

lecture 2

$\text{Reach}(T) = \{s \in S \mid \exists s_0 \in I \wedge s_0 \rightarrow \dots \rightarrow s_n = s\} = \text{Post}^*(I)$

program graph: digraph with conditions on the edges, $\text{PG} = (\text{Locations}, \text{Actions}, \text{Effect}, \text{transition relation}, \text{Initial locations}, \text{initial})$

lecture 3

safety (something bad never happens): $\neg F(\text{formula})$, liveness (something good will happen): $\text{GF}(\dots)$, persistence (ensure property holds forever): $\text{FG}(\dots)$, unconditional fairness: for all $i \wedge_i \text{GF}(\dots)$, strong fairness: for all $i \wedge_i \text{GF}(\dots) \rightarrow \text{GF}(\dots)$, weak fairness: for all $i \wedge_i \text{FG}(\dots) \rightarrow \text{GF}(\dots)$

temporal operators: \bigcirc, U, G, F

derived operators: $F\phi = \text{TU}\phi, G\phi = \neg F\neg\phi, \phi W\psi = (\phi U\psi) \vee G\phi, \phi R\psi = \neg(\neg\phi U\neg\psi)$

$\text{Words}(\phi) = \{w = a_0 a_1 a_2 \dots \in (2^p)^\omega \mid w \models \phi\}, \pi \in \text{Paths}(T): \pi \models \phi \leftrightarrow \pi \models \neg\phi, \text{TS } T \models \phi \leftarrow T \models \neg\phi$
($\text{Traces}(T) \in \text{Words}(\neg\phi)$)

lecture 4

NFA NBA concatenation: $I = I^1$ if $I^1 \cap F^1$ is empty, $I^1 \cup I^2$ otherwise; transition: $(q, A) = \delta^1(q, A)$ if $q \in Q^1$ and $\delta^1(q, A) \cap F^1$ is empty; $\delta^1(q, A) \cup I^2$ if $q \in Q^1$ and $\delta^1(q, A) \cap F^1$ is not empty; $\delta^2(q, A)$ if $q \in Q^2$
 ω -operator for NFA: add transitions from q_{new} to all states directly reachable by q_0 ; for all states in F , if $\delta(q, q_f, \alpha)$ then add $\delta(q, q_{\text{new}}, \alpha)$; the new set $I' = F' = I$; remove useless states

$L(\text{NBA}) \neq \emptyset$ iff there is an accepting state on a reachable cycle

Accepting run for NBA: sequence of states such that $q_i \in F$ for infinitely many indices i , $L(\text{NBA}) = \{\text{all words for which there is an accepting run}\}$

lecture 5

$\text{Closure}(\phi) = \text{set of all sub-formulas and their negation}$; a set of sub-formulas $B \in \text{Closure}$ is elementary if B is logically and locally consistent as well as maximal; states for a GNBA for LTL are all elementary sets, if ϕ is in B B is initial, the initial sets are those with an Until or the second part of an Until

Persistence checking: compute reachable SCCs and check if one contains a state satisfying $\neg\phi$ OR construct T and $A_{\neg\phi}$ in parallel and simultaneously construct the reachable fragment of the product via nested DFS

lecture 6

State formula: $\Phi = T[a|\Phi \wedge \Psi | \neg\Phi | \exists\phi | \forall\phi]$; path formula: $\phi = \bigcirc\Phi | \Phi U \Psi$

CTL derived operators: $\exists\Diamond\phi = \exists(TU\phi), \forall\Diamond\phi = \forall(TU\phi), \exists\Box\phi = \neg\forall\Diamond\neg\phi, \forall\Box\phi = \neg\exists\Diamond\neg\phi$

$\text{Sat}_T(\Phi) = s \in S \mid s \models \Phi; s \in S \models \Phi \leftrightarrow s \models \neg\Phi, \text{TS } T \models \Phi \leftarrow T \models \neg\Phi$

lecture 7

define boolean function with truth table, can only use \wedge, \vee, \neg , negating BDD: replace 0 and 1 constant value leaves, from decision tree to BDD: merge/remove useless subtrees, merge equivalent nodes

lecture 8

Symbolic model checking via BDDs: encode states as bitvectors, represent transitions as boolean functions (=switching functions) = characteristic function $\chi_R(s) = 1$ if $s \in R$ and 0 otherwise, represent labeling via $\chi_{\text{Sat}(s)}$; at the end of the model checking process check that $\neg\chi_I \vee f_{\text{Sat}(\Phi)} = 1$

lecture 10

Given a circuit with automaton A , check if there exists a function μ such that the language of $Ax\mu$ is empty

AIGER: aag M I L O A; every variable and AND-gate is represented by an even index $i \geq 2$, negations by $i+1$; $i \leq 2M+1$ for every index i

Games played on automata: states Q just keep track of latch values, Σ corresponds to valuations of the inputs, δ respects the latch next-step functions

LTL to games: construct NBA for ϕ , determinize it to a DPA and attempt to synthesize a strategy for player in the parity game