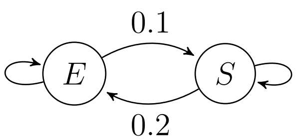
You are more than welcome to openly discuss these problems. You don't need to hand in these problems. The home assignments are graded only through quizzes. [FF] means [For Fun] and questions with this mark will not enter into the quizzes.

Home assignment 1

Enters in the quizzes during the week 2: 8-13 september.

1. The Cat can be only in two states: Sleeping (S) and Eating (E). Cat's mood depends only on the previous state. The transition probabilities are given below:



   (a) Compute the missing probabilities on the graph.
   (b) Write down the transition matrix.
   (c) Compute $\mathbb{P}(X_3 = \text{Eating} \mid X_0 = \text{Eating})$.

2. Cowboy Joe enters the Epsilon Bar and orders one pint of beer. He drinks it and orders one pint more. And so on and so on and so on... The problem is that the barmaid waters down each pint with probability 0.2 independently of other pints. Joe does not like watered down beer. He will blow the Epsilon Bar to hell if two or more out of the last three pints are watered down.

   We point out that Joe never drinks less than 3 pints in a bar.

   Let Y_t be the indicator that the pint number t was watered down. Consider the Markov chain $S_t = (y_{t-2}, y_{t-1}, y_t)$. For example, $S_t = (100)$ means that the pint number $t-2$ was watered down while pints number $t-1$ and t are good.

   (d) What are the possible values of S_3 and their probabilities?
   (e) Write down the transition matrix of this Markov chain.
   (f) [FF] What is the expected number of pints of beer Joe will drink?

3. Pavel Durov starts at the point $X_0 = 1$ on the real line. Each minute he moves left with probability 0.4 or right with probability 0.6 independently of past moves. The points 0 and 3 are absorbing. If Pavel reaches 0 or 3 he stays there forever. Let X_t be the coordinate of Pavel after t minutes.

   (a) Write down the transition matrix of this Markov chain.
   (b) Calculate the distribution of X_3 [list all values of the random variable X_3 and estimate the probabilities].

4. Consider two identical hedgehogs starting at the vertices A and B of a polygon $ABCD$. Each minute each hedehog simulteneously and independently chooses to go clockwise to the adjacent point, to go counter-clockwise to the adjacent point or to stay at his location.

   Thus the brotherhood of two brave hedgehogs can be in three states: in one vertex, in two adjacent vertices, in two non-adjacent vertices.

(a) Draw the graph for the brotherhood Markov chain and calculate all transition probabilities.

(b) Write down the transition matrix of the brotherhood Markov chain.

(c) What is the probability that they will be in one vertex after 3 steps?