MTH 225: Discrete Structures for Computer Science 1

Daily Preparation, Module 7B: The binomial coefficient

Due by: 11:59pm ET, Tuesday, October 20

Estimated time requirement: About 45-60 minutes for the whole assignment. *If you have worked on this assignment for 30 minutes and you're not at least halfway done, DON'T work any further* — instead, *stop and ask for help* on the `#dailyprep` channel on CampusWire. Remember these are graded just on completeness and effort — try to be right and understand everything, but don't get bogged down if you get stuck. Just give a good effort and move on, and ask a question.

Overview

In Module 7B we introduce a concept of fundamental importance to discrete math: the **binomial coefficient**. This is a number that represents the number of ways to count certain kinds of arrangements — in fact, it counts arrangements of objects that you wouldn't expect to be the same, but actually are. We'll see that the following situations have the same number of arrangements:

- The number of ways to form a string of bits of length n whose bits add up to k
- The number of k-element subsets of an n-element set
- The coefficient on the kth term of the expression $(x+y)^n$
- And more!

In the process of developing this concept we'll get our first real look at a **recurrence relation**, which helps us break complex counting problems down into simpler situations that are dealt with *recursively*.

What you will learn

Learning Targets addressed in this module:

- **C.1 (Core)**: I can use the additive and multiplicative principles and the Principle of Inclusion and Exclusion to formulate and solve counting problems.
- **C.2**: I can calculate a binomial coefficient and correctly apply the binomial coefficient to formulate and solve counting problems.

BEFORE your class meeting, use the Resources for Learning (below) to learn how to do the following:

- (*Review*) State the cardinality of $P(A)$ given a finite set A. (That is, state the number of subsets that a

finite set has.)

- Find the number of k-element subsets of an n-element set for small values of k and n.
- Explain the meaning of the notation $\mathbf{B}_k^n$ and use a recurrence relation to calculate the value of $|\mathbf{B}_k^n|$ for small values of k and n.
- Explain why $|\mathbf{B}_k^n|$ and the number of k-element subsets of an n-element set will always be the same number.
- Define the binomial coefficient $\binom{n}{k}$ in terms of counting the number of n-bit strings of length k, the number of k-element subsets of an n-element set, and the coefficient of the $x^k y^{n-k}$ term of $(x+y)^n$.

DURING AND AFTER your class meeting, you will learn how to do the following:

- Use the recurrence relation for the binomial coefficient to construct Pascal's Triangle.
- Compute the value of $\binom{n}{k}$ using Pascal's Triangle.
- Apply the binomial coefficient to solve counting problems involving selecting subsets out of a set of objects.

Resources for Learning

It's recommended that for this lesson, you watch the videos first and then read through the text afterwards.

Video: For this lesson you have some videos made especially for MTH 225 and for this topic:

- Counting subsets and bit strings (6:11) https://gvsu.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=590a19b1-7b0b-44c9-818c-ac550161301a
- Counting the k-element subsets of a set (6:57) https://gvsu.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=35ee1a2a-ce66-4698-8d4c-ac570130b61d
- Counting bit strings of weight k (10:28) https://gvsu.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=1a6ca4e3-e704-4705-8852-ac570130c5d6
- Why counting subsets and counting bit strings are the same (7:14) https://gvsu.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=9d94f238-a463-41a7-87a4-ac570135de6c
- Introducing the binomial coefficient (9:01) https://gvsu.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=a94702d8-bfa2-4876-b2ae-ac57013768fd

Text: Once you've watched those videos, then go and read through Section 1.2 from the Levin text.

You are free to search for and use other resources in addition to, or instead of the above, as long as you can work the exercises below.

Exercises

The exercises are on the following Google Form: https://bit.ly/30Zpnsa

Submission, grading, and getting help

Submitting your work: Your work is to be done on Classkick using the link/code above. Classkick saves your work as you go, so there's nothing to submit – just do the work and you're good.

How this is graded: Daily Prep assignments are graded on the basis of *completeness and effort*: If your submission has **all parts completed** (no blank entries, even if left blank accidentally) and **a good-faith effort to provide a correct solution or explanation is given** (no responses of "I don't know" or "I didn't understand") and **the work is submitted on time**, it gets a "check". Otherwise it gets an "x". If you are stuck on an item, you're expected to ask questions and give your best effort.

Getting help on this assignment: *You may work with others on this assignment, but you may not copy each others' answers.* Evidence of copying will be treated as academic dishonesty. You may also ask questions on the #dailyprep channel on CampusWire, but you may not ask simply to be given the answers; giving and receiving answers on CampusWire will be treated as academic dishonesty.