② Bepaal de AO van $y.y'' + y'^2 = 2y.y'$

$y.y'' + y'^2 = 2y.y'$: 2^{de} orde DVG die x niet expliciet bevat $F(y, y', y'') = 0$

(1) $y' = p(y) = dy/dx \Rightarrow y'' = \dfrac{dp(y)}{dx} = \dfrac{dp(y)}{dy} \cdot \dfrac{dy}{dx} = p.p'$

$y.p.p' + p^2 = 2y.p$

$\Downarrow$ valt uiteen in :

$* \; p = 0 \xrightarrow[(2)]{p=y'} y' = 0 \Rightarrow \boxed{y = C}$

$* \; y.p' + p = 2y$: DVG v/d 1^{ste} orde en 1^{ste} graad

$\Downarrow$

$\boxed{p' + p.\underbrace{\left(\frac{1}{y}\right)} = \underbrace{2}}$ (*) : lineaire DVG in p en p'
$p' + p.\underbrace{Q(y)} = \underbrace{R(y)}$

(1) substitutie van $p = u.v$ in (*) :

$$u'v + \underbrace{u.v' + u.v.\left(\frac{1}{y}\right)} = 2 \qquad (**)$$

(2) kies een functie v zodat de coëfficiënt van u nul is :

$$v' + v\left(\frac{1}{y}\right) = 0 \xrightarrow[v' = dv/dy]{} \int \frac{dv}{v} = -\int \frac{dy}{y} \Rightarrow \ln|v| = -\ln|y| + C_0 = \ln\left|\frac{1}{y}\right| + C_0$$

$\rightarrow$ kies bv. $C_0 = 0$: $\boxed{v = \frac{1}{y}}$ (***)

(3) substitutie van (***) in (**)

$$u' \cdot \frac{1}{y} = 2 \xrightarrow[u' = du/dy]{} \int du = \int 2y\,dy \Rightarrow \boxed{u = y^2 + C_1} \quad (****)$$

(4) substitutie van (***) en (****) in $p = u.v$

$$p = \frac{y^2 + C_1}{y}$$

(2) $p = y'$

$$y' = \frac{y^2 + C_1}{y} \xrightarrow[y' = dy/dx]{} \int \frac{y}{y^2 + C_1} dy = \int dx \Rightarrow \frac{1}{2} \ln|y^2 + C_1| = x + C_2$$

$$\Rightarrow \ln|y^2 + C_1| = 2x + 2C_2 \Rightarrow y^2 + C_1 = e^{2x + 2C_2} = e^{2C_2} \cdot e^{2x} = C_3 e^{2x}$$

$$\Rightarrow y^2 = C_3 e^{2x} - C_1 \Rightarrow \boxed{y^2 = C_3 e^{2x} + C_4}$$

Aangezien $y = C$ een bijzonder geval is van $y^2 = C_3 e^{2x} + C_4$ (nl. $C_3 = 0$) is:

$$\boxed{\underline{AO}: \; y^2 = C_3 e^{2x} + C_4}$$

(11) Bepaal de AO van $y''^2 = 4xy'' - 4y'$

$y''^2 = 4xy'' - 4y'$: 2de orde DVG die y niet expliciet bevat : $F(x, y', y'') = 0$

(1) $y' = p(x) = dy/dx \Rightarrow y'' = \frac{d\,p(x)}{dx} = p'$

$p'^2 = 4xp' - 4p$: DVG v/d 1ste orde en 2de graad

$\Downarrow$

$\boxed{x = \frac{p'}{4} + \frac{p}{p'}}$ $\rightarrow$ oplosbaar naar x $[x = G(p, p')]$

(1) substitutie van $p' = k$ in $x = G(p, p')$:

$\boxed{x = \frac{k}{4} + \frac{p}{k}}$ (*) $[x = G(p, k)]$

(2) afleiden van (*) naar p $\rightarrow$ 1. totale differentiaal nemen; 2. delen door dp

$$\frac{dx}{dp} = \frac{1}{k}\frac{dp}{dp} + \left(\frac{1}{4} - \frac{p}{k^2}\right)\frac{dk}{dp} \qquad \left[\frac{dx}{dp} = \frac{\partial G(p,k)}{\partial p}\frac{dp}{dp} + \frac{\partial G(p,k)}{\partial k}\frac{dk}{dp}\right]$$

$$\Downarrow \quad \frac{dx}{dp} = \frac{1}{p'} = \frac{1}{k} \quad \& \quad \frac{dk}{dp} = k'$$

$$\frac{1}{k} = \frac{1}{k} + \left(\frac{1}{4} - \frac{p}{k^2}\right)k' \Rightarrow \boxed{\left(\frac{1}{4} - \frac{p}{k^2}\right)k' = 0}$$ DVG v/d 1ste orde en 1ste graad

valt uiteen in:

* $k' = 0 \Rightarrow k = C_1$

$\Downarrow$ (3) substitutie in (*)

$$x = \frac{C_1}{4} + \frac{p}{C_1} \Rightarrow p = C_1 x - \frac{C_1^2}{4}$$

(2) $p = y' = \frac{dy}{dx}$

$$\int dy = \int\left(C_1 x - \frac{C_1^2}{4}\right)dx \Rightarrow y = \frac{C_1}{2}x^2 - \frac{C_1^2}{4}x + C_0$$ $C_2 = 4C_0$

$$\Rightarrow \boxed{4y = 2C_1 x^2 - C_1^2 x + C_2}$$

* $\frac{1}{4} - \frac{p}{k^2} = 0 \Rightarrow k^2 = 4p \Rightarrow k = \pm 2\sqrt{p}$

$\Downarrow$ (3) substitutie in (*)

$$x = \pm\frac{\sqrt{p}}{2} \pm \frac{p}{2\sqrt{p}} = \pm\frac{\sqrt{p}}{2} \pm \frac{\sqrt{p}}{2} \Rightarrow x = \pm\sqrt{p} \Rightarrow p = x^2$$

(2) $p = y' = \frac{dy}{dx}$

$$\int dy = \int x^2 dx \Rightarrow y = \frac{x^3}{3} + C$$

$$\Rightarrow \boxed{y = \frac{x^3}{3} + C}$$

12 Bepaal de PO van $y'' + 2xy'^2 = 0$ waarvoor $y(1)=1$ en $y'(1)=1$

$y'' + 2xy'^2 = 0$: 2^{de} orde DVG die y niet expliciet bevat: $F(x, y', y'') = 0$

(1) $y' = p(x) = dy/dx \Rightarrow y'' = \frac{dp(x)}{dx} = p'$

$p' + 2xp^2 = 0$: DVG v/d 1ste orde en 1ste graad

$\left(p' = \frac{dp}{dx}\right)$ scheiden v/d veranderlijken

$$-\int \frac{dp}{p^2} = \int 2x\,dx \Rightarrow \frac{1}{p} = x^2 + C_1 \Rightarrow p = \frac{1}{x^2+C_1}$$

(2) $p = y'$

$$y' = \frac{1}{x^2+C_1}$$

$y'(1) = 1 = \frac{1}{1+C_1} \Rightarrow C_1 = 0$

$$y' = \frac{1}{x^2}$$

$y' = \frac{dy}{dx}$

$$\int dy = \int \frac{dx}{x^2}$$

$$y = -\frac{1}{x} + C_2$$

$y(1) = 1 = -1 + C_2 \Rightarrow C_2 = 2$

$$\boxed{y = -\frac{1}{x} + 2}$$