HOW GAUSS, MARKOV AND PYTHAGORAS MET: GEOMETRY IN ECONOMETRICS

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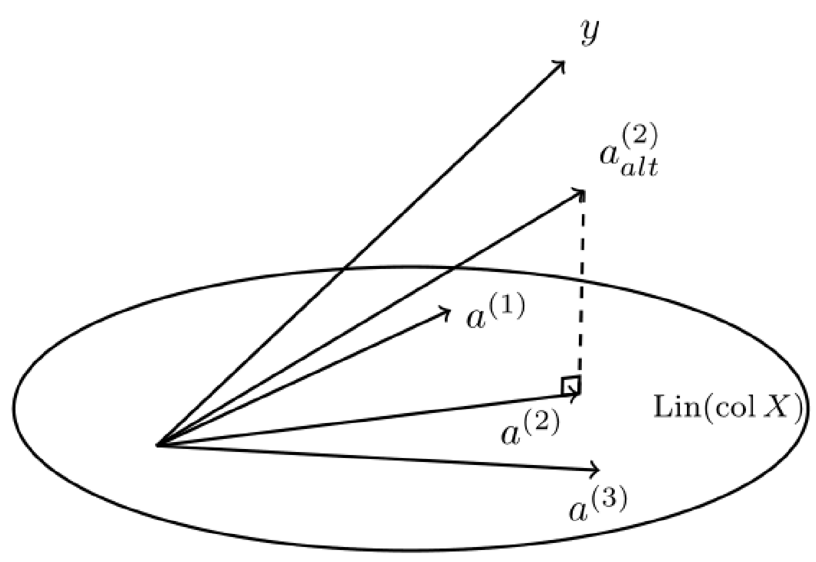
Basic definitions

$$\langle X, Y\rangle = Cov(X,Y)$$

$$\|X\|^2 = Var(X)$$

$$\frac{\langle X, Y\rangle}{\sqrt{\|X\|^2\|Y\|^2}} = Corr(X,Y)$$

Gauss-Markov theorem



$$\hat\beta_{OLS} = A^T y$$

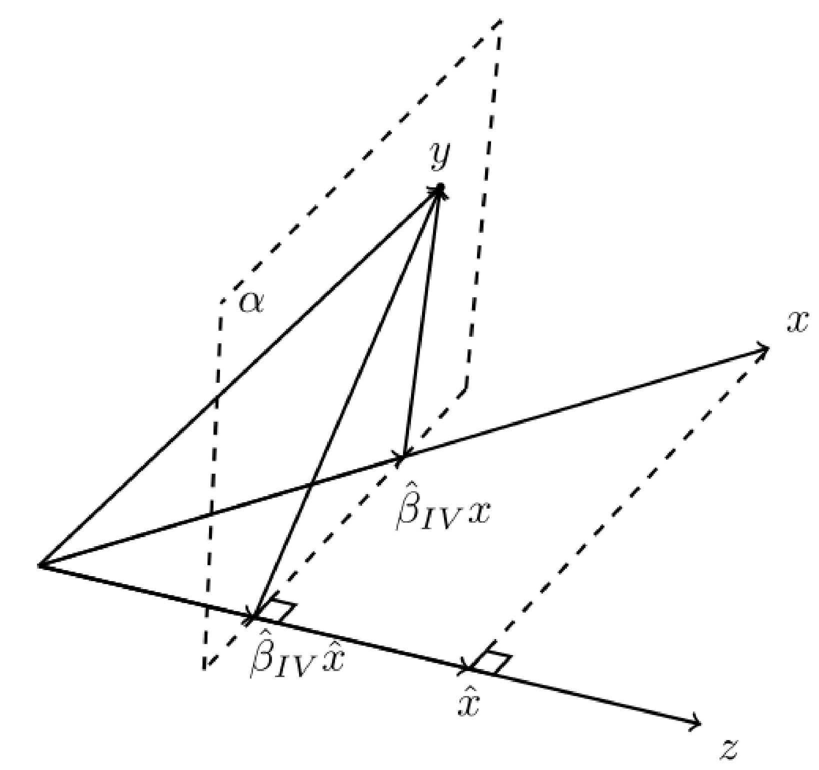
$$\hat\beta_{alt} = A^T_{alt} y$$

$$(A^T_{alt} - A^T) \perp X$$

$$Var\left(\hat\beta^{(2)}_{OLS}\right) = \sigma^2\|a\|^2$$

$$Var\left(\hat\beta^{(2)}_{alt}\right) = \sigma^2\|a_{alt}\|^2$$

Instrumental variables



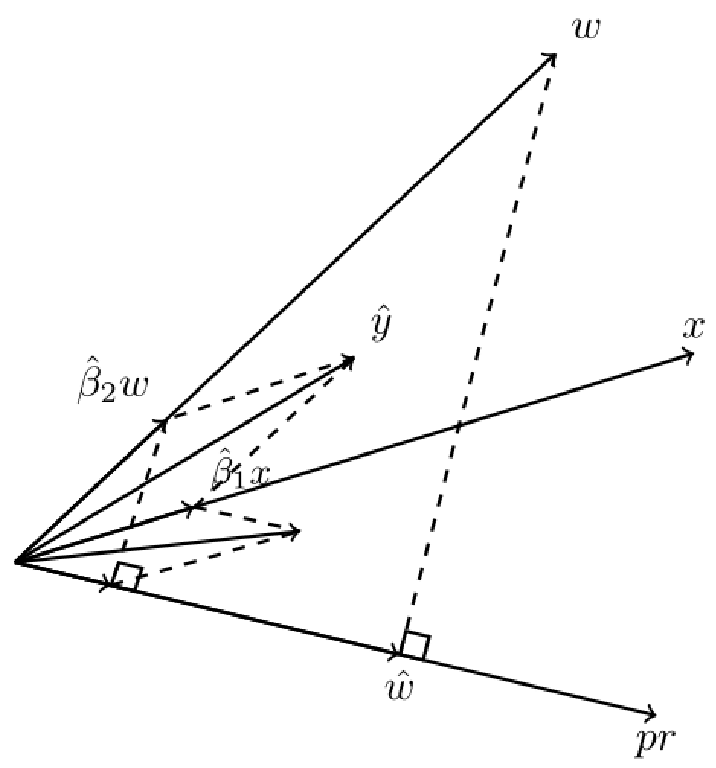
Model: $y = \beta x + u$

z is an instrument,

x is an endogeneous variable.

The picture shows equivalence of 2SLS procedure and oblique projection.

Proxy variables



Model: $y = \beta_1 x + \beta_2 w + u$

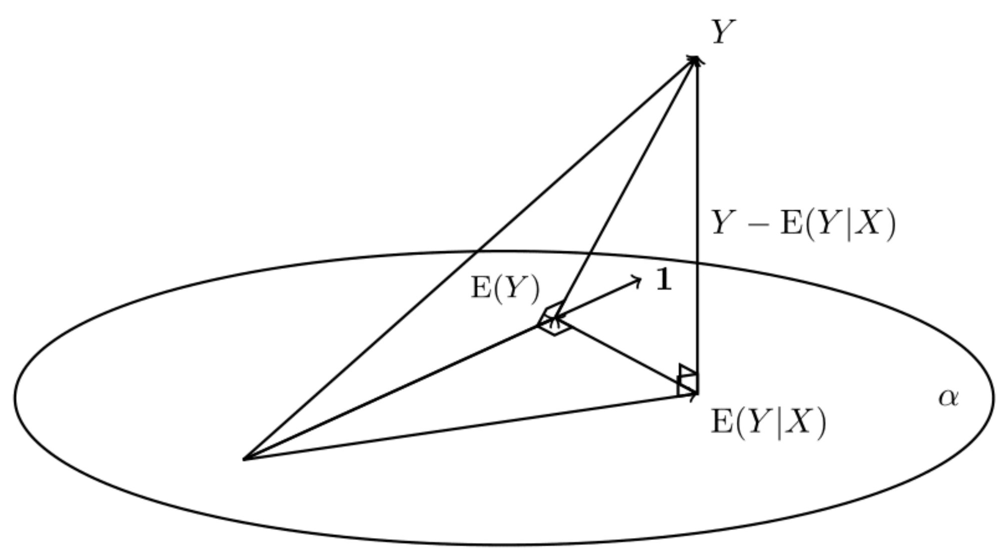
w is an unobserved variable,

pr is a proxy for w.

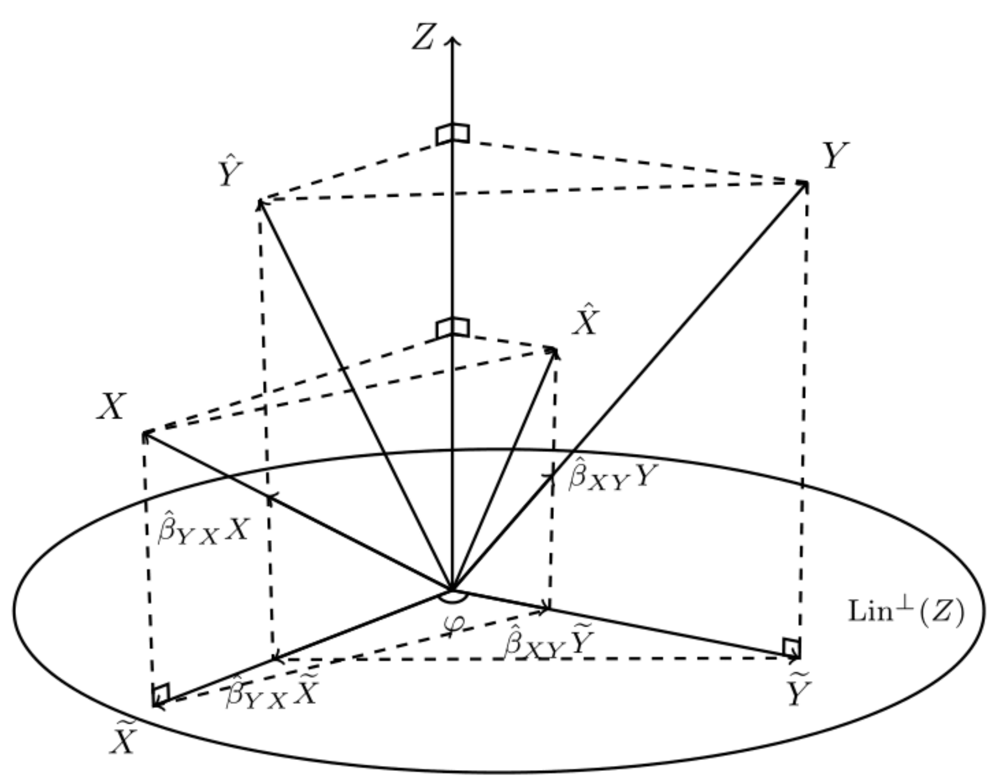
The picture illustrates the consistency of $\hat\beta_1$.

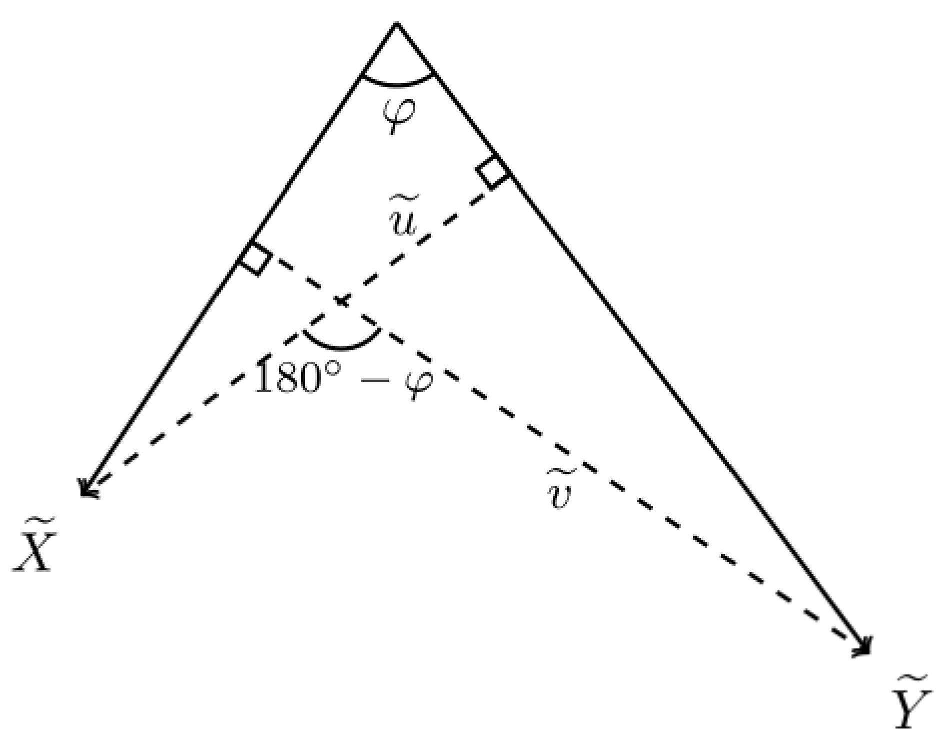
The law of iterated expectations

$$E(E(X|Y)) = E(X)$$

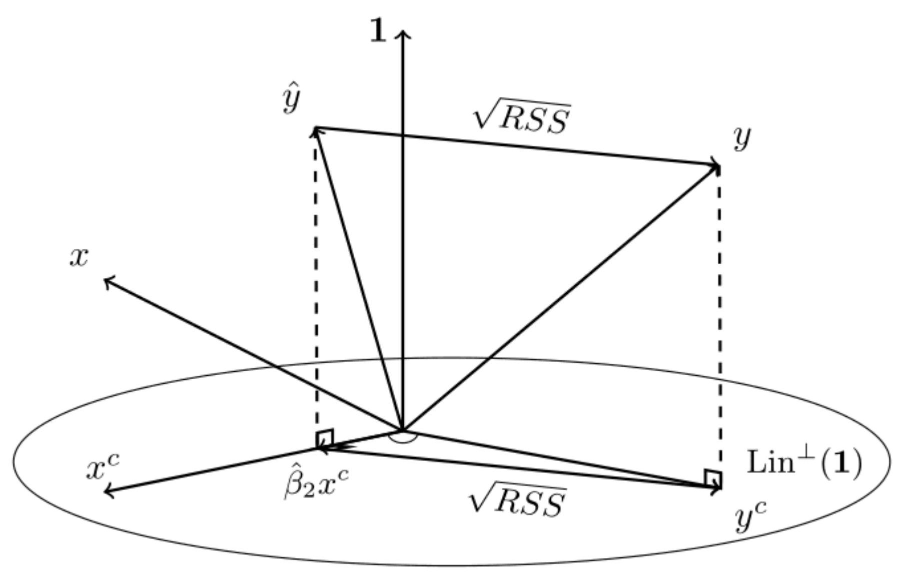


Partial correlation

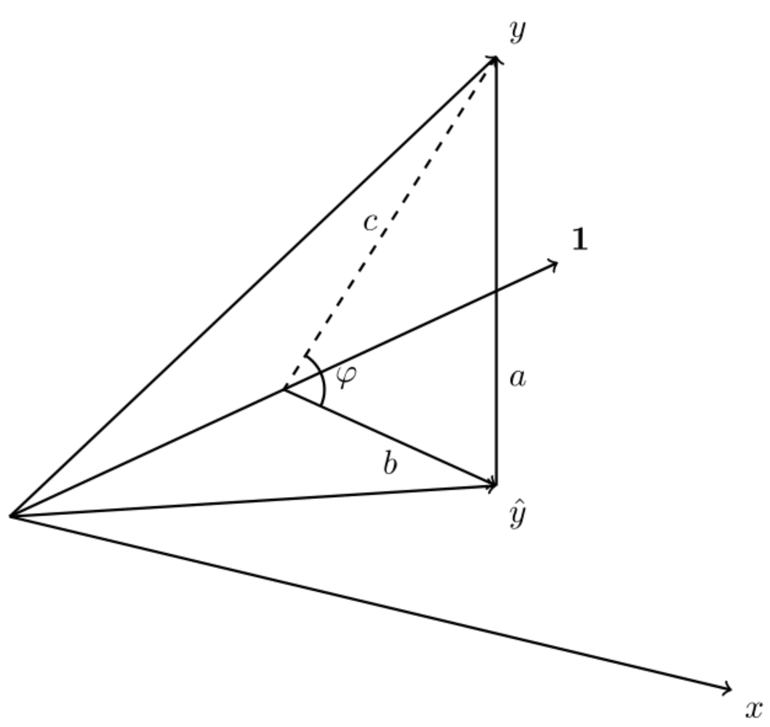




t-test



F-test



$$t = \frac{\hat\beta_2|x^c|}{\sqrt{RSS}} = ctg\varphi\cdot\sqrt{n-2}$$

$$F = \frac{(RSS_R - RSS_{UR})/q}{RSS_{UR}/(n-k_{UR})} = ctg^2\varphi\cdot\frac{n-k_{UR}}{q}$$

$$a = \sqrt{RSS_{UR}}$$

$$b = \sqrt{RSS_R - RSS_{UR}}$$

$$c = \sqrt{RSS_R}$$