Home assignment 1

Deadline: 2024-09-16

1. Each day Elon Musk solves econometrics problems and creates posts in X. Let y_i be the number of solved problems and x_i be the number of posts in X. You have 3 observations: $x_1 = 2$, $y_1 = 5$, $x_2 = 1$, $y_2 = 10$, $x_3 = 3$, $y_3 = 4$.

    (a) Find $\hat{\beta}$ if fitted values are given by $y_i = \hat{\beta} x_i$.

    (b) Find $\hat{\beta}_0$ and $\hat{\beta}_1$ if fitted values are given by $y_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$.

    (c) Find $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\beta}_2$ if fitted values are given by $y_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + \hat{\beta}_2 x_i^2$.

2. Simplify as much as possible the following expressions:

$$A = \sum_{i=1}^{n}(x_i - \bar{x})\bar{x}, \quad B = A = \sum_{i=1}^{n}(x_i - \bar{x})\bar{y}, \quad C = \sum(x_i - \bar{x})^2 + n\bar{x}^2.$$

3. Consider simple regression model with $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$. You have n observations (x_1, y_1), ..., (x_n, y_n) and you estimate $\hat{\beta}_0$ and $\hat{\beta}_1$ using OLS.

   What will happen with $\hat{\beta}_0$ and $\hat{\beta}_1$ in each of the following cases?

    (a) You add n more observations given by $(x_{n+i} = -x_i, y_{n+i} = y_i)$ for $i = 1, 2, ..., n$ to the original dataset.

    (b) You add one new observation $(y_{n+1} = \bar{y}, x_{n+1} = \bar{x})$ to the original dataset.

    (c) You copy every observation from the original dataset twice.

   Hint: you may start by guessing the answer with an experiment, but the proof is required :)