Exercises on Chapter 5

Beau De Clercq

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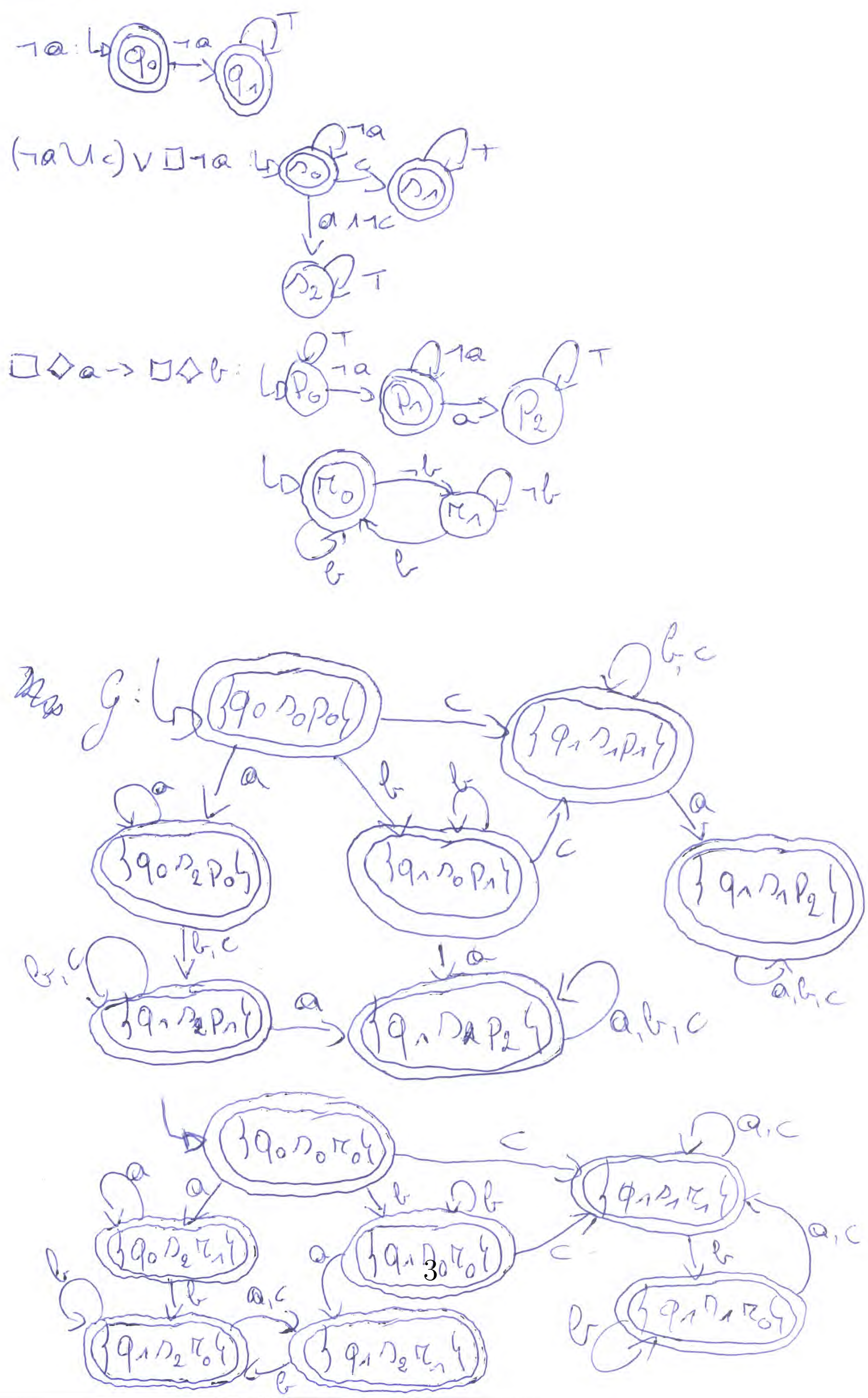
Ex. 5.16

Question

Depict a GNBA G over the alphabet $\Sigma = 2^{\{}a, b, c\}$ such that
$L_\omega = Words((\Box\Diamond a \rightarrow \Box\Diamond b) \wedge \neg a \wedge (\neg a W c))$

Answer

5.16



Ex. 5.17

Question

Let $\psi = \Box(a \leftrightarrow \bigcirc\neg a)$ and $AP = \{a\}$.

- Show that ψ can be transformed into the following equivalent basic LTL formula $\varphi = \neg[\text{true} \cup (\neg(a \wedge \bigcirc\neg a) \wedge \neg(\neg a \wedge \neg\bigcirc\neg a))]$.
- Compute all elementary sets with respect to $closure(\varphi)$.
- Construct the GNBA G_φ with $L_\omega(G_\varphi) = Words(\varphi)$. To that end:
  - Define its set of initial states and its acceptance component.
  - For each elementary set B, define $\delta(B, B \cap AP)$.

Answer

5.17 a) $\square(a \leftrightarrow \bigcirc\neg a) = \square((a \rightarrow \bigcirc\neg a) \wedge (\bigcirc\neg a \rightarrow a))$

$$= \square((\neg a \vee \bigcirc\neg a) \wedge (\neg(\bigcirc\neg a) \vee a))$$

$$= \neg\lozenge\neg((\neg a \vee \bigcirc\neg a) \wedge (\neg(\bigcirc\neg a) \vee a))$$

$$= \neg(T \,\mathsf{U}\, \neg((\neg a \vee \bigcirc\neg a) \wedge (\neg(\bigcirc\neg a) \vee a))$$

$$= \neg(T \,\mathsf{U}\, (\neg(a \wedge \bigcirc\neg a) \wedge \neg(\neg a \wedge \neg\bigcirc\neg a)))$$

$$= \varphi$$

b) elementary sets:

~~B_1 $\{T, a$~~ $\quad B_1: \{T, a, \bigcirc\neg a, a \wedge \bigcirc\neg a\}$

~~$B_2: \{\neg T\}$~~ $\quad B_2: \{\neg T, a, \bigcirc\neg a, a \wedge \bigcirc\neg a\}$

~~B_3 $\{T, a, \bigcirc\neg a,$~~ $\quad B_3: \{T, \neg a, \neg\bigcirc\neg a, \neg a \wedge \neg\bigcirc\neg a\}$

~~B_4~~ $\quad B_4: \{\neg T, \neg a, \neg\bigcirc\neg a, \neg a \wedge \neg\bigcirc\neg a\}$

~~B_5~~ $\quad B_5: \{T, \neg a, \neg\bigcirc\neg a, \neg(\neg a \wedge \neg\bigcirc\neg a)\}$

~~B_6~~ $\quad B_6: \{\neg T, \neg a, \neg\bigcirc\neg a, \neg(\neg a \wedge \neg\bigcirc\neg a)\}$

c) $Q = \{B_1, B_2, B_3, B_4, B_5, B_6\}$

$I = \{B_1, B_3, B_5\}$

$F = \emptyset$

$\delta(B_1, a) = \{B_3, B_5\}$

$\delta(B_2, a) = \{B_4, B_6\}$

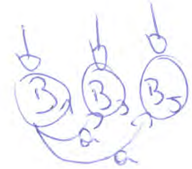
$\delta(B_3, a) = \{B_5\}$

$\delta(B_4, a) = \{B_6\}$

$\delta(B_5, a) = \{\}$

$\delta(B_6, a) = \{\}$

G:



Ex. 5.18

Question

Let $AP = \{a\}$ and $\varphi = (a \wedge \bigcirc a) \cup \neg a$ an LTL formula over AP.

- Compute all elementary sets with respect to φ.
- Construct the GNBA G_φ such that $L_\omega(G_\varphi) = Words(\varphi)$.

Answer

5.18| a) closure(φ) = $\{$ $a, \neg a,$
$\bigcirc a, \neg \bigcirc a,$
$(a \wedge \bigcirc a), \neg(a \wedge \bigcirc a)\}$

$B_1 : \{a, \bigcirc a, a \wedge \bigcirc a\}$

$B_2 : \{\neg a, \bigcirc a, \neg(a \wedge \bigcirc a)\}$

$B_3 : \{\neg a, \neg \bigcirc a, \neg(a \wedge \bigcirc a)\}$

$B_4 : \{a, \neg \bigcirc a, \neg(a \wedge \bigcirc a)\}$

$B_5 : \{a, \bigcirc a, \neg(a \wedge \bigcirc a)\}$

b) $\mathcal{G}_\varphi$:

Ex. 5.20

Question

We consider the LTL formula $\varphi = \Box(a \to (\neg b \cup (a \wedge b)))$ over the set $AP = \{a, b\}$ of atomic propositions and we want to check $TS \models \varphi$ for TS outlined below.

- To check $TS \models \varphi$, convert $\neg\varphi$ into an equivalent LTL formula ψ which is constructed according to the following grammar:
  $\phi ::= \text{true} | a | b | \phi \wedge \phi | \neg\phi | \bigcirc\phi | \phi \cup \phi$.
  Then construct $closure(\psi)$.
- Give the elementary sets w.r.t. $closure(\psi)$!
- Construct the GNBA G_ψ.
- Construct an NBA $A_{\neg\varphi}$ *directly* from $\neg\varphi$, i.e., without relying on G_ψ. (Four states suffice).
- Construct $TS \otimes A_{\neg\varphi}$.

Answer

Ex. 5.23

Question

Which of the following LTL formulae φ_i are representable by a determinstic Büchi automaton?

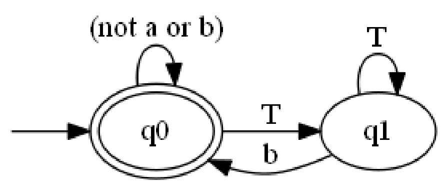
$$\varphi_1 = \Box(a \to \Diamond b), \varphi_2 = \neg\varphi_1$$

Explain your answer.

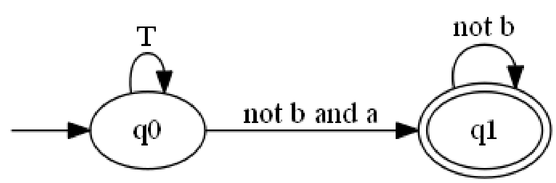
Answer

Both formulae are representable by a DBA: both φ_1 and φ_2 have the same closure.

φ_1 :



φ_2 :



Ex. 5.24

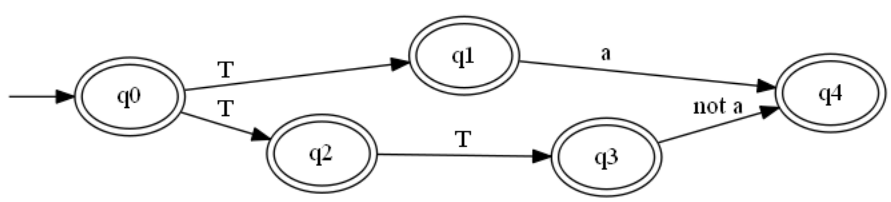
Question

Check for the following LTL formula whether they are (i) satisfiable, and/or (ii) valid:

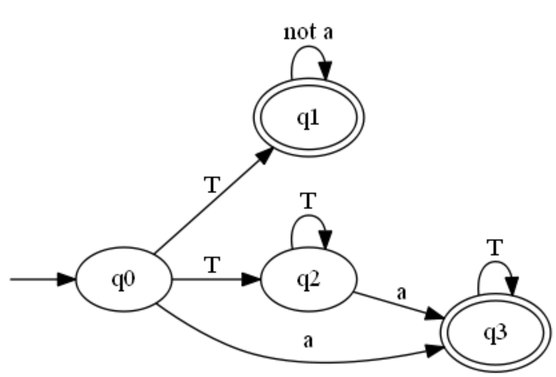
- $\bigcirc\bigcirc a \implies \bigcirc a$
- $\bigcirc(a \vee \Diamond a) \implies \Diamond a$
- $\Box a \implies \neg\bigcirc(\neg a \wedge \Box\neg a)$
- $(\Box a) \cup (\Diamond b) \implies \Box(a \cup \Diamond b)$
- $\Diamond b \implies (a \cup b)$

Answer

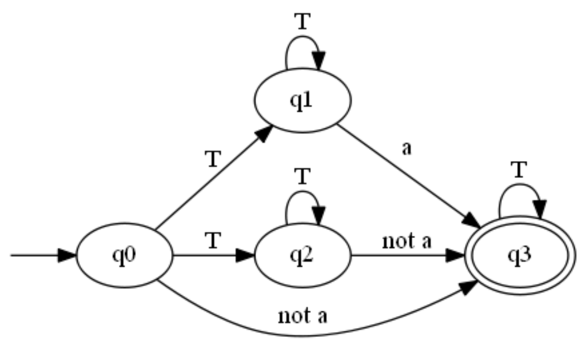
- Both satisfiable and valid



- Both satisfiable and valid



- Both satisfiable and valid



- Both satisfiable and valid
- Both satisfiable and valid