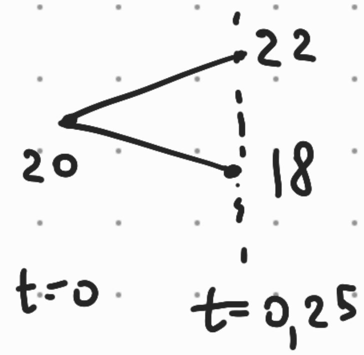
Task 1

You have:

- stocks:



- 1 derivative $c(K=21)$
- Risk-neutral market
- $r = 0{,}12$

N stocks – ?

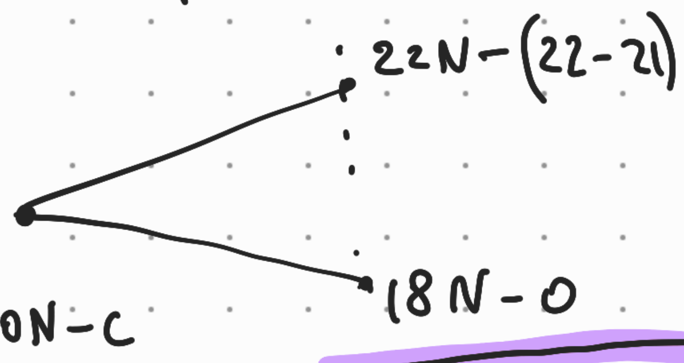
c – ?

I use definition Π_t^s – profit for time t in prices of time s (inflation) :(

No money, how to gain some profit?

Let's sell call w. strike price 21

[You can buy 1 stock and pay only 21 \$]



$$x^0 (\underbrace{1+r}_{\text{interest}})^t = x^t$$

$$x^0 = \frac{x^t}{\underbrace{(1+r)^t}_{\text{discount}}}$$

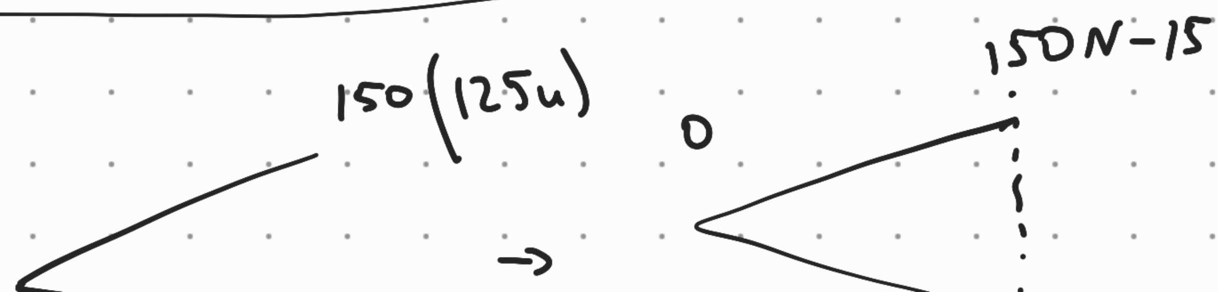
- no risk for you: $22N - 1 = 18N$

  $4N = 1 \Rightarrow N = 0{,}25$

- $\Pi_{0,25} = 22 \cdot 0{,}25 - 1 = 4{,}5$
- no profit: $\Pi^0_{0,25} = \Pi_0 \Rightarrow 20N - c = \frac{\Pi_1}{(1+r)^{0,25}} = \frac{4{,}5}{1{,}12^{0,25}}$

$$c = 20 \cdot 0{,}25 - \frac{4{,}5}{1{,}12^{0,25}} = 5 - 4{,}25 = 0{,}75$$

Task 2



(I)

(2) 125 $\quad 100(125d) \quad 125N - C \quad 100N$

$C(K=135)$ $\qquad$ $150N - 15 = 100N$

$r = 8\%$ $\qquad$ $50N = 15 \quad N = 0{,}3$

- $125N - C = \dfrac{100N}{1+0{,}08} \Rightarrow C = 125 \cdot 0{,}3 - \dfrac{30}{1{,}08} = 9{,}72$

(II)

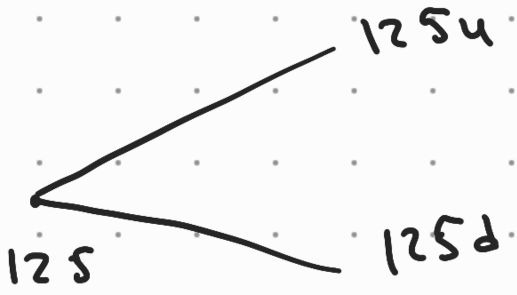
- Here C_t is a martingale : no profit

$$E^*(C_t) = \frac{p^*[S_u - K]_+}{(1+r)^t} = \boxed{C_0 = 9{,}72}$$

p^* – risk-neutral prob

a martingale

$$E^*\left(\frac{S_t}{(1+r)^t}\right) = \text{const} = S_0$$



$$125 = 125p^*\frac{u}{1+r} + 125\frac{d}{1+r}(1-p^*)$$

$$\frac{up^*}{1+r} - \frac{dp^*}{1+r} + \frac{d}{1+r} = 1$$

$$\boxed{p^* = \frac{1+r-d}{u-d}}$$

- $p^* = \dfrac{1{,}08 - \frac{100}{125}}{\frac{150-100}{125}} = \dfrac{1{,}08 - 0{,}8}{0{,}4} = 0{,}7$

- $c_0 = E^*(C_t) = \dfrac{0{,}7 \cdot 15}{1+r} + \dfrac{0{,}3 \cdot 0}{1+r} = \dfrac{15 \cdot 0{,}7}{1{,}08} = 9{,}72$

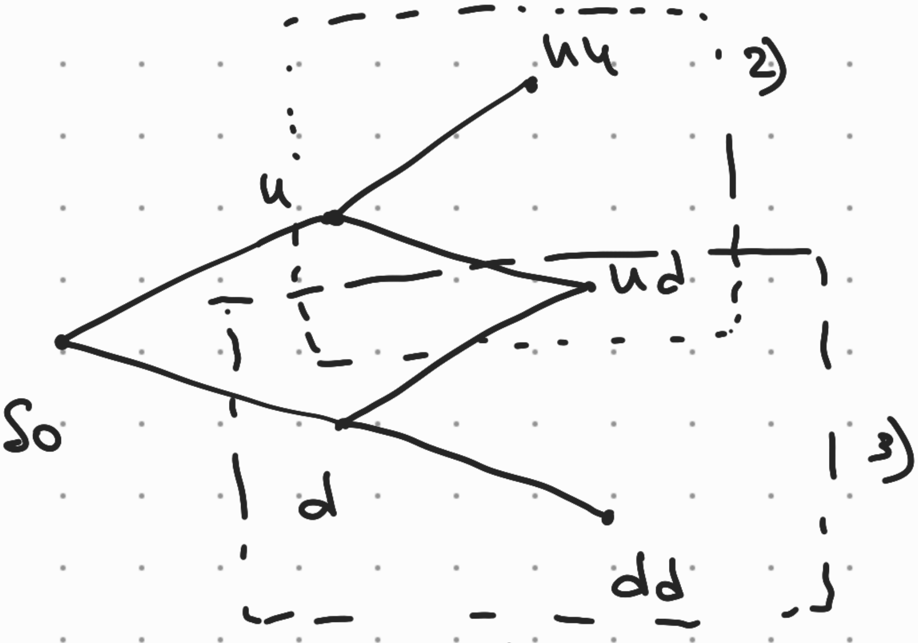
$\boxed{\text{Task 3}}$

$T = 2$
$S_0 = 100$
$u = 1{,}25$
$d = 0{,}75$
$K = 100$
$r = 0{,}05$

c/p -> t = 1

$\boxed{\text{deriv. price?}}$

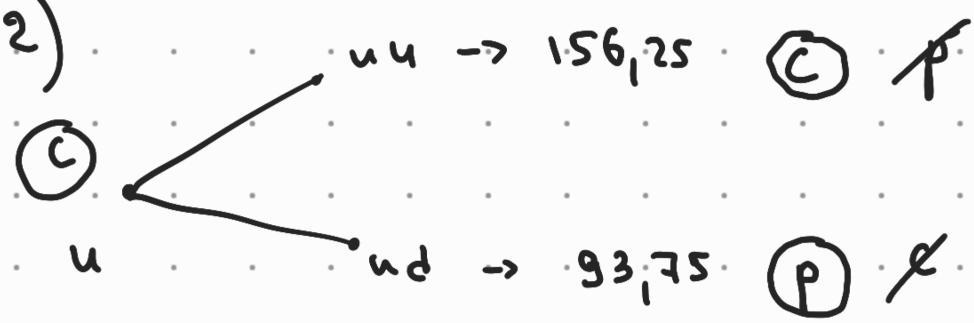
you can choose option type (put or call) at t = 1



1) $p^* = \dfrac{1+r-d}{u-d} = \dfrac{1{,}05-0{,}75}{1{,}25-0{,}75} = \dfrac{0{,}3}{0{,}5} = 0{,}6$

"Let's solve from leaves"

2)

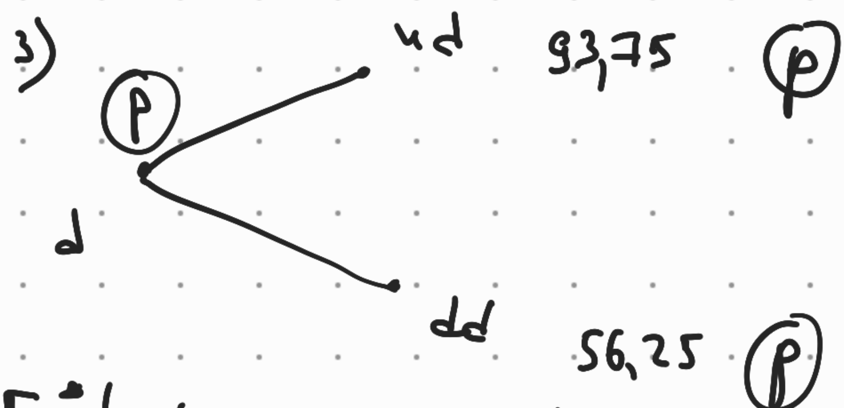


$E^*(c_1) = \dfrac{p^* \cdot 56{,}25}{1+r} = \dfrac{0{,}6 \cdot 56{,}25}{1{,}05} = 32{,}14$

V

$E^*(p_1) = \dfrac{(1-p^*) \cdot 6{,}25}{1+r} = \dfrac{0{,}4 \cdot 6{,}25}{1{,}05} = 2{,}38$

3)

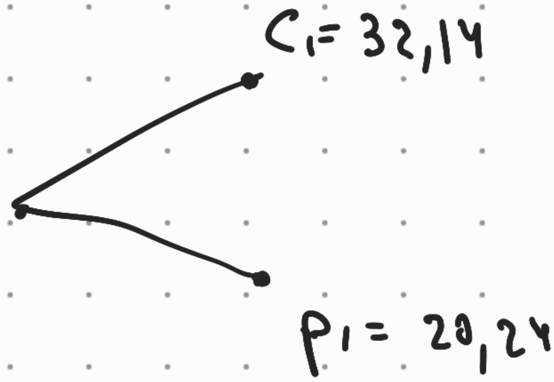


$E^*(p_1) = p^* \cdot 6{,}25 + (1-p^*)\, 43{,}75$

$$\ldots = \frac{\ldots}{1{,}05} = 20{,}24$$

$$E^*(G) = 0$$

4)



$$E^*(der_0) = p^* \frac{E^*(C_1)}{(1+r)} + (1-p^*)\frac{E^*(P_1)}{1+r} =$$

$$= \left[\frac{(p^*)^2\, 56{,}25}{(1+r)^2} + \frac{(1-p^*)^2 \cdot 43{,}75}{(1+r)^2} + \frac{p^*(1-p^*)6{,}25}{(1+r)^2} = \right.$$

$$= \frac{0{,}6 \cdot 32{,}14}{1{,}05} + \frac{0{,}4 \cdot 20{,}24}{1{,}05} = 26{,}08$$

All options above with known time of exposure were European

$\boxed{\text{Task 4}}$ American option

$S_0 = 100$

$u = 1{,}23$

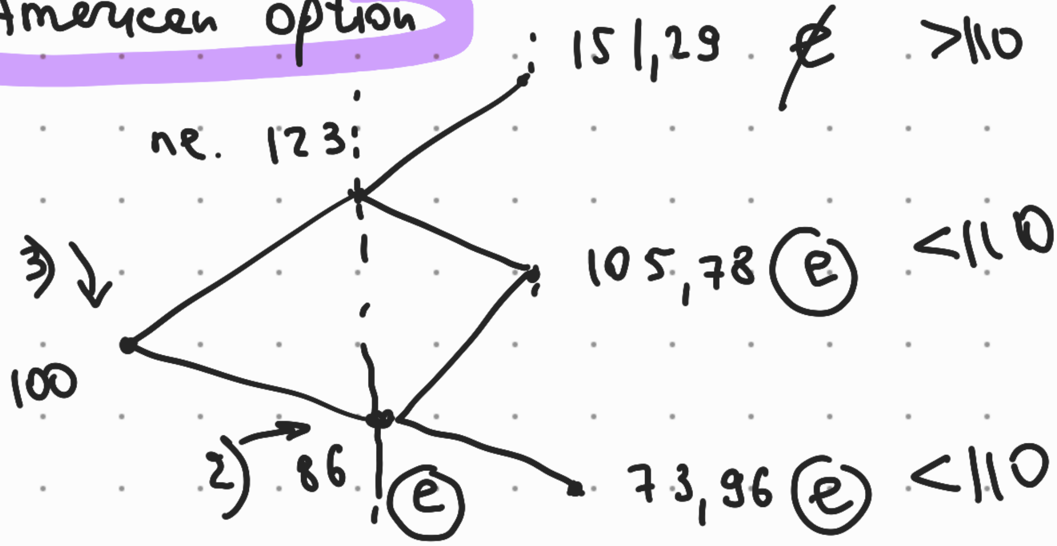
$d = 0{,}86$

$r = 0{,}03$

$K = 110$

$T = 2$

$P^A - ?$



$$1)\ p^* = \frac{1+r-d}{u-d} = \frac{1{,}03-0{,}86}{1{,}23-0{,}86} = 0{,}46$$

$$2)\ = (P_1^A) \ldots \quad p^* \cdot 4{,}72 + (1-p^*) \cdot 36{,}04$$

can be exposured at any time

$$2)\ E\left(\frac{\Pi^1}{1+r}\right) = \frac{[illegible]}{1+0,03} =$$

$$= \frac{0,46 \cdot 4,22 + 0,54 \cdot 36,04}{1,03} = 20,78$$

$$86 \to e \to \Pi_1 = 110-86=24$$

$$86 \to ne \to \Pi_2^1 = 20,78 \checkmark$$

$\Rightarrow$

$$3)\quad 100 \to e \to \Pi_0 = 10$$

$$E\left(\frac{p_0^A}{1+r}\right) = p^* \cdot \frac{(1-p^*) \cdot 4,22}{(1,03)^2} + \frac{(1-p^*) \cdot 24}{1,03} =$$

$$= \frac{0,46 \cdot 0,54 \cdot 4,22}{1,03^2} + \frac{0,54 \cdot 24}{1,03} = 0,99 + 12,58 =$$

$$= 13,57$$

$$100 \to ne \to \Pi_{1|2}^0 = 13,57$$

Ans: 13,57