

1. [10] The process (u_t) is a white noise with $\text{Var}(u_t) = \sigma^2$. Consider the process

$$y_t = (2 + 3t)u_1 + (3 + 2t)u_t.$$

- (a) [4] Find $\mathbb{E}(y_t)$ and $\text{Var}(y_t)$.
 - (b) [4] Find $\text{Cov}(y_t, y_s)$.
 - (c) [2] Is the process (y_t) stationary?
2. [10] Consider the stationary solution of the equation $y_t = 5 + 0.7y_{t-1} + u_t - 0.5u_{t-1}$, where (u_t) is a white noise process with variance 40.
- (a) [4] If possible rewrite this solution as $AR(\infty)$ process.
 - (b) [4] If possible rewrite this solution as $MA(\infty)$ process.
 - (c) [2] Find $\text{Cov}(u_t, \Delta y_s)$ for this solution.
3. [10] Consider the equation $y_t = 6 + 0.4y_{t-1} - 0.12y_{t-2} + u_t + 0.7u_{t-1}$, where (u_t) is a white noise.
- (a) [1] How many non-stationary solutions does this equation have?
 - (b) [4] How many stationary solutions of $MA(\infty)$ form with respect to (u_t) does this equation have?
 - (c) [3] Can we rewrite the stationary solution in $AR(\infty)$ form with respect to (u_t) ?
 - (d) [2] Find $\mathbb{E}(y_t)$ for the stationary solution.
4. [10] Let (y_t) be the solution of the equation $y_t = 3t + 2y_{t-1} - y_{t-2} + u_t$, where (u_t) are independent and normally distributed $\mathcal{N}(0; 9)$ and y_0 is a constant.
- (a) [5] Find 95% confidence for y_{101} given that $y_{100} = 3$ and $y_{99} = 4$.
 - (b) [5] Find 95% confidence for y_{102} given that $y_{100} = 3$ and $y_{99} = 4$.
5. [10] For the stationary solution of the equation $y_t = 5 + 0.4y_{t-1} - 0.01y_{t-2} + u_t$, where (u_t) is a white noise process.
- (a) [5] Find the first three values of the autocorrelation function ρ_1, ρ_2, ρ_3 .
 - (b) [5] Find all values of the partial autocorrelation function ϕ_{kk} .
6. [10] Let (y_t) be a stationary solution of $(1 + 0.5L)y_t = (1 + 0.5F)u_t$, where L is the lag operator, F is the forward operator and (u_t) is a white noise with $\text{Var}(u_t) = 1$.
- (a) [5] Find α and β in the representation $y_t = \alpha y_{t+1} + \beta y_t + \dots$.
 - (b) [5] Find the autocovariance function of (u_t) .