Exercises on LTL

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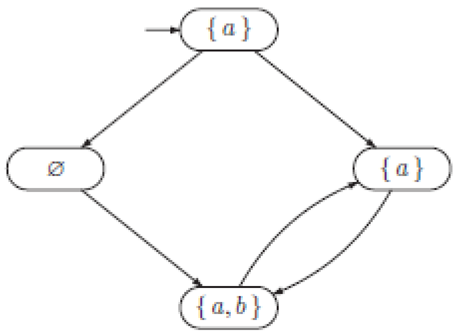
Ex. 3.1

Question

Give the traces on the set of atomic propositions $\{a, b\}$ of the following transition system:



Answer

$\{a\}(\{\phi\} \vee \{a\})(\{a, b\}\{a\})^{\omega}$

Ex. 3.2

Question

On page 97, a transformation is described of a transition system TS with possible terminal states into an equivalent" transition system TS^* without terminal states. Questions:

- Give a formal definition of this transformation $TS \mapsto TS^*$.
- Prove that the transformation preserves trace-equivalence, i.e., show that if TS_1, TS_2 are transition systems (possibly with terminal states) such that $\text{Traces}(TS_1) = \text{Traces}(TS_2)$, then $\text{Traces}(TS_1^*) = \text{Traces}(TS_2^*)$.

Answer

- $\forall s \in S$: if s is terminal add s_{stop} to S^* and update A^* and $\rightarrow^*$ accordingly.
- If $Traces(T_1) = Traces(T_2)$ then their finite and infinite traces concide $\forall t \in Traces(T_1)$: either t is infinite or $t\{\}^\omega \in Traces(T_1^*)$

Ex. 3.5

Question

Consider the set AP of atomic propositions defined by AP = $\{x = 0, x > 1\}$ and consider a nonterminating sequential computer program P that manipulates the variable x.
Formulate the following informally stated properties as LT properties:

- false
- initially x is equal to zero
- initially x differs from zero
- initially x is equal to zero, but at some point x exceeds one
- x exceeds one only finitely many times
- x exceeds one infinitely often
- the value of x alternates between zero and two
- true

Answer

- $\perp$
- x=0
- $\neg(x = 0)$
- $(x = 0) \wedge \Diamond(x > 1)$
- $\Diamond\Box(\neg x > 1)$
- $\Box\Diamond(x > 1)$
- $\Box\Diamond(x = 0) \wedge \Box(x = 0 \rightarrow \bigcirc(x = 2)) \wedge x = 0$
- $\top$

Ex. 3.6

Question

Consider the set AP $= \{A, B\}$ of atomic propositions. Formulate the following properties as LT properties and characterize each of them as being either an invariance, safety property, or liveness property, or none of these.

- A should never occur,
- A should occur exactly once,
- A and B alternate infinitely often,
- A should eventually be followed by B.

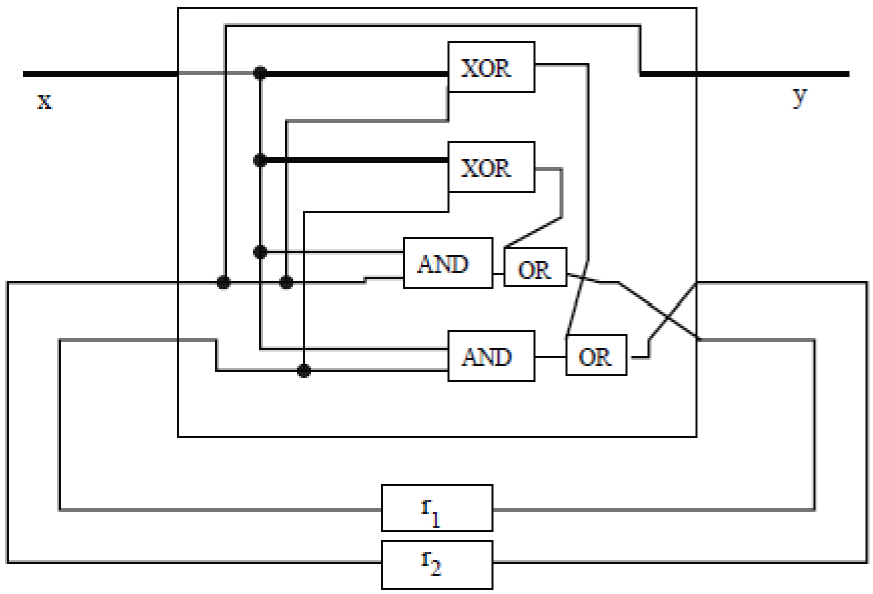
Answer

- $\neg\Diamond A$
  - Safety (A should never happen)
- $B\mathcal{U}A \bigcirc (\Box B)$
  - Invariant (purely state-based)
- $\Box(A \cap B)$
  - Invariant (purely state-based)
- $\Diamond(A \rightarrow B)$
  - Liveness (B will eventually happen)

Ex. 3.7

Question

Consider the following sequential hardware circuit:



The circuit has input variable x, output variable y, and registers r_1 and r_2 with initial values $r_1 = 0$ and $r_2 = 1$. The set AP of atomic propositions equals $\{x, r_1, r_2, y\}$. Besides, consider the following informally formulated LT properties over AP:
P1 : Whenever the input x is continuously high (i.e., x=1), then the output y is infinitely often high.
P2 : Whenever currently $r_2 = 0$, then it will never be the case that after the next input, $r_1 = 1$.
P3 : It is never the case that two successive outputs are high.
P4 : The configuration with $x = 1$ and $r_1 = 0$ never occurs.

- Give for each of these properties an example of an infinite word that belongs to P_i. Do the same for the property $(2^{AP})^\omega \backslash P_i$, i.e., the complement of P_i.
- Determine which properties are satisfied by the hardware circuit that is given above.

- Determine which of the properties are safety properties. Indicate which properties are invariants.
  - For each safety property P_i, determine the (regular) language of bad prefixes.
  - For each invariant, provide the propositional logic formula that specifies the property that should be fulfilled by each state.

Answer

- P1: $((1|0)111)^\omega$
  P2: $(0000)^\omega$
  P3: $(1111\square \rightarrow 0111)^\omega$
  P4: $(((01)|(11)|(00))11)^\omega$
- P1, P2 and P3 are satisfied, P4 is not.
- Safety properties:
  - P2
  - P3
- Invariants: P4

Ex. 3.8

Question

Let LT properties P and P' be equivalent, notation $P \cong P'$, if and only if $\text{pref}(P) = \text{pref}(P')$. Prove or disprove: $P \cong P'$ if and only if $\text{closure}(P) = \text{closure}(P')$.

Answer

$\text{pref}(P)$ can be seen as the finite part of $\text{closure}(P)$ and $\text{pref}(P')$ can be seen as the finite part of $\text{closure}(P')$.

Ex. 3.9

Question

Show that for any transition system TS, the set $\text{closure}(\text{Traces}(TS))$ is a safety property such that $TS \models \text{closure}(\text{Traces}(TS))$.

Answer

Ex. 3.10

Question

Let P be an LT property. Prove: pref(closure(P)) = pref(P).

Answer

closure(P) = $\{\sigma \in (2^{AP})^\omega | pref(\sigma) \subseteq pref(P)\}$
$pref(\sigma) = \{\hat{\sigma} \in (2^{AP})^* | \hat{\sigma}$ is a finite prefix of $\sigma)\}$
$\implies$ pref(P) is the finite part of pref(closure(P)).

Ex. 3.11

Question

Let P and P' be liveness properties over AP. Prove or disprove the following claims:

- $P \cup P'$ is a liveness property,
- $P \cap P'$ is a liveness property.

Answer the same question for P and P' being safety properties.

Answer

- $P \cup P'$ is a liveness property: P and P' will both evnetually happen
- $P \cap P'$ is not a liveness property: only part of P and P' will eventually happen
- $P \cup P'$ is a safety property: neither P or P' will ever happen
- $P \cap P'$ is a safety property: parts of P and P' will eventually happen