

## Home assignment 1

Deadline: 2024-09-16

1. Each day Elon Musk solves econometrics problems and creates posts in X. Let  $y_i$  be the number of solved problems and  $x_i$  be the number of posts in X. You have 3 observations:  $x_1 = 2$ ,  $y_1 = 5$ ,  $x_2 = 1$ ,  $y_2 = 10$ ,  $x_3 = 3$ ,  $y_3 = 4$ .
  - (a) Find  $\hat{\beta}$  if fitted values are given by  $y_i = \hat{\beta}x_i$ .
  - (b) Find  $\hat{\beta}_0$  and  $\hat{\beta}_1$  if fitted values are given by  $y_i = \hat{\beta}_0 + \hat{\beta}_1x_i$ .
  - (c) Find  $\hat{\beta}_0$ ,  $\hat{\beta}_1$  and  $\hat{\beta}_2$  if fitted values are given by  $y_i = \hat{\beta}_0 + \hat{\beta}_1x_i + \hat{\beta}_2x_i^2$ .
2. Simplify as much as possible the following expressions:

$$A = \sum_{i=1}^n (x_i - \bar{x})\bar{x}, \quad B = \sum_{i=1}^n (x_i - \bar{x})\bar{y}, \quad C = \sum_{i=1}^n (x_i - \bar{x})^2 + n\bar{x}^2.$$

3. Consider simple regression model with  $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1x_i$ . You have  $n$  observations  $(x_1, y_1), \dots, (x_n, y_n)$  and you estimate  $\hat{\beta}_0$  and  $\hat{\beta}_1$  using OLS.

What will happen with  $\hat{\beta}_0$  and  $\hat{\beta}_1$  in each of the following cases?

- (a) You add  $n$  more observations given by  $(x_{n+i} = -x_i, y_{n+i} = y_i)$  for  $i = 1, 2, \dots, n$  to the original dataset.
- (b) You add one new observation  $(y_{n+1} = \bar{y}, x_{n+1} = \bar{x})$  to the original dataset.
- (c) You copy every observation from the original dataset twice.

Hint: you may start by guessing the answer with an experiment, but the proof is required :)

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