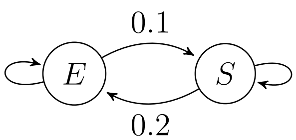
You are more than welcome to openly discuss these problems. You don't need to hand in these problems. The home assignments are graded only through quizzes. [FF] means [For Fun] and questions with this mark will not enter into the quizzes.

Home assignment 1

Enters in the quizzes during the week 2: 8-13 september.

1. The Cat can be only in two states: Sleeping (S) and Eating (E). Cat's mood depends only on the previous state. The transition probabilities are given below:



   (a) Compute the missing probabilities on the graph.
   (b) Write down the transition matrix.
   (c) Compute $\mathbb{P}(X_3 = \text{Eating} \mid X_0 = \text{Eating})$.

2. Cowboy Joe enters the Epsilon Bar and orders one pint of beer. He drinks it and orders one pint more. And so on and so on and so on... The problem is that the barmaid waters down each pint with probability 0.2 independently of other pints. Joe does not like watered down beer. He will blow the Epsilon Bar to hell if two or more out of the last three pints are watered down.

   We point out that Joe never drinks less than 3 pints in a bar.

   Let Y_t be the indicator that the pint number t was watered down. Consider the Markov chain $S_t = (y_{t-2}, y_{t-1}, y_t)$. For example, $S_t = (100)$ means that the pint number $t-2$ was watered down while pints number $t-1$ and t are good.

   (d) What are the possible values of S_3 and their probabilities?
   (e) Write down the transition matrix of this Markov chain.
   (f) [FF] What is the expected number of pints of beer Joe will drink?

3. Pavel Durov starts at the point $X_0 = 1$ on the real line. Each minute he moves left with probability 0.4 or right with probability 0.6 independently of past moves. The points 0 and 3 are absorbing. If Pavel reaches 0 or 3 he stays there forever. Let X_t be the coordinate of Pavel after t minutes.

   (a) Write down the transition matrix of this Markov chain.
   (b) Calculate the distribution of X_3 [list all values of the random variable X_3 and estimate the probabilities].

4. Consider two identical hedgehogs starting at the vertices A and B of a polygon $ABCD$. Each minute each hedehog simulteneously and independently chooses to go clockwise to the adjacent point, to go counter-clockwise to the adjacent point or to stay at his location.

   Thus the brotherhood of two brave hedgehogs can be in three states: in one vertex, in two adjacent vertices, in two non-adjacent vertices.

(a) Draw the graph for the brotherhood Markov chain and calculate all transition probabilities.

(b) Write down the transition matrix of the brotherhood Markov chain.

(c) What is the probability that they will be in one vertex after 3 steps?

Home assignment 2

Enters in the quizzes during the week 3: 15-20 september. During the quiz you may save time by writing only the system of equations and clearly stating which unknown is the answer to the question.

1. I throw a fair dice indefinitely often. If it shows 1 then 1 rouble is added to the hat and the game continues. If it shows 2 then I immediately get 2 roubles and the game stops. If it shows 3 then the sum in the hat is burned and the game continues. If it shows 4 then I immediately get 4 roubles and the game continues. It it shows 5 then the sum in the hat is burned and the game stops. If it shows 6 nothing happens and the game continues.

   (a) What is the expected duration of the game?

   (b) What is the expected total payoff?

2. Winnie-the-Pooh starts at zero on the real line. Every minute he moves one step to the left with probability 0.2, one step to the right with probability 0.3 or skips. If he reaches points (-1) or (3) he stays there forever.

   (a) What is the expected duration of the game?

   (b) What is the probability that he will eventually rest at (-1)?

3. Alice throws a fair coin until the sequence HTH appears.

   (a) What is the expected duration of the game?

   (b) What is her expected payoff if she gets 5 roubles for each head and 3 roubles for each tail?

4. [For Fun] Consider three games:

Game A: You toss a biased coin with probability 0.48 of H. You get $+1$ dollar for H and -1 dollar for T.

Game B: If your wellfare is divisible by three you toss a coin that lands on H with probability 0.09. If your wellfare is not divisible by three you toss a coin that lands on H with probability 0.74. You get $+1$ dollar for H and -1 dollar for T.

Game C: You toss an unbiased coin. If it lands on H you play Game A. If it lands on T you play Game B.

Your initial capital is 10000\$.

   (a) Generate and plot two random trajectories of your wellfare if you play Game A 10^6 times.

   (b) Generate and plot two random trajectories of your wellfare if you play Game B 10^6 times.

   (c) Generate and plot two random trajectories of your wellfare if you play Game C 10^6 times.