Pseudo-code

Beau De Clercq

November 2019

1 Ex. 1 p6

1.1 Question

What is the computational complexity of determining whether u can reach v in a given graph? Can you describe an efficient algorithm to solve this problem?

1.2 Answer

Strongly Connected Components:

- Perform a DFS on the graph $G = (V, E)$.
- Construct the graph $G' = (V, E')$ where $(u, v) \in E'$ iff $(v, u) \in E$.
- Perform a DFS on G' (the SCC's will be produced in this step).

Complexity: $O(|V| + |E|)$.

2 Ex. 2 p8

2.1 Question

Is the language L = $\{w | \#_a(w) \geq \#b(w)\}$, where $\#_\sigma(w)$ denotes the number of times the symbol σ occurs in w, an ω-regular language?

2.2 Answer

No: counting is not possible.

3 Ex. 3 p9

3.1 Question

Are the words a^ω and ab^ω in the language of the automaton from Figure 1 in the following cases?

- with reachability acceptance condition and T = $\{q_0\}$
- with safety acceptance condition and U = $\{q_2\}$
- with Bchi acceptance condition and B = $\{q_2\}$
- with co-Bchi acceptance condition and B = $\{q_2\}$
- with parity acceptance condition

3.2 Answer

4 Ex. 4 p11

4.1 Question

Consider the sequence $\alpha = 1, -1, 1, 1, -1, 1, 1, 1, 1, ...$ where the i-th -1 is followed by 2^i occurrences of 1. What are the values of $\underline{\mathrm{MP(a)}}$ and $\overline{MP(a)}$?

4.2 Answer

5 Ex. 5 p12

5.1 Question

With $\aleph = 3^\omega$ and $\lambda = \frac{3}{4}$, what is the value of $\mathrm{DS}_\lambda(\aleph)$?

5.2 Answer

6 Ex. 6 p12

6.1 Question

Using the meanpayoff example, prove that the parity and co-Bchi payoff functions are Borel when $\triangleright = \geq$ and $a = 1$.

6.2 Answer

7 Ex. 7 p16

7.1 Question

In the game from Figure 2, does Eve have a strategy to ensure her mean-payoff value is non-negative?

7.2 Answer

8 Ex. 8 p16

8.1 Question

Describe the Mealy machine that corresponds to the strategy Eve for the game in Figure 2 which consists in playing from v_0 to v_1 every other time v_0 is visited and to v_0 otherwise.

8.2 Answer

9 Ex. 9 p17

9.1 Question

Describe the product of the strategy from the previous exercise and the game.

9.2 Answer

10 Ex. 10 p21

10.1 Question

Is "staying in v_0 forever" a worst-case optimal strategy for Eve in the game from Figure 2 with the mean-payoff function?

10.2 Answer

11 Ex. 11 p21

11.1 Question

Describe best-case optimal strategies for both players in the game from Figure 2 with the mean-payoff function. What is the co-operative value of the game?

11.2 Answer

12 Ex. 12 p22

12.1 Question

Consider the game from Figure 4 and suppose that Adam wants to ensure at most two vertices distinct are visited. From which vertices can he win against Eve? Describe a strategy of his that witnesses the fact. What if we allow at most three vertices?

12.2 Answer

13 Ex. 13 p27

13.1 Question

Give a proof by induction of Theorem 10 for Eve based on the definition of the attractor sets.

13.2 Answer

14 Ex. 14 p28

14.1 Question

Prove Theorem 11.

14.2 Answer

15 Ex. 15 p28

15.1 Question

Prove Theorem 12 using the fact that positional strategies suffice for both players in reachability games.

15.2 Answer

16 Ex. 16 p28

16.1 Question

Using the fact that R is exactly the set of vertices from which a player wins a reachability game (and the other one loses a safety game), prove that $G \setminus R$ contains no sinks. Prove that $G \setminus S$ also contains no sinks.

16.2 Answer

17 Ex. 17 p29

17.1 Question

Prove the upper bound on the running time of the algorithm.

17.2 Answer

18 Ex. 18 p30

18.1 Question

Given a (co-)Bchi game, construct a parity game with the same arena such that both players win from a vertex for the original objective if and only if they win from it for the parity objective. (Tip: you should not use more than three priorities)

18.2 Answer

19 Ex. 19

19.1 Question

Based on how many recursive calls Zielonkas algorithm makes on (co-)Bchi games, conclude that determining the winner of such games is decidable in polynomial time.

19.2 Answer

20 Ex. 20

20.1 Question

Compute all distinct attractor sets for both players in the game from Figure 5. From which vertices can each player guarantee to win the game?

20.2 Answer

21 Ex. 21

21.1 Question

Run Zielonkas algorithm to determine the vertices from which each player can guarantee to win the parity game from Figure 6.

21.2 Answer