③ Bepaal de AO en SO van $y = 2xy' - y.y'^2$

$y = 2xy' - y.y'^2$ DVG van 1ste orde en 2de graad

→ moeilijk oplosbaar naar y'

→ oplosbaar naar x:

$$x = \frac{y}{2y'} + \frac{y.y'}{2} \qquad [x = G(y,y')]$$

(1) substitutie van $y' = p$ in $x = G(y,y')$:

$$\boxed{x = \frac{y}{2p} + \frac{y.p}{2}} \quad (*) \qquad [x = G(y,p)]$$

(2) afleiden van (*) naar y → 1. totale differentiaal nemen → 2. delen door dy

$$\frac{dx}{dy} = \left(\frac{1}{2p} + \frac{p}{2}\right)\cancel{dy} + \left(\frac{-y}{2p^2} + \frac{y}{2}\right)\frac{dp}{dy} \qquad \left[\frac{dx}{dy} = \frac{\partial G(y,p)}{\partial y}.\cancel{dy} + \frac{\partial G(y,p)}{\partial p}\frac{dp}{dy}\right]$$

$$\Downarrow \quad \frac{dx}{dy} = \frac{1}{y'} = \frac{1}{p} \;\&\; \frac{dp}{dy} = p'$$

$$\frac{1}{p} = \frac{1}{2p} + \frac{p}{2} + \left(\frac{-1}{2p^2} + \frac{1}{2}\right)yp' \Rightarrow \frac{p}{2} - \frac{1}{2p} + \left(\frac{1}{2} - \frac{1}{2p^2}\right)yp' = 0$$

$$\Rightarrow \left(\frac{1}{2} - \frac{1}{2p^2}\right)p + \left(\frac{1}{2} - \frac{1}{2p^2}\right)yp' = 0 \Rightarrow \boxed{\left(\frac{1}{2} - \frac{1}{2p^2}\right)(p + y.p') = 0}$$

valt uiteen in: ⇙ DVG v/d 1ste orde en 1ste graad

$*$ $\frac{1}{2} - \frac{1}{2p^2} = 0 \Rightarrow p^2 = 1$

$p = 1$ $\qquad$ $p = -1$

(3) (*) ⇓ substitutie in $x = G(y,p)$ ⇓

$$x = \frac{y}{2} + \frac{y}{2} = y \qquad x = -\frac{y}{2} - \frac{y}{2} = -y$$

$$\Rightarrow \boxed{\text{SO: } x^2 = y^2}$$

$*$ $p + yp' = 0 \xRightarrow{p' = dp/dy} \int \frac{dp}{p} = -\int \frac{dy}{y} \Rightarrow \ln|p| = \underbrace{-\ln|y|}_{=\ln\left|\frac{1}{y}\right|} + \underbrace{C_1}_{=\ln|C|} = \ln\left|\frac{C}{y}\right|$

$\Rightarrow p = \frac{C}{y}$ $\qquad$ (3) substitutie in $x = G(y,p)$ (*)

$$x = \frac{y^2}{2C} + \frac{C}{2} \Rightarrow 2Cx = y^2 + C^2$$

$$\Rightarrow \boxed{\text{AO: } y^2 = 2Cx - C^2}$$

7 Bepaal de AO en SO van $\cos y' = y - xy'$

$\cos y' = y - xy'$ DVG van 1ste orde en onbepaalde graad

$\rightarrow$ oplosbaar naar y:

$$y = \cos y' + xy' \qquad [y = G(x,y')]$$

(1) substitutie van $y' = p$ in $y = G(x,y')$:

$$\boxed{y = \cos p + xp} \quad (*) \qquad [y = G(x,p)]$$

(2) afleiden van (*) naar x $\rightarrow$ 1. totale differentiaal nemen
$\rightarrow$ 2. delen door dx

$$\frac{dy}{dx} = p\frac{\cancel{dx}}{\cancel{dx}} + (-\sin p + x)\frac{dp}{dx} \qquad \left[\frac{dy}{dx} = \frac{\partial G(x,p)}{\partial x}\cdot\frac{\cancel{dx}}{\cancel{dx}} + \frac{\partial G(x,p)}{\partial p}\cdot\frac{dp}{dx}\right]$$

$$\Downarrow \quad \frac{dy}{dx} = y' = p \;\&\; \frac{dp}{dx} = p'$$

$$p = p + (x - \sin p)p' \Rightarrow \boxed{(x - \sin p)p' = 0}$$ DVG v/d 1ste orde en 1ste graad

valt uiteen in:

* $x - \sin p = 0 \Rightarrow \sin p = x \Rightarrow p = \text{Bgsin}\, x$

$p \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos p = +\sqrt{1-\sin^2 p} = \sqrt{1-x^2}$ (grondformule)

(3) $\Downarrow$ substitutie in $y = G(x,p)$ (*)

$$\boxed{\underline{SO}: \; y = x\,\text{Bgsin}\, x + \sqrt{1-x^2}}$$

* $p' = 0 \Rightarrow p = C$

(3) $\Downarrow$ substitutie in $y = G(x,p)$ (*)

$$\boxed{\underline{AO}: \; y = Cx + \cos C}$$