HOW GAUSS, MARKOV AND PYTHAGORAS MET: GEOMETRY IN ECONOMETRICS

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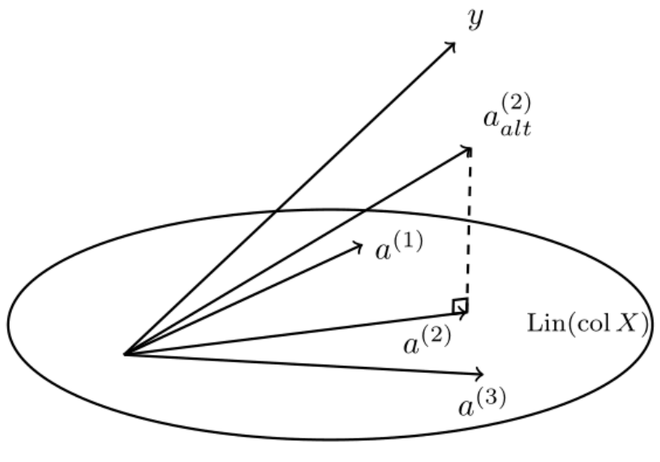
Basic definitions

$$\langle X, Y\rangle = Cov(X, Y)$$

$$\|X\|^2 = Var(X)$$

$$\frac{\langle X, Y\rangle}{\sqrt{\|X\|^2\|Y\|^2}} = Corr(X, Y)$$

Gauss-Markov theorem

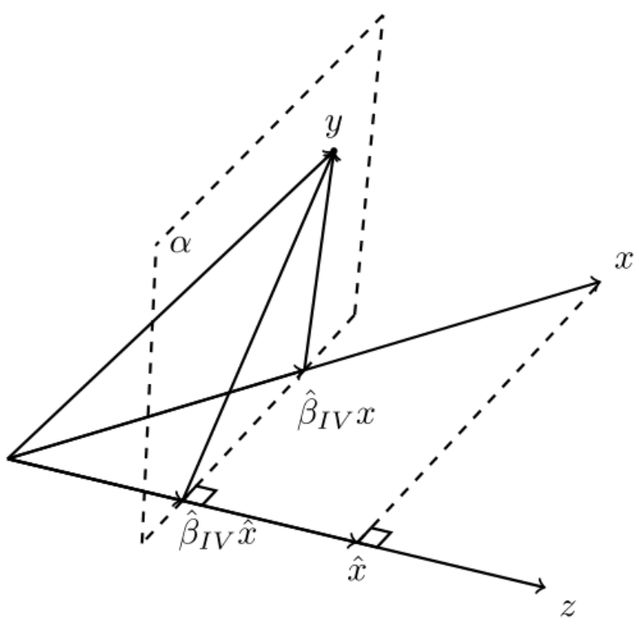


$$\hat\beta_{OLS} = A^T y$$
$$\hat\beta_{alt} = A^T_{alt} y$$

$$\left(A^T_{alt} - A^T\right) \perp X$$

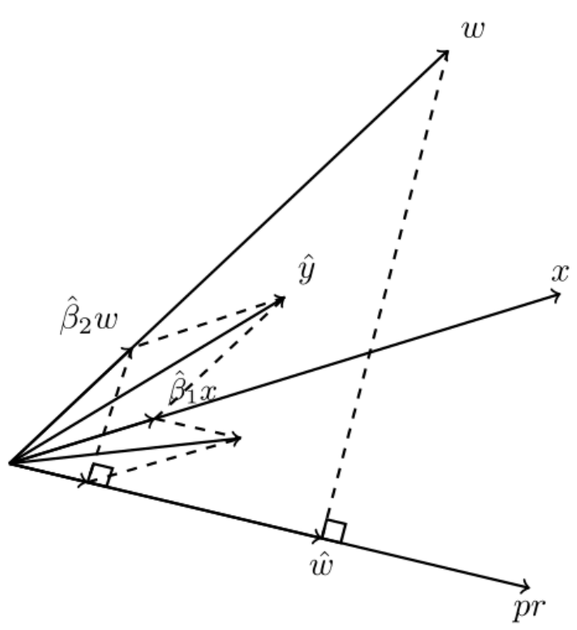
$$Var\left(\hat\beta^{(2)}_{OLS}\right) = \sigma^2\|a\|^2$$
$$Var\left(\hat\beta^{(2)}_{alt}\right) = \sigma^2\|a_{alt}\|^2$$

Instrumental variables



Model: $y = \beta x + u$
z is an instrument,
x is an endogeneous variable.
The picture shows equivalence of 2SLS procedure and oblique projection.

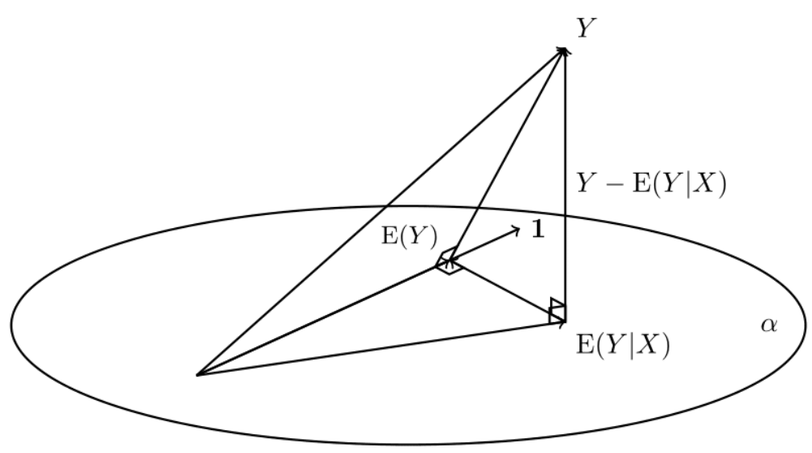
Proxy variables



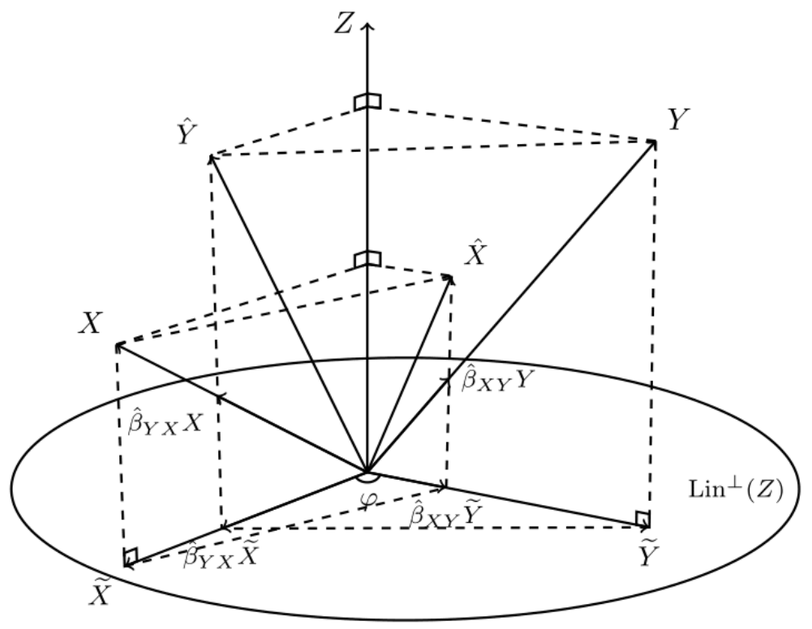
Model: $y = \beta_1 x + \beta_2 w + u$
w is an unobserved variable,
pr is a proxy for w.
The picture illustrates the consistency of $\hat\beta_1$.

The law of iterated expectations

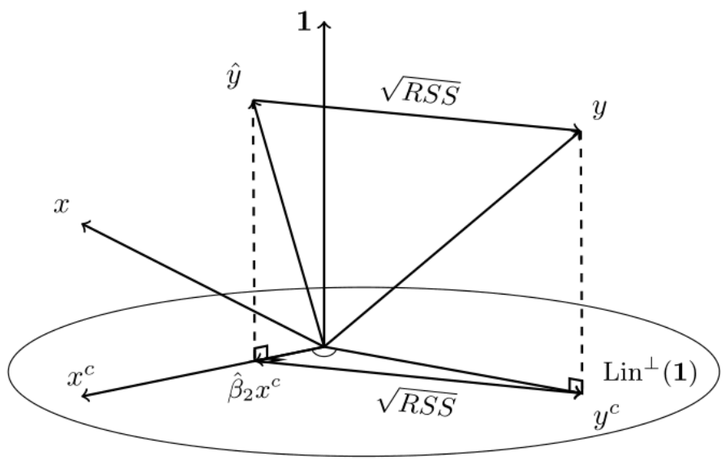
$$E\left(E(X|Y)\right) = E(Y)$$



Partial correlation



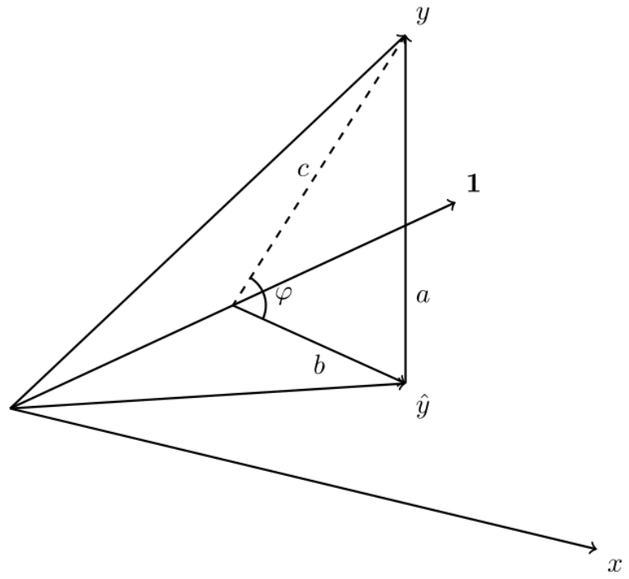
t-test



$$t = \frac{\hat\beta_2|x^c|}{\sqrt{RSS}} = ctg\varphi$$

$$F = \frac{\left(RSS_R - RSS_{UR}\right)/q}{RSS_{UR}/\left(n - k_{UR}\right)} = ctg^2\varphi$$

F-test



$$a = \sqrt{RSS_{UR}}$$
$$b = \sqrt{RSS_R - RSS_{UR}}$$
$$c = \sqrt{RSS_R}$$