

Project

1. Consider the framework of Black and Scholes model. Let $S_0 = 100$, $\sigma = 0.1$, $r = 0.05$, $\mu = 0.1$.

Consider a European type claim that pays you $X_2 = \sqrt{S_2}$ at time moment $T = 2$.

- (a) Simulate 500 trajectories of share price. Plot only 3 of them.
You are free to choose any reasonably small length for time sub-intervals.
- (b) Calculate the arbitrage free price X_0 using two approaches: theoretical expected value pricing equation and average over 500 simulations.
- (c) How many shares should be in replicating portfolio at time moment $t \in [0; 2]$?
Hint: you need to calculate the arbitrage free price X_t and the derivative $\partial X_t / \partial S_t$.
- (d) For each of 500 share price trajectories calculate the number of shares in the replicating portfolio.
Plot 3 trajectories of the number of shares in the replicating portfolio.
- (e) Plot 3 trajectories of the replicating portfolio price.
- (f) Plot the scatter plot of 500 final replicating portfolio price versus X_2 .
Does the portfolio really replicate the claim? Why?

2. Consider a two dimensional stochastic process

$$\begin{cases} dX_t = aX_t dt - Y_t dW_t \\ dY_t = aY_t dt + X_t dW_t \end{cases}$$

with initial conditions $X_0 = 1$ and $Y_0 = 0$.

- (a) Simulate 3 trajectories of this process and plot them on a plane.
- (b) How would you describe the behaviour of this process in plain words?

Notes

You should answer questions with text and plots and add the code in appendix.

The report should not exceed **10 pages**.

You may do the project alone or in small groups of two or three students.

The report should be upload as one pdf file.

Deadline - 2022-04-28, 23:59.