Weekly Practice 7 key

1. Since this is a sequence of tasks that has two steps, we find the number of ways to do each step and then **multiply** the results. We get

$$4 \times 6 = 24$$

total routes.

2. The answer here depends on whether the word is even length or odd length.

Even length: Split the word in half. The second half of the word is completely determined by the first half (for example the last letter has to be the same as the first letter, the next-to-last letter has to be the same as the second letter, etc.), so choosing a letter in the left half is also a choice of letters in the right half. There are $n/2$ letters in the first half of the word and we have a free chose of 26 letters for each position. So the total number of even-length palindromes is $26^{n/2}$.

Odd length: Words of odd length have a middle letter (for example the middle letter of "kayak" is "y"). We have a free choice of 26 letters for the one in the middle, leaving $n-1$ letters not in the middle. The letters on the right half of the word have to be the same as the letters on the left half as described above. There are $\frac{n-1}{2}$ letters in the left half and we have a free choice of 26 letters for those. This gives us $26^{(n-1)/2}$ ways to choose the left (and right) halves, and 26 ways to pick the middle one. Picking the left half then picking the middle is a sequence of tasks, so we multiply the number of ways to do each task. This gives us a total of:

$$26^{(n-1)/2} \cdot 26 = 26^{(n+1)/2}.$$

3. Choosing a single shirt here involves a sequence of three choices: gender, color, and size. There are 2 choices for gender, 12 for color, and 3 for size. Since those choices are independent of each other, the result is just the product: $2 \times 12 \times 3 = 72$.

4. (a) $2^{16} = 65536$ as discussed in the videos and in class.
   (b) $\binom{16}{5} = 4368$ as discussed in the videos and in class.
   (c) $\binom{16}{11} = 4368$ as discussed in the videos and in class. Note this is the same as the previous part.
   (d) Ending the bitstring in 00 means there are 14 bits we can choose, and there's a free choice of 2 possibilities for each bit. So we get $2^{14} = 16384$.
   (e) There are 2^{14} 16-bit strings ending in 00 (see above) and also 2^{14} that begin with 11 by the same logic. We also need to count the number of 16-bit strings that *both* start with 11 *and* end in 00: there are 2^{12} of those since this involves a free choice of 0 or 1 for the middle 12 bits. So the total count is $2^{14} + 2^{14} - 2^{12} = 28672$.
   (f) The number of such bitstrings is zero, because it's impossible to get the bits to add up to 15 if two of them are forced to be 0's.
   (g) If the final two bits are forced to be 0's, then the rest of the bit string consists of 14 bits with 4 1's. The number of 14-bit strings with a weight of 4 like this is $\binom{14}{4} = 1001$ (a decimal integer, not a bit string!).