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One-factor Stochastic Models Of Interest Rates

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Contents

Chapter 1

Introduction: History of Stochastic Calculus and Aims of this Paper

Predictions and forecasts are very important parts of today's financial world. They make possible for financial agents to gain extra returns and profits, on the basis of the fact that they know approximate future value or possible behavior of some asset. There are a huge variety of possible ways for forecasting and one of them is based on Stochastic Calculus.
Basically this is the way when a researcher, believes that it is possible to build models and forecasts, on the basis of some historical data. Hence, one of the most important assumptions is that markets are not efficient, which means that it is possible to make extra profit or return using historical data. There exist a lot of papers that actually proves that market are actually efficient, and thus there is no way to get extra returns or profits systematically. However, in this paper we will not focus on the Efficient Market Hypothesys, and we assume that we can make some certain predicitions on the basis of historical data. Now lets turn to some historical facts and reasons and possible ways where the stochastic calculus is a necessary thing.
In 1827, English Botanist Robert Brown noted that pollen grains under a microscope seemed to move in a random pattern. Than in the 19th century

this phenomenon was developed by Louis Bachelier as a modeling tool for financial price process. Shortly afterward, Einstein also explained how the random motion worked, calling it "Brownian motion." In 1944, Kiyoshi Ito used Brownian motion as the foundation for stochastic calculus. In this paper you will not, that a notation for Brownian Motion is W_t. Today, Stochastic calculus is a very important part of, and as was said below, is used as a model to describe and predict the behavior of variations in market prices for financial assets over time. Actually, markets today operate largely according to models that are based on stochastic calculus, with computer systems that make decisions, also on the basis of them.
In this paper we will focus on the efficiency of ceveral among these models, comparing it with each other. These Models are Vasicek(1977), Cox Ingersoll Rolls(1985), Dothan model(1978), and Merton model(1973).

Chapter 2

Stochatic equations And Its Derivation

2.1 Merton Model(1973)

The first model, which will be considered is the simplest one:

$$dr_t = \alpha dt + \delta dW_t$$

where $\alpha, \delta > 0$

$r_t = r_0 + \int_0^t \alpha ds + \int_0^t \delta dW_s = r_0 + \alpha t + \delta W_t \Rightarrow r_t = r_u + \alpha (t - u) + \delta (W_t - W_u)$
let $u = t$ and $t = t + 1 \Longrightarrow$

$$\Longrightarrow r_{t+1} - r_t = \alpha + \underbrace{\delta (W_{t+1} - W_t)}_{\xi} \tag{2.1}$$

From the formula (2.1) follows that the increment of a interest rate is constant over time.

$$\Delta r = const + \xi$$

Also as $\xi = \delta (W_{t+1} - W_t) \Longrightarrow \xi \sim N(0, \delta)$, and the final form of the Merton model becomes:

$$\Delta r = const$$

2.2 The Vasicek Model(1977)

The VAsicek model a little bit more complex and it has the following equation:

$$dr_t = \alpha\left(\beta - r_t\right)dt + \delta dW_t \tag{2.2}$$

Before we turn to derivation of a solution of this model, we should indicate one of the very important advantages of this model, and this advantage is that this model is analiticaly tracktable. If we look on the equation (2.2) we can see that β is long-run mean interest rate, and if $\beta \leq r_t$, than expression inside of the brackets becomes negative, which pushes down the interest rate to the mena interest rate level, with a sensitivity equal to α. Now lets tuen to derivation. In order to solve this differential equation lets multiply both sides by $e^{\alpha t}$. After that we get:

$$e^{\alpha t}dr_t = e^{\alpha t}\alpha\left(\beta - r_t\right)dt + e^{\alpha t}\delta dW_t$$

Now lets open the brackets:

$$e^{\alpha t}dr_t + e^{\alpha t}\alpha r_t dt = e^{\alpha t}\alpha\beta + e^{\alpha t}\delta dW_t \Longrightarrow$$

$$\Longrightarrow d\left(e^{\alpha t}r_t\right) = e^{\alpha t}\alpha\beta + e^{\alpha t}\delta dW_t \Longrightarrow$$

$$\Longrightarrow e^{\alpha t}r_t = r(0) + \int_0^t e^{\alpha s}\alpha\beta ds + \delta\int_0^t e^{\alpha s}dWs$$

Now lets divide both sides by $e^{\alpha t}$:

$$r_t = r_0 e^{-\alpha t} + \alpha\beta\int_0^t e^{-\alpha(t-s)}ds + \delta\int_0^t e^{-\alpha(t-s)}dWs$$

However if we want to estimate the value of interest rate in moment t, not starting from the poit 0, but from some moment u, the equation becomes:

$$r_t = r_u e^{-\alpha(t-u)} + \alpha\beta\int_u^t e^{-\alpha(t-s)}ds + \delta\int_u^t e^{-\alpha(t-s)}dWs$$

So the solution to the equation (2.2) is above. Now lets make this model discrete. In order to do that lets substract r_u from both sides, and than asume that $u = t - 1$:

$$r_t - r_{t-1} = r_{t-1}e^{-\alpha} - r_{t-1} + \alpha\beta \int_{t-1}^{t} e^{-\alpha(t-s)}ds + \delta \int_{t-1}^{t} e^{-\alpha(t-s)}dWs \Longrightarrow$$

$$\Longrightarrow r_t - r_{t-1} = r_{t-1}\left(e^{-\alpha} - 1\right) + \alpha\beta \int_{t-1}^{t} e^{-\alpha(t-s)}ds + \underbrace{\delta \int_{t-1}^{t} e^{-\alpha(t-s)}dWs}_{\xi}$$

So what we have here is that at the right hand side we have increment of a interest rate, while at the left hand side we have $e^{-\alpha} - 1$, which is constant and the first integral is also equal to some constant. In other words $\alpha\beta \int_{t-1}^{t} e^{-\alpha(t-s)} = const$. The last term xi is normaly distributed with parameters: $\xi \sim N\left(0, \delta^2 \int_{t-1}^{t} e^{-2a(t-s)}ds\right)$. It means that Vasicek model in discrete case has the followinf form:

$$\Delta r = const + ar_{t-1},$$

where $\Delta r = r_t - r_{t-1}$

2.3 Cox-Ingersoll-Ross Model(1985)

The CIR model is the youngest one, as it was produced by Cox-Ingersoll-Ross in 1985. The equation of it is below:

$$dr_t = \beta\left(\alpha - r_t\right)dt + \delta r_t^{1/2}dW_t \tag{2.3}$$

Here α, β , δ are non-negative constants. The problem is that CIR model doesn't have an analytical explicit solution to its corresponding stochastic equation, which means that the way of making this model discrete for further analysis could not be shown. However, we can be sure that this stochastic equation has positive unique solution, and it could be proven, but this is beyond the scope of this paper,and therefore there will be no proof for this fact, and we will accept it as axiom and also accept that in the discrete case CIR model have the following form:

$$r_{t+1} = \left(1 - \varphi\right)\theta + \varphi r_t + \delta r_t^{1/2}\xi_{t+1}$$

Now we need to make this model linear, in some sense. In order to do that we need to subtract r_t from both sides and than divide both parts by $r_t^{1/2}$ and we will get the following equation:

$$\frac{(r_{t+1} - r_t)}{r_t^{1/2}} = \frac{(1-\varphi)}{\theta r_t^{1/2}} + (\varphi - 1)\, r_t^{1/2} + \delta\xi_{t+1}$$

As ξ_{t+1} has expectation equal to 0, or in other words $E\left(\xi_{t+1}\right) = 0$, than the CIR model has the following form:

$$\frac{\Delta r}{r_t^{1/2}} = a r_t^{1/2} + b\frac{1}{r_t^{1/2}}$$

where a,b constants such that

$$b = \frac{(1-\varphi)}{\theta}, a = (\varphi - 1)$$

2.4 Dothan Model(1978)

Another model that we are going to consider is Dothan model, stochastic differential equation of which has the following form:

$$dr_t = \sigma r_t dW_t, \text{ where } r(0) = r_0$$

Now lets derive the analytical expression of solution to this differential equation.As r_0, δ are positive constants than the solution for this equation is

$$r_t = r_0 + \int_0^t \sigma r_s dW_s$$

However it is not the final answer, in order to make it complete lets assume that $f(x) = \log x$, now if we take first and second derivative of $f(x)$ we get: $f'(x) = \frac{1}{x}$ and $f"(x) = -\frac{1}{x^2}$.
Using Ito Lemma, we get:

$$\log r_t = \log r_0 + \int_0^t \frac{1}{r_s} r_s \sigma dW_s - \frac{1}{2}\int_0^t \frac{1}{r_s^2} r_s^2 \sigma ds = \log r_0 + \int_0^t \sigma dW s - \frac{1}{2}\int_0^t \sigma^2 ds \Longrightarrow$$

$$\log r_t = \log r_0 + \sigma W_t - \frac{1}{2}\sigma^2 t \Longrightarrow \quad r_t = r_0 e^{\left(-\frac{1}{2}\sigma^2 t + \sigma W_t\right)}.$$

Now for $u \leq t$:

$$\log r_u = \log r_0 + \int_0^u \sigma dW_s - \frac{1}{2}\int_0^u \sigma^2 ds$$

$$\log r_t = \log r_0 + \int_u^t \sigma dW_s + \int_0^t \sigma dW_s - \frac{1}{2}\int_u^t \sigma^2 ds - \frac{1}{2}\int_0^t \sigma^2 ds \Longrightarrow$$

$$\log r_u = + \int_u^t \sigma dW_s - \frac{1}{2}\int_u^t \sigma^2 ds = \log r_u + \sigma\left(W_t - W_u\right) - \frac{1}{2}\sigma^2\left(t-u\right) \Longrightarrow$$

$$r_t = r_u e^{\left(-\frac{1}{2}\sigma^2(t-u)\right) + \sigma(W_t - W_u)}$$

Now lets make this model discrete for further analysis. Let's devide both parts by r_u and than take log from both parts:

$$\log \frac{r_t}{r_u} = -\frac{1}{2}\sigma^2\left(t-u\right) + \sigma\left(W_t - W_u\right)$$

Now assume that $u = t - 1$:

$$\log r_t - \log r_{t-1} = \frac{1}{2}\sigma^2 + \underbrace{\sigma\left(W_t - W_{t-1}\right)}\xi$$

So finally we have that $\frac{1}{2}\sigma^2$ is some constant and $\sigma\left(W_t - W_{t-1}\right)$ is normaly distributed with the following parameters:$\xi \sim N\left(0, \sigma\right)$. It means that Dothan model in discrete case has the following form:

$$\log r_t - \log r_{t-1} = const$$

Chapter 3

The analysis of the stability of coefficients

As was stated in introduction, the aim of these models is to predict possible behaviour of short-term interest rates, for instance federal funds rate. In order to check, how efficient they are in this field we need to determine on the basis of what size of sample we will do it, and what is more important, we need to check how models behave in different periods of time. For these purposes we will use a special script in gretl console. What this script actually does, is that it takes particular observations that we want, and starting this observation he takes next ceveral observations and run regression. The number of next observartions is also determened by ourselves. In other words we could take for instance $i = 1, 2, 3$, where i is number of observation, than we can take next one hundred observations and run a regression of size 101. As a result there will 3 regresions, with all parameters that we need.
Still, even in this script we need to determine what values to chooose, and what size of smaples we wnat to see. This question is pretty intuitive, or in other words there is no precise criterions for choosing it. However there exist one thing that could help us to determine what size of samples is better to test. As we know September of 2008 is a starting date of crisis, and in our data we have observations starting from the June of 1954 to .Thus, we choose such size of sample that covers period of time from september 2008

to 2012, and check samples of the same size starting from the year 1954. As a result we will get 651 regressions of size 41 for each model, which will help us to analyse the behaviour of coefficients.Below we have all equations and graphs that shows the behaviour of model, when we take samples of size 41, starting from the 1954,and the last observation is september 2008.

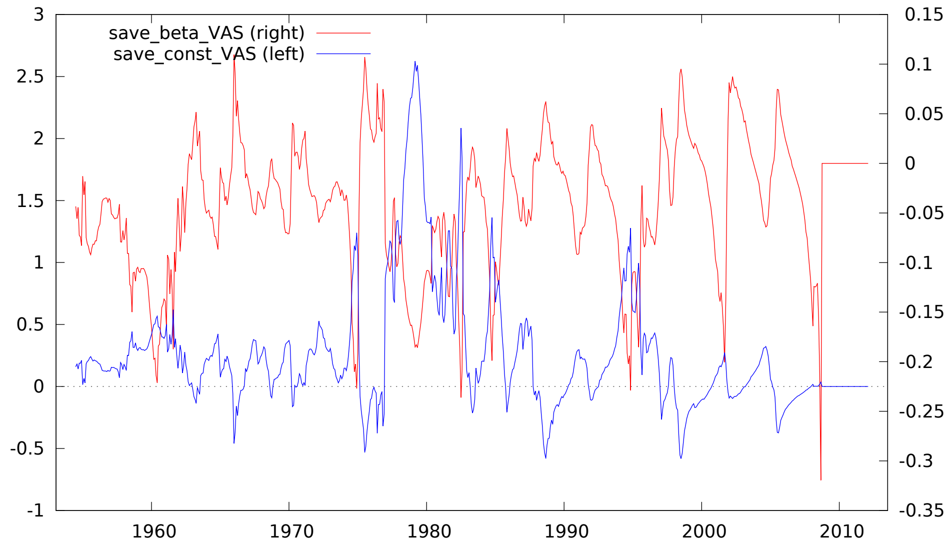

Figure 3.1: Vasicek

Lets start our analysis from the Vasicek model and before that lets see what equation it has:

$$\Delta r = const + ar_{t-1}$$

On the the Figure 3.1 the red line represents the behaviour of the beta coefficient of this model. We can see that in terms of changes in sign or in other words changes in trends this coeficient is relatively stable, as the change of sign happends only 3 times: approximately in 1976(it means that the strating observation was taken from this year), in 1983 and in 2009.Reasons of these changes could be very different and we cant tell exactly why and beacuse of

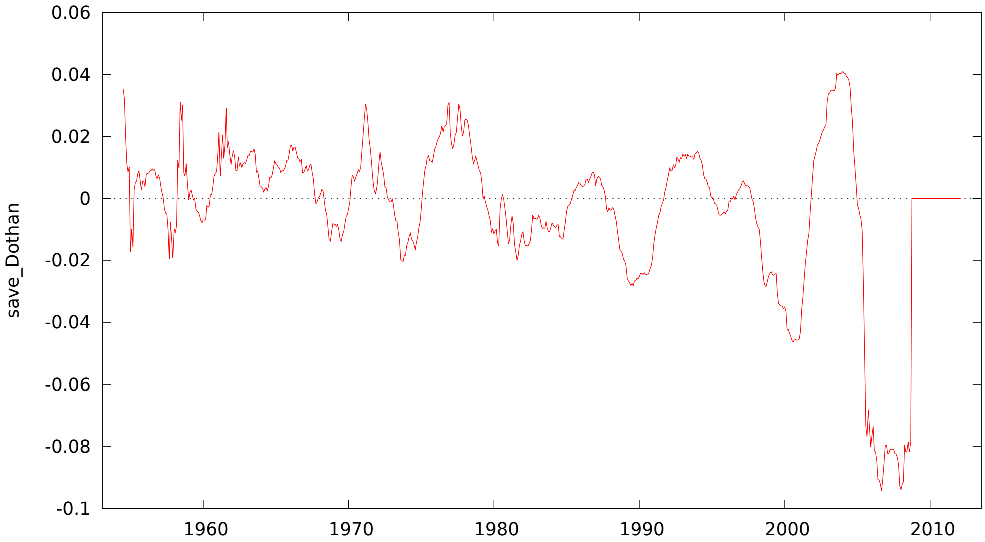

Figure 3.2: Dothan

what facts they have happended, but we can defenetely assume what probably could be the reason. For 1976 it could be the fact, that three years earlier, Fischer Black and Myron Scholes published their groundbreaking paper in which they derived famous Black-Scholes formula, and therefore behaviour of market changed, and as a result the behaviour of model changed. Another change in sign in 2009, could be explained as a result of financial crisis, because markets in crisis works and behave differently, therefore the behaviour of model also changes.Another part of the Vasicek model, in discrete case is a constant, which itself less important than beta coefficient, but still we can analyse it (it is represented by the blue line on the figure 3.1). The constant in this model is not that stable, it changes its sign about 23 times, but its preatty obvious that, unlike beta coeficient the spread of values that are taken by it is less, than by beta, actually we could compare them, because all information could be found on the graph. For beta the range is preatty big, from -0.8 to aproximately 2.6, for the constant the range is from -0,6 to aproximately 2,55. It seems that they have almost equal ranges, however if we look on the graph, constsnt has such a big range only in period of time from 1975 to 1985, while in other years the values that it takes fluctuate

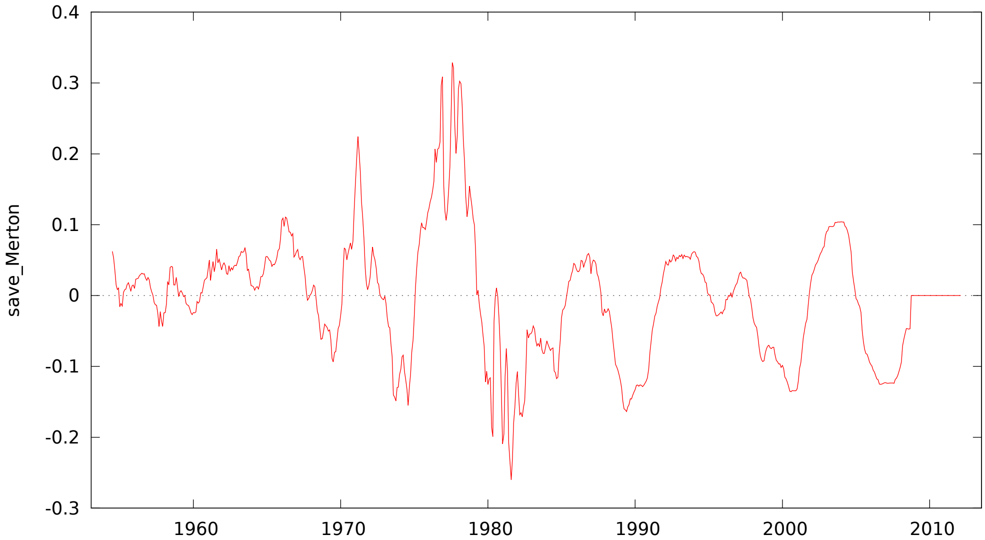

Figure 3.3: Merton

around the zero. So in other words the beta coefficient is stable in terms of trend or sign, but it is not that stable in terms of range of values that it takes, because this criteria is better for constant in thos model, as amoung all periods of time it has smaller spread of values.

Now lets turn to the Dothan model.The behaviour of this model is shown on the Figure 3.2. From the Chapter 2 we know, that this model has the following equation:

$$\log r_t - \log r_{t-1} = const$$

So unlike Vasicek model we need to analyse behaviour of only coefficient in case of Dothan model. So, as can be seen from the graph, this coefficient changes its sign aproximately 23 times, and it takes values from -0.1 to 0.06, which is smaller, than the range of both coefficients of Vasicek model. However, unlike beta coefficient in Vasicek model, it changes its sign a lot of times. So, the behaviour of Dothan model is preatty similar with a behaviour, of constant in Vasicek model, except the fact, that it has smaller range of values. One of the changes of sign was in 2007 and than untill 2009 there was a deep fall in value of coefficient, down to aproximately -0,09. The

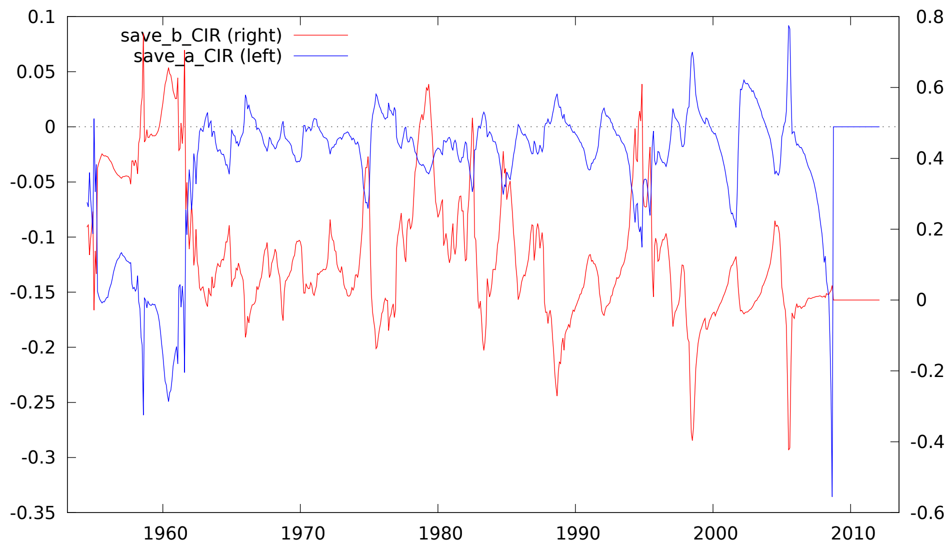

Figure 3.4: CIR

reason again could be financial crisis, which changes the behaviour of markets, which in its term become a reason of changes in model.
The next model could be found on the Figure 3.3. This is Merton model, and it has very similar equation to Dothan model:

$$\Delta r = const$$

The Merton model has almost the same behaviour, as the Dothan does. Unstable in sign, as it changes it almost 22 times, the range of values is from aproximately -0,28 to 0,32, which is less than range of Vasicek model, but clearly more than Dothan has. And again, in 2006-2007 changes its sign anf than the value of coefficient fall down to -0,28, which is minimal value on the graph. This is also a possible consequence of financial crisis.
The last model is CIR. It has a very specific equation, and itself very different from other models

$$\frac{\Delta r}{r_t^{1/2}} = a r_t^{1/2} + b \frac{1}{r_t^{1/2}}$$

As can be seen from the equation, it has two coefficients a and b. On the graph (Figure 3.4) the red line shows the behaviour of coefficient b, while the blue one represents a. Coefficient a has a lot of points in which it changes its sign, the range of of values of this coefficient is from aproximately -0,34 to 0,08. The coeffient b is very close in its behaviour to beta coefficient of Vasicek model, except the fact, that it does not change its sign in 2009, more over it doesnt change sign starting from aproximately 1997. The range of coefficient b is a little bit less than coefficient a has: from approximately -0,29 to 0,08. So what we have is that both coefficients of CIR model have smaller range than, for instance Vasicek has, also the behavior of coefficient b could be considered as almost the same as the behavior of constant in Vasicek model, and behaviour of coefficients of Dothan and Merton, whereas the behaviour of coefficient a in CIR model is closer to behavior of beta coefficient of Vasicek model. In order to clearly see all the differences and similarities of the behaviour these four model lets illustrate them on the one graph (Figure 3.5). From the Figure 3.5 we can see that the behavior of b

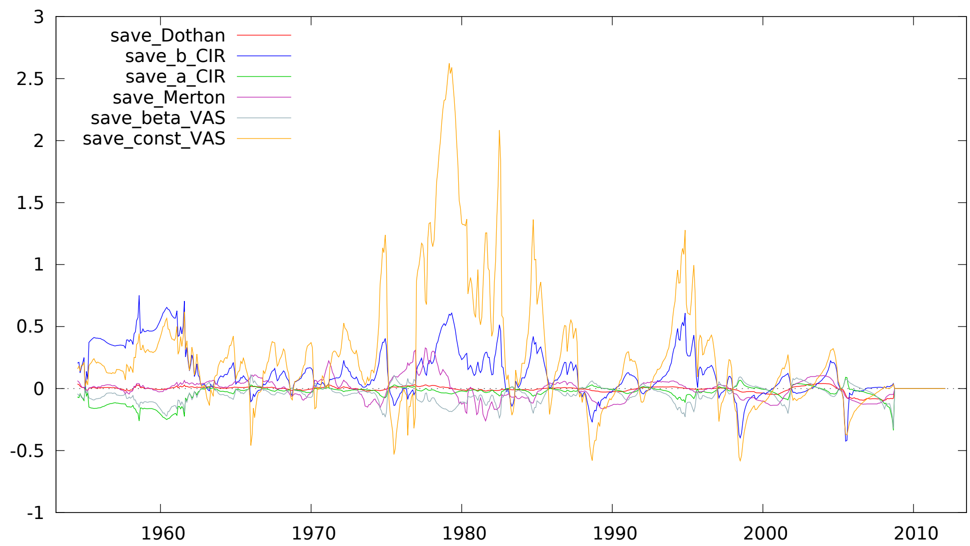

Figure 3.5: CIR, Vasicek, Merton, Dothan

coefficient of CIR and costant from Vasicek has almost the same shape on the graph, with only one difference which is scale (Vasicek clearly has wider range). However, other models and coefficients don't have such similiraties in a shape of graph, more over from this figure could be seen that the behaviour of models is preatty different from each other in any period of time.

Chapter 4

Test of efficiency of One Factor Stochastic Equations

In this chapter we are going to analyse efficiency of these models. As these modes are used to predict the behaviour of short-tern interest rates, we will need a relevant database. Therefore for further analysis we will take monthly Federal Funds Rate, as its term structure is exactly the same as we need.
In order to test the efficiency of these models we will take the sample from our data, which actually incliudes observations from February 1990 to February 2000, and on the basis of this ten years we will try to predict interest rates from May 2000 to February 2012 . The first step will be to get all information about each model, and than to construct forecasts on the basis of it and compare the deviations of forecasts among this models.

4.1 Vasicek

The first model to examine will be the Vasicek model. We have derived its discrete case in a chapter 2, and it has the following form:

$$\Delta r = const + ar_{t-1}$$

Below we have a table of summary statistics about these model:

Model 9: OLS, using observations 1990:02–2000:02 ($T = 121$)
Dependent variable: dr

	Coefficient	Std. Error	t-ratio	p-value
const	0.126733	0.0623872	2.0314	0.0444
r_t	−0.0286335	0.0117078	−2.4457	0.0159

Mean dependent var	−0.020661	S.D. dependent var	0.181033
Sum squared resid	3.744534	S.E. of regression	0.177388
R^2	0.047858	Adjusted R^2	0.039857
$F(1, 119)$	5.981360	P-value(F)	0.015921
Log-likelihood	38.57579	Akaike criterion	−73.15158
Schwarz criterion	−67.56000	Hannan–Quinn	−70.88063
$\hat{\rho}$	0.441067	Durbin–Watson	1.111077

Another important part is forecasts that could be produced, using coefficients that we have derived:
The Figure 6.1 in Appendix represents graphically all actual and predicted values. The summary statistics on forecasting is below :

Forecast evaluation statistics

Mean Error	−0.095726
Mean Squared Error	0.050789
Root Mean Squared Error	0.22536
Mean Absolute Error	0.16282
Bias proportion, U^M	0.18042
Regression proportion, U^R	0.033996
Disturbance proportion, U^D	0.78558

4.2 Merton Model

Now lets turn to the Merton model. In the previous chapter we have derived the discrete form of the stochastic equation:

$$\Delta r = const$$

Now lets estimate all parameters of the model and get all information about forecasting:

Model 10: OLS, using observations 1990:02–2000:02 (T = 121)
Dependent variable: dr

	Coefficient	Std. Error	t-ratio	p-value
const	−0.0206612	0.0164575	−1.2554	0.2118

Mean dependent var	−0.020661	S.D. dependent var	0.181033
Sum squared resid	3.932747	S.E. of regression	0.181033
R^2	0.000000	Adjusted R^2	0.000000
Log-likelihood	35.60880	Akaike criterion	−69.21761
Schwarz criterion	−66.42182	Hannan–Quinn	−68.08213
$\hat{\rho}$	0.455015	Durbin–Watson	1.087662

The Figure 6.2 that illustrates all predicted and actual values could be found in the Apendix, here is just the summary of it:
Forecast evaluation statistics

Mean Error	−0.018436
Mean Squared Error	0.040586
Root Mean Squared Error	0.20146
Mean Absolute Error	0.12191
Bias proportion, U^M	0.0083746
Regression proportion, U^R	1.0171e-31
Disturbance proportion, U^D	0.99163

4.3 Dothan Model

This is the third model that we are going to analyse, and from the Chapter 2 we have derived its discrete form:

$$\log r_t - \log r_{t-1} = const$$

Estimated parameters of the model, on the basis of sample from February 1990 to February 2000:

Model 11: OLS, using observations 1990:02–2000:02 ($T = 121$)
Dependent variable: lndr

	Coefficient	Std. Error	t-ratio	p-value
const	−0.00299232	0.00340029	−0.8800	0.3806

Mean dependent var	−0.002992	S.D. dependent var	0.037403
Sum squared resid	0.167880	S.E. of regression	0.037403
R^2	0.000000	Adjusted R^2	0.000000
Log-likelihood	226.4165	Akaike criterion	−450.8330
Schwarz criterion	−448.0372	Hannan–Quinn	−449.6975
$\hat{\rho}$	0.387679	Durbin–Watson	1.220764

The summary statistics of Figure 6.3 (summary of deviations of actual values from real) is below: Forecast evaluation statistics

Mean Error	−0.025121
Mean Squared Error	0.025419
Root Mean Squared Error	0.15943
Mean Absolute Error	0.08601
Bias proportion, U^M	0.024827
Regression proportion, U^R	5.8463e-34
Disturbance proportion, U^D	0.97517

4.4 CIR

From the Chapter 2 we have derived a discrete form of this model, what is important is that this model is different from others, as it doesn't have an explicit analytical solution. Another important fact is it is not linear:

$$\frac{\Delta r}{r_t^{1/2}} = a\frac{1}{r_t^{1/2}} + br_t^{1/2}$$

Here is the estimated parameters of the model:

Model 12: OLS, using observations 1990:02–2000:02 ($T = 121$)
Dependent variable: CIR

	Coefficient	Std. Error	t-ratio	p-value
a	−0.0105423	0.00543013	−1.9414	0.0546
b	0.0471629	0.0269051	1.7529	0.0822

Mean dependent var	−0.002299	S.D. dependent var	0.037122
Sum squared resid	0.160671	S.E. of regression	0.036745
R^2	0.032139	Adjusted R^2	0.024006
$F(2, 119)$	1.975787	P-value(F)	0.143175
Log-likelihood	229.0717	Akaike criterion	−454.1433
Schwarz criterion	−448.5518	Hannan–Quinn	−451.8724
$\hat{\rho}$	0.388442	Durbin–Watson	1.217391

And summary statistics on forecasting from the Figure 6.4 from Appendix:

Forecast evaluation statistics

Mean Error	-0.059043
Mean Squared Error	0.02308
Root Mean Squared Error	0.15192
Mean Absolute Error	0.097943
Bias proportion, U^M	0.15104
Regression proportion, U^R	0.079567
Disturbance proportion, U^D	0.76939

4.5 Comparison of models

Previously we have derived all necessary statistics about each model and also tried to do forecast on the basis of these information. In this chapter we will compare these model in order to find out which of then is actually better in what it supposed to do, merely in forecasting.We will compare each model on the basis of several parameters: Mean Absolute error, Root Mean Squared Error and Mean Squared Error. We have derived values of these parameters in previous sections of this chapter, below the table, that contains all necessary information:

Parameters	Vasicek	Dothan	Merton	CIR
Mean Absolute Error	0.16282	0,08601	0,12191	0,097943
Root Mean Squared Error	0.22536	0,15943	0,20146	0,15192
Mean Squared Error	0.05089	0,025419	0,040586	0,02308

As can be seen feom the table above, the least Mean Absolute Error has Dothan Model, while Root Mean Squared Error and Mean Squared Errors smaller in CIR model. It means that Dothan is better if we use first parameter as a criterion, however if we use second one or third, the best Model is CIR. But what should be said here is that, everything is not that strict. It is true that Dothan and CIR has better parameters that others, but the difference between models parameters is very smaller, which in its term means that we chould conclude that they are almost equally efficient in forecastin of the short term interest rates.

Chapter 5

Conclusion

As was stated in the introduction, the main aim of this paper was to test how efficient one factor models in prediction of behaviour of interest rates. However, actully in this paper we have considered more than just efficiency of these models. As adition to the main aim of this paper, we have considered the derivation of each model, both in continious and discrete cases, and than checked stability of coefficient.

The first key point here will be the derivation of each model. All models that we have derived and analysed were linear except Dothan and CIR. The first one amoung these two is exponential, while the second is more complex than any from other three models. As we have found out the CIR, and it is one of its complexity, doesn have explicit analitical solution. And we have found out that the main advantage of Vasicek model is that it is analitically tracktable:

The second key point will be the stability of each model. The analysis shows that all models are unstable, and what is more important their behaviour is not the same, morever it is completely different. It means, that actually markets change its tendencies and behaviour and models adapt to it. And what is more important these adaptations, as we have considered in Chapter 3, are preatty consistent. A very good example of it is that in 2005-2009 all models change its trends to negative.

To sum up, this paper has shown that models are almost equally efficient in prediction of interest rates. Another important thing is that, all of these models are not very accurate in forecasts, and therefore due to bias in its predictions, its imposible to generate extra returns on the basis of them. Which means that the efficient market hypothesis holds. However, all our conclusions are based only on the Federal Funds Data base, which means that for further and more relevant conclusions another databases should be analysed.

Chapter 6

Appendix

Figure 6.1: Forecast on the basis of Vasicek model

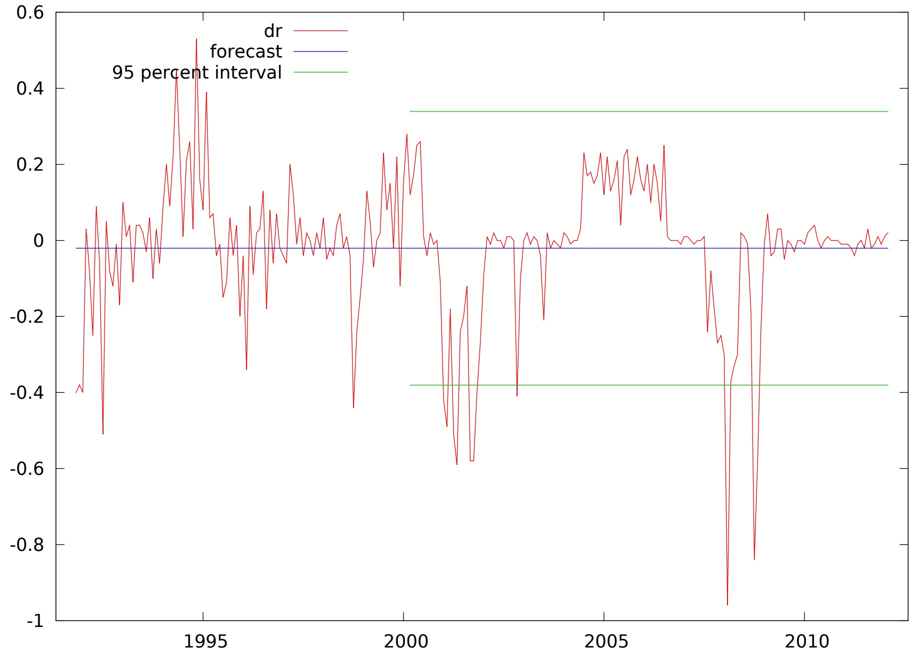

Figure 6.2: Forecast on the basis of Merton model

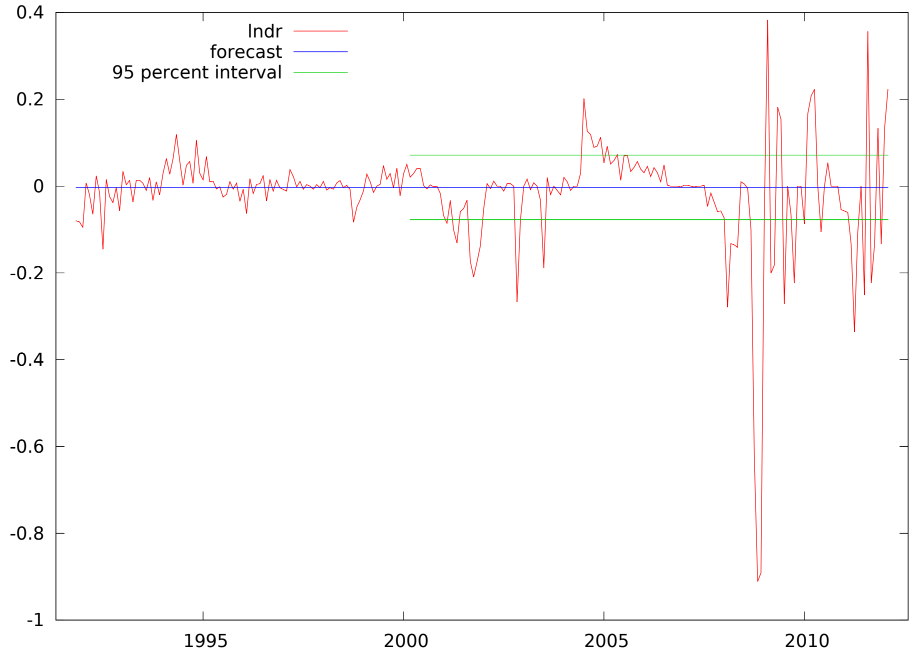

Figure 6.3: Forecast on the basis of Dothan model

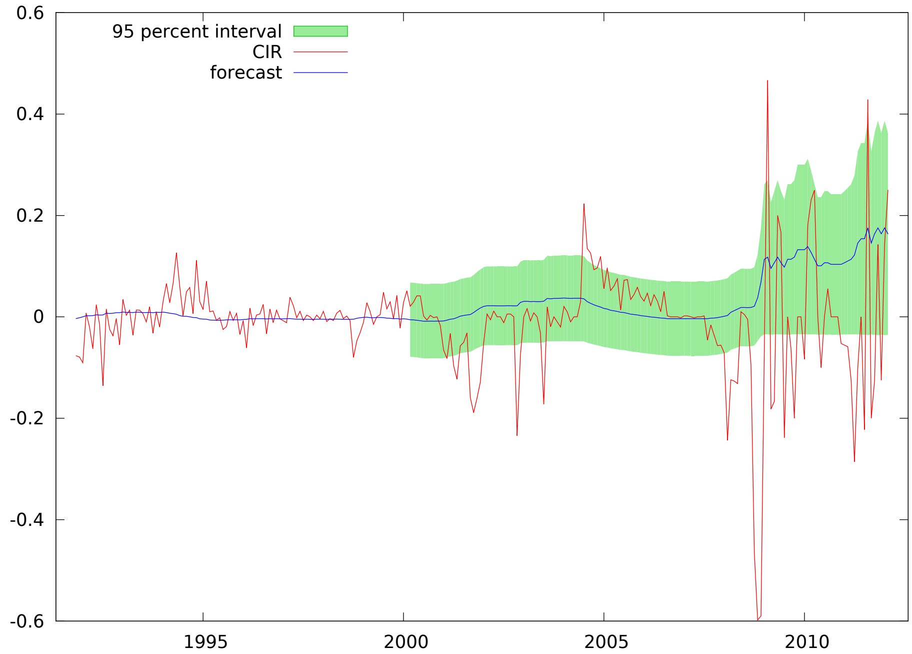

Figure 6.4: Forecast on the basis of CIR model

Bibliography

[1] Kerry Back *A Course in Derivative Securities Introduction to Theory and Computation* 2005: Department of Finance,Mays Business School, Texas A&M University

[2] Damiano Brigo, Fabio Mercurio *Interest Rate Models-Theory and Practice* Springer-Verlag Berlin Heidelberg 2001, 2006

[3] Steven Shreve *Stochastic Calculus and Finance* July 25, 1997

[4] Fima C.Klebaner *Introduction To Stochastic Calculus With Applications* 2005: Imperial College Press.

[5] Dervis Bayazit *Yield Curve Estimation And Prediction With Vasicek Model* June 2004 : The Middle East Technical University.

[6] Yeliz Yolcu *One-Factor Interest Rate Models: Analytic Solutions And Approximations* January 2005 : The Middle East Technical University

Derivation of solutions of all models in continuous cases are taken from these two papers above (Yield Curve Estimation And Prediction With Vasicek Model and One-Factor Interest Rate Models: Analytic Solutions And Approximations)