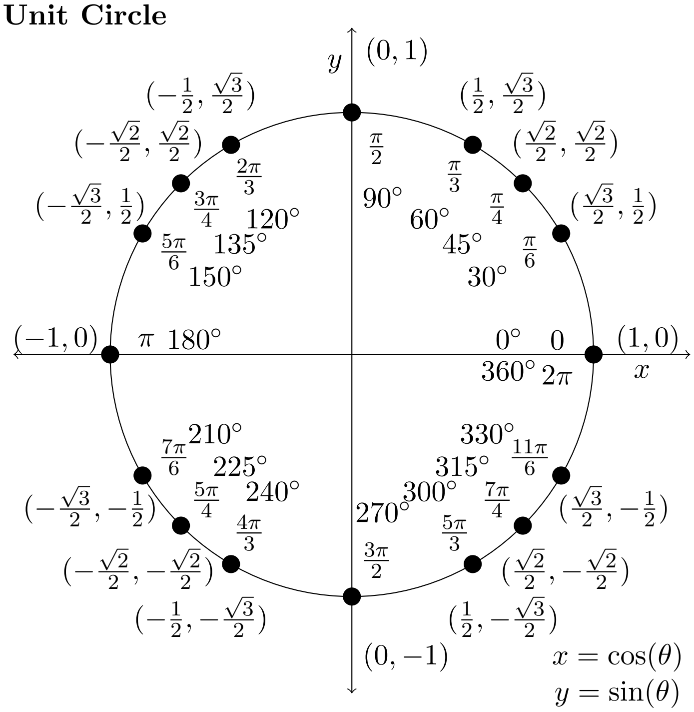
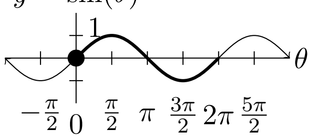
Unit Circle

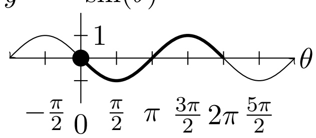


Basic Graphs

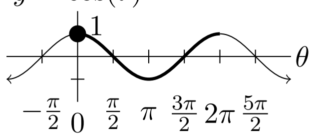
$y = \sin(\theta)$



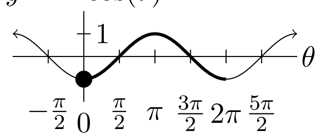
$y = -\sin(\theta)$



$y = \cos(\theta)$



$y = -\cos(\theta)$



Basic Definitions and SOH CAH TOA

$$\sin(\theta) = \frac{\text{Opp.}}{\text{Hyp.}} \qquad \cos(\theta) = \frac{\text{Adj.}}{\text{Hyp.}} \qquad \tan(\theta) = \frac{\text{Opp.}}{\text{Adj.}}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Shifts and Reflections

$$\cos(\theta + 2\pi) = \cos(\theta) \qquad \sin(\theta + 2\pi) = \sin(\theta)$$
$$\cos(\theta + \pi) = -\cos(\theta) \qquad \sin(\theta + \pi) = -\sin(\theta)$$
$$\cos(\theta - \frac{\pi}{2}) = \sin(\theta) \qquad \sin(\theta + \frac{\pi}{2}) = \cos(\theta)$$
$$\cos(-\theta) = \cos(\theta) \qquad \sin(-\theta) = -\sin(\theta)$$
$$\tan(\theta + \pi) = \tan(\theta) \qquad \tan(-\theta) = -\tan(\theta)$$

Pythagorean Identities

$$* \ \cos^2(\theta) + \sin^2(\theta) = 1$$
$$1 + \tan^2(\theta) = \sec^2(\theta)$$
$$\cot^2(\theta) + 1 = \csc^2(\theta)$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

Law of Sines

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Sum and Difference

$$* \ \cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$
$$* \ \sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$
$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$
$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

Product to Sum

$$\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2}$$
$$\sin(\alpha)\sin(\beta) = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2}$$
$$\sin(\alpha)\cos(\beta) = \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{2}$$
$$\cos(\alpha)\sin(\beta) = \frac{\sin(\alpha + \beta) - \sin(\alpha - \beta)}{2}$$
$$\tan(\alpha)\tan(\beta) = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{\cos(\alpha - \beta) + \cos(\alpha + \beta)}$$

Sum to Product

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$
$$\cos(\alpha) - \cos(\beta) = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$
$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$
$$\sin(\alpha) - \sin(\beta) = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

Double Angle

$$\begin{aligned} \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= 2\cos^2(\theta) - 1 \\ &= 1 - 2\sin^2(\theta) \end{aligned}$$
$$\sin(2\theta) = 2\cos(\theta)\sin(\theta)$$
$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

Note: The three identities marked (*) can be used to prove the vast majority of the other trigonometric identities.

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Triple Angle

$$\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$$
$$\sin(3\theta) = -4\sin^3(\theta) + 3\sin(\theta)$$
$$\tan(3\theta) = \frac{3\tan(\theta) - \tan^3(\theta)}{1 - 3\tan^2(\theta)}$$

Half Angle

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$
$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$
$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}} = \frac{\sin(\theta)}{1+\cos(\theta)} = \frac{1-\cos(\theta)}{\sin(\theta)} = \csc(\theta) - \cot(\theta)$$

Rectangular to Polar

$$r = \sqrt{x^2 + y^2}$$
$$\tan(\theta) = \frac{y}{x}$$

Note: $\theta = \tan^{-1}(\frac{y}{x})$ for $x > 0$, but $\theta = \pi + \tan^{-1}(\frac{y}{x})$ for $x < 0$.

Polar to Rectangular

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

Euler's Formula

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

Range of Inverse Functions

$$\theta = \cos^{-1}(t), \qquad 0 \le \theta \le \pi$$
$$\theta = \sin^{-1}(t), \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$
$$\theta = \tan^{-1}(t), \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

Composition

$$\cos(\sin^{-1}(t)) = \sqrt{1-t^2} \qquad \cos(\tan^{-1}(t)) = \frac{1}{\sqrt{1+t^2}}$$
$$\sin(\cos^{-1}(t)) = \sqrt{1-t^2} \qquad \sin(\tan^{-1}(t)) = \frac{t}{\sqrt{1+t^2}}$$
$$\tan(\cos^{-1}(t)) = \frac{\sqrt{1-t^2}}{t} \qquad \tan(\sin^{-1}(t)) = \frac{t}{\sqrt{1-t^2}}$$

Solving

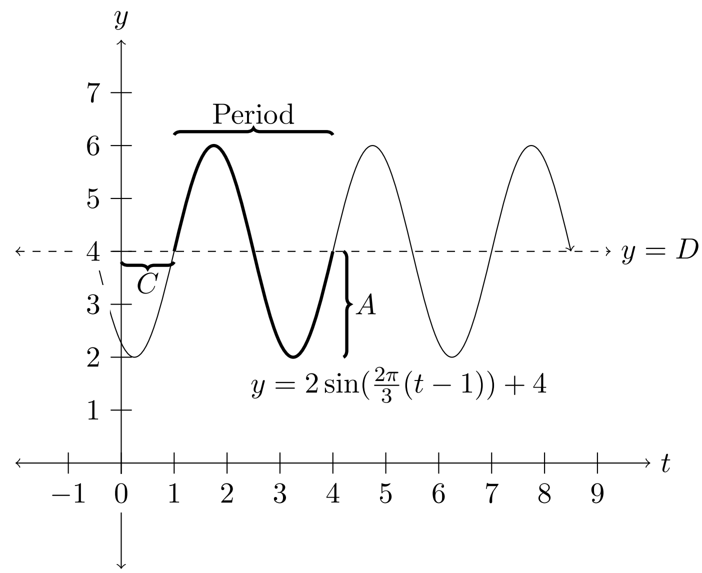
$$\cos(\theta) = t \implies \theta = \cos^{-1}(t) + 2\pi k \quad \text{or } \theta = -\cos^{-1}(t) + 2\pi k$$
$$\sin(\theta) = t \implies \theta = \sin^{-1}(t) + 2\pi k \quad \text{or } \theta = \left[\pi - \sin^{-1}(t)\right] + 2\pi k$$
$$\tan(\theta) = t \implies \theta = \tan^{-1}(t) + \pi k$$

Note: Using $\theta = [2\pi - \cos^{-1}(t)] + 2\pi k$ in place of $\theta = -\cos^{-1}(t) + 2\pi k$ is acceptable.

Converting to $A\sin(Bt + \phi)$

$$A\sin(Bt+\phi) = a_1\sin(Bt) + a_2\cos(Bt)$$
$$\tan(\phi) = \frac{a_2}{a_1}, \quad A = \sqrt{a_1^2 + a_2^2}$$

Form of Sin Functions



$$y = A\sin(B(t-C)) + D$$

A = Amplitude $\qquad B = \dfrac{2\pi}{\text{Period}}$

C = Horizontal Shift $\qquad D$ = Midline

$B \cdot C$ = Phase Shift

Complementary and Supplementary

- Complementary angles add up to 90° or $\frac{\pi}{2}$ radians.
- Supplementary angles add up to 180° or π radians.

Ambiguous Case

- The ambiguous case is Side-Side-Angle (SSA).
- This typically matters when using Law of Sines.

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