ок?

20.2 $X_1, X_2, X_3 \ldots X_n$ ~ независимы Unif[0;1]

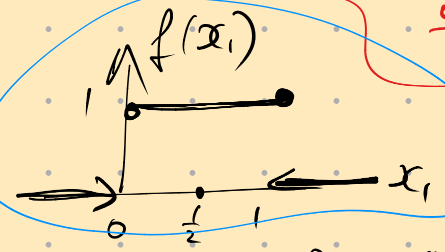
а) $\bar{X}_{100} \sim ?$ (примерно)

$$\bar{X}_{100} = \frac{X_1+X_2+\ldots+X_{100}}{100}$$ — примерно N

$$\frac{X_1+\ldots+X_n - E(X_1+\ldots X_n)}{\sqrt{Var(X_1+\ldots+X_n)}} \xrightarrow{dist} N(0;1)$$

$\bar{X}_{100} \approx N(?;?)$

$E(\bar{X}_{100})$? $Var(\bar{X}_{100})$?



$X_1 \sim Unif[0;1]$

$$E(X_1) = \frac{1}{2}$$

$$Var(X_1) \overset{?}{=} E(X_1^2) - (E(X_1))^2 = E(X_1^2) - \left(\frac{1}{2}\right)^2$$

$$E(X_1^2) = \int_0^1 x^2 \cdot 1\, dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$Var(X_1) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$!!

$$E(\bar{X}_{100}) = E\left(\frac{X_1+\ldots+X_{100}}{100}\right) = \frac{E(X_1)+E(X_2)+\ldots+E(X_{100})}{100} = \frac{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\ldots+\frac{1}{2}}{100} = \frac{1}{2}$$

$$Var(\bar{X}_{100}) = Var\left(\frac{X_1+\ldots+X_{100}}{100}\right) = \frac{1}{100^2} Var(X_1+\ldots+X_{100}) =$$

$\boxed{< Var(X_1)}$ ~~[illegible]~~

$$= \frac{1}{100^2}\left(Var X_1 + Var X_2 + \ldots + Var X_{100} \cancel{+ 2Cov(X_1,X_2) + \ldots + 2Cov(X_{99},X_{100})}\right)$$

X_i незав.

$Var(X_i) = \frac{1}{12}$

$$= \frac{1}{100^2} \cdot 100 \cdot \frac{1}{12} = \frac{1/12}{100}$$

$$\bar{X}_{100} \approx N\left(0.5; \frac{1}{1200}\right)$$

$$P(\bar{X}_{100} > 0{,}51) \overset{?}{=} \;\ldots$$

ответ:
→ с ф-цией распр-ия $F(\cdot)$ для $N(0;1)$

на комп.	по табл

стандарт.

$$\approx N(\mu; \sigma^2) \xrightarrow{?} W \approx N(0;1)$$

$$W = \frac{\;\; - E(\;\;)}{\sqrt{Var(\;\;)}}$$

отнять и поделить!

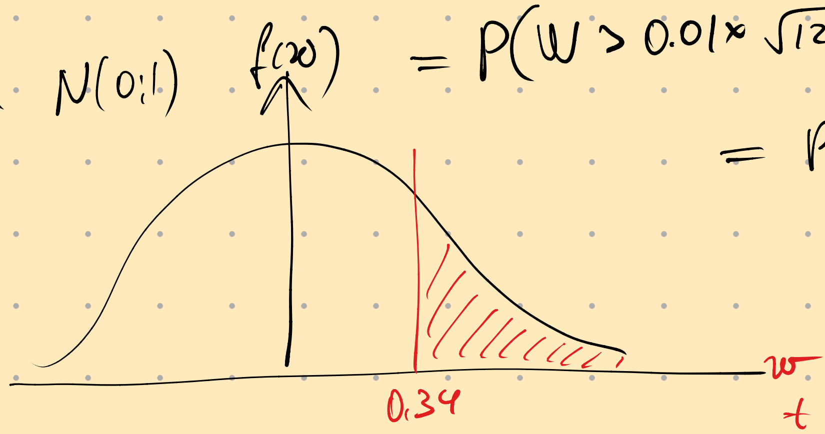
$$\frac{\bar{X}_{100} - \frac{1}{2}}{\sqrt{1/1200}} \approx N(0;1)$$

$$P(\bar{X}_{100} > 0{,}51) = P\left(\frac{\bar{X} - \frac{1}{2}}{\sqrt{1/1200}} > \frac{0.51 - 0.5}{\sqrt{1/1200}}\right) =$$

$$= P(W > 0.01 \times \sqrt{1200}) = P(W > \sqrt{12}/10) =$$

$$= P(W > 0{,}34) = 1 - F(0{,}34) = 0{,}36$$

плотн $N(0;1)$



$$\boxed{F(t) = P(W \le t)}$$

$$F(t) = \int_{-\infty}^{t} f(u)\,du = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{u^2}{2}\right) du$$

scipy → stats → norm → cdf
→ табл

$$E(\bar{X}_{100} \mid \bar{X}_{100} > 0{,}4) \overset{?}{=}$$

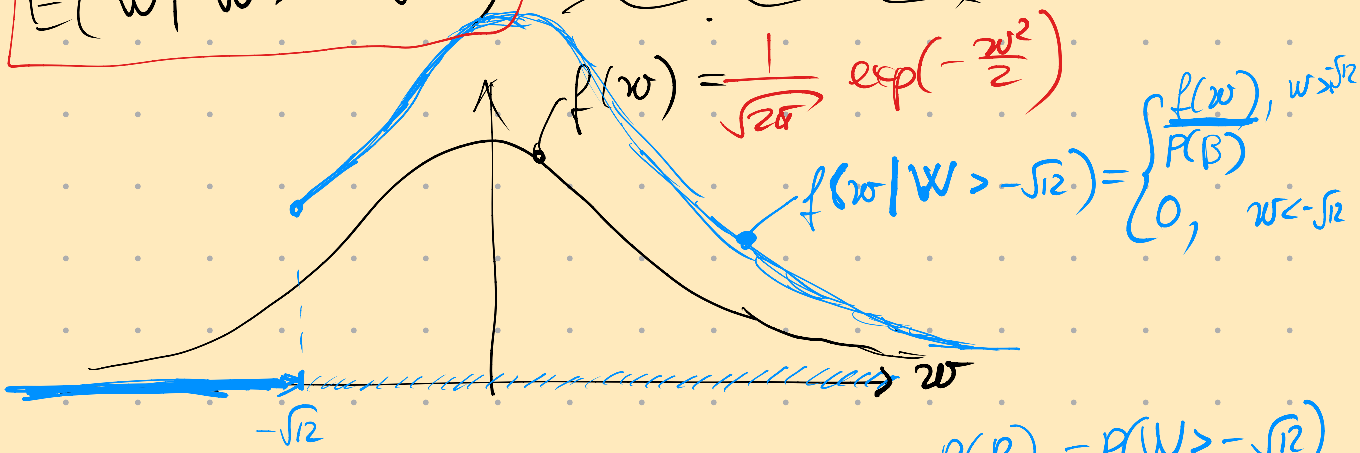
$$W = \frac{\bar{X}_{100} - \frac{1}{2}}{\sqrt{1/1200}}$$

$$\boxed{\bar{X}_{100} = W \cdot \sqrt{1/1200} + \frac{1}{2}}$$

$$= E\left(W \cdot \sqrt{1/1200} + \frac{1}{2} \;\middle|\; W \cdot \sqrt{1/1200} + \frac{1}{2} > 0{,}4\right) =$$

$$= \frac{1}{2} + \sqrt{1/1200} \cdot E\left(W \mid W > -0{,}1 \cdot \sqrt{1200}\right);$$

$$\boxed{E(W \mid W > -\sqrt{12})} = E(W \mid W > -3{,}46) =$$



$$P(B) = P(W > -\sqrt{12})$$

$$= \int_{-\infty}^{\infty} w \cdot f(w \mid W > -\sqrt{12})\, dw = \int_{-\sqrt{12}}^{+\infty} w \cdot \frac{f(w)}{P(W > -\sqrt{12})} \cdot dw \overset{\star}{=}$$

$\boxed{\text{аналог}}$

w	-3	-1	1	2	3
$P(W = w)$	$0{,}1$	$0{,}2$	$0{,}1$	$0{,}1$	$0{,}5$
$P(W = w \mid W > 0)$	0	0	$\frac{0{,}1}{0{,}7}$	$\frac{0{,}1}{0{,}7}$	$\frac{0{,}5}{0{,}7}$

$\boxed{\text{делить на } P(W>0)}$

$P(W>0) = 0{,}7$

$$P(A \mid B) = \frac{P(A \text{ AND } B)}{P(B)}$$

$$E(W) = 0$$

$$E(W \mid B) \overset{\boxed{\text{опр.}}}{=} \frac{E(W \cdot I_B)}{P(B)}$$

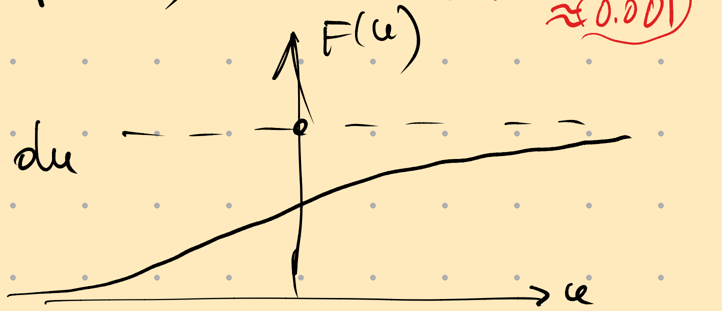
$$E(W \mid W > -\sqrt{12}) =$$

$$\star = \frac{1}{P(W > -\sqrt{12})} \cdot \int_{-\sqrt{12}}^{\infty} w \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-w^2/2\right) dw =$$

$$= \frac{1}{1 - F(-\sqrt{12})} \cdot \left(-\frac{1}{\sqrt{2\pi}} \exp\left(-w^2/2\right)\right)\Bigg|_{-\sqrt{12}}^{+\infty} =$$

$$= \frac{1}{1-F(-\sqrt{12})} \cdot \frac{1}{\sqrt{2\pi}} \exp(-6) = \frac{1}{F(\sqrt{12})} \cdot \frac{1}{\sqrt{2\pi}} \exp(-6) \approx$$
$$\approx 0.001$$

$$F(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp(-u^2/2)\, du$$

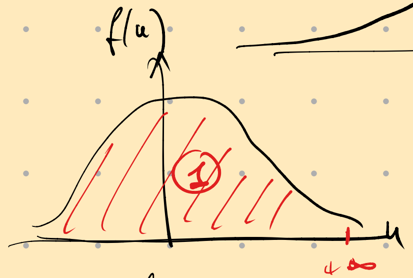


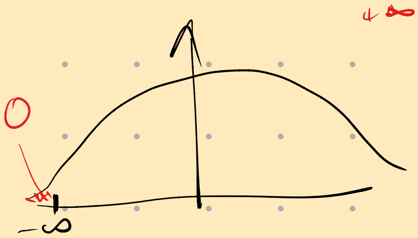
$F(+\infty) = 1$

$F(-\infty) = 0$

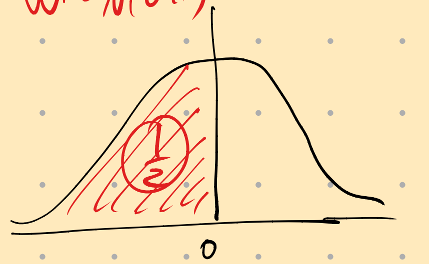
$F(0) = \frac{1}{2}$

$\boxed{F \uparrow \text{мон-но}}$

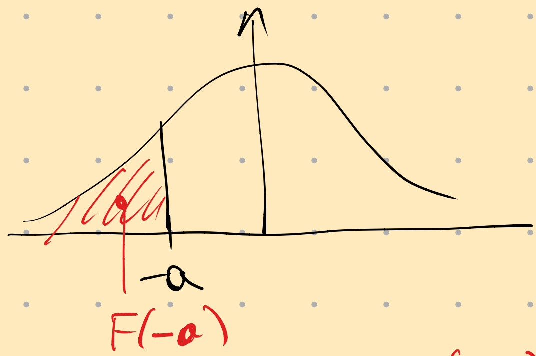


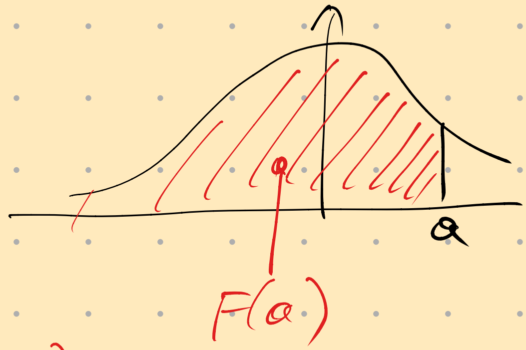


$W \sim N(0;1)$



$P(W \leq 0) = F(0) = \frac{1}{2}$





$F(-a) + F(a) = 1$

20.2 б) $S_n = X_1 + \ldots + X_{100}$

n-велико a? $P(S_n < a) = 0{,}65$

$S_n \approx N(?;?)$ [по Ц.П.Т]

$E(S_n) = E(X_1 + \ldots + X_{100}) = 100 \cdot \frac{1}{2} = 50$

$Var(S_n) = Var(X_1 + \ldots + X_{100}) = 100 \cdot \frac{1}{12} = \frac{100}{12}$

$$W = \frac{S_n - 50}{\sqrt{\frac{100}{12}}} \approx N(0;1)$$

$$P(S_n < a) = P\left(\frac{S_n - 50}{\sqrt{100/12}} < \frac{a-50}{\sqrt{100/12}}\right) = 0.65$$

f = cum. distr.

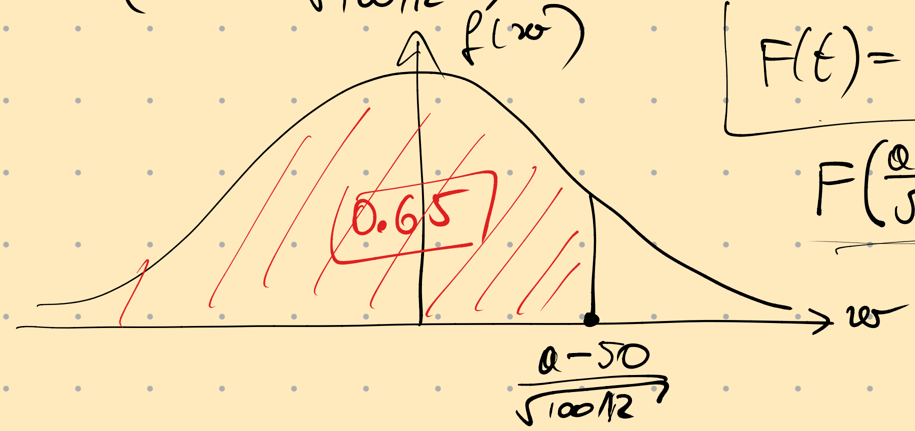
function

$\boxed{\text{ф-ция распр.}}$

$$P\left(W < \frac{a-50}{\sqrt{100/12}}\right) = 0{,}65$$

$$\boxed{F(t) = P(W \le t)}$$

$$F\left(\frac{a-50}{\sqrt{100/12}}\right) = 0{,}65$$



$$\frac{a-50}{\sqrt{100/12}} = F^{-1}(0{,}65)$$

scipy → stats → norm → ppf

ppf = percentile probab. function
= quantile function

таблицы для F

[сотые]

[десят] ← [0,65]

$$F^{-1}(0{,}65) = 0{,}39$$

$$\frac{a-50}{\sqrt{100/12}} = 0{,}39 \Rightarrow \ldots !!$$

[20.5] вер-сть „орла" 0.63. в 100 бросках выборочная
а) какова вер-сть, что ... истинной вер-сти
доля „орлов" будет отлич-ся от
менее, чем на 0.07?

[Пример] 3 раза: [Орел, Орел, Решка]   выб доля орлов $= \boxed{\frac{2}{3}}$ ← С.В.

Y – выб. доля „орлов" в 100 бросках.

$$P(|Y - 0.63| < 0.07) \ ?$$

X_i → 1, если i-ый бросок – „орел",
→ 0, если i-ый бросок – „решка"

$$Y = \frac{X_1 + \ldots + X_{100}}{100} \approx N(?;\ ?)$$

$\boxed{\text{Шаг 1}}$ $E(X_1)$, $Var(X_1)$?

$\boxed{\text{Шаг 2}}$ $E(Y)$? $Var(Y)$?

$\boxed{\text{Шаг 3}}$ $Y \to N(0;1)$ стандарт.

t	0	1
$P(X_i = t)$	0,37	0,63

$\boxed{\text{Шаг 1}}$:

$$E(X_i) = 0 \cdot 0{,}37 + 1 \cdot 0{,}63$$

$$Var(X_i) = \boxed{\text{инт} > 0}$$

$$= E(X_i^2) - (E(X_i))^2 =$$

$$X_i \to 0,\ X_i \to 1 \qquad \boxed{X_i = X_i^2}$$

$$= E(X_i) - (E(X_i))^2 =$$

$$= 0{,}63 - 0{,}63^2 = 0{,}2331$$

$\boxed{\text{Шаг 2}}$

$$Y = \frac{X_1 + \ldots + X_{100}}{100} \qquad E(Y) = \frac{E(X_1) + \ldots + E(X_{100})}{100} =$$

$$= \frac{0{,}63 + 0{,}63 + \ldots + 0{,}63}{100} =$$

$$= 0{,}63$$

$$Var(Y) = Var\left(\frac{X_1 + \ldots + X_{100}}{100}\right) = [\text{уже решали}] = \frac{1}{100^2} \cdot 100 \cdot 0{,}2331$$

$$= \frac{0{,}2331}{100}$$

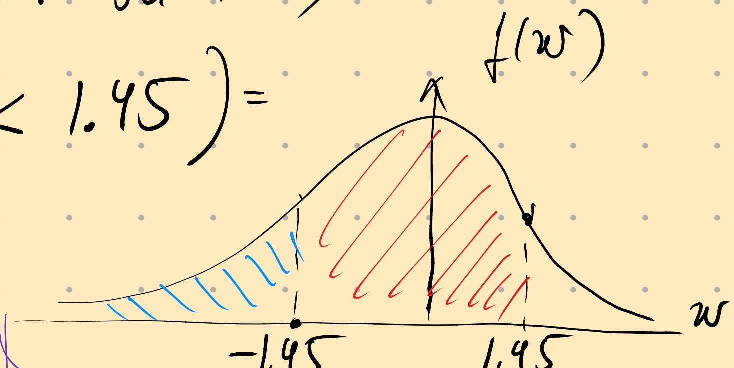
$$Y \approx N\left(0.63;\ \frac{0{,}2331}{100}\right) \xrightarrow{\text{станд.}} W \approx N(0;1)$$

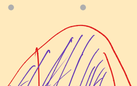
$$W = \frac{Y - 0{,}63}{\sqrt{0{,}2331/100}}$$

$$P(|Y - 0{,}63| < 0{,}07) =$$

$$= P\left(\left|\frac{Y - 0{,}63}{\sqrt{0{,}2331/100}}\right| < \frac{0{,}07}{\sqrt{0{,}2331/100}}\right) =$$

$$= P(|W| < 1.45) =$$





$$= F(1{,}45) - F(-1{,}45) = F(1{,}45) - (1 - F(1{,}45)) =$$

$P(W \le 1{,}45)$ $\quad$ $P(W \le -1{,}45)$

$$= \boxed{2F(1{,}45) - 1} \approx$$

$$\approx 0{,}85$$