Entry Game (Midterm 2013)

Let's consider two very special cases which can be solved using backward induction:

1. $\alpha = 0$. In this spetial case one may consider the game with only two strategies for the first player. Using backward induction: II fights (F), I stays (S).
2. $\alpha = 1$. In this spetial case one may consider the game with only two strategies for the first player. Using backward induction: II quits (Q), I enters (E).

Interesting case $0 < \alpha < 1$. First player has 4 strategies (EE, ES, SE, SS), first letter corresponds to the efficient case. If the player I is efficient then it is optimal to enter no matter how the second player plays. Indeed, 0.5 or 1 is better than 0. So strategies SS and SE are strictly dominated by ES and EE correspondingly.

Let's build the matrix of the game (without strictly dominated strategies):

Case ES,F: $\alpha(0.5, -0.5) + (1-\alpha)(0, 1) = (\frac{\alpha}{2}, \frac{2-3\alpha}{2})$

Case EE,F: $\alpha(0.5, -0.5) + (1-\alpha)(-0.5, 0.5) = (\frac{2\alpha-1}{2}, \frac{1-2\alpha}{2})$

Case ES,Q: $\alpha(1, 0) + (1-\alpha)(0, 1) = (\alpha, 1-\alpha)$

Case EE,Q: $1, 0$

I \ II	F	Q
ES	$\frac{\alpha}{2}, \frac{2-3\alpha}{2}$	$\alpha, 1-\alpha$
EE	$\frac{2\alpha-1}{2}, \frac{1-2\alpha}{2}$	$1, 0$

If $\alpha \geq 0.5$ then EE, Q is the only pure NE. In this case beliefs are given by probabilities $\alpha, 1-\alpha$.

If $\alpha < 0.5$ no pure NE exists. Let's find mixed NE.

Indifference condition for the first player:

$$\frac{\alpha}{2}q + \alpha(1-q) = \frac{2\alpha-1}{2}q + (1-q)$$

and $q = 2/3$.

Indifference condition for the first player:

$$p\frac{2-3\alpha}{2} + (1-p)\frac{1-2\alpha}{2} = p(1-\alpha)$$

and $p = \frac{1-2\alpha}{1-\alpha}$.

The total probability of entering for the Ist player is $\alpha + (1-\alpha)(1-p) = \alpha + \alpha = 2\alpha$. So the beliefs are $(0.5, 0.5)$.