Mixed effects data: interactions and main effects

1 **Random graph**
Erdős-Rényi or Scale-free

2 **Adjacency matrix**

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Degree vector
$$v_d = [1, 1, 1, 1, 3, 1, 4]$$

Functional features
$$F = [2, 4, 7]$$

3 **Correlation matrices**

$$P_{ij}^{\text{ctrl}} = \begin{cases} \rho^{\text{hi}} + \varepsilon_{ij} & A_{ij} = 1 \\ \rho^{\text{lo}} + \varepsilon_{ij} & A_{ij} = 0 \end{cases}$$

$$P_{ij}^{\text{case}} = \begin{cases} b^{\text{int}} + \varepsilon_{ij} & A_{ij} = 1 \ \& \ (i \in F \text{ or } j \in F) \\ P_{ij}^{\text{ctrl}} & \text{otherwise} \end{cases}$$

Controls

$P^{\text{ctrl}} =$

- High
- Null
- Diagonal

Cases

$P^{\text{case}} =$

- Low
- Null
- Diagonal

4 **Cholesky decomposition**

$$P^{\text{ctrl}} = U^{\text{ctrl}} \left(U^{\text{ctrl}}\right)^{\mathsf{T}}$$

$$P^{\text{case}} = U^{\text{case}} \left(U^{\text{case}}\right)^{\mathsf{T}}$$

5 **Interaction data**

$$Y^{\text{ctrl}} = X^{\text{ctrl}} \left(U^{\text{ctrl}}\right)^{\mathsf{T}} \ , \ x_{ij}^{\text{ctrl}} \sim \mathcal{N}(0,1)$$

$$Y^{\text{case}} = X^{\text{case}} \left(U^{\text{case}}\right)^{\mathsf{T}} \ , \ x_{ij}^{\text{case}} \sim \mathcal{N}(0,1)$$

$$X^{\text{int}} = \begin{bmatrix} Y^{\text{ctrl}} \\ \hline Y^{\text{case}} \end{bmatrix}$$

7 **Mixed effects data**

$$X = \begin{bmatrix} X^{\text{int}} & X^{\text{main}} \end{bmatrix}$$

6 **Design matrix**

$$X^{\text{main}} = (BY)^{\mathsf{T}} + E$$

$$X^{\text{main}} = \begin{bmatrix} b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \\ \vdots & \vdots & \vdots \\ b_1 & b_2 & b_3 \\ \hline & & \\ & 0 & \end{bmatrix} + E$$

Binary outcome

Cases   Controls
$$Y = [\overbrace{1,1,\ldots,1}, \overbrace{0,0,\ldots,0}]$$

Effect sizes
$$B^{\mathsf{T}} = [b_1, b_2, b_3]$$
$$b_1, b_2, b_3 \sim \mathcal{N}(0, b^{\text{main}})$$

Gaussian noise
$$\varepsilon_{ij} \sim \mathcal{N}(0,1)$$