

# Stochastic processes and applications: Seminar #6

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**i** | **Info:** Do not be afraid to solve unfinished tasks at home!

## Conditional expectations

### Task 1

$X$  is a random variable following Poisson distribution with parameter  $\lambda = 1$ ; random variable  $Y$  is a random variable uniformly distributed on  $[1, 2]$ . Assuming that  $X$  and  $Y$  are independent, find  $E(XY|X)$ ,  $\text{Cov}(XY, X)$ ,  $\text{Var}(XY|X)$ ,  $\text{Var}(XY)$ .

### Task 2

Let  $X$  and  $Y$  be two jointly continuous random variables with joint probability density function

$$f(x, y) = \begin{cases} x + y & \text{if } x, y \in [0, 1] \\ 0 & \text{otherwise.} \end{cases}$$

Find  $f(x)$ ,  $f(y|x)$ ,  $E(Y|X)$ ,  $\text{Var}(Y|X)$ .

### Task 3

Random variables  $X$  and  $Y$  are independent and exponentially distributed with parameter  $n$ .

#### Question 1

- (a) Find  $P(X|X + Y)$ ,  $E(X|X + Y)$ .
- (b) How the answer will change, if  $X$  and  $Y$  are following Poisson distribution with parameter  $\lambda$ ?
- (c) Find  $E(X|X + Y)$  if  $X \sim \text{Exp}(n_1)$ ,  $Y \sim \text{Exp}(n_2)$ .



#### Sources:

1. Demeshev B., Problems on stochastic analysis <https://github.com/bdemeshev>
2. Brzeźniak Z. and Zastawniak T., Basic stochastic processes