You are more than welcome to openly discuss these problems. You don't need to hand in these problems. The home assignments are graded only through quizzes. Questions with [For Fun] mark will not enter the quizzes.

Home assignment 1

Enters in the quizzes during the week 3: 26 January.

1. The semi-annual y_t is modelled by $ETS(AAA)$ process:

$$\begin{cases} u_t \sim \mathcal{N}(0; 4) \\ s_t = s_{t-2} + 0.1u_t \\ b_t = b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + b_{t-1} + s_{t-2} + u_t \end{cases}$$

   (a) Given that $s_{100} = 2$, $s_{99} = -1.9$, $b_{100} = 0.5$, $\ell_{100} = 4$ find 95% prëdictive interval for y_{102}.

   (b) In this problem particular values of parameters are specified. How many parameters are estimated in semi-annual $ETS(AAA)$ model before real forecasting?

2. The $ETS(AAdN)$ model is given by the system

$$\begin{cases} u_t \sim \mathcal{N}(0; 16) \\ b_t = 0.9b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + 0.9b_{t-1} + 0.1u_t \\ y_t = \ell_{t-1} + 0.9b_{t-1} + u_t \end{cases}$$

   with $\ell_{100} = 20$ and $b_{100} = 2$.

   (a) Find the 95% predictive interval for y_{101}.

   (b) Find conditional probability $\mathbb{P}(y_{102} > 30 \mid \ell_{100}, b_{100})$.

   (c) Approximately find the best point forecast for y_{10000}.

   (d) Find the 95% predictive interval for b_{10000}.

3. The semi-annual y_t is modelled by $ETS(AAA)$ process:

$$\begin{cases} u_t \sim \mathcal{N}(0; 4) \\ s_t = s_{t-2} + 0.1u_t \\ b_t = b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + b_{t-1} + s_{t-2} + u_t \end{cases}$$

   Given that $s_0 = 2$, $s_{-1} = -2$, $b_0 = 0.5$, $\ell_0 = 4$ decompose $y_1 = 3$, $y_2 = 6$, $y_3 = 4$ into trend, seasonal component and random shocks.

Home assignment 1

Enters in the quizzes during the week 5: 9 February.

1. Consider an $MA(2)$ process defined by the equation:

$$y_t = 5 + u_t + 2u_{t-1} + 4u_{t-2},$$

where (u_t) is a white noise process with variance $\mathbb{V}\mathrm{ar}(u_t) = \sigma^2$.

   (a) Find the expected value $\mathbb{E}(y_t)$.
   (b) Find the autocorrelation function $\rho(k) = \mathbb{C}\mathrm{orr}(y_t, y_{t-k})$.
   (c) Is the process (y_t) stationary?
   (d) Construct a 95% confidence interval for y_{101}, given $u_{100} = 1$ and $u_{99} = -1$.

2. Consider an $MA(1)$ process defined by the equation:

$$y_t = \mu + u_t + bu_{t-1},$$

where (u_t) is a white noise process with zero mean and variance σ^2.

Assume we observe the following three consecutive realizations of this process:

$$y_1 = 2, \quad y_2 = 0.5, \quad y_3 = 1.5.$$

   (a) Using the method of moments, set up the system of equations by equating $\mathbb{E}(y_t)$, $\mathbb{E}(y_t^2)$ and $\mathbb{E}(y_t y_{t-1})$ with their sample counterparts. Derive estimates for the parameters μ, and σ^2.
   (b) Based on your estimates, is the process invertible? Justify your answer.

3. Suppose it is known that the autocorrelation function of an MA process takes the following values:

$$\rho(1) = -\frac{13}{19}, \quad \rho(2) = -\frac{91}{190}, \quad \rho(k) = 0 \text{ for } k \geq 3.$$

   (a) What is the order of this MA process?
   (b) Is there enough information to recover all parameters of the model generating this process?
   (c) Recover the parameters if possible.