We assume that (W_t) is a Wiener process and $Z \sim \mathcal{N}(0;1)$.

Expected values and variances

$$\mathrm{E}(Z^{2k+1}) = 0.$$

For example, $\mathrm{E}(Z^7) = 0$.

$$\mathrm{E}(Z^{2k}) = (2k-1)\cdot(2k-3)\cdot\ldots\cdot 5\cdot 3\cdot 1.$$

For example, $\mathrm{E}(Z^6) = 5\cdot 3\cdot 1 = 15$.

$$MGF_Z(u) = \exp\bigl(0.5u^2\bigr).$$

For example, $\frac{d^2}{dt^2}MGF_Z(0) = \mathrm{E}(Z^2)$.

$$\mathrm{E}(W_t^{2k+1}) = 0.$$

For example, $\mathrm{E}(W_9^{11}) = 0$.

$$\mathrm{E}(W_t^{2k}) = t^k(2k-1)\cdot(2k-3)\cdot\ldots\cdot 5\cdot 3\cdot 1.$$

For example, $\mathrm{E}(W_9^8) = 9^4\cdot 7\cdot 5\cdot 3\cdot 1$.

$$MGF_{W_t}(u) = \exp\bigl(0.5tu^2\bigr).$$

For example, $\frac{d^2}{dt^2}MGF_{W_t}(0) = \mathrm{E}(W_t^2)$.

Three stochastic integrals

$$\int_0^t 1\,dW_u = W_t$$

For example, $\int_0^5 3\,dW_u = 3W_5$.

$$\int_0^t W_t\,dW_u = \frac{W_t^2 - t}{2}$$

For example, $\int_0^5 8W_u\,dW_u = 4W_5^2 - 20$.

$$\int_0^t \exp\left(aW_u - \frac{1}{2}a^2u\right)dW_u = \frac{1}{a}\left(\exp\left(aW_t - \frac{1}{2}a^2t\right) - 1\right)$$