You are more than welcome to openly discuss these problems. You don't need to hand in these problems. The home assignments are graded only through quizzes. Questions with [For Fun] mark will not enter into the quizzes.

Home assignment 1

Enters in the quizzes during the week 3: 26 January.

1. The semi-annual y_t is modelled by $ETS(AAA)$ process:

$$\begin{cases} u_t \sim \mathcal{N}(0; 4) \\ s_t = s_{t-2} + 0.1u_t \\ b_t = b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + b_{t-1} + s_{t-2} + u_t \end{cases}$$

   (a) Given that $s_{100} = 2$, $s_{99} = -1.9$, $b_{100} = 0.5$, $\ell_{100} = 4$ find 95% prëdictive interval for y_{102}.

   (b) In this problem particular values of parameters are specified. How many parameters are estimated in semi-annual $ETS(AAA)$ model before real forecasting?

2. The $ETS(AAdN)$ model is given by the system

$$\begin{cases} u_t \sim \mathcal{N}(0; 16) \\ b_t = 0.9b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + 0.9b_{t-1} + 0.1u_t \\ y_t = \ell_{t-1} + 0.9b_{t-1} + u_t \end{cases}$$

   with $\ell_{100} = 20$ and $b_{100} = 2$.

   (a) Find the 95% predictive interval for y_{101}.

   (b) Find conditional probability $\mathbb{P}(y_{102} > 30 \mid \ell_{100}, b_{100})$.

   (c) Approximately find the best point forecast for y_{10000}.

   (d) Find the 95% predictive interval for b_{10000}.

3. The semi-annual y_t is modelled by $ETS(AAA)$ process:

$$\begin{cases} u_t \sim \mathcal{N}(0; 4) \\ s_t = s_{t-2} + 0.1u_t \\ b_t = b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + b_{t-1} + s_{t-2} + u_t \end{cases}$$

   Given that $s_0 = 2$, $s_{-1} = -2$, $b_0 = 0.5$, $\ell_0 = 4$ decompose $y_1 = 3$, $y_2 = 6$, $y_3 = 4$ into trend, seasonal component and random shocks.