1. Zmey Gorynych worries about his weight. Each year he makes weight measurements. He is very happy when his weight is below 1000 kilograms, does not record the measurement precisely and eats much more young beautiful princesses.

   Here are his recordings for the last 5 years $y = (\leq 1000, 1200, 1500, 1300, \leq 1000)$.

   Assume that his weight measurements are uniform on $[0, a]$ and independent.

   (a) [7] Estimate unknown a using maximum likelihood.

   (b) [3] Estimate the probability that the next year Zmey Gorynych will be happy about his weight and hence will eat many young beautiful princesses.

2. Roman Bokhyan and Rene Khobua[1] praise the econometrics course, even though they don't attend lectures. The amount of praise y_t they collectively give on day t depends on the quality of the lecture x_t and random factors in Roman and Rene's lives $r_t \sim \mathcal{N}(0, \sigma^2)$, $y_t = \beta_1 + \beta_2 x_t + r_t$.

   If the praise exceeds an upper threshold y_h, the lecturer Boris becomes too arrogant. If the praise falls below a lower threshold y_l, the class teacher Maria gets very upset. Both outcomes are unacceptable for the course.

   (a) [3] What is the probability that for a given x_t, Roman and Rene will give an acceptable amount of praise?

   (b) [7] Find the expected amount of praise given a known x_t, conditional on the praise level being acceptable.

   Your answer may contain x_t, β_1, β_2, σ^2, standard normal density or cumulative distribution function.

3. Classify the processes x_t and y_t as stationary, trend stationary, difference stationary, integrated of order d. Here (u_t) is a white noise independent of (x_s) and (y_s) for $s < t$.

   (a) [5] $x_t = u_1 + 2u_2 + 3u_3 + 3u_4 + 3u_5 + \cdots + 3u_{t-3} + 3u_{t-2} + 2u_{t-1} + u_t$;

   (b) [5] $y_t = 6 + 3t + u_t + 0.5u_{t-1} + 0.5^2 u_{t-2} + 0.5^3 u_{t-3} + \ldots$.

[1] Look up who is this guy after the exam!

4. Consider the probit model
$$\mathbb{P}(y = 1 \mid z_1, z_2, q) = F(z_1\delta + \gamma z_2 q),$$
where F is a standard normal cdf and q is independent of z and has standard normal distribution. Here we omit the observation lower index i for simplicity.

   (a) [3] Find the marginal effect of z_2 for fixed z_1 and q.

   Let y^* be the latent variable of the probit model, $y^* = z_1\delta + \gamma z_2 q + u$.

   (b) [3] Find the conditional distribution of $\gamma z_2 q + u$ for fixed z_1 and z_2.

   (c) [4] Find $\mathbb{P}(y = 1 \mid z_1, z_2)$.

   Your answers may contain z_1, z_2, δ, γ, normal cdf or pdf.

5. Consider the following vector moving average process
$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 6 & 1 \\ 1 & 6 \end{pmatrix} \begin{pmatrix} v_t \\ w_t \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} v_{t-1} \\ w_{t-1} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} v_{t-2} \\ w_{t-2} \end{pmatrix}.$$
Here $u_t = (v_t, w_t)$ is a two-dimensional white noise with $\mathbb{E}(u_t) = 0$ and $\mathbb{V}\text{ar}(u_t) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

   (a) [3] Is the process (x_t) stationary? Is the process (y_t) stationary?

   (b) [4] Find the cross-covariance $\mathbb{C}\text{ov}(x_t, y_{t-1})$.

   (c) [3] Are the cross covariances $\mathbb{C}\text{ov}(x_{t-1}, y_t)$ and $\mathbb{C}\text{ov}(x_t, y_{t-1})$ always equal for the vector moving average process?

6. (from the LSE course) Let us consider the following model: $y_t = \alpha + \delta t + u_t$ where $u_t = \rho u_{t-1} + e_t$ and e_t is i.i.d. with zero mean and variance σ^2.

   (a) [2] Discuss the following statement: "If $\{u_t\}_{t=1}^T$ is stationary and weakly dependent then $\{y_t\}_{t=1}^T$ is trend-stationary (and weakly dependent)."

   (b) [3] Show that you can write the equation in the following form:
$$\Delta y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 t + e_t$$
   where $\beta_0 = \alpha(1 - \rho) + \rho\delta$, $\beta_1 = \rho - 1$, and $\beta_2 = \delta(1 - \rho)$.

   Hint: Subtract from the original model ρ times the model one period lagged.

   (c) [3] Show that under the null of a unit root, y_t is a random walk with drift.

   (d) [2] Discuss the Dickey-Fuller test in detail. (Provide the null and alternative hypotheses, test statistic, and rejection rule.)