Hypothesis testing

Task1 $\underset{y_i}{grades_i} = \beta_0 + \beta_1 \underset{x_i}{homeworks_i} + \beta_2 \underset{z_i}{seminars_i} + u_i$

all missed

$n=5$ $\quad y_i = \begin{pmatrix} 9 \\ 8 \\ 10 \\ 6 \\ 7 \end{pmatrix} \quad X = \begin{pmatrix} 1 & 4 & 7 \\ 1 & 3 & 5 \\ 1 & 6 & 8 \\ 1 & 2 & 4 \\ 1 & 4 & 6 \end{pmatrix}$

1) $E(u_i|X)=0$

2). $Var(u|X)=\sigma_u^2 I_n$

3) $(X^TX)^{-1}$ existst

$$\hat{\beta} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = (X^TX)^{-1}X^Ty$$

$$(X^TX)^{-1} = \begin{pmatrix} 7,5 & 2,3 & -2,7 \\ 2,3 & 1,4 & -1,3 \\ -2,7 & -1,3 & 1,3 \end{pmatrix} \rightarrow \hat{\beta} = \begin{pmatrix} 2,37 \\ -0,14 \\ 1,02 \end{pmatrix}$$

$$\star \begin{cases} \widehat{grades_1} = \hat{\beta}_0 + \hat{\beta}_1 hw_1 + \hat{\beta}_2 sem_1 \\ \widehat{grades_2} = \hat{\beta}_0 + \hat{\beta}_1 (1+hw_1) + \hat{\beta}_2 sem_1 \end{cases}$$

$\Delta \widehat{grades} = \hat{\beta}_1 \cdot 1$ $\quad \hat{\beta}_1$: if $hw_1 \uparrow$ by 1unit c.p. (all other regressors are constant) then $\hat{\beta}_1$ points

---

$\hat{\beta} + \sqrt{Var(\hat{\beta})}$

$$Var(\hat{\beta}|X) = \frac{Var(u|X)}{(X^TX)} = Var(\underset{\sigma_u^2 I_n}{u|X}) \cdot (X^TX)^{-1} = \sigma_u^2 (X^TX)^{-1}$$

usually uknown

$$\widehat{Var}(\hat{\beta}|X) = \hat{\sigma}_{\hat{u}}^2 (X^TX)^{-1}$$

SER $\rightarrow$ st. error of regress

$$\boxed{\hat{\sigma}^2_{\hat{u}} = \frac{1}{n-k}\sum \hat{u}_i^2} \equiv \frac{1}{n-k}\sum(\hat{u}_i - \bar{\hat{u}})^2$$

unbiased

$$= \frac{SSRes}{n-k} = \frac{SSR}{n-3}$$

if $\exists$ const, then $\sum \hat{u}_i = 0$

$y_i - \hat{y}_i$

$$\widehat{Var}(\hat{\beta}|X) = \begin{bmatrix} 7 & 2,2 & -2,5 \\ 2,2 & 1,3 & -1,2 \\ -2,5 & -1,2 & 1,2 \end{bmatrix}$$

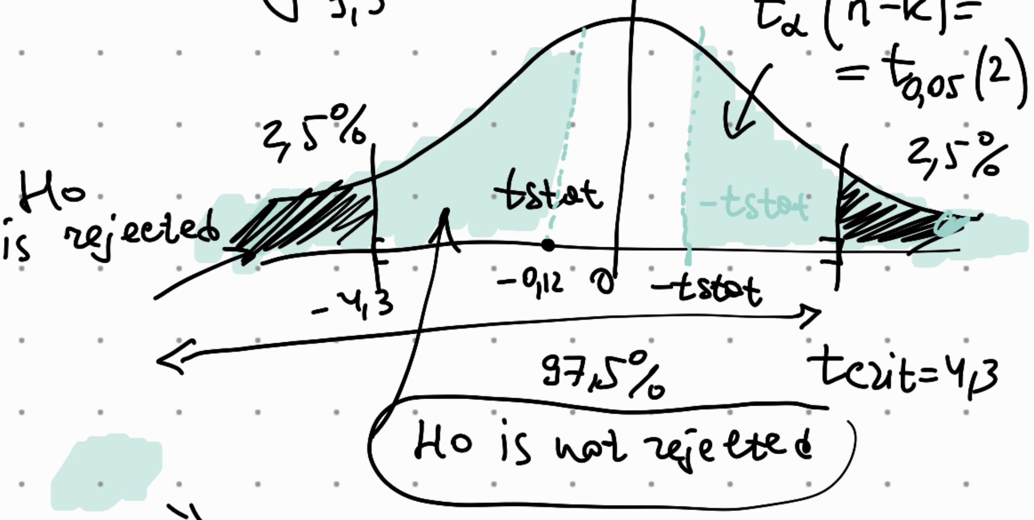
$$\begin{bmatrix} var(\hat{\beta}_0) & cov(\hat{\beta}_0,\hat{\beta}_1) & cov(\hat{\beta}_0,\hat{\beta}_2) \\ \cdot & var(\hat{\beta}_1) & cov(\hat{\beta}_1,\hat{\beta}_2) \\ \cdot & \cdot & var(\hat{\beta}_2) \end{bmatrix}$$

$$\begin{cases} H_0: & \beta_1 = 0 \\ H_1: & \beta_1 \neq 0 \end{cases}$$

1) $t_{stat} = \dfrac{\hat{\beta}_1 - 0}{SE(\hat{\beta}_1)} = \dfrac{\hat{\beta}_1}{\sqrt{\widehat{Var}(\hat{\beta}_1|X)}} =$

$$= \frac{-0,14}{\sqrt{1,3}} = -0,12$$

$t_\alpha(n-k) = t_{0,05}(2)$



p-value $\quad \beta_1 = 0$

p-value $= 90\% > \alpha = 5\%$

Ho is not rejected.

$$CI_{95\%} = [\hat{\beta}_1 - t_{crit,\alpha} \cdot SE(\hat{\beta}_1) \;;$$

$$\hat{\beta}_1 + t_{crit,\alpha} \cdot SE(\hat{\beta}_1)] =$$

$$= [\,0,14 - 4,3\sqrt{1,3}\;;$$

$$= \left[-0{,}14 - 4{,}3\cdot\sqrt{1{,}3}\ ;\ -0{,}14 + 4{,}3\cdot\sqrt{1{,}3}\right] =$$

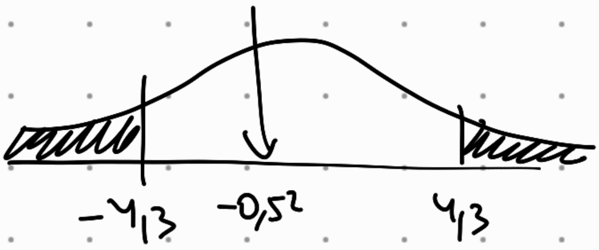
$$= [-5\ ;\ 4{,}9]$$

$H_0:\ \beta_1 = \beta_2$ $\Rightarrow$ $H_0:\ \beta_1 - \beta_2 = 0$

$H_1:\ \beta_1 \neq \beta_2$ $\quad$ $H_1:\ \beta_1 - \beta_2 \neq 0$

$$t_{stat} = \frac{\hat\beta_1 - \hat\beta_2 - 0}{SE(\hat\beta_1 - \hat\beta_2)} = \frac{\hat\beta_1 - \hat\beta_2}{\sqrt{\widehat{Var}(\hat\beta_1 - \hat\beta_2)}} = \frac{\hat\beta_1 - \hat\beta_2}{\sqrt{\widehat{Var}\,\hat\beta_1 + \widehat{Var}\,\hat\beta_2 - 2\,\widehat{cov}(\hat\beta_1, \hat\beta_2)}} = \frac{-0{,}14 - 1}{\sqrt{1{,}4 + 1{,}3 - + 2\cdot 1{,}3}} =$$

$$= -0{,}52$$



H_0 is not rej.

$\beta_1 = \beta_2$

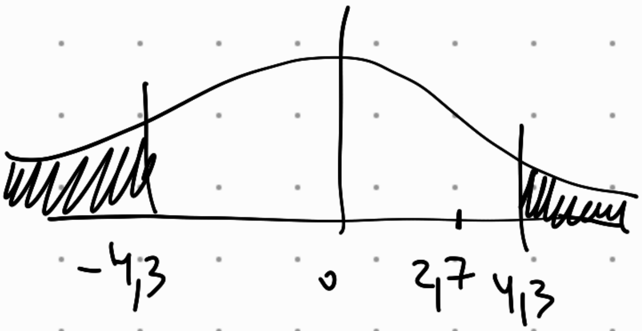
p-value = 65% > 5%

$H_0:\ \beta_1 + \beta_2 = 0$

$H_1:\ \beta_1 + \beta_2 \neq 0$

$$t_{stat} = \frac{\hat\beta_1 + \hat\beta_2}{\sqrt{\widehat{Var}(\hat\beta_1 + \hat\beta_2)}} =$$

$$= \frac{\hat\beta_1 + \hat\beta_2}{\sqrt{\widehat{Var}\,\hat\beta_1 + \widehat{Var}\,\hat\beta_2 + 2\,\widehat{cov}(\hat\beta_1, \hat\beta_2)}} =$$



$$= 2{,}7$$

H_0 is not rejected

p-value = 11% > 5%