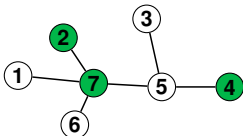


# Correlated data with interactions

1

**Random graph**  
Erdős-Rényi or Scale-free



2

**Adjacency matrix**

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

**Degree vector**

$$v_d = [1, 1, 1, 1, 3, 1, 4]$$

**Functional features**

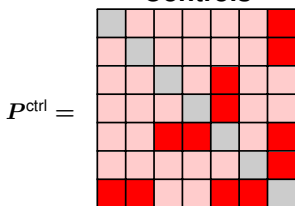
$$F = [2, 4, 7]$$

3

**Correlation matrices**

$$P_{ij}^{\text{ctrl}} = \begin{cases} \rho^{\text{hi}} + \varepsilon_{ij} & A_{ij} = 1 \\ \rho^{\text{lo}} + \varepsilon_{ij} & A_{ij} = 0 \end{cases}$$

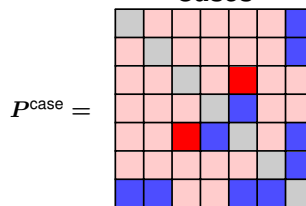
**Controls**



■ - High  
■ - Null  
■ - Diagonal

$$P_{ij}^{\text{case}} = \begin{cases} b^{\text{int}} + \varepsilon_{ij} & A_{ij} = 1 \text{ \& } (i \in F \text{ or } j \in F) \\ P_{ij}^{\text{ctrl}} & \text{otherwise} \end{cases}$$

**Cases**



■ - Low  
■ - Null  
■ - Diagonal

4

**Cholesky decomposition**

$$P^{\text{ctrl}} = U^{\text{ctrl}} (U^{\text{ctrl}})^T$$

$$P^{\text{case}} = U^{\text{case}} (U^{\text{case}})^T$$

5

**Interaction data**

$$Y^{\text{ctrl}} = X^{\text{ctrl}} (U^{\text{ctrl}})^T, \quad x_{ij}^{\text{ctrl}} \sim \mathcal{N}(0, 1)$$

$$Y^{\text{case}} = X^{\text{case}} (U^{\text{case}})^T, \quad x_{ij}^{\text{case}} \sim \mathcal{N}(0, 1)$$

$$X = \begin{bmatrix} Y^{\text{ctrl}} \\ \vdots \\ Y^{\text{case}} \end{bmatrix}$$