and B must satisfy in order to be the inverse of each other.

I got this question correct.

Question 3c (Received 3/5 points)

Suppose we have a set of points P in 3-dimensional space centered at $q = (0, 1, 0)$. Write the set of necessary elementary transformations to scale P by a factor of 2 in the x-direction and 3 in the y-direction and to rotate such that its new center is $q' = (0, 0, 1)$.

$R_{x=90^\circ} T_{0,1,0} S_{2,3,0} T_{0,-1,0}(P)$

Question 4a (Received 3/10)

Draw the camera model centered at c and with a field of view α, a near plane and a far plane located at a distance n and f from c. Label the axes, the location of the near and far plan (e.g. w =...). Provide the coordinates of the points in the viewing pyramid that map to (0, 0, 1), (0, 0, -1), (0, 1, 1), and (1, 0, 1) in the image space.

The following table delineates the four points in question in image space (original) and view space (translated):

point	view	image
p	$(0, n \cdot \tan(\frac{\alpha}{2}), -n)$	(0, 1, 1)
q	$(n \cdot \tan(\frac{\alpha}{2}), 0, -n)$	(1, 0, 1)
r	$(0, 0, -f)$	(0, 0, -1)
s	$(0, 0, -n)$	(0, 0, 1)

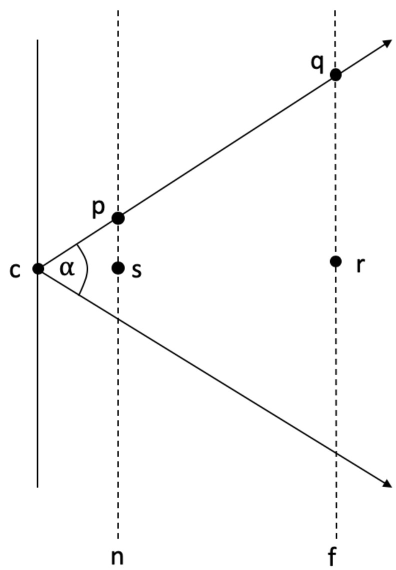

Figure 2: The view pyramid.

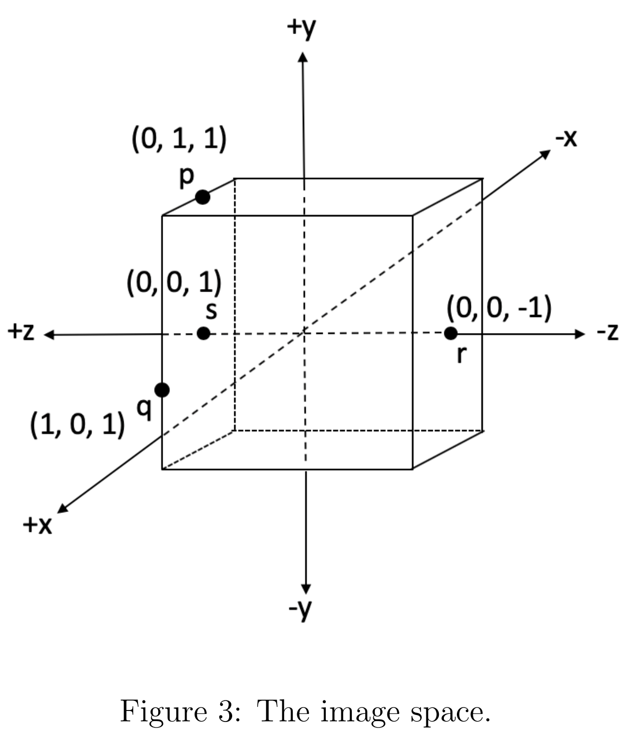

Figure 3: The image space.

Question 4b (Received 1/10 points)

We derived the viewing transform by solving 3 equations and made the assumption that the element corresponding to the x-component (top-left-hand corner of the matrix) is $\cot(\frac{\alpha}{2})$, because the viewing pyramid is symmetrical.

$$V = \begin{bmatrix} \cot(\frac{\alpha}{2}) & 0 & 0 & 0 \\ 0 & \cot(\frac{\alpha}{2}) & 0 & 0 \\ 0 & 0 & \frac{n+f}{f-n} & \frac{2f\cdot n}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Show that this is true by solving a 4^{th} equation.

$$V = \begin{bmatrix} \cot(\frac{\alpha}{2}) & 0 & 0 & 0 \\ 0 & \cot(\frac{\alpha}{2}) & 0 & 0 \\ 0 & 0 & \frac{n+f}{f-n} & \frac{2f\cdot n}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$V \begin{bmatrix} n \cdot \tan(\frac{\alpha}{2}) & 0 & -n \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cot(\frac{\alpha}{2}) & 0 & 0 \\ 0 & 0 & \frac{n+f}{f-n} & \frac{2f\cdot n}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} n \cdot \tan(\frac{\alpha}{2}) \\ 0 \\ -n \\ 1 \end{bmatrix} = \begin{bmatrix} n \cdot \tan(\frac{\alpha}{2}) \\ 0 \\ -n \cdot \frac{n+f}{f-n} + \frac{2f\cdot n}{f-n} \\ n \end{bmatrix}$$

$$\Pi \begin{bmatrix} n \cdot \tan(\frac{\alpha}{2}) \\ 0 \\ -n \cdot \frac{n+f}{f-n} + \frac{2f\cdot n}{f-n} \\ n \end{bmatrix} = \begin{bmatrix} \tan(\frac{\alpha}{2}) \\ 0 \\ \frac{-n-f}{f-n} + \frac{2f}{f-n} \\ 1 \end{bmatrix} = \begin{bmatrix} \tan(\frac{\alpha}{2}) \\ 0 \\ \frac{f-n}{f-n} \\ 1 \end{bmatrix} = \begin{bmatrix} \tan(\frac{\alpha}{2}) \\ 0 \\ 1 \end{bmatrix}$$

We need this matrix to be

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$

so V[0][0] must be $\cot(\frac{\alpha}{2})$.

Question 5a (Received 5/5 points)

When deriving the camera transform, we computed an element as $e_{11} = \frac{(<1,0,0>) \times \vec{v} \cdot \vec{w}}{(\vec{u} \times \vec{v}) \cdot \vec{w}}$. Explain what is occurring in the equation and what this ultimately allows us to do.

I got this question correct.

Question 5b (Received 5/5 points)

Suppose we defined the camera frame without setting a near and a far plane. Explain what would happen when projecting our points to image space.

I got this question correct.

Question 5c (Received 5/10 points)

Suppose we have an object defined by P centered at $p = (p_x, p_y, p_z)$ in world frame $F_w =< \vec{x}, \vec{y}, \vec{z}, O >$. Express a set of points P' after undergoing scaling by a factor of s in all directions and a translation such that it is now centered at (q_x, q_y, q_z). You may use the shorthand notations to denote each matrix (e.g. T refers to translation, R refers to rotation).

$P' = \Pi V C T_q T_O^{-1} S_s T_O(P)$