In this text (W_t) denotes the standard Wiener process*.

1. [10] We keep our promises!
   (a) [3] What is the distribution of $3W_7 + W_8$?
   (b) [5] What is the conditional distribution of $(3W_7 + W_8 \mid W_1 = 3)$?
   (c) [2] Find the probability $\mathbb{P}(W_1 W_7 + W_8 > W_1 \mid W_1 = 3)$ in terms of a standard normal cdf $F()$.
2. [10] Consider the processes $X_t = t^2 + W_t t + \int_0^t (W_u^3 + W_u)\,dW_u$.
   (a) [2] Find dX_t.
   (b) [1] Is (X_t) a martingale? Why?
   (c) [2 + 5] Find $\mathbb{E}(X_t)$ and $\mathbb{V}\mathrm{ar}(X_t)$.
3. [10] Let $M_t = h(t) \cdot \cos(2W_t)$.
   (a) [6] Find a non-zero function $h(t)$ such that M_t is a martingale.
   (b) [4] Find $\mathbb{E}(\cos(2W_t))$.
4. [10] Let (S_T) be a symmetric random walk with $S_0 = 0$. The process Y_t is given by $Y_t = S_t - t$. The stopping time τ is given by $\tau = \min\{t \mid Y_t^2 = 100\}$.
   (a) [3] If possible find the value of α such that $M_t = \exp(\alpha Y_t)$ is a martingale.
   (b) [3] Find the distribution of Y_τ.
   (c) [4] Find the expected value $\mathbb{E}(\tau)$.
5. [10] Consider a two-period binomial tree model with an initial share price $S_0 = 100$. The up and down share price multipliers are $u = 2$ and $d = 0.5$. Risk-free interest rate is $r = 10\%$ in the first period. The central bank will increase the interest rate in the second period exactly to $r = 30\%$.

   The option is a European put with strike price $K = 300$ and maturity $T = 2$.
   (a) [4] Find the risk neutral probabilities.
   (b) [6] Find the arbitrage free price X_0 of this option.
6. [10] Let N_t be a Poisson process with $\mathbb{P}(N_7 - N_5 = 0) = \exp(-5)$.
   (a) [4] Find the rate λ of the process N_t.
   (b) [6] Find the constant b such that $M_t = 2N_t + bt$ is a martingale.

*Броуновское движение [всё ещё пока не запрещено на территории РФ]