1. Using 500 observations James Bond has estimated a very simple regression model, $\hat{y}_i = 3x_i$. The standard error of $\hat{\beta}$ is equal to 1. The standard error of regression is equal to 4.

   (a) Build 95% confidence interval for β;

   (b) Build 95% predictive interval for y given $x = 2$;

2. James Bond estimates regression of the number of ice cream bought, y_i, on the rain dummy-variable, r_i (1 — rainy day, 0 — no rain).

$$y_i = \beta_1 + \beta_2 r_i + u_i$$

   The available data is summarised in the table below:

Day type	rainy day	no rain
Number of observations	200	200
$\sum y_i$	800	400
$\sum y_i^2$	10000	10000

   (a) Find $\hat{\beta}_1$ and $\hat{\beta}_2$;

   (b) Estimate $\text{Var}(\hat{\beta}_2)$;

   (c) Find 95% confidence interval for β_2.

3. James Bond has estimated 3 models, using the same set of 500 observations:

Model	Equation	R^2
A	$y_i = \beta_1 + u_i$	?
B	$y_i = \beta_1 + \beta_2 x_i + u_i$	0.5
C	$y_i = \beta_1 + \beta_2 x_i + \beta_3 x_i^2 + u_i$	0.7

   (a) Test model A against model B at 5% significance level.

   (b) Test model B against model C at 5% significance level.

   (c) Which model would you prefer?

   $F_{a,b}$ distribution 5% critical values are given below:

	$b = 100$	$b = 200$	$b = 500$
$a = 1$	3.94	3.89	3.86
$a = 2$	3.09	3.04	3.01
$a = 3$	2.70	2.65	2.62
$a = 200$	1.34	1.26	1.21

4. Consider the model $y_i = \beta_1 + \beta_2 x_i + \beta_3 z_i + u_i$. James Bond has sorted 500 available observation by the variable x and has estimated the original model on 4 datasets:

Sample	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	RSS
$i = 1, \ldots, 500$	0.96	2.25	3.44	52.70
$i = 1, \ldots, 200$	1.07	2.46	2.40	5.55
$i = 201, \ldots, 300$	1.32	1.01	2.88	11.69
$i = 301, \ldots, 500$	1.04	2.56	4.12	16.00

The error term u_i has normal distribution.

Apply the Goldfeld-Quandt test for heteroskedasticity at 5% significance level. Clearly state H_0, H_a and your conclusion.

5. James Bond assumes that observations y_i are independent and probability distribution for y_i is given by the table

y_i	$y_i = 1$	$y_i = 2$	$y_i = 3$
probability	a	$2a$	$1 - 3a$

James Bond has collected 400 observations: 100 observations are equal to 1, 100 observations equal to 2, and 200 observations equal to 3.

Estimate unknown a using maximum likelihood;

6. James Bond estimated logit model $\mathbb{P}(y_i = 1) = \Lambda(1 + 2x_i)$ using 500 observations. Estimates are $\hat{\beta}_1 = 0.7$ and $\hat{\beta}_2 = 3$. Estimate of coefficient covariance matrix is

$$\begin{pmatrix} 0.04 & 0.01 \\ 0.01 & 0.09 \end{pmatrix}$$

   (a) Test the hypothesis H_0: $\beta_2 = 0$ at 5% significance level.

   (b) Estimate marginal effect of x_i on the probability $\mathbb{P}(y_i = 1)$ for $x_i = -0.5$.

   (c) Build 95% confidence interval for the true value of marginal effect.