AEP 8: Proving binomial coefficient identities

Overview

In this AEP, you'll apply the concept of mathematical induction to prove an identity about binomial coefficients.

Learning Targets associated with this AEP:

- **P.2 (Core)**: Given a statement to be proven by (weak) induction, I can state and prove the base case, state the inductive hypothesis, and outline the proof.

Remember, AEPs do not have fixed deadlines; simply work on this item until you are ready to submit it. But remember the **Two Items Per Week Rule.**

Technology Background

No tech background needed or used on this one.

AEP Description and Tasks

What this AEP is about

The binomial coefficient $\binom{n}{k}$ not only solves lots of counting problems, it has interesting properties of its own. We've seen a few of these **binomial coefficient identities** already:

- For all natural numbers n and k greater than zero and $n \geq k$, $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$. (This was the basic recurrence relation that defines the binomial coefficient; think of it as an "identity" or a property that it has.)
- For all natural numbers n and k with $n \geq k$, $\binom{n}{k} = \binom{n}{n-k}$. (We proved this using a combinatorial proof and symmetry back in Module 8.)
- For all natural numbers n, $\sum_{i=0}^{n} \binom{n}{i} = 2^n$. (Also proven in Module 8 using a combinatorial proof, also using algebra.)

These can all be thought of as **propositions** about the binomial coefficient, and since they are propositions using the natural numbers, you might wonder if we could use mathematical induction to prove these or other similar identities. And you'd be right!

Setup (do this first)

No real setup here, but you will need to review the concepts of mathematical induction, particularly the three-step framework that this method uses.

Tasks for this AEP

Pick exactly one of the following identities and write a clear and correct proof using mathematical induction. (Please note, these are all true identities.)

1. For all natural numbers $n > 0$, $\sum_{k=1}^{n} k\binom{n}{k} = n \cdot 2^{n-1}$.
2. For all natural numbers n, $\sum_{k=0}^{n} \binom{n}{k} 2^k = 3^n$.
3. For all natural numbers n, $\sum_{k=0}^{n} \binom{n-k}{k} = f_n$ where f_n is the nth Fibonacci number (remember we start indexing at 0, so $f_0 = 1, f_1 = 1, f_2 = 2, \ldots$). Also: for this one, assume that $\binom{a}{b} = 0$ if $b > a$. For example $\binom{2}{4} = 0$.

Here is a document with some demonstrations of why these are true: https://drive.google.com/file/d/1dDTf_ooG6spbyQ2ilhztqraHYxhoCL3P/view?usp=sharing These aren't proofs! These are just demonstrations that show the propositions at work.

Your proof should pass the test of critical analysis:

- The base case, inductive hypothesis, and proof step must be clearly stated and executed.
- There must be no significant or fatal errors or omissions in your proof.
- Your proof must be clearly written and understandable to a human audience. **Pretend your audience is your classmates and that you are teaching the class with your proof.** If you cannot honestly say that students in the class would clearly understand your argument just by reading your proof, then work on making it clearer.

See below for grading criteria; it's a little different this time.

Assignment Expectations and Grading Criteria

Specific grading criteria for this AEP:

- You will earn a grade of **M** if your work correctly establishes the base case, correctly states the inductive hypothesis, and correctly outlines the remainder of the proof following the inductive hypothesis. That is, **you'll receive an M on this AEP if you simply demonstrate that you have satisfied Learning Target P.2.**
- You will earn an **M** in this way **even if you do not submit a complete proof, but just the framework.** Ordinarily incomplete work is not acceptable, but the framework will count this time.
- Additionally, if you earn an **M** on this AEP, **it will also automatically provide you with a "check" for Learning Target P.2**. If you have earned a check already through a Checkpoint or oral assessment on Learning Target P.2, then an **M** on this AEP will put you at Mastery level on that target.
- You will earn a grade of **E** if, and only if, you do all the work that would earn an **M** (see above) and also submit a complete, correct, and clear proof according to the standards outlined above.

As always, AEPs are graded using the “EMRN” rubric found in the syllabus. Please note, it is the case with all AEP’s that **your grade is primarily based on your explanations and writing, and only secondarily on the precision and correctness of your computations.** Correct computations with insufficient explanation will need to be revised and may receive an “N” grade.

Also, **significant incompleteness will result in a grade of “N”.** This includes the following:

- **Giving answers with no explanations.** As mentioned above, you are being graded on explanations and writing, not so much on answers. Submissions that contain items where there is an answer with no explanation or insufficient explanation, will be graded “N” and returned without comment.
- Leaving items blank (even accidentally)
- Leaving large gaps in computations (skipping important steps)
- Giving only partial attempts at tasks (for example, working down to a certain point in a solution and then stopping because you need help)

A grade of “E” is given if all of the above expectations are met, and the work is of superior quality in terms of the clarity of explanations and work, appearance of the writeup, and precision of the mathematics.

Submitting your work

AEP submissions must be typewritten and saved as either a PDF or MS Word file. No part of your submission may involve handwriting; work that is submitted that contains handwriting will be graded N and returned without feedback. This includes electronic handwritten documents, for example using a stylus and a note-taking app. To type up your work, you can use MS Word or Google Docs (both of which have equation editors for mathematical notation) or any other computer-based math typesetting tool. Just make sure you save your work as a Word document or PDF (no `.odt`, `.rtf`, or other file extensions are allowed).

If you are doing the programming option: Submit your code in a Jupyter notebook using Colab as outlined in earlier AEPs and elsewhere: Put your code in a notebook and then share a link to the notebook with me — do not “invite” me because the email will go into spam — and make sure that the permissions are set to “Everyone with the link can edit”. Then include the link in the writeup for the rest of your work.

When you have your work typed up, double-check it for neatness, correctness, and clarity. Then, go to Blackboard, then **Assignments**, then **AEP**, then **AEP 8**. Clicking on the text “AEP 7” will take you to a place on Blackboard where you can upload your work. All grading and revisions of labs are done entirely on Blackboard. **Do not email your work to the professor** – only Blackboard submissions are accepted.

Getting Help

Please note that according to the syllabus, for AEP’s **“no interactions at all with another person or with unauthorized sources on the internet is allowed.”** Violations of this rule include *any* consultation with other students or former students, including Math Center tutors; using work from another student or former student; submitting the problem set to an online help site such as Chegg or Coursehero; or asking for help in an online forum. All such violations will be treated as academic dishonesty and will result in a grade of “N” and being banned from revising the work.

You **may** ask me (Talbert) for help on this assignment in the form of **specific mathematical or technical questions**. If I cannot answer a question because it would give too much away, I’ll tell you so. **However please note: I will not “look over your work” before you submit it to give you feedback on the overall submission**; the expectations are clearly laid out above, so just follow those directions and submit your best work, and you’ll be allowed to revise it if needed.

You can ask technology related questions to anyone at any time. For example if you need help with Desmos, or with figuring out how to type up your work, there are no restrictions on that. I recommend the `#tech` channel on Campuswire so that you’ll reach a large audience.