In this text (W_t) denotes the standard Wiener process*.

1. [10] We keep our promises!
   (a) [3] What is the distribution of $W_7 + 2W_8$?
   (b) [5] What is the conditional distribution of $(W_7 + 2W_8 \mid W_1 = 2)$?
   (c) [2] Find the probability $\mathbb{P}(W_7 + 2W_8 > 1 \mid W_1 = 2)$ in terms of a standard normal cdf $F()$.
2. [10] Consider processes $X_t = \int_0^t W_u^3 dW_u$ and $Y_t = \int_0^t W_u^4 du$.
   (a) [3 + 3] Find $\mathbb{E}(X_t + Y_t)$ and $\mathbb{V}\text{ar}(X_t)$.
   (b) [4] Find $\mathbb{C}\text{ov}(X_t, W_t)$.
3. [10] Let $X_t = (W_t + g(t)) \exp(-W_t - t/2)$ with $X_0 = 0$.
   (a) [4 + 2] Find dX_t and write X_t as a sum of two integrals.
   (b) [4] Find at least one function $g(t)$ such that X_t is a martingale.

*Броуновское движение [ещё пока не запрещено на территории РФ]

4. [10] It's the midterm exam. Five students are sitting in the last row. The student in the middle of the last row is the only one who brought a calculator.

   Every minute his calculator moves one seat to the right (+1) or one seat to the left (-1) with equal probabilities.

   Let's X_n be the coordinate of the calculator at time n with $X_0 = 0$.

   (a) [4] Check whether $M_n = X_n^2 - n$ is a martingale.

   (b) [6] Find the average time for the calculator to reach the end of the row (left or right).

5. [10] Consider a two-period binomial tree model with an initial share price $S_0 = 100$. The up and down share price multipliers are $u = 2$ and $d = 0.5$. Risk-free interest rate is $r = 10\%$ in the first period. The central bank will increase the interest rate in the second period exactly to $r = 20\%$.

   The option is a European call with strike price $K = 300$ and maturity $T = 2$.

   (a) [4] Find the risk neutral probabilities.

   (b) [6] Find the arbitrage free price X_0 of this option.

6. [10] The assistant has not checked the home assignments in time. Hence teachers are receiving student complaints according to a Poisson process with daily rate $\lambda = 3$ messages per hour from $7:00$ to $23:00$ and nightly rate $\lambda = 1$ message per hour from $23:00$ to $7:00$.

   Let X be the number of messages from $6:00$ to $9:00$ and Y — number of messages from $7:00$ to $9:00$.

   (a) [2+2+2] Find $\mathbb{E}(X)$, $\mathbb{V}\text{ar}(X)$ and $\mathbb{P}(X = 2)$.

   (b) [4] Find the conditional distribution of Y given that $X = 2$.