Видно / Слышно? !!

16.18

ф. распр.

$$F(x,y) = \begin{cases} 1-\exp(-x)-\exp(-y)+\exp(-x-y), & x,y \ge 0 \\ 0, & \text{иначе} \end{cases}$$

снизить размерн.

а) $P(X>0.5)$?

$= 1 - P(X \le 0.5) =$

$= 1 - (1-\exp(-0.5)) =$

$= \exp(-0.5) \approx 0.61$

$$\boxed{F(x,y) = P(X \le x, Y \le y)}$$

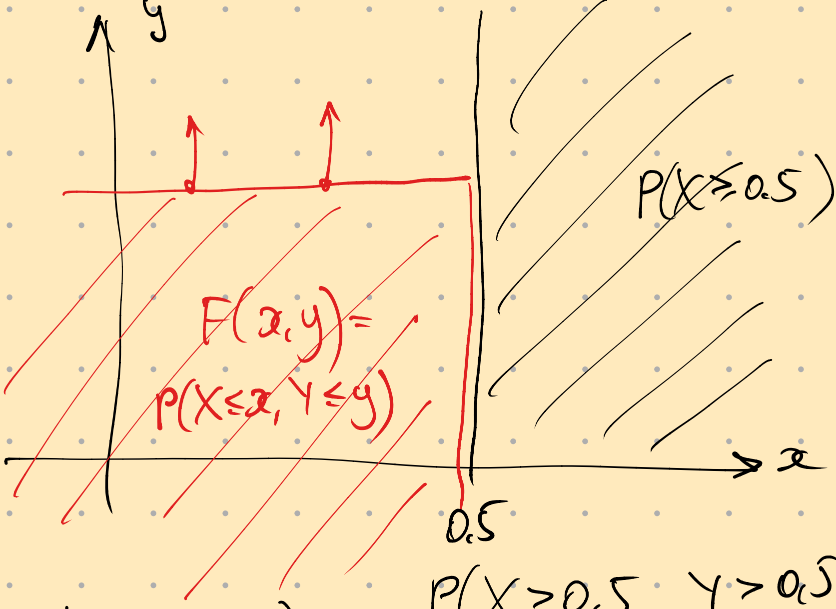
$F_X(x) = P(X \le x) =$

$= P(X \le x, Y \le \infty) =$

$= F(x, +\infty) =$

$= \begin{cases} 1-\exp(-x), & x \ge 0 \\ 0, & \text{иначе} \end{cases}$

рассуждение на диаграмме Эйлера



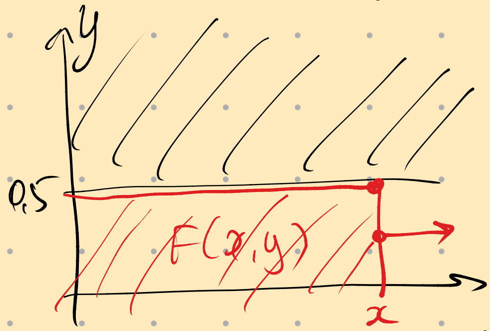
$P(X \ge 0.5) =$

$= 1 - F(0.5, \infty)$

a) $P(X>0.5 \mid Y>0.5) = \dfrac{P(X>0.5,\ Y>0.5)}{P(Y>0.5)}$

$P(Y>0.5) =$

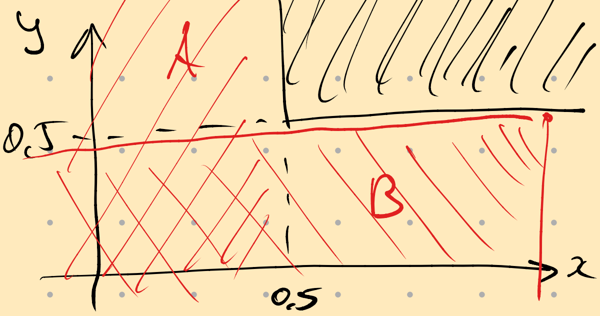
$= 1 - F(+\infty, 0.5)$



$P(X>0.5 \text{ и } Y>0.5) = 1 - \big(P(X \le 0.5, Y \le +\infty) + P(X \le \infty, Y \le 0.5)\big) +$

$+ P(X \le 0.5, Y \le 0.5)$

$$\boxed{\begin{array}{l}\text{если независимы,} \\ \text{то} \\ = P(X>0{,}5)\cdot P(Y>0{,}5)\end{array}}$$



$$= 1 - F(0{,}5,\infty) - F(\infty, 0{,}5) + F(0{,}5, 0{,}5)$$

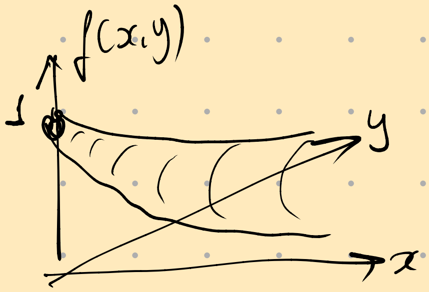
$$P(X>0{,}5,\, Y>0.5) = 1 - (1-\exp(-0{,}5) + 1 - \exp(-0{,}5)) + 1 - 2\exp(-0{,}5) + \exp(-1) =$$

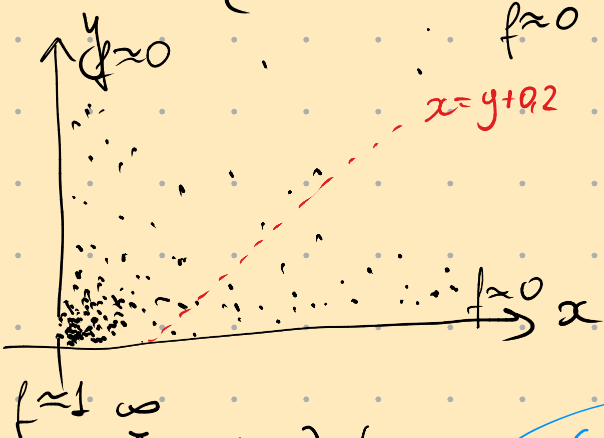
$$= \exp(-1) \approx 0{,}36$$

$$F = \begin{cases} 1-\exp(-x)-\exp(-y)+\exp(-x-y), & x\geq 0,\ y\geq 0 \\ 0, & \text{иначе} \end{cases}$$

$$P(X>0{,}5 \mid Y>0{,}5) = \frac{\exp(-1)}{\exp(-0{,}5)} = \exp(-0{,}5) \approx 0{,}61$$

в) $$f(x,y) = \frac{\partial^2 F}{\partial x \partial y} = \begin{cases} \exp(-x-y), & x\geq 0,\ y\geq 0 \\ 0, & \text{иначе} \end{cases}$$





$$f_X(x) \overset{?}{=} \begin{cases} \int_{-\infty}^{\infty} f(x,y)\,dy \\ \frac{\partial F_X(x)}{\partial x} = \frac{\partial(1-\exp(-x))}{\partial x} = \begin{cases} \exp(-x), & \text{при } x\geq 0 \\ 0, & \text{иначе} \end{cases} \end{cases}$$

$$P(X=x) = \sum_y P(X=x, Y=y)$$

$$P(X=0{,}7) = \int_{0{,}7}^{0{,}7} f(x)\,dx = 0$$

a) $P(X=Y+0{,}2) = 0$

е) $$f(x,y) = \begin{cases} \exp(-x-y), & x\geq 0,\ y\geq 0 \\ 0, & \text{иначе} \end{cases}$$

$$f_x(x) = \begin{cases} \exp(-x), & x\geq 0 \\ 0 & \text{иначе} \end{cases}$$

$$f_Y(y) = \frac{\partial F_Y(y)}{\partial y} = \frac{\partial F_{X,Y}(+\infty, y)}{\partial y} = \begin{cases} \exp(-y) & y \geq 0 \\ 0, & \text{иначе} \end{cases}$$

зависимы ли X и Y ?

Критерий → для любых $F(x,y) = F_X(x) \cdot F_Y(y)$

→ для СВ с плотн : $f(x,y) = f_X(x) \cdot f_Y(y)$

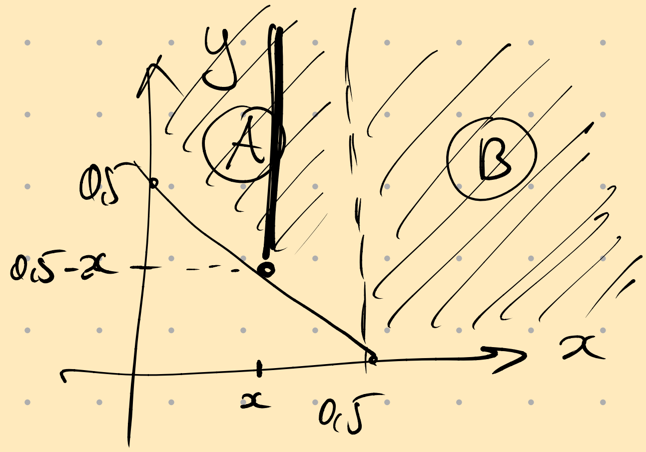
дискр. $P(X=x, Y=y) = P(X=x) \cdot P(Y=y)$

не зависимы!

2) $P(X+Y>0{,}5) =$

$= P(X \leq 0{,}5, X+Y>0{,}5) +$

$+ \underbrace{P(X > 0{,}5)}_{1 - F_X(0{,}5) \approx 0{,}61}$



$$P(X \leq 0{,}5, Y > 0{,}5 - X) = \int_{x=0}^{0{,}5} \int_{y=0{,}5-x}^{+\infty} f(x,y)\, dy\, dx =$$

$$= \int_{x=0}^{0{,}5} \int_{y=0{,}5-x}^{\infty} \exp(-x) \exp(-y)\, dy\, dx =$$

$$= \int_{x=0}^{0{,}5} \exp(-x) \cdot \left(\left. \frac{\exp(-y)}{-1} \right|_{y=0{,}5-x}^{\infty} \right) dx =$$

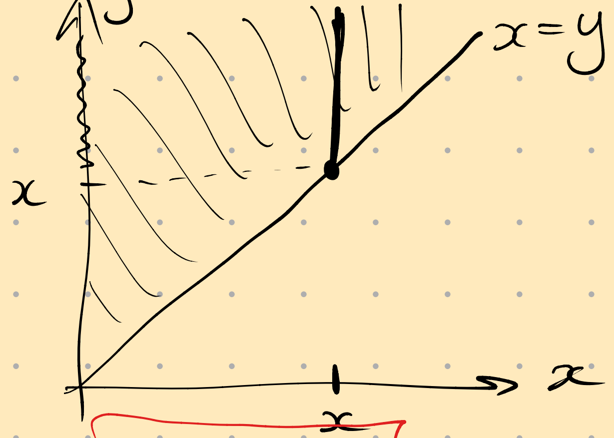
$$= \int_{x=0}^{0{,}5} \exp(-x) \cdot \exp(-0{,}5+x)\, dx =$$

$$= \int_{x=0}^{0{,}5} \exp(-0{,}5)\, dx = 0{,}5 \cdot \exp(-0{,}5) \approx 0{,}30$$

$P(X \le Y) =$

по симметрии $(f(x,y) = f(y,x))$

$\boxed{\frac{1}{2}}$



$$\int_{x=0}^{+\infty} \int_{y=x}^{+\infty} f(x,y)\, dy\, dx = \dots \frac{1}{2}$$

$f(x,y) = \begin{cases} \exp(-x-y) \\ 0 \; \dots \end{cases}$

$$\int_{y=0}^{+\infty} \int_{x=0}^{y} f(x,y)\, dx\, dy = \dots = \frac{1}{2}$$

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g) $E(X) \overset{?}{=}$

$$\int_{-\infty}^{\infty} x \cdot f(x)\, dx = \int_0^{\infty} x \cdot \exp(-x)\, dx = \dots = 1$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot f(x,y)\, dx\, dy = \int_0^{\infty}\int_0^{\infty} x \cdot \exp(-x-y)\, dx\, dy$$

$E(X \cdot Y) \overset{?}{=}$

$E(X^2) \overset{?}{=}$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot y \cdot f(x,y)\, dx\, dy = \int_0^{\infty}\int_0^{\infty} x\, y \exp(-x-y)\, dx\, dy$$

аналог дискр: $E(XY) = \sum_{x,y} x \cdot y \cdot P(X=x, Y=y)$

как повезло: X и Y незав. $\Leftrightarrow$ $E(g(X) \cdot h(Y)) = E(g(X)) \cdot E(h(Y))$

$$E(X \cdot Y) = E(X) \cdot E(Y) = \int_0^{\infty} x \cdot f_X(x)\, dx \cdot \int_0^{\infty} y \cdot f_Y(y)\, dy$$

$= 1 \cdot 1 = 1$

x^2 1 4 9

$$E(X^2) = \begin{cases} \int\limits_{-\infty}^{\infty} x^2 f_X(x)\,dx = \ldots = 1 \\ \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} x^2 \cdot f(x,y)\,dx\,dy = \ldots \end{cases}$$

x	1	2	3
вер	0,3	0,2	0,5

$$E(X^2) = \sum_x x^2 \cdot P(X=x) = \sum_q q \cdot P(Q=q)$$

$Q = X^2$

$$\mathrm{Cov}(X,Y) = E(X \cdot Y) - E(X)\cdot E(Y) = \ldots$$

$$\mathrm{Corr}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\cdot \mathrm{Var}(Y)}} = \ldots$$

$$\mathrm{Var}(X) = E(X^2) - (E(X))^2 = \ldots \qquad \mathrm{Var}(X) = \mathrm{Cov}(X,X)$$

16.23 а

Пусть $f(x,y) = \begin{cases} 4xy, & \text{если } x,y \in [0;1] \\ 0, & \text{иначе.} \end{cases}$

$$\begin{pmatrix} X \\ Y \end{pmatrix} \to \begin{pmatrix} S \\ R \end{pmatrix} \qquad S = X+Y, \quad R = X/(X+Y) \qquad f_{S,R}(s,r)\ ?$$

$$f_{S,R}(s,r) = f_{X,Y}(?,?) \cdot |\det J| \qquad J = \frac{\partial\, old}{\partial\, new}$$

$$\begin{cases} S = X+Y \\ R = \dfrac{X}{X+Y} \end{cases} \overset{?}{\to} \begin{pmatrix} X \\ Y \end{pmatrix}?$$

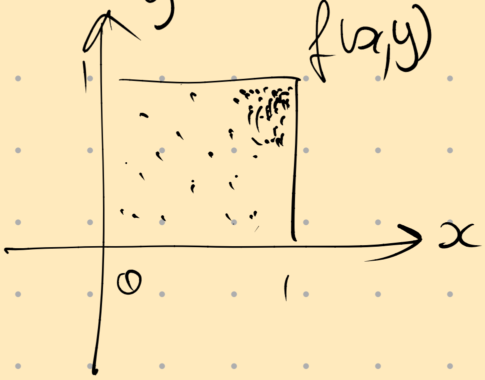
$$X = R\cdot S$$
$$Y = S - RS$$

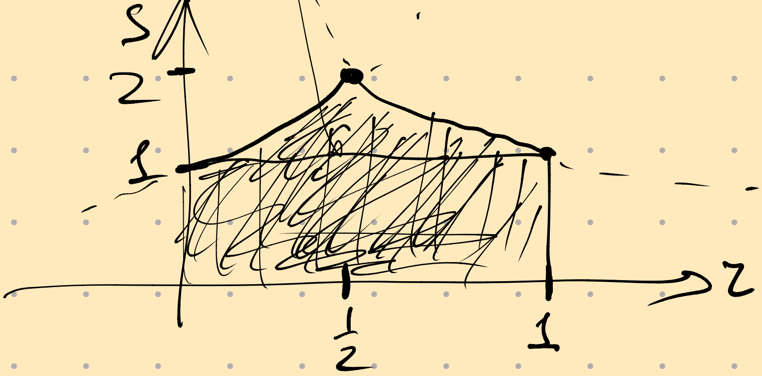
$$J = \begin{array}{c} \text{по } s \quad \text{по } r \\ \begin{pmatrix} R & S \\ 1-R & -S \end{pmatrix} \end{array} \begin{array}{l} \text{от } X \\ \text{от } Y \end{array}$$

$$\det J = -RS - S\cdot(1-R) = -S$$

$$f_{S,R}(s,r) = \begin{cases} 4\cdot(rs)\cdot(s-rs)\cdot s \\ 0, & \text{иначе.} \end{cases}$$

$rs \in [0;1]$
$s - rs \in [0;1]$

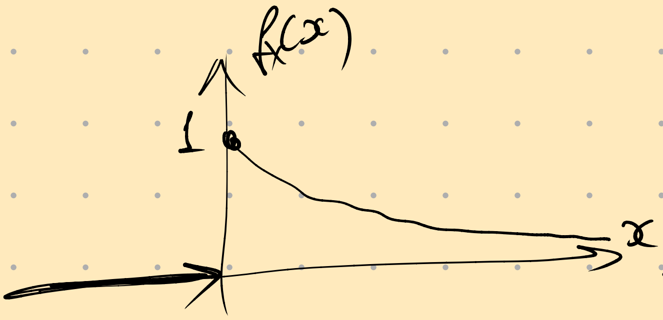


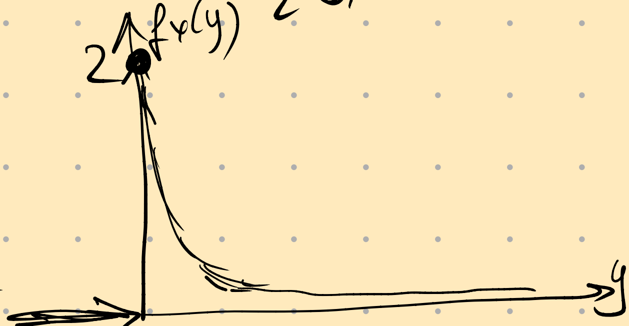


Упр. СВ X и Y независимы

$X \sim Expo(\lambda = 1)$ $\qquad$ $Y \sim Expo(\lambda = 2)$

$$f_X(x) = \begin{cases} 1 \cdot \exp(-1 \cdot x), & x \geq 0 \\ 0, & \text{иначе} \end{cases} \qquad f_Y(y) = \begin{cases} 2\exp(-2x), & x \geq 0 \\ 0, & \text{иначе.} \end{cases}$$





$$S = X + Y \qquad \underline{f_S(s)\ ?}$$

аналогия с дискр.

$$P(S = s) = \sum_x P(X = x) \cdot P(Y = s - x)$$

формула свёртки

$$f_S(s) = \int_{-\infty}^{\infty} f_X(x) \cdot f_Y(s-x)\, dx = \qquad \boxed{x \geq 0 \quad s - x \geq 0}$$

$$= \int_{x=0}^{x=s} f_X(x) \cdot f_Y(s-x)\, dx =$$

$$= \int_0^s 1 \cdot \exp(-x) \cdot 2 \cdot \exp(-2(s-x))\, dx =$$

$$= 2\int_0^s \exp(-2s) \cdot \exp(x)\, dx =$$

$$= 2\exp(-2s) \cdot \exp(x)\Big|_{x=0}^{s} =$$

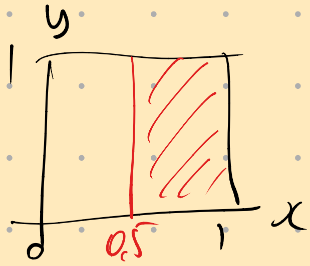
$$= 2\exp(-2s) \cdot (\exp(s) - 1) =$$

$$f_S(s) = \begin{cases} 2\exp(-s) - 2\exp(-2s), & s \geq 0 \\ s < 0 \end{cases}$$

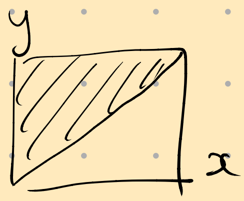
16.17

$$f(x,y) = \begin{cases} 4xy, & x \in [0;1],\ y \in [0;1] \\ 0, & \text{иначе.} \end{cases}$$

$$P(X > 0{,}5) = \int_{y=0}^{1} \int_{x=0{,}5}^{1} 4xy\,dx\,dy = \ldots$$



$$P(X = Y + 0{,}5) = 0$$



$$P(X \leq Y) = \begin{cases} \to \displaystyle\int_{y=0}^{1} \int_{x=0}^{y} 4xy\,dx\,dy \\ \to \text{по симметрии } x \Leftrightarrow y = P(Y \leq X) = \frac{1}{2} \end{cases}$$