Promo-code activation :)

You have two options!

You can try to solve all the proposed problems and submit your solutions as a pdf file. In this case I will honestly grade them.

You can write a promo-code #beta_hot. In this case I will ignore your solutions and grade your exam as 4/10.

Problems

1. James Bond estimates the model $y_t = \beta x_t + u_i$ with OLS. We know that u_t are independent and identically distributed with unknown heteroskedasticity structure. We also know that $x_t = 1$ for odd t and $x_t = 2$ for even t. The total number of observations n is 600.

	all odd t	all even t
$\sum y_t$	300	200
$\sum \hat{u}_t^2$	1000	800

   (a) Find $\hat{\beta}_{ols}$

   (b) Calculate ordinary and robust standard errors, $se(\hat{\beta}_{ols})$ and $se_{HC}(\hat{\beta}_{ols})$;

   (c) Construct 95% ordinary and robust confidence intervals for β.

2. James Bond estimates the model $y_t = \beta x_t + u_i$ with OLS. We know that u_t are normal $\mathcal{N}(0; x_t \cdot \sigma^2)$ and independent. We also know that $x_t = 1$ for odd t and $x_t = 2$ for even t. The total number of observations n is even.

   (a) Find expected value and variance of $\hat{\beta}_{ols}$.

   (b) Construct the most efficient unbiased linear estimator $\hat{\beta}_{best}$ and find its variance.

   (c) Calculate probability limit of $\hat{\beta}_{ols}$. Is $\hat{\beta}_{ols}$ consistent?

3. James Bond assumes that observations y_i are independent and probability distribution for y_i is given by the table

y_i	$y_i = 1$	$y_i = 2$	$y_i = 3$
probability	a	b	$1 - a - b$

   James Bond has collected 400 observations: 120 observations are equal to 1, 80 observations equal to 2, and 200 observations equal to 3.

   (a) Estimate unknown a and b using maximum likelihood;

   (b) Construct 95% confidence interval for a and 95% confidence interval for $a + b$.

   (c) Using likelihood ratio statistic test the hypothesis $a = b$. You may assume that the critical value of the statistic is 4 but you should clearly state the degrees of freedom.

4. Using 400 observation James Bond estimated the logistic regression for probability of passing econometrics exam using x_i — number of study hours as predictor:

$$\hat{\mathbb{P}}(\text{pass}) = \Lambda(0.15 - 0.025x_i^2 + 0.9x_i),$$

The estimate of coefficient covariance matrix is:

$$\begin{pmatrix} 0.0004 & -0.0001 & 0 \\ -0.0001 & 0.0001 & 0 \\ 0 & 0 & 0.0009 \end{pmatrix}$$

   (a) Estimate the number of study hours that maximizes the probability of passing the exams. What is the maximal probability of passing the exam?

   (b) Construct approximate 95% confidence interval for the number of study hours that maximizes the probability of passing the exams. Hint: use Taylor expansion to obtain a linear function of $\hat{\beta}$.

5. James Bond would like to check whether the econometrics final exam score, y_i, varies for students that dinner at Khatchapuri. Once again x_i is the number of study hours.

He estimates the same model using four datasets:

dataset	equation	RSS	observations
all students	$\hat{y}_i = 40 + 2x_i$	14000	600
two or more Khatchapuri dinners	$\hat{y}_i = 60 + 4x_i$	4000	200
exactly one Khatchapuri dinner	$\hat{y}_i = 50 + 3x_i$	3000	200
no Khatchapuri dinner	$\hat{y}_i = 37 + 1.8x_i$	2000	200

   (a) Using appropriate test check whether the econometrics final exam score varies for three type of students.

   (b) Which unique equation should James Bond estimate to obtain all given beta estimates at once?

You may assume that $F_{critical}$ for $\alpha = 0.05$ is approximately equal to 2.4. However you should clearly state the degrees of freedom.

6. The last problem :)

James Bond would like to estimate the model $y_i = \beta x_i + u_i$, where y_i — is the econometrics final exam centered result, x_i is the centered number of study hours.

The problem is that James has only data on self-reported $x_i^* = x_i \cdot f_i$ where f_i is the famous Russian exageration factor. We know that $x_i \sim \mathcal{N}(0; 9)$, $u_i \sim \mathcal{N}(0; \sigma^2)$, f_i is uniform on $[0; 4]$.

James Bond also has estimates of the famous Russian exageration factor, f_i^* for each student. Here $f_i^* = f_i + \nu_i$ where $\nu_i \sim \mathcal{N}(\delta, \sigma_\nu^2)$. The parameters δ and σ_ν^2 are unknown.

Variables (x_i), (u_i), (ν_i) and (f_i) are independent. The dataset is a random sample.

   (a) Find the probability limit of $\hat{\beta}$ in the regression $\hat{y}_i = \hat{\beta} \cdot x_i^*$.

   (b) Provide explicit formulas or algorithms for consistent estimators of all parameters: ν, σ_ν^2, β, σ^2.