Exercises on Chapter 6

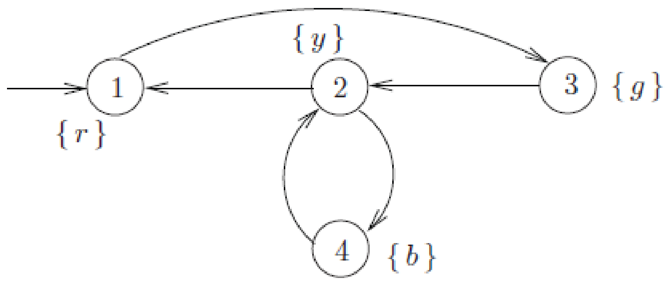
Beau De Clercq

November 2019

Ex. 6.1

Question

Consider the following transition system over AP $= \{b, g, r, y\}$:



The following atomic propositions are used: r (red), y (yellow), g (green), and b (black). The model is intended to describe a traffic light that is able to blink yellow. You are requested to indicate for each of the following CTL formulae the set of states for which these formulae hold:

(a) $\forall\Diamond\, y$

(b) $\forall\Box\, y$

(c) $\forall\Box\,\forall\Diamond\, y$

(d) $\forall\Diamond\, g$

(e) $\exists\Diamond\, g$

(f) $\exists\Box\, g$

(g) $\exists\Box\,\neg\, g$

(h) $\forall(b \,\mathsf{U}\, \neg\, b)$

(i) $\exists(b \,\mathsf{U}\, \neg\, b)$

(j) $\forall(\neg\, b \,\mathsf{U}\, \exists\Diamond\, b)$

(k) $\forall(g \,\mathsf{U}\, \forall(y \,\mathsf{U}\, r))$

(l) $\forall(\neg\, b \,\mathsf{U}\, b)$

Answer

- $\{1, 2, 3, 4\}$
- $\{\}$
- $\{1, 2, 3, 4\}$
- $\{1, 3\}$

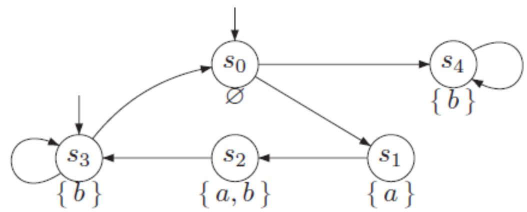
- $\{1, 2, 3, 4\}$
- $\{\}$
- $\{2, 4\}$
- $\{1, 2, 3, 4\}$
- $\{1, 2, 3, 4\}$
- $\{1, 2, 3, 4\}$
- $\{1\}$
- $\{4\}$

Ex. 6.2

Question

Consider the following CTL formulae and the transition system TS outlined on the right:

$$\begin{aligned}
\Phi_1 &= \forall(a \,\mathsf{U}\, b) \vee \exists \bigcirc (\forall\Box\, b) \\
\Phi_2 &= \forall\Box\, \forall(a \,\mathsf{U}\, b) \\
\Phi_3 &= (a \wedge b) \rightarrow \exists\Box\, \exists \bigcirc \forall(b \,\mathsf{W}\, a) \\
\Phi_4 &= (\forall\Box\, \exists\Diamond\, \Phi_3)
\end{aligned}$$



Determine the satisfaction sets $\mathrm{Sat}(\phi_i)$ and decide whether $TS \models \phi_i (1 \leq i \leq 4)$.

Answer

- ϕ_1: $TS \not\models \phi_1$ (checked with PRISM), $\mathrm{Sat}(\phi_1) = \{s_0, s_1\}$
- ϕ_2: $TS \not\models \phi_2$ (checked with PRISM), $\mathrm{Sat}(\phi_2) = \{s_1, s_2, s_3\}$
- ϕ_3: $TS \models \phi_3$ (checked with PRISM), $\mathrm{Sat}(\phi_3) = \{s_0, s_2, s_3\}$
- ϕ_4: $TS \models \phi_4$ (checked with PRISM), $\mathrm{Sat}(\phi_4) = \{s_0, s_2, s_3\}$

Ex. 6.3

Question

Which of the following assertions are correct? Provide a proof or a counterexample.

- If $s \models \exists \Box a$, then $s \models \forall \Box a$.
- If $s \models \forall \Box a$, then $s \models \exists \Box a$.
- If $s \models \forall \Diamond a \vee \forall \Diamond b$, then $s \models \forall \Diamond (a \vee b)$.
- If $s \models \forall \Diamond (a \vee b)$, then $s \models \forall \Diamond a \vee \forall \Diamond b$.
- If $s \models \forall (aUb)$, then $s \models \neg(\exists(\neg bU(\neg a \wedge \neg b)) \vee \exists \Box \neg b)$.

Answer

- Incorrect: the first $\models$ allows paths that not always see a while the second one only contains such paths (so if $s \models expr_1$ then $s \not\models expr_2$).
- Correct: $expr_1$ always sees a on all paths and $expr_2$ states that there are paths that always see a, so if $s \models expr_1$ it also holds that $s \models expr_2$.
- Correct: both expressions have the same meaning.
- Correct: both expressions have the same meaning.
- Correct: both expressions have the same meaning.

Ex. 6.4

Question

Let ϕ and ψ be arbitrary CTL formulae. Which of the following equivalences for CTL formulae are correct?

(a) $\forall\bigcirc\forall\Diamond\Phi \equiv \forall\Diamond\forall\bigcirc\Phi$

(b) $\exists\bigcirc\exists\Diamond\Phi \equiv \exists\Diamond\exists\bigcirc\Phi$

(c) $\forall\bigcirc\forall\Box\Phi \equiv \forall\Box\forall\bigcirc\Phi$

(d) $\exists\bigcirc\exists\Box\Phi \equiv \exists\Box\exists\bigcirc\Phi$

(e) $\exists\Diamond\exists\Box\Phi \equiv \exists\Box\exists\Diamond\Phi$

(f) $\forall\Box(\Phi \Rightarrow (\neg\Psi \wedge \exists\bigcirc\Phi)) \equiv (\Phi \Rightarrow \neg\forall\Diamond\Psi)$

(g) $\forall\Box(\Phi \Rightarrow \Psi) \equiv (\exists\bigcirc\Phi \Rightarrow \exists\bigcirc\Psi)$

(h) $\neg\forall(\Phi \mathsf{U} \Psi) \equiv \exists(\Phi \mathsf{U} \neg\Psi)$

(i) $\exists((\Phi \wedge \Psi) \mathsf{U} (\neg\Phi \wedge \Psi)) \equiv \exists(\Phi \mathsf{U} (\neg\Phi \wedge \Psi))$

(j) $\forall(\Phi \mathsf{W} \Psi) \equiv \neg\exists(\neg\Phi \mathsf{W} \neg\Psi)$

(k) $\exists(\Phi \mathsf{U} \Psi) \equiv \exists(\Phi \mathsf{U} \Psi) \wedge \exists\Diamond\Psi$

(l) $\exists(\Psi \mathsf{W} \neg\Psi) \vee \forall(\Psi \mathsf{U} \text{ false}) \equiv \exists\bigcirc\Phi \vee \forall\bigcirc\neg\Phi$

(m) $\forall\Box\Phi \wedge (\neg\Phi \vee \exists\bigcirc\exists\Diamond\neg\Phi) \equiv \exists X\neg\Phi \wedge \forall\bigcirc\Phi$

(n) $\forall\Box\forall\Diamond\Phi \equiv \Phi \wedge (\forall\bigcirc\forall\Box\forall\Diamond\Phi) \vee \forall\bigcirc(\forall\Diamond\Phi \wedge \forall\Box\forall\Diamond\Phi)$

(o) $\forall\Box\Phi \equiv \Phi \vee \forall\bigcirc\forall\Box\Phi$

Answer

- Correct
- Correct

- Incorrect
- Correct
- Incorrect
- Incorrect
- Incorrect
- Correct
- Incorrect
- Incorrect
- Correct
- Incorrect
- Correct
- Incorrect
- Correct

Ex. 6.7

Question

Transform the following CTL formulae into ENF and PNF. Show all intermediate steps.

$$\Phi_1 = \forall\big((\neg a)\,\mathsf{W}\,(b \rightarrow \forall\bigcirc c) \big)$$

$$\Phi_2 = \forall\bigcirc \big(\exists((\neg a)\,\mathsf{U}\,(b \wedge \neg c)) \vee \exists\Box\forall\bigcirc a \big)$$

Answer

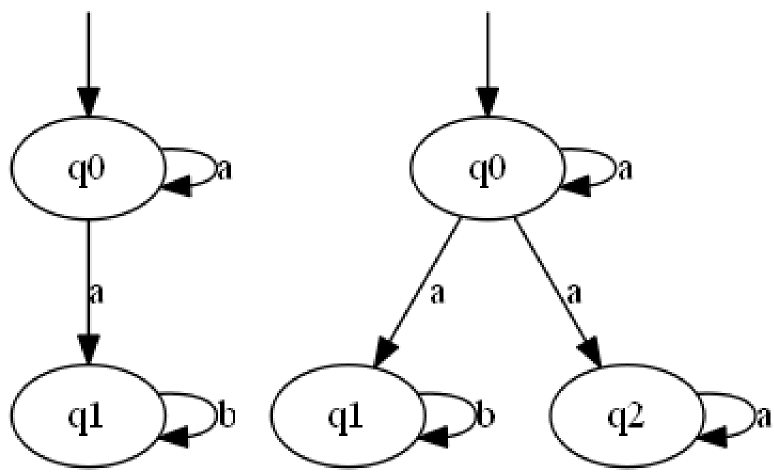
- ENF: $\neg\exists(((\neg a) \wedge \neg(b \implies \neg\exists\bigcirc\neg c)U(\neg(\neg a) \wedge \neg(b \implies \neg\exists\bigcirc\neg c))))$
  PNF: allready in PNF
- ENF: $\neg\exists\bigcirc\neg(\exists((\neg a)U(b \wedge \neg c)) \vee \exists\Box\neg\exists\bigcirc\neg a$
  PNF: allready in PNF

Ex. 6.8

Question

Provide two finite transition systems TS_1 and TS_2 (without terminal states, and over the same set of atomic propositions) and a CTL formula ϕ such that Traces(TS_1) = Traces(TS_2) and $TS_1 \models \phi$, but $TS_2 \not\models \phi$.

Answer



Ex. 6.9

Question

Answer

Ex. 6.13

Question

Answer

Ex. 6.14

Question

Check for each of the following formula pairs (ϕ_i, φ_i) whether the CTL formula ϕ_i is equivalent to the LTL formula φ_i. Prove the equivalence or provide a counterexample that illustrates why $\phi_i \not\equiv \varphi_i$.

(a) $\Phi_1 = \forall\Box\forall\bigcirc a.$ and $\varphi_1 = \Box\bigcirc a$

(b) $\Phi_2 = \forall\Diamond\forall\bigcirc a$ and $\varphi_2 = \Diamond\bigcirc a.$

(c) $\Phi_3 = \forall\Diamond(a \wedge \exists\bigcirc a)$ and $\varphi_3 = \Diamond(a \wedge \bigcirc a).$

(d) $\Phi_4 = \forall\Diamond a \vee \forall\Diamond b$ and $\varphi_4 = \Diamond(a \vee b).$

(e) $\Phi_5 = \forall\Box(a \rightarrow \forall\Diamond b)$ and $\varphi_5 = \Box(a \rightarrow \Diamond b).$

(f) $\Phi_6 = \forall(b \,\mathsf{U}\, (a \wedge \forall\Box b))$ and $\varphi_6 = \Diamond a \wedge \Box b.$

Answer

- Equivalent (absorption law)
- Equivalent (absortpion law)
- Equivalent: the $\forall$ in ϕ can be dropped (because of the $\Diamond$) which leaves $\Diamond(a \wedge \exists\bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$. Both expressions state there needs to be a path that has a followed by another a.
- Equivalent: stating that for all paths you will eventually see a or eventually see b is the same as stating that eventually you will see a or b.
- Equivalent: ϕ_5 states that for all paths it always holds that seeing an a implies that you will always eventually see a b. This is the same meaning φ_5 has in LTL.
- Not equivalent: ϕ_6 only contains paths *bbbbb.....abbbbbbbbb....* where φ_6 also satisfies $\neg b \neg b ... \neg babbbbbbb......$

1 Appendix

1.1 PRISM code (ex 6.2)

```
dtmc

module ex2
// local state
s : [0..4];
a : bool;
b : bool;

[] s=0 -> (s'=1) & (a'=true);
[] s=0 -> (s'=4) & (b'=true);
[] s=1 -> (s'=2) & (a'=true);
[] s=1 -> (s'=2) & (b'=true);
[] s=2 -> (s'=3) & (b'=true) & (a'=false);
[] s=3 -> (s'=3) & (b'=true);
[] s=3 -> (s'=0) & (b'=false);
[] s=4 -> (s'=4) & (b'=true);

endmodule

init
(s=0 | s=3) & a=false & b=false
endinit
```

1.2 PRISM properties (ex 6.2)

```
(A[(a=true)U(b=true)]) | (E[X(A[G(b=true)])])

A[G(A[(a=true)U(b=true)])]

((a=true) & (b=true)) => E[G(E[X(A[(b=true)W(a=true)])])]

A[G(E[F(((a=true) & (b=true)) => E[G(E[X(A[(b=true)W(a=true)])])])])]
```