1. [10] The process (u_t) is a white noise with $\mathbb{V}\text{ar}(u_t) = \sigma^2$. Consider the process

$$y_t = (2 + (-1)^t)u_1 + (3 + (-1)^t)u_2.$$

   (a) [4] Find $\mathbb{E}(y_t)$ and $\mathbb{V}\text{ar}(y_t)$.

   (b) [4] Find $\mathbb{C}\text{ov}(y_t, y_s)$.

   (c) [2] Is the process (y_t) stationary?

2. [10] Consider the stationary solution of the equation $y_t = 2 + 0.9y_{t-1} + u_t - 0.5u_{t-1}$, where (u_t) is a white noise process with variance 60.

   (a) [4] If possible rewrite this solution as $AR(\infty)$ process.

   (b) [4] If possible rewrite this solution as $MA(\infty)$ process.

   (c) [2] Find $\mathbb{C}\text{ov}(u_t, y_s)$ for this solution.

3. [10] Consider the equation $y_t = 7 + 0.4y_{t-1} - 0.13y_{t-2} + u_t + 2u_{t-1}$, where (u_t) is a white noise.

   (a) [1] How many non-stationary solutions does this equation have?

   (b) [4] How many stationary solutions of $MA(\infty)$ form with respect to (u_t) does this equation have?

   (c) [3] Can we rewrite the stationary solution in $AR(\infty)$ form with respect to (u_t)?

   (d) [2] Find $\mathbb{E}(y_t)$ for the stationary solution.

4. [10] Let (y_t) be the solution of the equation $y_t = 2y_{t-1} - y_{t-2} + u_t$, where (u_t) are independent and normally distributed $\mathcal{N}(0; 9)$ and y_0 is a constant.

   (a) [5] Find 95% confidence for y_{101} given that $y_{100} = 3$ and $y_{99} = 4$.

   (b) [5] Find 95% confidence for y_{102} given that $y_{100} = 3$ and $y_{99} = 4$.

5. [10] For the stationary solution of the equation $y_t = 0.3y_{t-1} - 0.02y_{t-2} + u_t$, where (u_t) is a white noise process.

   (a) [5] Find the first three values of the autocorrelation function ρ_1, ρ_2, ρ_3.

   (b) [5] Find all values of the partial autocorrelation function ϕ_{kk}.

6. [10] Let (y_t) be $MA(1)$ process.

   (a) [5] What are the possible values of $\rho_1 = \mathbb{C}\text{orr}(y_t, y_{t-1})$?

   (b) [5] What are the possible values of the partial correlation $\phi_{22} = \text{pCorr}(y_t, y_{t-2}; y_{t-1})$?