Task 1 $\quad \log w_i = \alpha_1 + \alpha_2 S_i + \varepsilon_i$

$\hat{\alpha}_2 > 0$ & signif

- omitted relevant z_i

problem: $cov(S_i, z_i) \neq 0$

$cov(\log w_i, z_i) \neq 0$

z_i is connected with w_i

z_i: country$_i$; exp$_i$; abilities$_i$

- $\to \log w_i = \alpha_1 + \alpha_2 S_i + \alpha_3 C_i + u_i$ $\quad E(u_i \mid S_i, C_i) = 0$

$\Downarrow$

$cov(u_i, S_i) = 0$

$cov(u_i, C_i) = 0$

$\to \log w_i = \alpha_1 + \alpha_2 S_i + \varepsilon_i$

$$\hat{\alpha}_2 = \frac{\sum(\log w_i - \overline{\log w})(S_i - \bar{S})}{\sum(S_i - \bar{S})^2} = \frac{\$ cov(\log w_i, S_i)}{\$ var(S_i)} =$$

$$= \frac{\text{sample } cov(\log w_i, S_i)}{\text{sample } var(S_i)} \xrightarrow[p]{WLN} \frac{cov(\log w_i, S_i)}{var(S_i)}$$

population cov, var

$$E(\hat{\alpha}_2) = \frac{cov(\log w_i, S_i)}{var S_i} = \frac{cov(\alpha_1 + \alpha_2 S_i + \varepsilon_i, S_i)}{var(S_i)} =$$

$$= \frac{cov(\alpha_1, S_i) + \alpha_2 cov(S_i, S_i) + cov(\varepsilon_i, S_i)}{var(S_i)} =$$

$$= \alpha_2 + \frac{cov(\varepsilon_i, S_i)}{var(S_i)} = \alpha_2 + \frac{cov(u_i + \alpha_3 \cdot C_i, S_i)}{var(S_i)} =$$

$$= \alpha_2 + \underbrace{\frac{cov(u_i, S_i)}{var(S_i)}}_{=0} + \alpha_3 \frac{cov(C_i, S_i)}{var(S_i)} = \alpha_2 + \text{bias}_{>0}$$

$cov(C_i, S_i) > 0$,

$\alpha_3 > 0$

- $\hat{\alpha}_2 > 0$ $\quad$ signif

$\hat{\alpha}_2$: bias

signif: not useful

Task 2

$$\widehat{\ln price} = -8{,}43 + 1{,}33 \ln A_i - 0{,}17 AR_i$$
$$\qquad (0{,}6) \quad (0{,}09) \quad (0{,}13)$$

$n = 430$, $R^2 = 0{,}336$ $\quad SE(\hat\beta_2)$

a) Test joint sign.

$$\ln price_i = \beta_0 + \beta_1 \ln A_i + \beta_2 AR_i + u_i \rightarrow SSR_{UR}$$

$H_0: \beta_1 = \beta_2 = 0$

$H_1:$ otherwise $\rightarrow$ tstat

Fstat

$\begin{cases} \beta_1 = 0 \\ \beta_2 = 0 \end{cases}$ $H_0: R^2_{UR} = R^2_R$

$H_1: R^2_{UR} > R^2_R$

$H_0: \beta_1 = \beta_2 \rightarrow \beta_1 - \beta_2 = 0$

$$tstat = \frac{\hat\beta_1 - \hat\beta_2}{SE(\hat\beta_1 - \hat\beta_2)} = \frac{\hat\beta_1 - \hat\beta_2}{\sqrt{\widehat{Var}(\hat\beta_1 - \hat\beta_2)}} = \frac{\hat\beta_1 - \hat\beta_2}{\sqrt{\widehat{Var}\,\hat\beta_1 + \widehat{Var}\,\hat\beta_2 - 2\widehat{cov}(\hat\beta_1, \hat\beta_2)}}$$

$$\ln price_i = \beta_0 + v_i \rightarrow SSR_R =$$

$$\boxed{Fstat = \frac{(SSR_R - SSR_{UR}) \setminus 2}{SSR_{UR} \setminus 430 - 3}}$$

$\rightarrow q$ # restri.

$n-k$

$$\left(R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{\sum \hat u_i^2}{\sum (y_i - \bar y)^2}\right. \Rightarrow SSR = (1 - R^2) \cdot SST \Rightarrow$$

$$\Rightarrow Fstat = \frac{1 - R^2_R - (1 - R^2_{UR})}{1 - R^2_{UR}} \cdot \frac{427}{2} = \boxed{\frac{R^2_{UR} - R^2_R}{1 - R^2_{UR}} \cdot \frac{427}{2}}$$

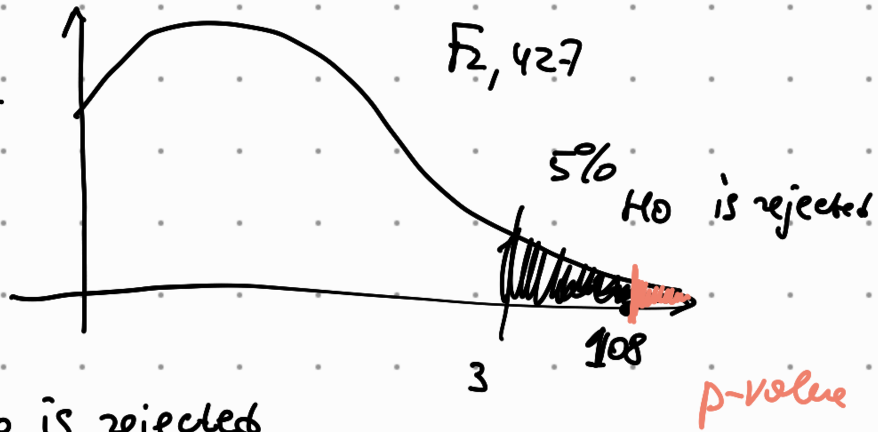
- $$Fstat = \frac{0{,}336 - R^2_R}{1 - 0{,}336} \cdot \frac{427}{2} = \frac{0{,}336}{1 - 0{,}336} \cdot \frac{427}{2} = 108$$

$$\ln price_i = \beta_0 + v_i \Rightarrow \hat\beta_0 = \bar y - \hat\beta_1 \bar x \Big|_{\hat\beta_1 = 0} = \bar y \Rightarrow \hat y = \bar y$$

$$\Rightarrow \quad SSE \quad \sum(\hat y_i - \bar y)^2 \quad \sum(\bar y - \bar y)^2$$

$$R^2_R = \frac{SSR}{SST} = \frac{0}{\sum(y_i-\bar{y})^2} = \frac{0}{\sum(y_i-\bar{y})^2} = 0$$

- $F_{stat} \overset{H_0}{\sim} F_{2,427}$ asum. GM + normal u_i + asy.



$108 > 3 = F_{crit,5\%} \Rightarrow H_0$ is rejected

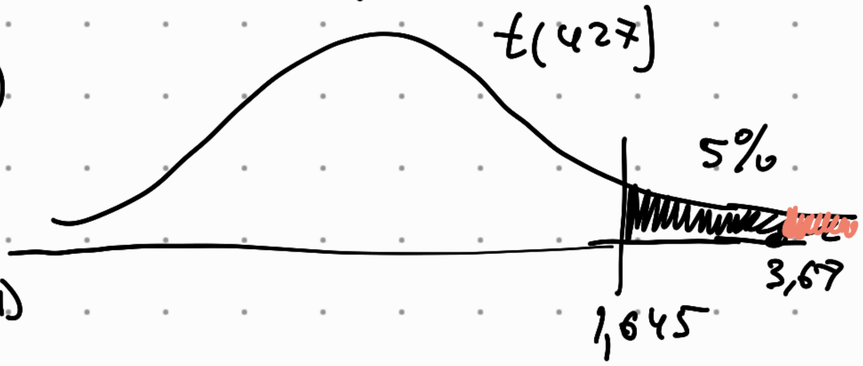
p-value $< \alpha = 5\%$

p-value $\approx 0{,}000$

b) $H_0: \beta_1 = 1$ $\quad t_{stat} = \dfrac{\hat{\beta}_1 - 1}{SE(\hat{\beta})} = \dfrac{1{,}33-1}{0{,}09} = 3{,}67$

$H_1: \beta_1 > 1$

$t_{stat} \overset{H_0}{\sim} t(427)$ ass GM-T + normal u or + asy $\sim N(0,1)$



$3{,}67 > 1{,}645 \Rightarrow H_0$ is rejected.

pvalue $< 5\%$

p-value $\approx 0{,}000$

Task3) $y_i = \beta_0 + \beta_1 x_i + u_i$

a) without restrictions:

$$\sum \hat{u}_i^2 = \sum(y_i - \hat{y})^2 = \sum\Big(y_i - \underbrace{b_0 - b_1 x_i}_{\hat{y}}\Big)^2 \rightarrow \min_{b_0, b_1}$$

with restrictions $\beta_1 = 1$

$$\sum \tilde{u}_i^2 = \sum(y_i - \tilde{y}_i)^2 = \sum(y_i - b_0 - x_i)^2 \rightarrow \min$$

$$-2\sum(y_i-\tilde\beta_0-x_i)=0 \Rightarrow \sum y_i - n\tilde\beta_0 - \sum x_i = 0$$

$$\bar y - \tilde\beta_0 - \bar x = 0$$

$$\boxed{\tilde\beta_0 = \bar y - \bar x}$$

b) $E(\tilde\beta_0) = E(\bar y - \bar x) = E\left(\frac{\sum y_i}{n}\right) - \bar x =$

$$= E\left(\frac{\sum(\beta_0+\beta_1 x_i + u_i)}{n}\right) - \bar x = E\left(\frac{n\beta_0+\beta_1\sum x_i+\sum u_i}{n}\right)-\bar x =$$

$$= \beta_0 + \beta_1\bar x + E\left(\frac{\sum u_i}{n}\right) - \bar x = \beta_0 + (\beta_1-1)\bar x + \underset{\substack{\| \\ 0}}{E u_i} =$$

$u_i \sim iid$ $\qquad$ ↑ assump

$$\Rightarrow E(\tilde\beta_0) = \beta_0 + (\beta_1-1)\bar x \Big|_{\beta_1=1} = \beta_0 \Rightarrow$$

$\Rightarrow \tilde\beta_0$ unbiased

$$E(\tilde\beta_0) = \beta_0 + \underbrace{(\beta_1-1)\bar x}_{\text{bias}}\Big|_{\bar x=0} = \beta_0 \to \text{unbiased}$$

Lemma: If $E(\hat\beta|x)=\beta$ (unbiased) and $Var(\hat\beta|x)\xrightarrow[n\to\infty]{}0 \Rightarrow$

$\Rightarrow \hat\beta \xrightarrow[p]{} \beta$. (consistent)

But: $\bar y \xrightarrow[p]{WLN} Ey_i = \mu_y$

$$\boxed{(\bar y)^2 \xrightarrow[p]{} \mu_y^2}$$

$$E(\bar y^2) = Var\,\bar y + E^2\bar y = \frac{\sigma_y^2}{n} + \mu_y^2 \neq \underline{\mu_y^2}$$

biased est. of μ_y^2

$\underline{\text{asym.}}$ unbiased $\left(\text{bias} = \frac{\sigma_y^2}{n} \to 0 \right)$

$$E(\hat{\beta}_1 | X) = \frac{s\,cov(y_i, x_i)}{s\,var(x_i)} \xrightarrow[p]{} \frac{cov(y_i, x_i)}{var(x_i)} =$$

$$\Rightarrow \frac{cov(y_i, x_i)}{var(x_i)} = \frac{cov(\beta_0 + \beta_1 x_i + u_i, x_i)}{var(x_i)} = \beta_1$$

$$\boxed{\hat{\beta}_1 \xrightarrow[p]{} \beta_1}$$