Short rules: one A4 cheatsheet is allowed, calculators are ok, offline, 120 minutes.

1. Consider approximating a hedgehog using two geometric shapes: a cone for the nose and a half-sphere for the rest of the body.

   Using a large dataset you have estimated the diameter $\hat{D} = 200$ mm and the length of the nose $\hat{h} = 25$ mm with $se(\hat{D}) = 1$ mm and $se(\hat{h}) = 5$ mm. The latter is higher since the hedgehog might bite. The estimators $\hat{D}$ and $\hat{h}$ are uncorrelated.

   

   The sphere volume is $V_s = \pi D^3/6$. The cone volume is $V_c = \pi D^2 h/12$.

   (a) [5] Find the standard error of the volume of the half-sphere hedgehog part using the delta method.

   (b) [5] Find the standard error of the hedgehog volume using the delta method.

2. The price level p_t and production q_t are endogeneous variables. Taxes a_t and income b_t are exogeneous. All variables are centered. The structural form is given by the system

$$\begin{cases} p_t = \alpha_1 q_t + \alpha_2 a_t + u_{1t} \\ q_t = \beta_1 p_t + \beta_2 b_t + u_{2t}. \end{cases}$$

   The estimates of the reduced form via equation-by-equation ols:

$$\begin{cases} p_t = 2a_t + 3b_t \\ q_t = -3a_t + 2b_t. \end{cases}$$

   (a) [6] Recover the estimates of the coefficients in structural form.

   (b) [4] Describe how you will estimate the standard errors of the structural form.

3. The price level p_t, production q_t and interest rate r_t are endogeneous variables. Taxes a_t and income b_t are exogeneous. All variables are centered. The structural form is given by the system

$$\begin{cases} p_t = \alpha_1 q_t + \alpha_2 a_t + u_{1t} \\ q_t = \beta_1 p_t + \beta_2 b_t + u_{2t} \\ r_t = \gamma_1 p_t + \gamma_2 q_t + \gamma_3 b_t + u_{3t} \end{cases}$$

   (a) [5] Check the order condition for each equation.

   (b) [5] Check the rank condition for each equation.

4. The probit model was estimated: $\hat{\mathbb{P}}(y_1 = 1 \mid x_i, d_i) = F(-0.3 + 0.2x_i + 0.1d_i)$.

   (a) [3] Forecast the odds of $y_1 = 1$ for $x_i = 1$, $d_i = 1$.

   (b) [3] Find the partial effect of changing the value of the dummy variable d_i from zero to one for $x_i = 1$.

   (c) [4] Find the marginal effect of changing the value of the variable x_i for $x_i = 1$ and $d_i = 1$.

   Hint: $\mathbb{P}(W \leq 0.1) = 0.54$ for a standard normal $W \sim \mathcal{N}(0; 1)$.

5. The probit model was estimated using 1000 observations. Standard errors are given in brackets.

$$\hat{\mathbb{P}}(y_1 = 1 \mid x_i, d_i) = F(\underset{(0.01)}{-0.3} + \underset{(0.02)}{0.2}\, x_i + \underset{(0.01)}{0.1}\, d_i), \quad AIC = 606.$$

   (a) [3] Provide a 95% confidence interval for β_x.

   (b) [4] Find the log-likelihood of the trivial probit model, $\hat{\mathbb{P}}(y_1 = 1 \mid x_i, d_i) = F(\hat{\beta}_0)$.

   (c) [3] Compare the initial model and the trivial model using likelihood ratio test at 5% significance level.

   Hint: 5% critical values are $\chi^2_1 = 3.84$, $\chi^2_2 = 5.99$, $\chi^2_3 = 7.81$, $\chi^2_4 = 9.49$.

6. A logit regression model and a Linear Probability Model (LPM) are used to explain the mortgage denial rates amongst a random sample of 1,234 young families in Britain, 35% of whom belong to a minority ethnicity group. There are 520 successful mortgage applications and 714 denied applications, with an average salary of £33,000 and an average deposit of £50,000. The regression results are as follows:

$$\underset{i}{\text{denial}} = \Lambda\left(\underset{(0.011)}{-0.032} - \underset{(0.052)}{0.103}\text{salary}_i + \underset{(0.054)}{0.240}\text{minority}_i - \underset{(0.044)}{0.095}\text{deposit}_i\right) \quad \text{(Logit)}$$

$$\widehat{\underset{i}{\text{denial}}} = \underset{(0.022)}{-0.103} - \underset{(0.152)}{0.263}\text{salary}_i + \underset{(0.024)}{0.062}\text{minority}_i - \underset{(0.144)}{0.295}\text{deposit}_i \quad \text{(LPM)}$$

   where $\Lambda(z) = \frac{\exp(z)}{1+\exp(z)}$ is the logistic function. The usual standard errors for the Logit model and the robust standard errors for the LPM are reported in parentheses.

   The variable denial_i equals to 1 if the mortgage application was denied, 0 otherwise. The variables salary_i and deposit_i are the total annual salary and the downpayment by family i in £10,000. The variable minority_i equals to 1 if the applicant belongs to a minority group, 0 otherwise.

   (a) [3] Describe the estimator underlying the logit model and discuss its properties. **Note**: clearly indicate the likelihood function used, but a detailed derivation of the estimator is not expected.

   (b) [2] Discuss the advantages and drawbacks of using the LPM, rather than the logit model, to model the mortgage denial rates.

   (c) [3] Discuss how you can test the null hypothesis that the *ceteris paribus* effect of salary on the probability of denial is equal to -0.25. Clearly state the alternative hypothesis, the test statistic, and rejection rule.

   (d) [2] Using the logit model, what is the marginal effect of being a minority on mortgage denial rate at the mean values of the explanatory variables. Discuss the limitation(s) of the marginal effect obtained and propose an alternative that overcomes such limitation(s).
   Note: there is no need to calculate the marginal effect using the alternative approach.