In this text (W_t) denotes the standard Wiener process[*].

1. [10] We keep our promises!

    (a) [3] What is the distribution of $3W_7 - W_8$?

    (b) [5] What is the conditional distribution of $(3W_7 - W_8 \mid W_1 = 3)$?

    (c) [2] Find the probability $\mathbb{P}(W_1 W_7 - W_8 > W_1 \mid W_1 = 3)$ in terms of a standard normal cdf $F()$.

2. [10] Consider the processes $X_t = t^2 - W_t t - \int_0^t (W_u^3 + W_u)\, dW_u$.

    (a) [2] Find dX_t.

    (b) [1] Is (X_t) a martingale? Why?

    (c) [2 + 5] Find $\mathbb{E}(X_t)$ and $\mathbb{V}\mathrm{ar}(X_t)$.

3. [10] Let $M_t = h(t) \cdot \sin(2W_t)$.

    (a) [6] Find a non-zero function $h(t)$ such that M_t is a martingale.

    (b) [4] Find $\mathbb{E}(\sin(2W_t))$.

4. [10] Let (S_t) be a symmetric random walk with $S_0 = 0$.

    What is the expected time to hit the line $S_t = t - 20$ on the (t, S_t)-plane?

5. [10] Consider a two-period binomial tree model with an initial share price $S_0 = 100$. The up and down share price multipliers are $u = 2$ and $d = 0.5$. Risk-free interest rate is $r = 10\%$ in the first period. The central bank will increase the interest rate in the second period exactly to $r = 50\%$.

    The option is a European put with strike price $K = 300$ and maturity $T = 2$.

    (a) [4] Find the risk neutral probabilities.

    (b) [6] Find the arbitrage free price X_0 of this option.

6. [10] Let N_t be a Poisson process with $\mathbb{V}\mathrm{ar}(N_7 - N_5) = 4$.

    (a) [4] Find the rate λ of the process N_t.

    (b) [6] Find the constant b such that $M_t = 3N_t + bt$ is a martingale.

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[*]Броуновское движение [до сих пор всё ещё не запрещено на территории РФ]