General notes

Exam will contain 6 problems. All the problems have equal weights. Exam duration is 120 minutes. Closed book, one A4 cheatsheet and simple calculator are allowed.

Demo variant «Infallibilitas»

1. The buses arrive at the bus stop according to the Poisson process. That means actually that my waiting times x_t for the bus every day are independent and exponentially distributed with unknown rate λ. Each day I arrive at the bus stop and I wait for no more than 10 minutes. If there is no bus during 10 minutes I leave the stop and go by foot.

   Here are my timings at the bus stop: $y = (5, 10, 6, 10, 7)$.

   (a) Estimate unknown λ using maximum likelihood.

   (b) Estimate the probability that I go by foot by maximum likelihood.

2. Pandas receive job proposal with wage $y_i^* = x_i^T\beta + u_i$, where x_i^T are characteristics of a panda, β — unknown parameters and u_i is normally distributed $u_i \sim \mathcal{N}(0; \sigma^2)$.

   Pandas never agree to work for less than $2 \cdot 10^5$ roubles per month. If the proposed wage is less than acceptable pandas prefer to attend econometrics classes.

   (a) What is the probability that a panda will accept a job offer?

   (b) Calculate the expected wage of panda that has accepted a job offer.

   Your answer may contain x_i, β, σ^2, standard normal density or cumulative distribution function.

3. Classify the processes x_t and y_t as stationary, trend stationary, difference stationary, integrated of order d. Here (u_t) is a white noise independent of (x_s) and (y_s) for $s < t$.

   (a) $x_t = t^2 + 6t + u_t + u_{2025}$;

   (b) y_t is described by the system

$$\begin{cases} y_t = \ell_{t-1} + b_{t-1} + u_t \\ b_t = b_{t-1} + 0.1u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + 0.4u_t \\ b_0 = \text{const}, \ell_0 = \text{const}. \end{cases}$$

4. Consider the system

$$\begin{cases} (x_t) \sim I(1), (y_t) \sim I(1) \\ y_t = 2 + 0.1y_{t-1} + 0.2x_t - 0.1x_{t-1} + u_t. \end{cases}$$

   Here (u_t) is a white noise independent of (x_s) and (y_s) for $s < t$.

   [10] Find the cointegration equation between x_t and y_t.

5. Consider the following equation

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 0.6 & 0.1 \\ 0.1 & 0.6 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} v_t \\ w_t \end{pmatrix}.$$

Here $u_t = (v_t, w_t)$ is a two-dimensional white noise with $\mathbb{E}(u_t) = 0$ and $\mathbb{V}\mathrm{ar}(u_t) = \begin{pmatrix} 1 & 0.2 \\ 0.2 & 1 \end{pmatrix}$.

(a) Does this equation has a stationary solution that is vector moving average with respect to (u_t)?

(b) Find $\mathbb{E}((x_t, y_t))$ and $\mathbb{V}\mathrm{ar}((x_t, y_t))$ for the stationary solution.

6. Something from LSE external exam;