

## Project

1. Consider the framework of Black and Scholes model. Let  $S_0 = 100$ ,  $\sigma = 0.1$ ,  $r = 0.05$ ,  $\mu = 0.1$ .

Consider a European type claim that pays you  $X_2 = \sqrt{S_2}$  at time moment  $T = 2$ .

- (a) Simulate 500 trajectories of share price. Plot only 3 of them.

You are free to choose any reasonably small length for time sub-intervals.

- (b) Calculate the arbitrage free price  $X_0$  using two approaches: theoretical expected value pricing equation and average over 500 simulations.

- (c) How many shares should be in replicating portfolio at time moment  $t \in [0; 2]$ ?

Hint: you need to calculate the arbitrage free price  $X_t$  and the derivative  $\partial X_t / \partial S_t$ .

- (d) For each of 500 share price trajectories calculate the number of shares in the replicating portfolio. Plot 3 trajectories of the number of shares in the replicating portfolio.

- (e) Plot 3 trajectories of the replicating portfolio price.

- (f) Plot the scatter plot of 500 final replicating portfolio price versus  $X_2$ .

Does the portfolio really replicate the claim? Why?

2. Consider a two dimensional process stochastic process

$$\begin{cases} dX_t = aX_t dt - Y_t dW_t \\ dY_t = aY_t dt + X_t dW_t \end{cases}$$

with initial conditions  $X_0 = 1$  and  $Y_0 = 0$ .

- (a) Simulate 3 trajectories of this process and plot them on a plane.

- (b) How would you describe the behaviour of this process in plain words?

## Notes

You should answer questions with text and plots and add the code in appendix.

The report should not exceed **10 pages**.

You may do the project alone or in small groups of two or three students.

The report should be upload as one pdf file.

Deadline - 2022-04-28, 23:59.