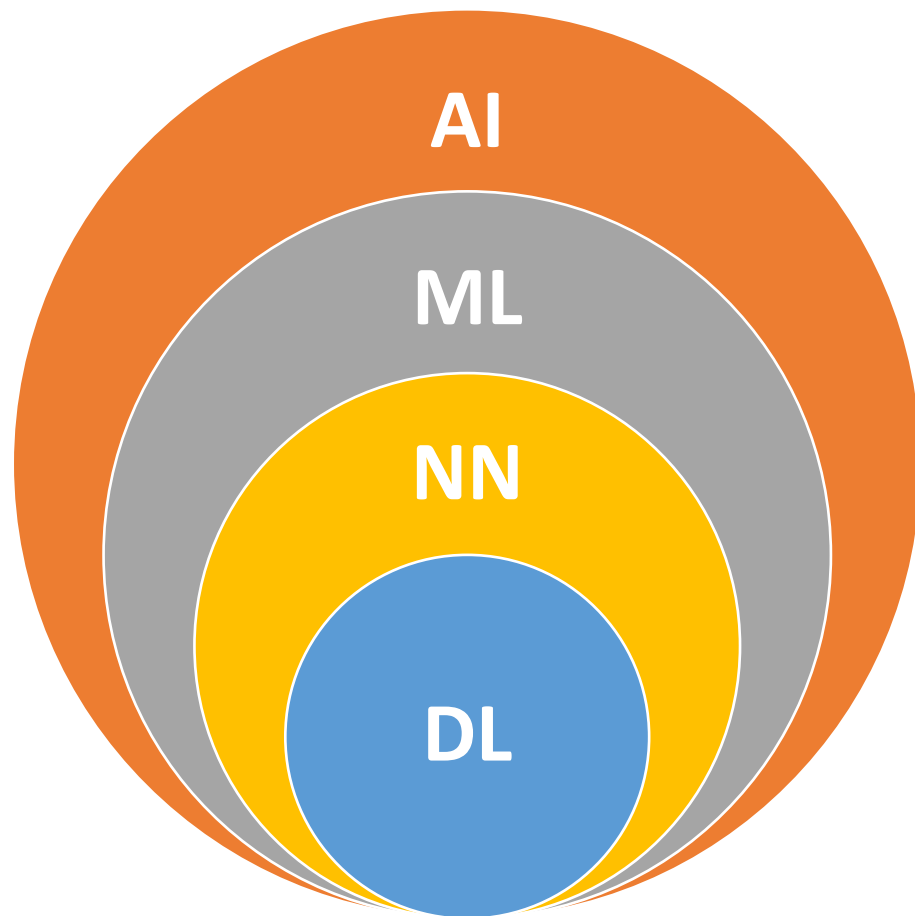


# Foundational Concepts of Artificial Intelligence-Based Modeling

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Information Technology Laboratory  
National Institute of Standards and Technology



- AI: Artificial intelligence is the overarching system.
- ML: Machine learning is a subset of AI.
- NN: Neural networks are the backbone of deep learning algorithms.
- DL: Neural network of more than three layers, including the inputs and the output.

[URL](#)

# 1. Why are we interested in Neural Networks?

# 1. Why are Scientists interested in Neural Networks?

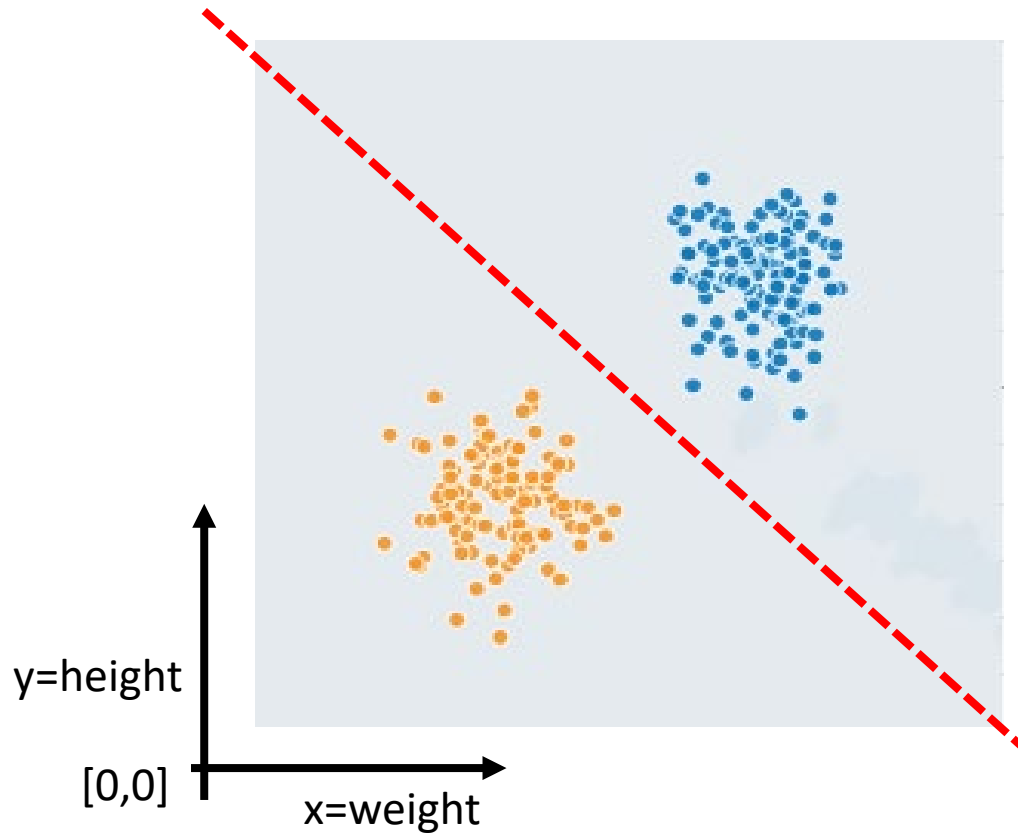
## **Relevance of AI to Metrology:**

- Neural Networks can classify pictures into classes →
  - Tasks: classification, object detection, segmentation, tracking, recognition
- Neural Networks can assist humans →
  - Learn from training data: labeled and partially unlabeled.
  - Learn from past models: a priori knowledge and simulations.
- Neural Networks can outperform humans →
  - Task metrics: overall accuracy (NN do not get tired) beyond visual detection limits and speed.
- Neural Networks can save us a lot of time →
  - Automation of tasks: processing TB of images, detecting outliers, and leading to discovery.

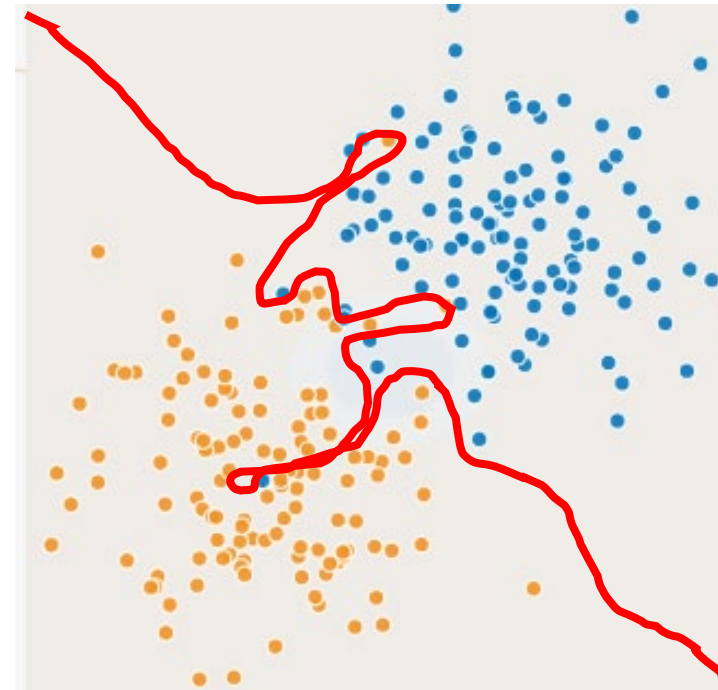
**2. How to create a complex boundary separating two classes?**

# Types of Boundaries

Linear boundary

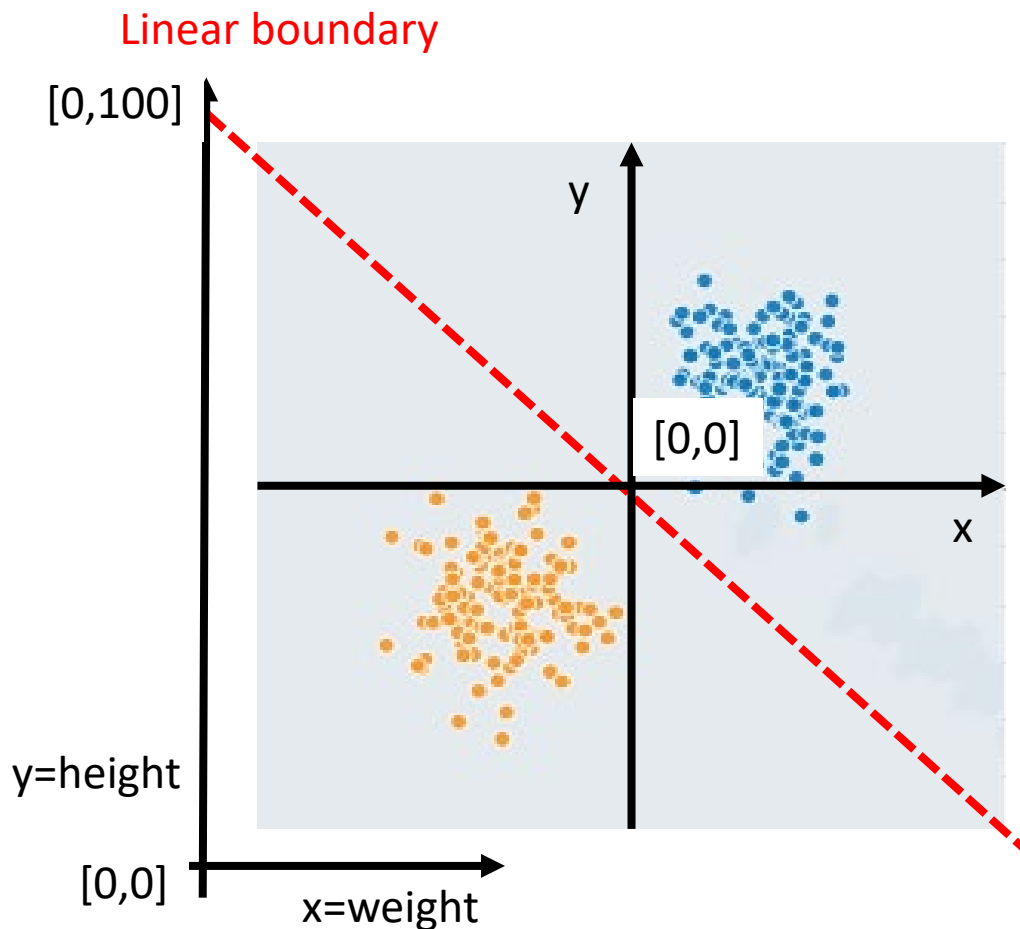


Non-linear boundary



How to describe and derive these boundaries automatically?

# Linear Boundaries



*Boundary:  $x + y = 100$*



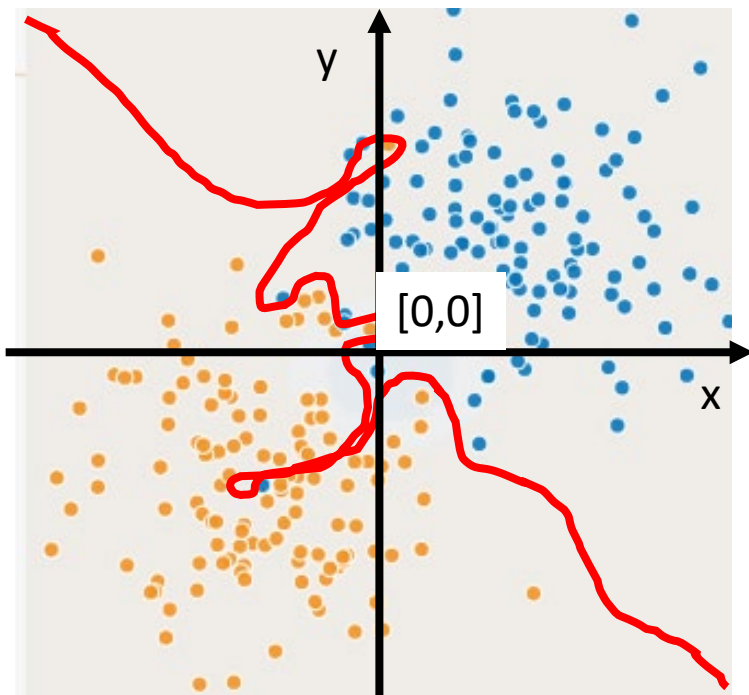
*Boundary:  $x + y = 0$*

*Math model: if  $x + y < 0$   
then orange else blue*

**How to describe and derive these boundaries automatically?**

# Nonlinear Boundaries

Non-linear boundary

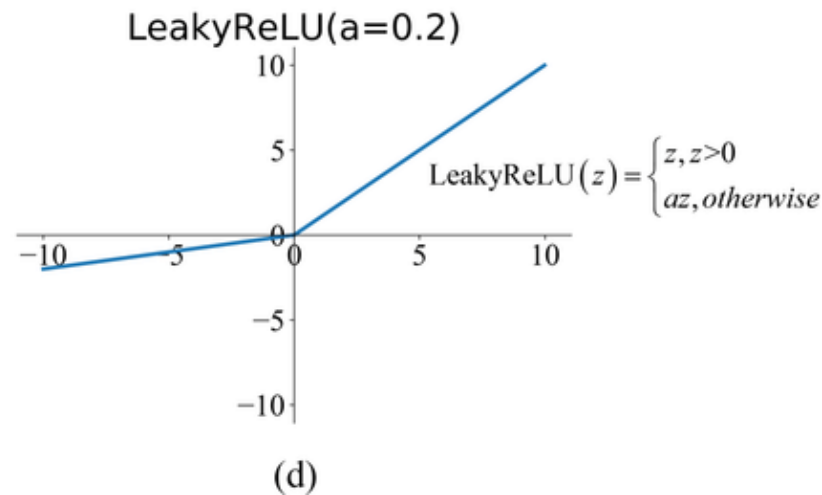
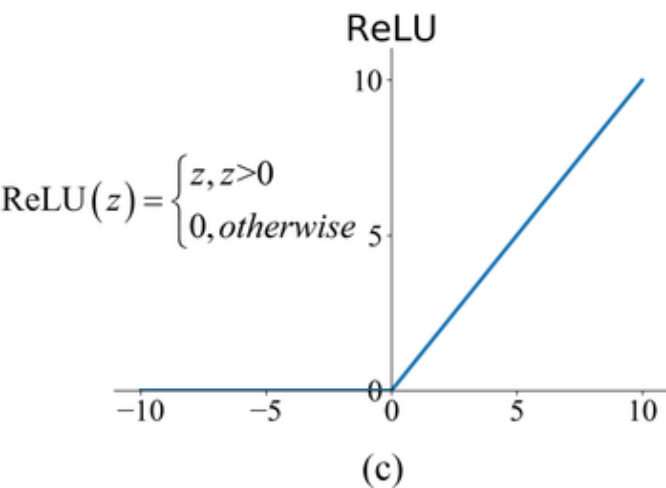
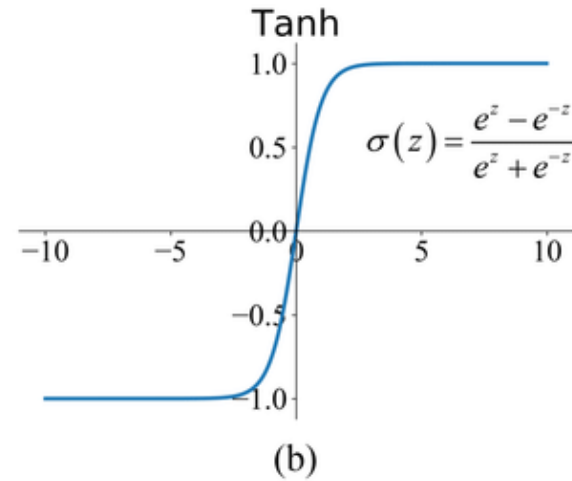
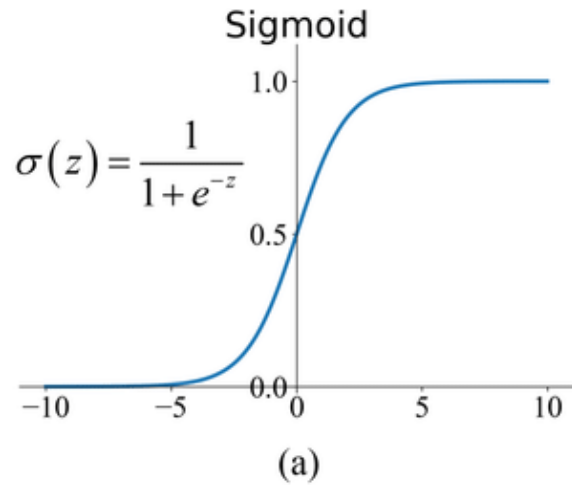


How to describe a nonlinear boundary using  $x$  and  $y$ ?

How to derive the boundary description automatically?



# Describe nonlinear boundary using x and y



Nonlinear operators: (a) Sigmoid, (b) Tanh, (c) ReLU, and (d) LReLU.

*Linear Boundary:*  
 $x + y = 0$



*Nonlinear Boundary:*  
 $\text{Tanh}(x + y) = 0$

Nonlinear operators on features

*Nonlinear Boundary:*  
 $(x^2 + y^2) = 100$

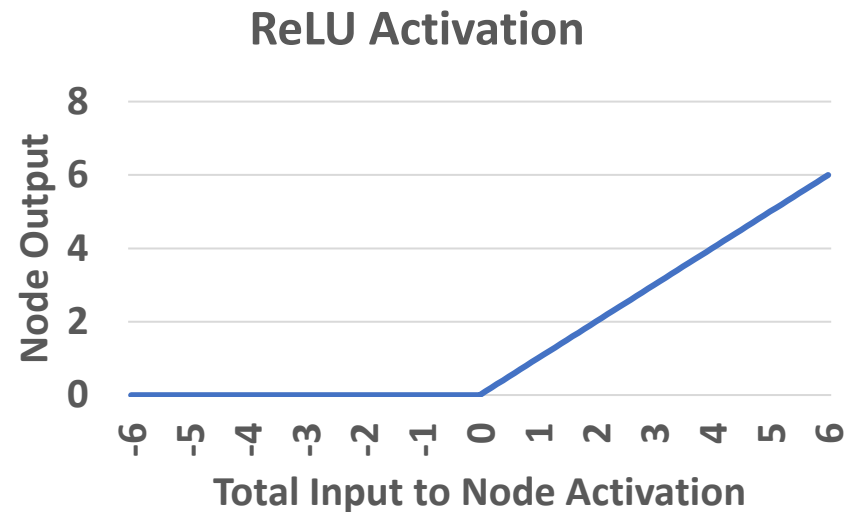
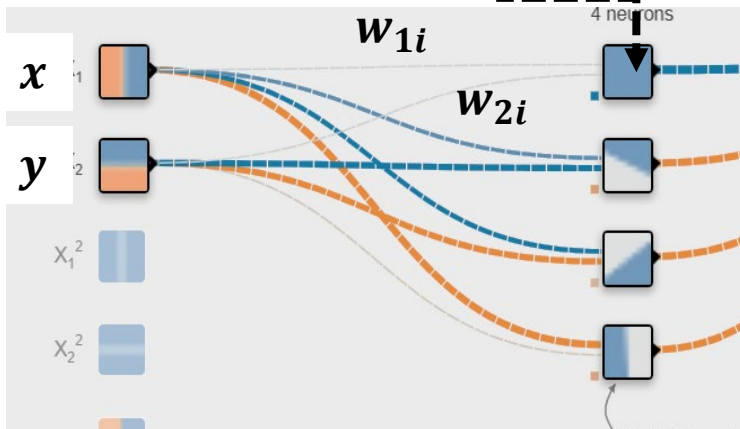
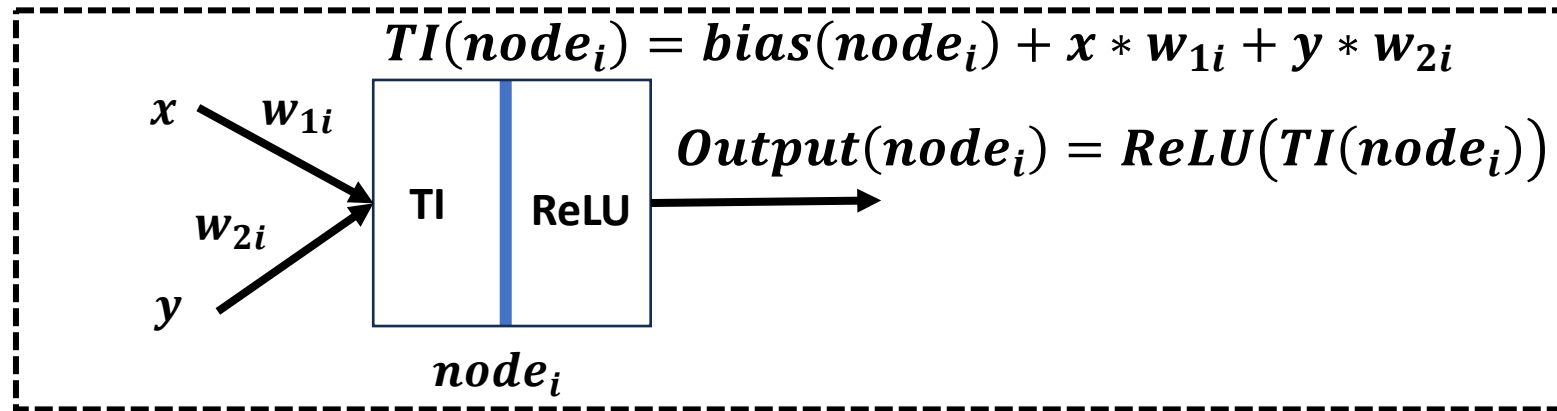
Nonlinear features

*Nonlinear Boundary:*  
 $\text{Tanh}(x^2 + y^2) = 100$

Nonlinear operators & nonlinear features

# Prepare NN model for non-linear boundary

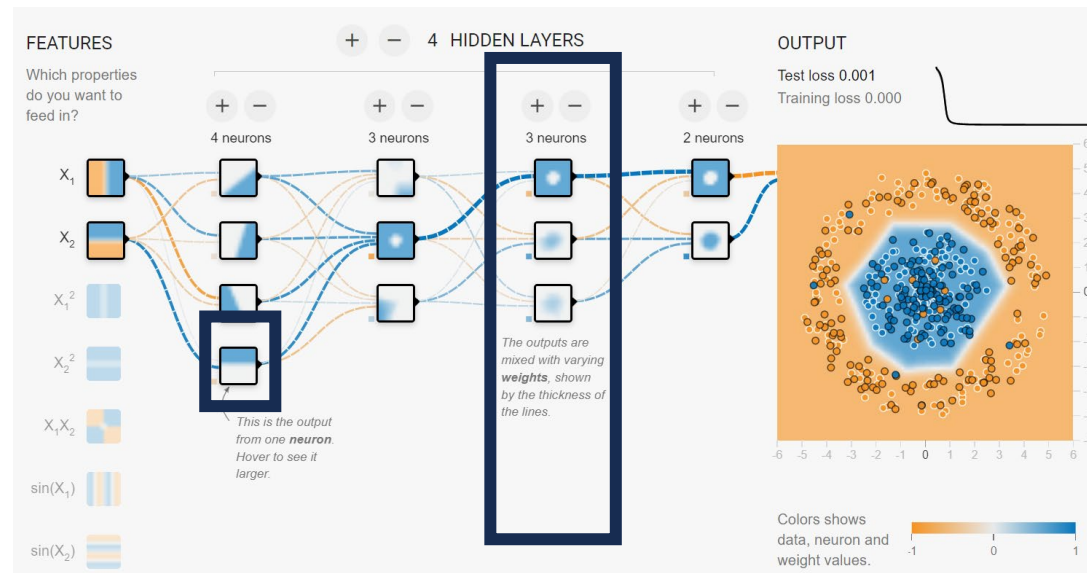
## Mathematical Model



# Derive boundary description automatically

1. Step - Configure Neural Network architecture (layers, nodes, connectivity)
2. Step – Prepare (input, output) pairs (images and labels)
3. Step – Initialize all weights in NN with random numbers
4. Step – Train weights so that the multiplications, additions, and nonlinear operators convert input to output (images of cats/dogs to labels of cats/dogs)
5. Step – Evaluate the accuracy of trained NN on test (input, output) pairs

**Input:**  
**(x, y) coordinates**



**Output:**  
**Label of (x, y) coordinates = {Orange, Blue}**

## 2. How Are Scientists Creating Complex Boundary?

### Relevance of AI to Metrology:

- Describe boundaries mathematically
  - → Boundaries are unknown in discoveries!
- Linear boundaries can be described directly with  $x$  and  $y$ 
  - → Linear regression is rarely accurate for complex biomedical phenomena!
- Nonlinear boundaries must be described with nonlinear transformations (nonlinear features and operators/activations)
  - → ReLU is the most frequently used in practice. The choice might depend on the convergence of the model training.
- Nonlinear boundary description can be derived automatically by training Neural Networks (NN) with training data.
  - → TBD next

# 3. How do you learn primitive Neural network concepts interactively?

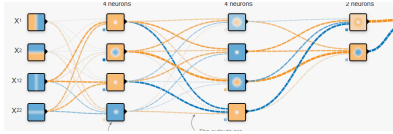
- <https://pages.nist.gov/nn-calculator/>
- Four simple exercises
  1. [Simple to complex NN architecture](#)
  2. [Linear to non-linear features](#)
  3. [Linear to non-linear activation functions](#)
  4. [Well-separated to inseparable \(interleaved\) classes](#)

# Learning about NNs Using a Web-Based “NN Calculator” **NIST**

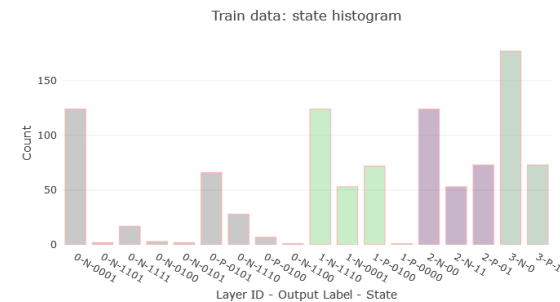
**Memory Operations:** MC, MR, M+, M-, MS, AVG

Data

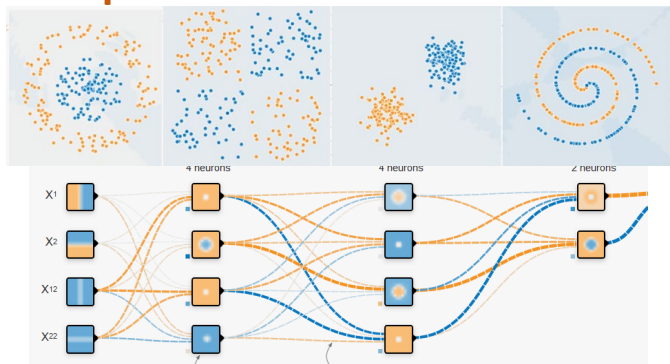
NN



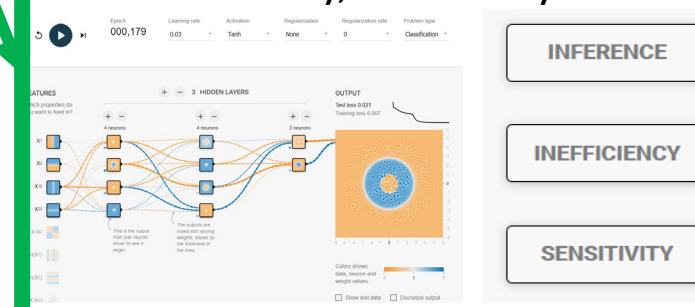
**Display:** NN, Data, Results



**Operands:** 2D data and NN



**NN Operators:** design, parametrize, train, infer, inefficiency, sensitivity



# NN Calculator Layout

NN & MEMORY

TRAIN

NONLINEAR OPERATOR

NN MC	NN MR
NN M+	NN M-
NN MS	NN AVG



Epoch  
000,069

Learning rate  
0.03

Activation  
Tanh

Regularization  
None

Regularization rate  
0

Problem type  
Classification

DATA PATTERN

FEATURES  
Which properties do  
you want to feed in?

DATA & MEMORY

D MC	D MR
D M+	D M-
D MS	D RG

Ratio of training to  
test data: 50%

Batch size: 10

Noise: 0

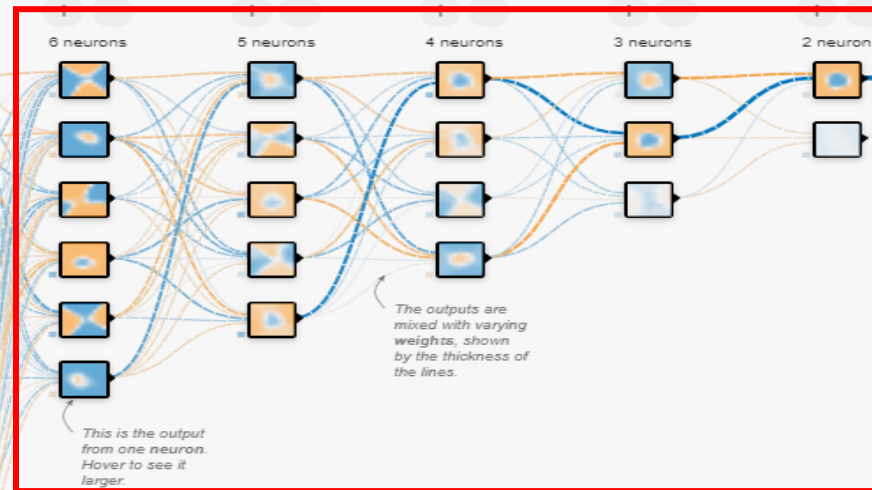
Trojan: 0

PARAMETERS

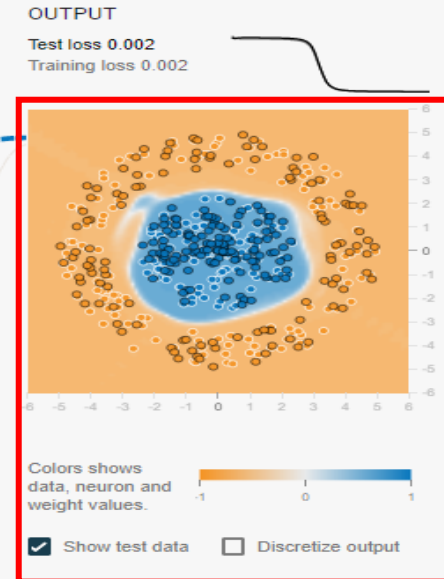
FEATURES

- $X_1$
- $X_2$
- $X_1^2$
- $X_2^2$
- $X_1 X_2$
- $\sin(X_1)$
- $\sin(X_2)$
- $n(X_1, X_2)$
- $\text{cir}(0, r)$
- $\text{add}(x, y)$

NEURAL NETWORK



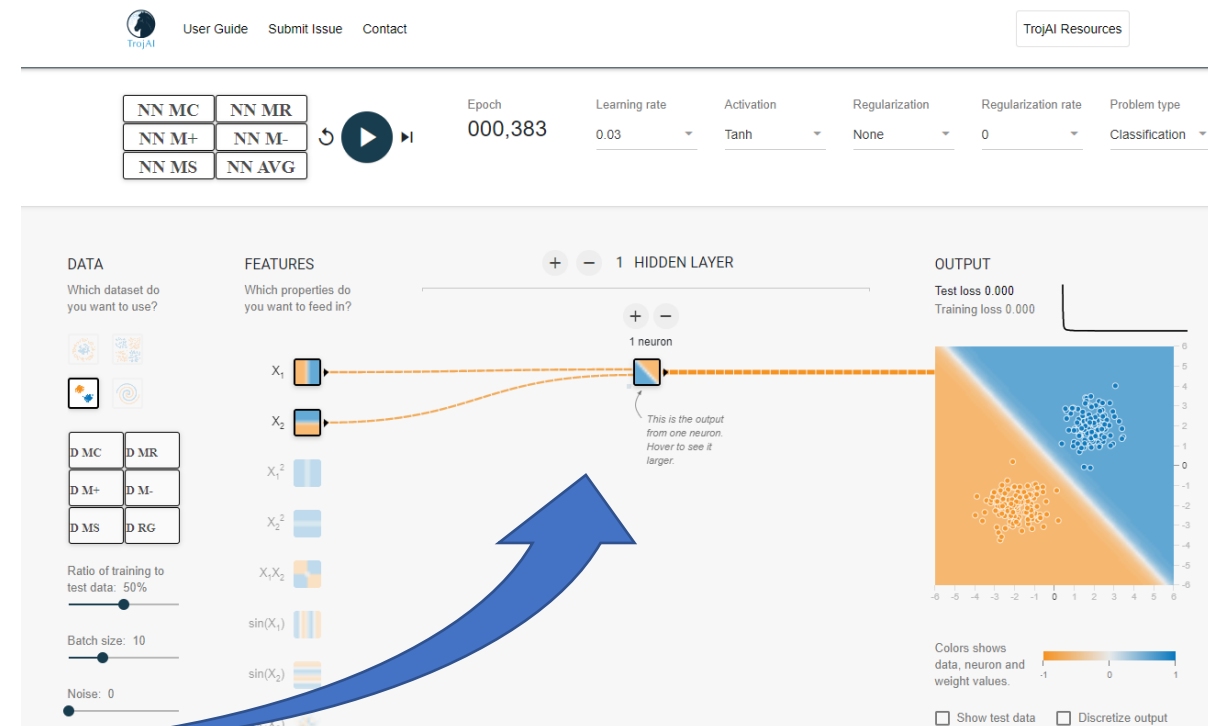
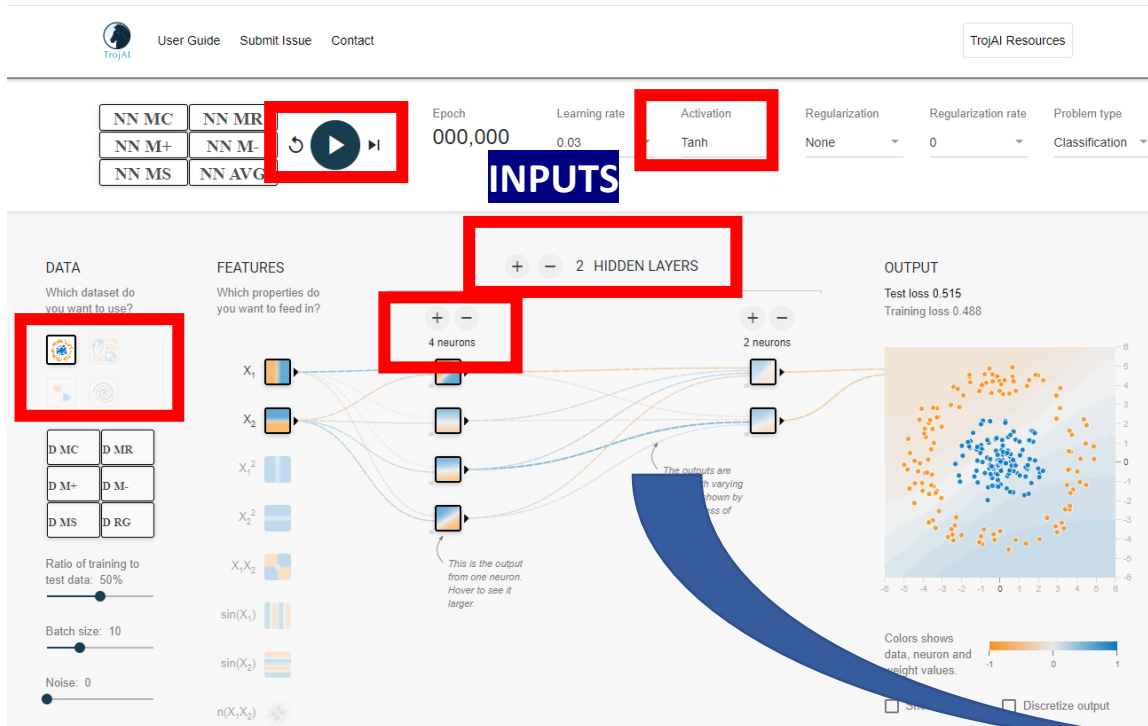
CLASS LABELS



NN calculator: <https://pages.nist.gov/nn-calculator/>

# EX #1: Modify the NN to a single node

1. Remove layers
2. Remove nodes
3. Select data pattern with cluster pattern
4. Select Linear activation function

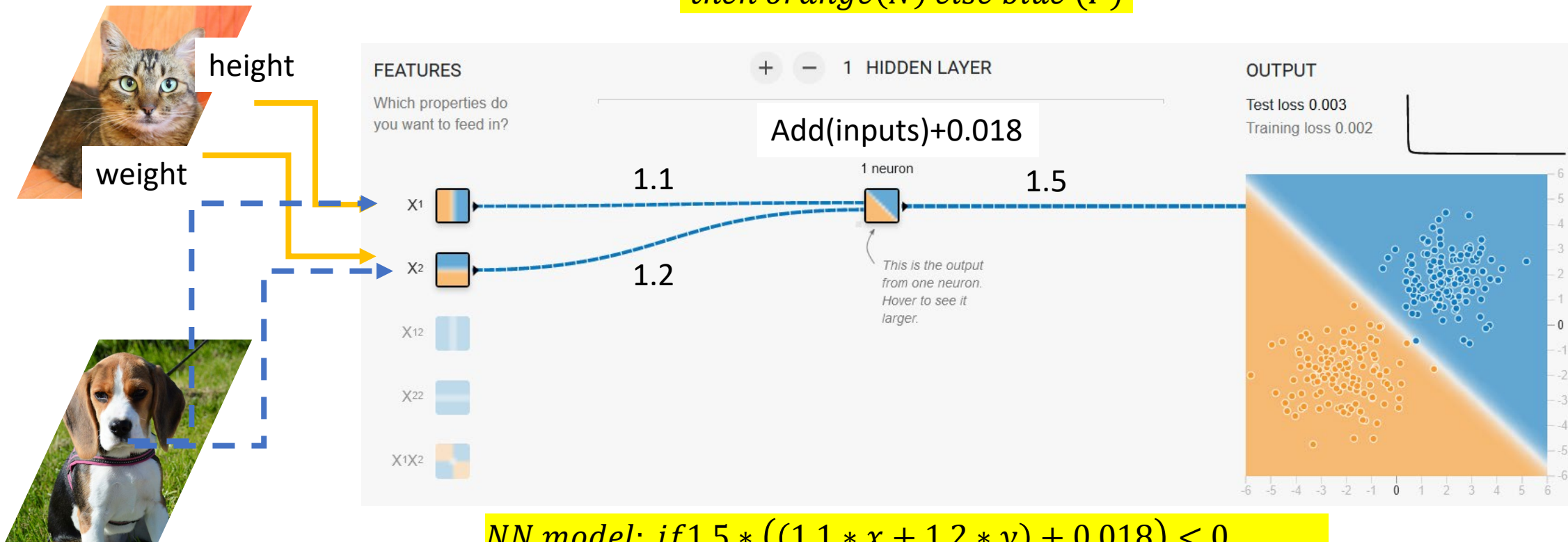




# Single node NN with Linear activation function

*Math model: if  $x + y < 0$   
then orange(N) else blue (P)*

**Linear Activation**



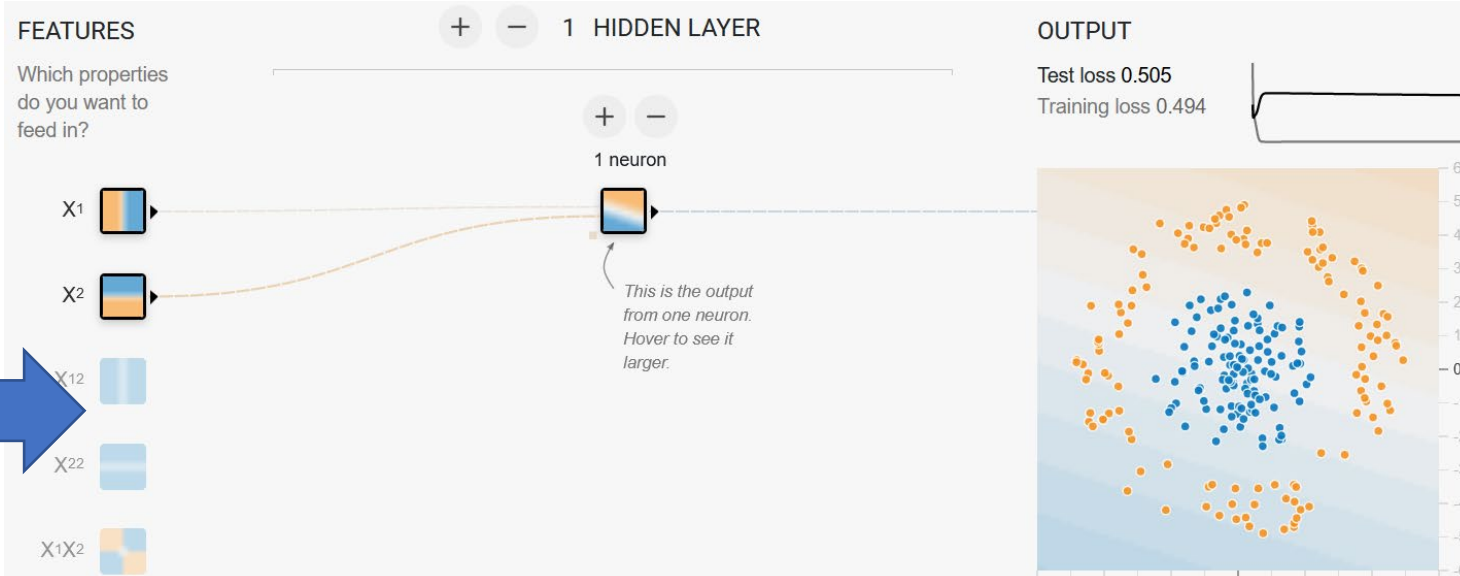
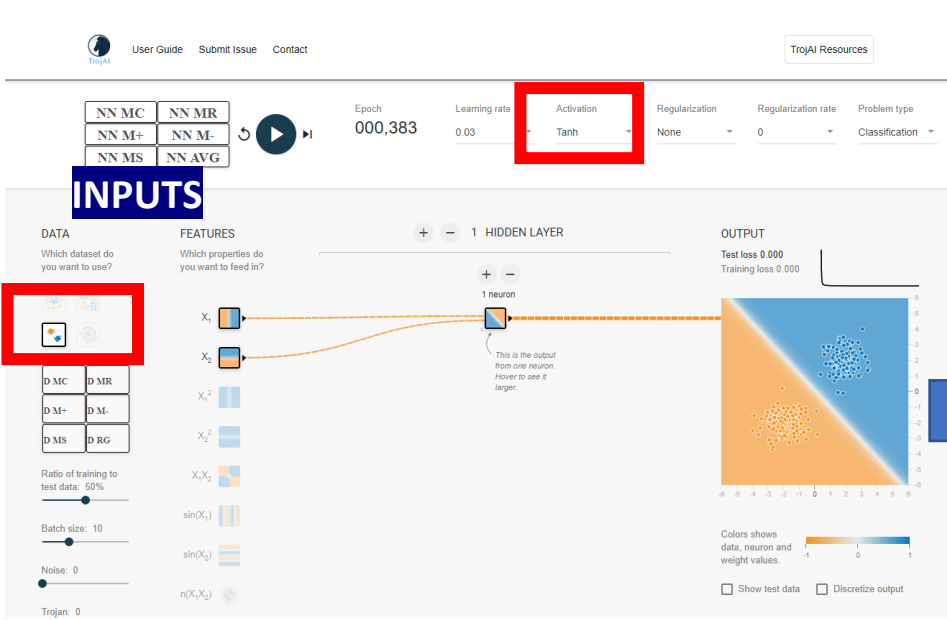
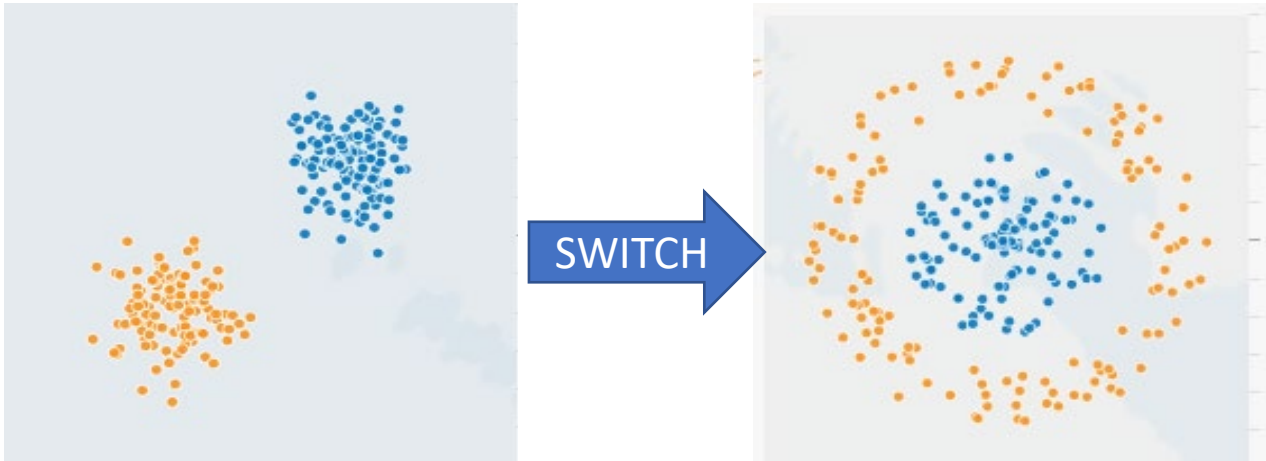
*NN model: if  $1.5 * ((1.1 * x + 1.2 * y) + 0.018) < 0$   
then orange (N) else blue (P)*

If  $1.65 * x + 1.8 * y + 0.027 < 0$  then orange (N) else blue P)

# Complexity of Data Patterns



- Switch data patterns
- Train

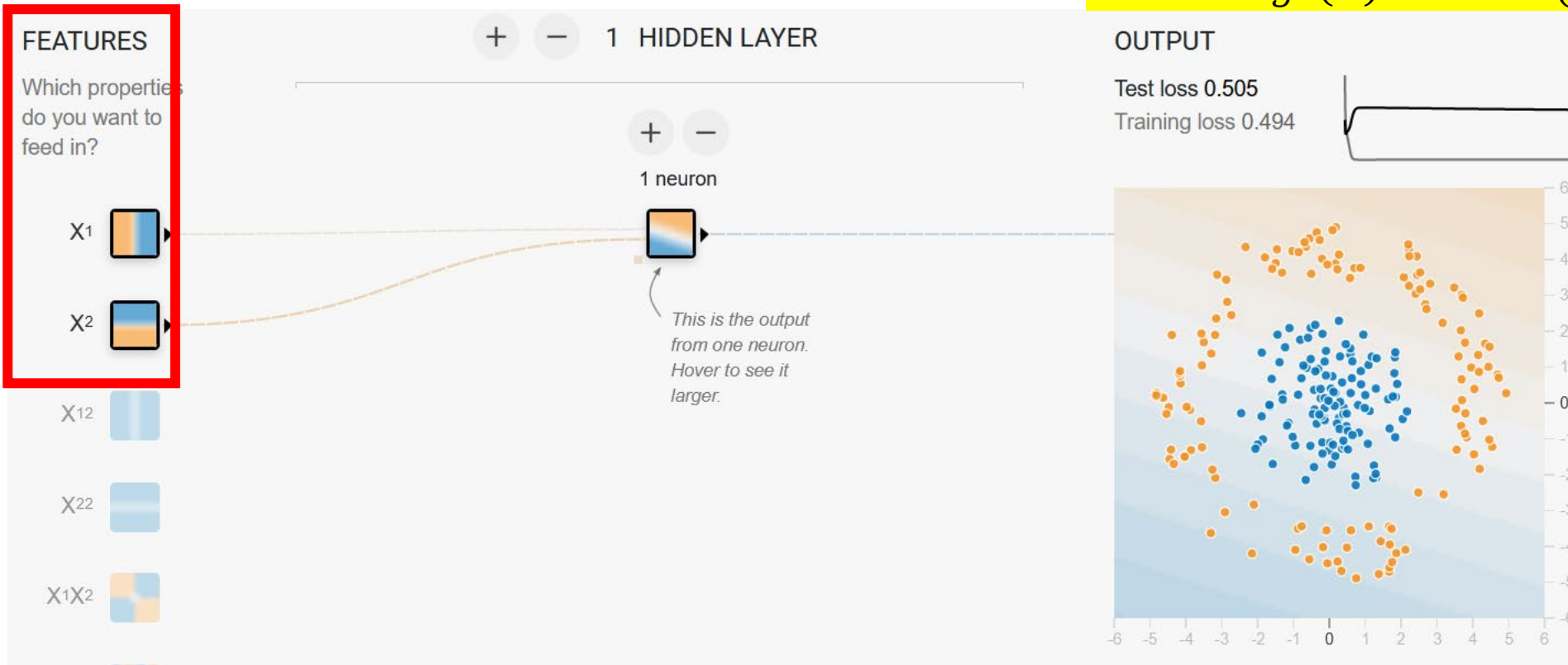


**NN model: FAILED**  
– could not come up with an accurate labeling formula based on  $x$  &  $y$

# Why did the model fail?

## Linear Activation

*Math: if  $x^2 + y^2 > r^2$   
then orange(N) else blue (P)*



*NN model: FAILED*

*– could not come up with an accurate labeling formula based on  $x$  &  $y$*

### Solution 1:

- Create non-linear features

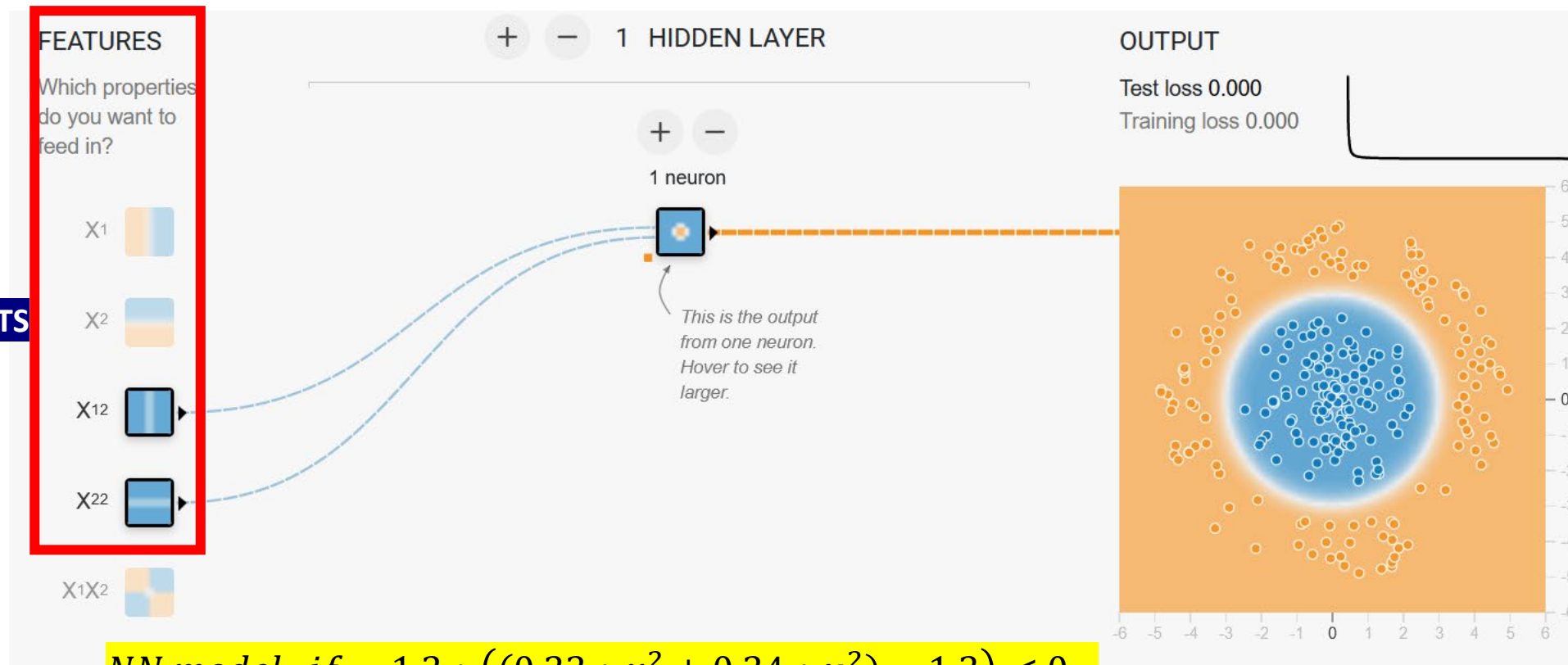
### Solution 2:

- Add 3 more nodes
- Switch to Tanh activation function

# EX #2: Non-Linear Features ( $x^2$ and $y^2$ )

*Math: if  $x^2 + y^2 > r^2$   
then orange(N) else blue (P)*

## Linear Activation



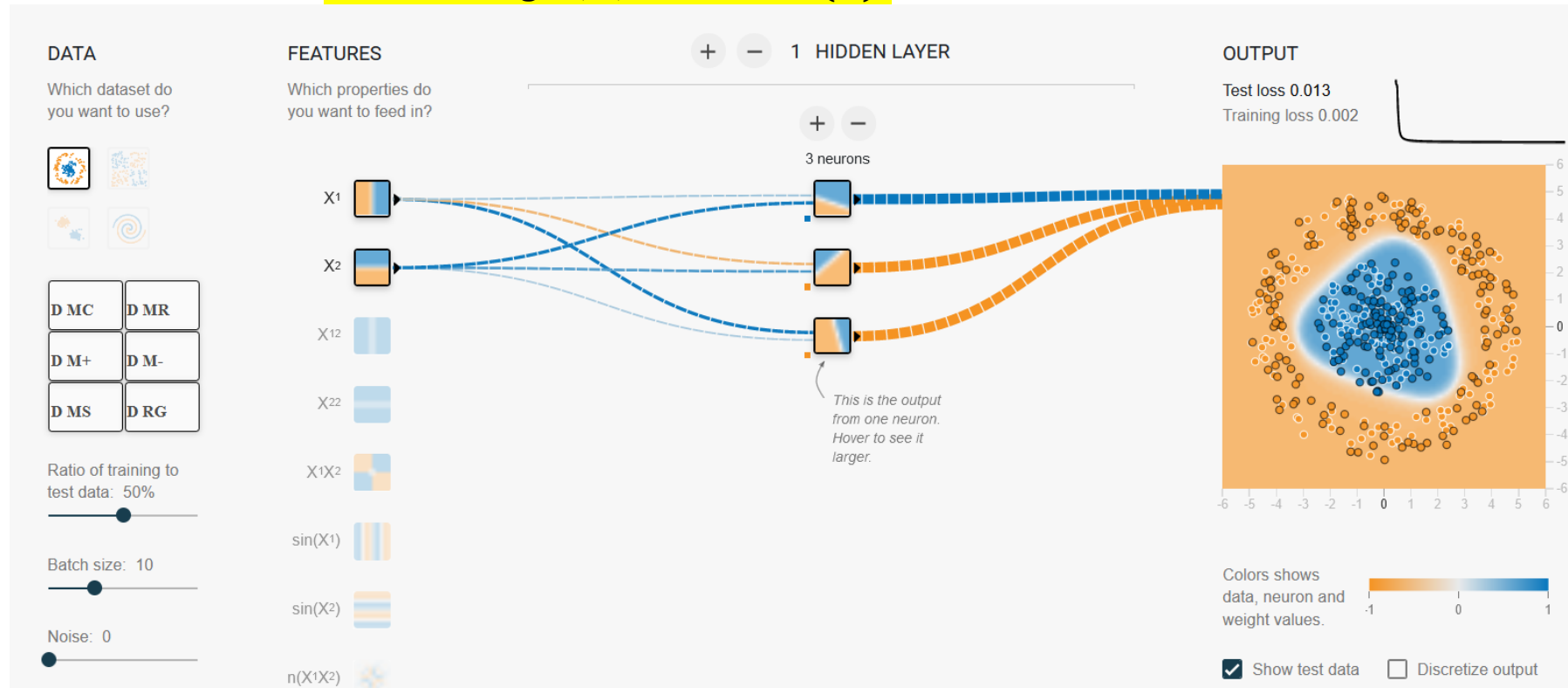
*NN model: if  $-1.3 * ((0.33 * x^2 + 0.34 * y^2) - 1.3) < 0$   
then orange(N) else blue (P)*

If  $-0.429 * x^2 - 0.442 * y^2 + 1.69 < 0$  then orange (N) else blue (P)  $\rightarrow$  radius  $\sim \text{Sqrt}(1.69/0.433) = 1.97$

# EX #3: Non-Linear Tanh Activation

*Math: if  $x^2 + y^2 > r^2$   
then orange(N) else blue (P)*

## Tanh Activation



Test loss 0.013  
Training loss 0.002

*NN model: if  $4 * \tanh(1.7 + 0.32 * x + 0.94 * y) - 4 * \tanh(-1.9 - 0.59 * x + 0.66 * y) - 4 * \tanh(-2.0 + 0.96 * x + 0.27 * y) < 0$   
then orange (N) else blue (P)*

# EX #4: Test well-separated and interleaved patterns

Select a dataset with cluster pattern

Use the “Noise” slider bar

