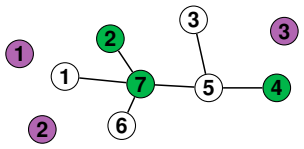
GWAS data: interactions and main effects

1 Random graph

Erdős-Rényi or Scale-free



2 Adjacency matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Degree vector

$$v_d = [1, 1, 1, 1, 3, 1, 4]$$

Functional features

$$F = [2, 4, 7]$$

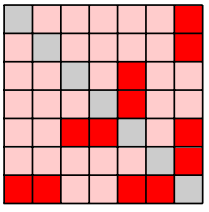
3 Correlation matrices

$$P_{ij}^{\text{ctrl}} = \begin{cases} \rho^{\text{hi}} + \varepsilon_{ij} & A_{ij} = 1 \\ \rho^{\text{lo}} + \varepsilon_{ij} & A_{ij} = 0 \end{cases}$$

$$P_{ij}^{\text{case}} = \begin{cases} b^{\text{int}} + \varepsilon_{ij} & A_{ij} = 1 \;\&\; (i \in F \text{ or } j \in F) \\ P_{ij}^{\text{ctrl}} & \text{otherwise} \end{cases}$$

Controls

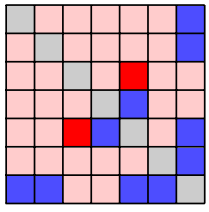
$P^{\text{ctrl}} =$



- High
- Null
- Diagonal

Cases

$P^{\text{case}} =$



- Low
- Null
- Diagonal

4 Cholesky decomposition

$$P^{\text{ctrl}} = U^{\text{ctrl}} \left(U^{\text{ctrl}}\right)^{\mathsf{T}}$$

$$P^{\text{case}} = U^{\text{case}} \left(U^{\text{case}}\right)^{\mathsf{T}}$$

5 Correlated continuous data

$$X^{\text{ctrl}} = Z^{\text{ctrl}} \left(U^{\text{ctrl}}\right)^{\mathsf{T}} \quad , \quad z_{ij}^{\text{ctrl}} \sim \mathcal{N}(0,1)$$

$$X^{\text{case}} = Z^{\text{case}} \left(U^{\text{case}}\right)^{\mathsf{T}} \quad , \quad z_{ij}^{\text{case}} \sim \mathcal{N}(0,1)$$

$$Y^{\text{norm}} = \begin{bmatrix} X^{\text{ctrl}} \\ \hline X^{\text{case}} \end{bmatrix}$$

$$Y^{\text{unif}} = \Phi\left(Y^{\text{cont}}\right)$$

6 Interaction data

$$f_a \sim \mathcal{U}(0.1, 0.4), \quad a = 1, 2, \ldots, 7$$

For $a = 1, 2, \ldots, 7$

  For each $i \in \mathcal{I}$

$$X_{ia}^{\text{int}} \sim B^{-1}\left(Y_{ia}^{\text{unif}}; n = 2, p = f_a\right)$$

7 Main effect data

Controls

$$p_{\text{ctrl}} = \frac{1 - b^{\text{main}}}{2}$$

For $a = 1, 2, 3$

  For each $i = 1, \ldots, m_{\text{ctrl}}$

$$X_{ia}^{\text{ctrl}} \sim \mathcal{B}\left(n = 2, p_{\text{ctrl}}\right)$$

Cases

$$p_{\text{case}} = \frac{1 + b^{\text{main}}}{2}$$

For $a = 1, 2, 3$

  For each $i = 1, \ldots, m_{\text{case}}$

$$X_{ia}^{\text{case}} \sim \mathcal{B}\left(n = 2, p_{\text{case}}\right)$$

$$X^{\text{main}} = \begin{bmatrix} X^{\text{ctrl}} \\ \hline X^{\text{case}} \end{bmatrix}$$

8 Mixed effects data

$$X = \left[\, X^{\text{int}} \;\middle|\; X^{\text{main}} \,\right]$$