Directions:

- Do only the Checkpoint problems that you need to take and feel ready to take. If you have already earned Mastery on a Learning Target, do not attempt a problem for that Target! You can skip a Target if you need more time to practice with it, and take it on the next round.
- Do not put any work on this form; do all your work on separate pages. You may either handwrite or type up your work.
- Clearly indicate which Learning Target you are attempting at the beginning of its solution; please also turn in solutions for learning targets in order (for example, do not turn in work for A.2 after work for SF.1). The easiest way to do this is to put each Learning Target on its own solution page and do not put more than one Learning Target on a single page.
- If you are handwriting, submit your work by **scanning your work** using a scanning app or scanning device; **do not just take a picture** but scan your work to a clear, legible, black and white PDF file of size less than 100 MB. Work submitted as an image file (JPG, PNG, etc.) will not be graded.
- Unless explicitly stated otherwise, you must show your work or explain your reasoning clearly on each item of each problem you do. Responses that consist of only answers with no work shown, or where the work is insufficient or difficult to read, or which have significant gaps or omissions (including parts left blank) will be given a grade of "x".
- Submit your work by uploading it as a PDF or Word file to the appropriate assignment area on Blackboard.

Learning Target A.1: *I can represent an integer in base 2, 8, 10, and 16.*

Perform all of the following conversions. Show all work and explain all reasoning.

1. 961 in decimal; convert to binary and hexadecimal.
2. 01010110 11010010 in binary; convert to decimal and octal. Note, this is one binary integer with 16 bits, not two with 8 bits each.
3. 8A1 in hexadecimal; convert to decimal.

Learning Target A.2 (Core): *I can add, subtract, multiply, and divide two integers written in binary.*

Perform all of the following computations in binary, without changing to base 10. Show all work and explain all reasoning.

1. `11110111 + 01110010`
2. `11010101 - 01101101`
3. `10110100` $\times$ `111`
4. `10101001` $\div$ `1001`

Learning Target A.3: *I can compute $a\%b$ given integers a and b and perform arithmetic mod n.*

Perform all of the following computations and either show your work or explain what you did in words.

1. $57322\%6$
2. $(-899)\%13$

3. $7491117143048354781 6\%2$
4. $(684+368)\%11$
5. $(6^{77})\%8$ using repeated squaring

Learning Target L.1: *I can use propositional variables and logical connectives to represent statements; and interpret symbolic logical statements in plain language.*

Let p, q, and r be the propositions:

p: Grizzly bears have been seen in the area.

q: Hiking is safe on the trail.

r: Berries are ripe along the trail.

Write these English propositions using p, q, and r and logical connectives (including negation).

1. Translate the following English sentences into symbolic logic expressions:
   (a) Berries are not ripe along the trail, but grizzly bears have been seen in the area.
   (b) If berries are ripe along the trail, then hiking is safe along the trail if and only if grizzly bears have not been seen in the area.
2. Translate the following symbolic logic expressions into clear English sentences:
   (a) $p \to (q \vee r)$
   (b) $\neg(p \wedge q) \to r$

Learning Target L.2 (Core): I can write the negation, converse, and contrapositive of a conditional statement and use DeMorgan's Laws to simplify symbolic logical expressions.

1. For each of the conditional statements below, write the converse, contrapositive, and negation. If the original statement is in symbols, your answers should be in symbols; if in words, the answer should be in clear English as well. For symbolic statements, do not just put $\neg$ in front of the original to form a negation, but instead use what we learned about negations of conditional statements to simplify. Likewise for English statements, don't just use "It is not the case that..." to form the negation.
   (a) If you passed MTH 201, you will take MTH 202.
   (b) $A \to (B \vee C)$
2. Use DeMorgan's laws to state the negations of each of the following. If the original statement is in symbols, your answers should be in symbols; if in words, the answer should be in clear English as well. For symbolic statements, do not just put $\neg$ in front of the original to form the negation, but instead use what we learned about negations of conditional statements to simplify. Likewise for English statements, don't just use "It is not the case that..." to form the negation.
   (a) David is neither a vegetarian nor a runner.
   (b) $A \vee (\neg B)$

Learning Target L.3: I can determine whether a quantified statement is true, false, or underdetermined, and state its negation.

Below are some statements; the domain of each is the set of all integers (positive, negative, and zero — but only integers). For each one, state whether the statement is TRUE, FALSE, or UNDERDETERMINED. For all the ones that are TRUE or FALSE, state the negation.

1. $\forall n(n^2 = 0)$

2. n is a triangular number
3. $\exists a(a^2 \text{ is a multiple of } 10)$
4. $\forall a \exists b(a + b = 0)$
5. $\exists a \exists b(ab = 1)$

Learning Target L.4 (Core): I can write the truth table for a logical statement.

Construct a truth table for each of the following propositions:

1. $p \vee (\neg p)$
2. $(p \wedge q) \rightarrow (p \vee q)$
3. $(p \wedge q) \wedge (\neg r)$

Learning Target L.5: I can determine if a statement is a tautology and whether two statements are logically equivalent.

1. Determine whether the statement $(p \wedge q) \rightarrow (p \vee q)$ is a tautology. Show your work and clearly indicate your answer.
2. Determine whether the following statements are logically equivalent or not. Show your work and clearly indicate your answer.
   (a) $p \rightarrow q$ and $q \rightarrow \neg p$

Learning Target SF.1 (Core): I can represent a set in roster notation and set-builder notation; determine if an object is an element of a set; and determine set relationships (equality, subset).

1. Write the following sets in roster notation:
   (a) $\{a \in \mathbb{N} : a^2 > 100\}$
   (b) $\{a^2 - 1 : a \in \mathbb{N}\}$
2. Write the following sets in set-builder notation. There may be more than one correct representation; but your representation must restate membership in the set in some way.
   (a) $\{1, 5, 9, 13, 17, \dots\}$
   (b) $\{4, 8, 16, 32, 64, \dots\}$
3. Define the following sets:

$$A = \{x, y, z, t\}$$
$$B = \{s, t, u, v, w, x\}$$
$$C = \{t, v, u, y\}$$
$$D = \{s, t, u, v\}$$

   Determine whether each of the statements is true or false. State your answer clearly. (You do not have to justify your answers, but they have to be correct.) **Notice**: "Meaningless" is no longer an option on this part; all of the items below are mathematically meaningful. Just state whether they are true or false.

   (a) $x \in D$
   (b) $D \subseteq B$
   (c) $\{x, v\} \subseteq C$
   (d) $D = C$

(e) $|D| = |C|$

Learning Target SF.2: I can perform operations on sets (intersection, union, complement, Cartesian product) and determine the cardinality of a set.

Let $A = \{1, 2\}$ and $B = \{n \in \mathbb{N} : n \text{ is a multiple of } 3\}$, and $C = \{2, 4, 6, 8\}$ and let the universal set be $\mathbb{N}$. Determine all of the following. If the item is a set, write the set in roster notation.

1. $A \cup C$
2. $A \cap B$
3. $A \times \{q, r, s\}$
4. $|C \setminus B|$

Learning Target SF.3 (Core): I can determine whether or not a given relation is a function, determine the domain and codomain of a function, and find the image and preimage of a point using a function.

Let $S = \{0, 1, 2, 3, 4\}$ and consider the following mappings from S to S. In each case, do the following:

- Determine whether the mapping is a function from S to S; if not, explain.
- Write the function in two-line notation; and
- State the range of the function (not the codomain, because the codomain of all of these is S).

1. $f(x) = x + 1$
2. $g(x) = (x + 1) \ \% \ 5$
3. $h(x) = \left\lfloor \dfrac{1}{2^x} \right\rfloor$ (Recall $y = \lfloor x \rfloor$ is the floor function.)

Learning Target SF.4: I can determine whether a function is injective, surjective, or bijective.

Consider the following functions. In each case, state whether the function is injective, whether it is surjective, or whether it is bijective. If any of these properties *fails* to hold, give a brief explanation why. If a property does hold, you do not need to explain why.

1. $f : \{0, 1, 2, 3, 4\} \to \{0, 1, 2, 3, 4\}$ given by $f(n) = \lfloor x/2 \rfloor$ (Recall $y = \lfloor x \rfloor$ is the floor function.)
2. $g : \{0, 1, 2, 3, 4\} \to \{0, 1, 2, 3\}$ given by $g(n) = n^2 \ \% \ 4$
3. $k : \mathbb{N} \to \mathbb{N}$ given by $k(x) = x + 10$

Learning Target C.1 (Core): I can use the additive and multiplicative principles and the Principle of Inclusion and Exclusion to formulate and solve counting problems.

Solve each of the following. State your answer and make your reasoning clear by showing complete work and giving a 1-3 sentence summary of what you did. **Answers without 1-3 sentence summaries will result in an "x" on the entire problem**.

1. A professor is buying breakfast for his students as a way of saying “thank you” for a good semester. Of the students in the class, 22 want cereal for breakfast, 11 want bacon, and 5 want both. How many students want either cereal or bacon?
2. An IT department is assigning computers to faculty members and attaches an ID number to each machine. The ID numbers consist of two letters of the English alphabet (there are 26 letters in the alphabet) followed by a 5-digit number, for example “XT09034”. There are no restrictions on using either letters or digits. How many different ID numbers of this format can be created?

3. In the previous question about ID numbers, how many can be created if letters cannot be repeated and the digit “0” cannot be used?

Learning Target C.2: I can calculate a binomial coefficient and correctly apply the binomial coefficient to formulate and solve counting problems.

1. Compute the exact value of each of the following binomial coefficients and explain what you did.
   (a) $\binom{11}{3}$
   (b) $\binom{100}{0}$
2. Solve each of the following. State your answer and make your reasoning clear by showing complete work and giving a 1-3 sentence summary of what you did. **Answers without 1-3 sentence summaries will result in an "x" on the entire problem.**
   (a) How many subsets of $\{r, s, t, u, v, w, x\}$ are there that contain exactly 2 elements?
   (b) How many bitstrings of length 16 are there, that have exactly 8 “0” bits?

Learning Target C.3 (Core): I can compute combinations and permutations and apply these to formulate and solve counting problems.

1. Compute the value of $P(30, 10)$.
2. Prof. Talbert has three kids (all different) and a minivan with three rows in it. How many ways are there to arrange the kids in the minivan so that there is exactly one kid in each row?
3. A club with 15 members needs to elect a president and vice president. How many ways are there to do so?

Learning Target C.4: I can use the "Stars and Bars" technique to formulate and solve counting problems.

Solve each of the following. State your answer and make your reasoning clear by showing complete work and giving a 1-3 sentence summary of what you did. **Answers without 1-3 sentence summaries will result in an "x" on the entire problem.**

1. Find the number of natural number solutions to the equation $x + y + z + w + t = 15$. (Remember that 0 is a natural number.)
2. Prof. Talbert cooks an entire package of bacon, consisting of 12 identical strips, for his kids for breakfast. How many ways are there to distribute the bacon to his kids so that each kid gets at least 2 strips?

Learning Target SR.1 (Core): I can generate several values in a sequence defined using a closed-form expression or using recursion.

Write the first 8 terms of the following sequences. Unless otherwise specified, begin indexing at 0. **Show work on at least part of your calculations, to demonstrate evidence that you know how to generate the sequences.**

1. $a_n = a_{n-1} + 3a_{n-2}, a_0 = 1, a_1 = 1$
2. $a_n = n^2 + 3$

Learning Target SR.2: I can use sigma notation to rewrite a sum and determine the sum of an expression given in sigma notation.

1. Compute the value of the following sums. Make your reasoning clear by showing work and/or explaining what you did (and don't just use a computer):
   (a) $\sum_{i=0}^{5} (i^2 - i)$
   (b) $\sum_{i=2}^{6} 3^i$
2. Write the following sum in correct sigma notation: $2 + 4 + 8 + 16 + 32$. More than one correct answer may be possible.

Learning Target SR.3 (Core): I can find closed-form and recursive expressions for arithmetic and geometric sequences and find their sums.

For each sequence below:

- State whether the sequence is arithmetic, geometric, or neither;
- If the sequence is arithmetic, state the common difference; if geometric, state the common ratio;
- Give a recursive definition;
- Give a closed formula; and
- Find the sum of the first 50 terms.

For the last item, you are more than welcome to *check* your work with a computer, but you must show work here — in particular, use the summing techniques that we learned in class. **Note: You must use the summing techniques we learned in class. You may not simply use a spreadsheet or use any formulas that were not directly discussed in class. Instances of this will result in an "x" on the problem.**

1. $2, 1.8, 1.62, 1.458, 1.3122, 1.18098, \ldots$
2. $2, 5, 8, 11, 14, 17, \ldots$

Learning Target SR.4: I can use iteration and characteristic roots to solve a recurrence relation.

Solve the recurrence relation:

$$a_n = 5a_{n-1} - 6a_{n-2}, \;\; a_0 = 1, a_1 = 3$$

using the method of characteristic roots. Make your reasoning clear by showing all your work in a clear and organized fashion, and use English to help explain what you're doing.

Learning Target P.2 (Core): Given a statement to be proven by (weak) induction, I can state and prove the base case, state the inductive hypothesis, and outline the proof.

Consider the following statement:

For all positive numbers n, $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$.

Suppose we were going to write a proof of this statement using (weak) mathematical induction.

1. Clearly state what you would need to prove in order to establish the base case.
2. Clearly state the inductive hypothesis that you would assume, including the correct quantifier.
3. Clearly explain what you would need to do next, once you have assumed the induction hypothesis, to complete the proof.