We assume that (W_t) is a Wiener process and $Z \sim \mathcal{N}(0;1)$, $F(x)$ is the standard normal distribution function, $F(x) = \mathbb{P}(Z \le x)$.

Expected values and variances

$$\mathbb{E}(Z^{2k+1}) = 0.$$

$$\mathbb{E}(Z^{2k}) = (2k-1)\cdot(2k-3)\cdot\ldots\cdot 5\cdot 3\cdot 1.$$

$$\mathbb{E}(Z\cdot I(Z>b)) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{b^2}{2}\right).$$

$$\mathbb{E}(Z\cdot I(Z<b)) = -\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{b^2}{2}\right).$$

$$\mathbb{E}(\exp(aZ)\cdot I(Z>b)) = F(a-b)\exp\left(a^2/2\right).$$

$$\mathbb{E}(\exp(aZ)\cdot I(Z<b)) = F(b-a)\exp\left(a^2/2\right).$$

$$MGF_Z(u) = \mathbb{E}(\exp(uZ)) = \exp\left(0.5u^2\right).$$

$$\mathbb{E}(W_t^{2k+1}) = 0.$$

$$\mathbb{E}(W_t^{2k}) = t^k(2k-1)\cdot(2k-3)\cdot\ldots\cdot 5\cdot 3\cdot 1.$$

$$\mathbb{E}(W_t\cdot I(W_t>b)) = \frac{\sqrt{t}}{\sqrt{2\pi}}\exp\left(-\frac{b^2}{2t}\right).$$

$$\mathbb{E}(W_t\cdot I(W_t<b)) = -\frac{\sqrt{t}}{\sqrt{2\pi}}\exp\left(-\frac{b^2}{2t}\right).$$

$$\mathbb{E}(\exp(aW_t)\cdot I(W_t>b)) = F\left(a\sqrt{t}-\frac{b}{\sqrt{t}}\right)\exp\left(ta^2/2\right).$$

$$\mathbb{E}(\exp(aW_t)\cdot I(W_t<b)) = F\left(\frac{b}{\sqrt{t}}-a\sqrt{t}\right)\exp\left(ta^2/2\right).$$

$$MGF_{W_t}(u) = \mathbb{E}(\exp(uW_t)) = \exp\left(0.5tu^2\right).$$

Three stochastic integrals

$$\int_0^t 1\, dW_u = W_t$$

$$\int_0^t W_t\, dW_u = \frac{W_t^2 - t}{2}$$

$$\int_0^t \exp\left(aW_u - \frac{1}{2}a^2 u\right) dW_u = \frac{1}{a}\left(\exp\left(aW_t - \frac{1}{2}a^2 t\right) - 1\right)$$