Correlated data with main effects

① Random graph
Erdős-Rényi or Scale-free

② Adjacency matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Degree vector
$v_d = [1, 1, 1, 1, 3, 1, 4]$

Functional features
$F = [2, 4, 7]$

③ Correlation matrix

$$P_{ij} = \begin{cases} \rho^{\text{hi}} + \varepsilon_{ij} & A_{ij} = 1 \\ \rho^{\text{lo}} + \varepsilon_{ij} & A_{ij} = 0 \end{cases}$$

$P =$

    ■ – High
    ■ – Null
    ■ – Diagonal

④ Cholesky decomposition
$$P = UU^{\mathsf{T}}$$

⑤ Design matrix
$$Z = (BY)^{\mathsf{T}} + E$$

$$Z = \left[\begin{array}{ccccccc} 0 & b_2 & 0 & b_4 & 0 & 0 & b_7 \\ 0 & b_2 & 0 & b_4 & 0 & 0 & b_7 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & b_2 & 0 & b_4 & 0 & 0 & b_7 \\ \hline & & & & & & \\ & & & 0 & & & \\ & & & & & & \end{array}\right] + E$$

Main effect data
$$X = ZU^{\mathsf{T}}$$

Binary outcome

$$Y = [\overbrace{1, 1, \ldots, 1}^{\text{Cases}}, \overbrace{0, 0, \ldots, 0}^{\text{Controls}}]$$

Effect sizes
$$B^{\mathsf{T}} = [0, b_2, 0, b_4, 0, 0, b_7]$$
$$b_2, b_4, b_7 \sim \mathcal{N}(0, b^{\text{main}})$$

Gaussian noise
$$\varepsilon_{ij} \sim \mathcal{N}(0, 1)$$