Directions:

- Do only the Checkpoint problems that you need to take and feel ready to take. If you have already earned Mastery on a Learning Target, do not attempt a problem for that Target! You can skip a Target if you need more time to practice with it, and take it on the next round.
- Do not put any work on this form; do all your work on separate pages. You may either handwrite or type up your work.
- Clearly indicate which Learning Target you are attempting at the beginning of its solution; please also turn in solutions for learning targets in order (for example, do not turn in work for A.2 after work for SF.1). The easiest way to do this is to put each Learning Target on its own solution page and do not put more than one Learning Target on a single page.
- If you are handwriting, submit your work by **scanning your work** using a scanning app or scanning device; **do not just take a picture** but scan your work to a clear, legible, black and white PDF file of size less than 100 MB. Work submitted as an image file (JPG, PNG, etc.) will not be graded.
- Unless explicitly stated otherwise, you must show your work or explain your reasoning clearly on each item of each problem you do. Responses that consist of only answers with no work shown, or where the work is insufficient or difficult to read, or which have significant gaps or omissions (including parts left blank) will be given a grade of "x".
- Submit your work by uploading it as a PDF or Word file to the appropriate assignment area on Blackboard.

Learning Target A.1: *I can represent an integer in base 2, 8, 10, and 16.*

Perform all of the following conversions. Show all work and explain all reasoning.

1. 511 in decimal; convert to binary and octal.
2. 511 in octal; convert to decimal and hexadecimal.
3. DF7 in hexadecimal; convert to decimal.

Learning Target A.2 (Core): *I can add, subtract, multiply, and divide two integers written in binary.*

Perform all of the following computations in binary, without changing to base 10. Show all work and explain all reasoning.

1. `10011010 + 11110010`
2. `11001100 - 01001101`
3. `11000110` $\times$ `011`
4. `11001100` $\div$ `111`

Learning Target A.3: *I can compute $a\%b$ given integers a and b and perform arithmetic mod n.*

Perform all of the following computations and either show your work or explain what you did in words.

1. $1293\%8$
2. $(-20)\%13$
3. $7491117143048354781 1\%2$

4. $(3^{25})\%10$ using repeated squaring

Learning Target L.1: *I can use propositional variables and logical connectives to represent statements; and interpret symbolic logical statements in plain language.*

Let p, q, and r be the propositions:

p: Snow is in the forecast.

q: Roads are safe for travel.

r: You should wear a coat.

Write these English propositions using p, q, and r and logical connectives (including negation).

1. Translate the following English sentences into symbolic logic expressions:
   (a) Roads are not safe for travel if snow is in the forecast.
   (b) If roads are not safe for travel or snow is in the forecast, you should wear a coat.
2. Translate the following symbolic logic expressions into clear English sentences:
   (a) $(q) \leftrightarrow (\neg r)$
   (b) $p \vee (q \wedge r)$

Learning Target L.2 (Core): I can write the negation, converse, and contrapositive of a conditional statement and use DeMorgan's Laws to simplify symbolic logical expressions.

1. For each of the conditional statements below, write the converse, contrapositive, and negation. If the original statement is in symbols, your answers should be in symbols; if in words, the answer should be in clear English as well. For symbolic statements, do not just put $\neg$ in front of the original to form a negation, but instead use what we learned about negations of conditional statements to simplify. Likewise for English statements, don't just use "It is not the case that..." to form the negation.
   (a) If the temperature is above 70, I will go outside.
   (b) $p \rightarrow (q \wedge r)$
2. Use DeMorgan's laws to state the negations of each of the following. If the original statement is in symbols, your answers should be in symbols; if in words, the answer should be in clear English as well. For symbolic statements, do not just put $\neg$ in front of the original to form the negation, but instead use what we learned about negations of conditional statements to simplify. Likewise for English statements, don't just use "It is not the case that..." to form the negation.
   (a) $p \wedge (q \vee r)$
   (b) Either the food is ready, or I need to cook and I need to go to the store.

Learning Target L.3: I can determine whether a quantified statement is true, false, or underdetermined, and state its negation.

Below are some statements; the domain of each is the set of all integers (positive, negative, and zero — but only integers). **New instructions**: For symbolic statements, the negation symbol ($\neg$ or $\sim$) must not appear in your answer. For English statements, don't just use "It is not the case that..." to form the negation. For each one, state whether the statement is TRUE, FALSE, or UNDERDETERMINED. For all the ones that are TRUE or FALSE, state the negation.

1. $\forall n(n \text{ is even})$
2. $\exists n(n \leq 0)$

3. $a^2 \% 5 = 0$
4. $\forall a \exists b (a + b < 0)$

Learning Target L.4 (Core): I can write the truth table for a logical statement.

Construct a truth table for each of the following propositions:

1. $p \to (\neg p)$
2. $p \wedge (\neg q)$
3. $(p \wedge q) \to (q \vee r)$

Learning Target L.5: I can determine if a statement is a tautology and whether two statements are logically equivalent.

1. Determine whether the statement $p \to (p \vee \neg p)$ is a tautology. Show your work and clearly indicate your answer.
2. Determine whether the following statements are logically equivalent or not. Show your work and clearly indicate your answer.
   (a) $(p \vee q) \to r$ and $(p \to r) \vee (q \to r)$

Learning Target SF.1 (Core): I can represent a set in roster notation and set-builder notation; determine if an object is an element of a set; and determine set relationships (equality, subset).

1. Write the following sets in roster notation:
   (a) $\{a \in \mathbb{N} : a^2 + 1 > 4\}$
   (b) $\{a^2 : a \in \mathbb{N} \text{ and } a \text{ is even}\}$
2. Write the following sets in set-builder notation. **New instructions**: There may be more than one correct representation; but your representation must restate membership in the set in some way.
   (a) $\{1, 5, 9, 13, 17, \dots\}$
   (b) $\{0.1, 0.01, 0.001, 0.0001, \dots\}$
3. Define the following sets:

$$A = \{x, y, z, t\}$$
$$B = \{s, t, u, v, w, x\}$$
$$C = \{t, y, z, x\}$$
$$D = \{s, t, u, v\}$$

   Determine whether each of the statements is true or false. State your answer clearly. (You do not have to justify your answers, but they have to be correct.)
   (a) $t \in A$
   (b) $B \subseteq A$
   (c) $\{x, t\} \subseteq C$
   (d) $A = C$
   (e) $|D| = |C|$

Learning Target SF.2 : I can perform operations on sets (intersection, union, complement, Cartesian product) and determine the cardinality of a set.

Let $A = \{10, 20, 30\}$ and $B = \{n \in \mathbb{N} : n \text{ is a multiple of } 5\}$, and $C = \{100, 99, 98\}$ and let the universal set be $\mathbb{N}$. Determine all of the following. If the item is a set, write the set in roster notation.

1. $A \cup B$
2. $C \cap B$
3. $A \setminus B$
4. $A \times \{x, y\}$
5. $|C \setminus A|$

Learning Target SF.3 (Core): I can determine whether or not a given relation is a function, determine the domain and codomain of a function, and find the image and preimage of a point using a function.

Let $S = \{0, 1, 2, 3, 4, 5\}$ and consider the following mappings from S to S. In each case, do the following:

- Determine whether the mapping is a function from S to S; if not, explain.
- Write the function in two-line notation; and
- State the range of the function (not the codomain, because the codomain of all of these is S).

1. $f(x) = \lceil \sqrt{x} \rceil$ (Recall $y = \lceil x \rceil$ is the ceiling function.)
2. $g(x) = (x^2 + 1) \ \% \ 6$
3. $h(x) = 2x$

Learning Target SF.4: I can determine whether a function is injective, surjective, or bijective.

Consider the following functions. In each case, state whether the function is injective, whether it is surjective, or whether it is bijective. If any of these properties *fails* to hold, give a brief explanation why. If a property does hold, you do not need to explain why.

1. $f : \{0, 1, 2, 3, 4\} \to \{0, 1, 2, 3, 4\}$ given by $f(n) = n \ \% \ 4$
2. $g : \{0, 1, 2, 3, 4\} \to \{0, 1, 2, 3\}$ given by $g(n) = n \ \% \ 4$
3. $k : \mathbb{Q} \to \mathbb{N}$ given by $k(x) = \lfloor x \rfloor$ (the floor function) (Recall $\mathbb{Q}$ is the set of rational numbers)

Learning Target C.1 (Core): I can use the additive and multiplicative principles and the Principle of Inclusion and Exclusion to formulate and solve counting problems.

Solve each of the following. State your answer and make your reasoning clear by showing complete work and giving a 1-3 sentence summary of what you did. **Answers without 1-3 sentence summaries will result in an "x" on the entire problem**.

1. An IT department is doing updates on 100 different computers. 60 of those run the Windows operating system and 52 of them run Linux. How many of them run both Windows and Linux?
2. Prof. Talbert is not a sharp dresser; he only has three shirts, four different pairs of pants, and two pairs of shoes. How many different outfits can be made from these items of clothing?
3. In the Southern United States, a popular kind of restaurant offers diners a choice of a meat and three side items. (They're called "meat and three's"). One such restaurant has five different kinds of meat available and 12 different side items. How many different dishes can be created from the menu, if you're not allowed to order the same side item twice?

Learning Target C.2: I can calculate a binomial coefficient and correctly apply the binomial coefficient to formulate and solve counting problems.

1. Compute the exact value of each of the following binomial coefficients and explain what you did.
   (a) $\binom{12}{8}$
   (b) $\binom{52}{5}$
2. Solve each of the following. State your answer and make your reasoning clear by showing complete work and giving a 1-3 sentence summary of what you did. **Answers without 1-3 sentence summaries will result in an "x" on the entire problem**.
   (a) How many subsets of $\{r, s, t, u, v, w, x\}$ are there that contain exactly 4 elements?
   (b) How many bitstrings of length 16 are there, that have exactly 8 "1" bits?

Learning Target C.3 (Core): I can compute combinations and permutations and apply these to formulate and solve counting problems.

1. Compute the value of $P(30, 10)$.
2. How many ways are there to rearrange the elements of a 5-element set?
3. A club with 15 members needs to elect a president, vice president, secretary, and treasurer. How many ways are there to do so?

Learning Target C.4: I can use the "Stars and Bars" technique to formulate and solve counting problems.

Solve each of the following. State your answer and make your reasoning clear by showing complete work and giving a 1-3 sentence summary of what you did. **Answers without 1-3 sentence summaries will result in an "x" on the entire problem**.

1. Find the number of natural number solutions to the equation $x + y + z + w = 30$. (Remember that 0 is a natural number.)
2. Find the number of ways to give 20 identical dollar bills to a group of 4 kids, if each kid must get at least one dollar.