

Stochastic processes and applications: Seminar #2

Maria Kirillova @makirill

HSE — October 4, 2024

1 Convergence

Task 1

Consider the sequence of random variables X_n from exponential distribution with parameter n .

Question 1

- (a) Write down the distribution function.
- (b) Find the probability limit for X_n

Task 2

Consider the random variable X and the sequence of random variables Y_n with $E(Y_n) = \frac{1}{n}$ and $\text{Var}(Y_n) = \frac{\sigma^2}{n}$.

Question 2

- (a) Prove that $X_n \xrightarrow{p} X$
- (b) Hint: you may need Triangle inequality and Chebyshev inequality.

Task 3

The random variables X_i are independent and uniformly distributed on $[0; 1]$. $Y_n = \min X_1, \dots, X_n$. For Y_n :

Question 3

- (a) find the almost sure limit;
- (b) find the probability limit;
- (c) find the distribution limit.



Sources:

1. Kelbert M., Sukhov Y., Probability and statistics in examples and problems
2. Cambridge course on Markov chains <http://www.statslab.cam.ac.uk>
3. Demeshev B., Problems on stochastic analysis <https://github.com/bdemeshev>