Short rules: one A4 cheatsheet is allowed, calculators are ok, offline, 120 minutes.

1. Using a large dataset Donald Trump has estimated two parameters, $\hat{a} = 1$, $\hat{b} = 2$ with $se(\hat{a}) = 0.2$, $se(\hat{b}) = 0.3$. Estimators are uncorrelated.

   (a) [6] Find the standard error of $\hat{\gamma} = \hat{a}^2\hat{b} + \hat{b}^2\hat{a}$.

   (b) [4] Find the 95% confidence interval for $\gamma = a^2b + b^2a$.

2. The price level $p_t$ and production $q_t$ are endogeneous variables. Taxes $a_t$ and income $b_t$ are exogeneous. All variables are centered. The structural form is given by the system

$$\begin{cases} p_t = \alpha_1 q_t + \alpha_2 a_t + u_{1t} \\ q_t = \beta_1 p_t + \beta_2 b_t + u_{2t}. \end{cases}$$

   The estimates of the reduced form via equation-by-equation ols:

$$\begin{cases} p_t = 2a_t + 4b_t \\ q_t = -3a_t + 2b_t. \end{cases}$$

   (a) [6] Recover the estimates of the coefficients in structural form.

   (b) [4] Describe how you will estimate the standard errors of the structural form.

3. The price level $p_t$, production $q_t$ and interest rate $r_t$ are endogeneous variables. Taxes $a_t$ and income $b_t$ are exogeneous. All variables are centered. The structural form is given by the system

$$\begin{cases} p_t = \alpha_1 q_t + \alpha_2 a_t + u_{1t} \\ q_t = \beta_1 p_t + \beta_2 b_t + u_{2t} \\ r_t = \gamma_1 p_t + \gamma_2 q_t + \gamma_3 b_t + u_{3t} \end{cases}$$

   (a) [5] Check the order condition for each equation.

   (b) [5] Check the rank condition for each equation.

4. The logit model was estimated: $\hat{\mathbb{P}}(y_1 = 1 \mid x_i, d_i) = F(-0.3 + 0.2x_i + 0.1d_i)$.
   (a) [3] Forecast the odds of $y_1 = 1$ for $x_i = 1$, $d_i = 1$.
   (b) [3] Find the partial effect of changing the value of the dummy variable $d_i$ from zero to one for $x_i = 1$.
   (c) [4] Find the marginal effect of changing the value of the variable $x_i$ for $x_i = 1$ and $d_i = 1$.

5. The probit model was estimated using 1000 observations with 200 observations where $y_i = 1$. Standard errors are given in brackets.

$$\hat{\mathbb{P}}(y_1 = 1 \mid x_i, d_i) = F(\underset{(0.01)}{-0.3} + \underset{(0.02)}{0.2}\, x_i + \underset{(0.01)}{0.1}\, d_i), \quad AIC = 606.$$

   (a) [3] Provide a 95% confidence interval for $\beta_x$.
   (b) [4] Find the log-likelihood of the trivial probit model, $\hat{\mathbb{P}}(y_1 = 1 \mid x_i, d_i) = F(\hat{\beta}_0)$.
   (c) [3] Compare the initial model and the trivial model using likelihood ratio test at 5% significance level.

   Hint: 5% critical values are $\chi^2_1 = 3.84$, $\chi^2_2 = 5.99$, $\chi^2_3 = 7.81$, $\chi^2_4 = 9.49$.

6. (from UOL past exams) It is postulated that a reasonable demand–supply model for the wine industry in Australia would be given by:

$$Q_t = \alpha_0 + \alpha_1 P_t^w + \alpha_2 P_t^b + \alpha_3 Y_t + \alpha_4 A_t + u_t \quad \text{(demand)}$$
$$Q_t = \beta_0 + \beta_1 P_t^w + \beta_3 S_t + v_t \quad \text{(supply)}$$

   where $Q_t$ = real per capita consumption of wine, $P_t^w$ = price of wine relative to CPI, $P_t^b$ = price of beer relative to CPI, $Y_t$ = real per capita disposable income, $A_t$ = real per capita advertising expenditure, and $S_t$ = storage cost. CPI is the Consumer Price Index.

   The endogenous variables in this model are $Q$ and $P^w$, and the exogenous variables are $P^b$, $Y$, $A$, and $S$.

   The variance of $u_t$ and $v_t$ are, respectively, $\sigma_u^2$ and $\sigma_v^2$, and $\mathbb{C}\text{ov}(u_t, v_t) = \sigma_{uv} \neq 0$. The errors do not exhibit any correlation over time.

   (a) [3] Provide the reduced form for $P_t^w$.
   (b) [2] The OLS estimation of the demand function, based on annual data from 1955–1975 ($T = 20$), gave the following results (all variables are in logs and figures in parentheses are $t$-ratios).

$$\hat{Q}_t = \underset{}{-23.651} + \underset{(-6.04)}{1.158} P_t^w \underset{(4.0)}{-\ 0.275} P_t^b + \underset{(-0.45)}{3.212} Y_t \underset{(4.5)}{-\ 0.603} A_t. \qquad (-1.3)$$

   All the coefficients except that of $Y$ have the wrong signs. The coefficient of $P^w$ (price elasticity of demand, $\alpha_1$) not only has the wrong sign but also appears significant.

   Explain why the OLS parameter estimator may give rise to these counter-intuitive results. You are expected to use your results in part (a) to support your answer.

   (c) [3] The supply equation is overidentified. Clearly explain this terminology. What distinguishes overidentification from exact identification and underidentification? Provide one set of assumptions that would render the supply equation exactly identified.
   (d) [2] Discuss how you should estimate the supply equation in light of the overidentification.