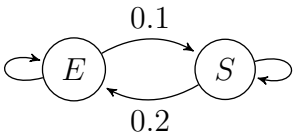
Be brave! You can use python. In this case just provide the code. You can use ChatGPT or any other LLM. In this case just provide the full prompt. Don't panic!

Home assignment 1

Deadline: 2024-09-23, 21:00.

1. The Cat can be only in two states: Sleeping (S) and Eating (E). Cat's mood depends only on the previous state. The transition probabilities are given below:



   (a) Compute the missing probabilities on the graph.

   (b) Write down the transition matrix.

   (c) Compute $\mathbb{P}(X_3 = \text{Eating} \mid X_0 = \text{Eating})$.

2. Cowboy Joe enters the Epsilon Bar and orders one pint of beer. He drinks it and orders one pint more. And so on and so on and so on... The problem is that the barmaid waters down each pint with probability 0.2 independently of other pints. Joe does not like watered down beer. He will blow the Epsilon Bar to hell if two or more out of the last three pints are watered down.

   We point out that Joe never drinks less than 3 pints in a bar.

   (a) What is the expected number of pints of beer Joe will drink?

   Let Y_t be the indicator that the pint number t was watered down. Consider the Markov chain $S_t = (y_{t-2}, y_{t-1}, y_t)$. For example, $S_t = (100)$ means that the pint number $t-2$ was watered down while pints number $t-1$ and t are good.

   (b) What are the possible values of S_3 and their probabilities?

   (c) Write down the transition matrix of this Markov chain.

   Note: questions (2b) and (2c) were updated!

3. Pavel Durov starts at the point $X_0 = 3$ on the real line. Each minute he moves left with probability 0.4 or right with probability 0.6 independently of past moves. The points 0 and 5 are absorbing. If Pavel reaches 0 or 5 he stays there forever. Let X_t be the coordinate of Pavel after t minutes.

   (a) Write down the transition matrix of this Markov chain.

   (b) Calculate the distribution of X_7 [list all values of the random variable X_7 and estimate the probabilities].

   Hint: you are free to use python for this problem :)

Home assignment 2

Deadline: 2024-09-27, 21:00.

1. [10 points] Consider two identical hedgehogs starting at the vertices A and B of a polygon $ABCD$. Each minute each hedehog simulteneously and independently chooses to go clockwise to the adjacent point, to go counter-clockwise to the adjacent point or to stay at his location.

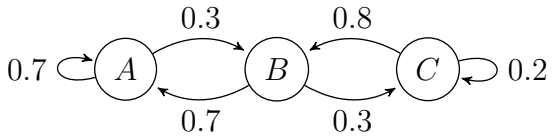
   Thus the brotherhood of two brave hedgehogs can be in three states: in one vertex, in two adjacent vertices, in two non-adjacent vertices.

   (a) Draw the graph for the brotherhood Markov chain and calculate all transition probabilities.

   (b) Write down the transition matrix of the brotherhood Markov chain.

   (c) What is the probability that they will be in one vertex after 3 steps?

2. [10 points] Consider the following Markov chain:



   (a) Find the stationary distribution of this Markov chain.

   The Markov chains starts at the vertex A. Let N be the first moment when the state C will be reached.

   (b) Find the expected value $\mathbb{E}(N)$.

   (c) Find the variance $\mathbb{V}\text{ar}(N)$.

3. [10 points] Bonnie and Clyde start at the points $(5, 0)$ and $(-5, 0)$ of the plane. Each minute each of them simulteneously and independently makes one step in one of the four possible directions (south, north, east, west).

   Each of them does n steps. Let X be the number of times they will be at the same point.

   (a) Estimate the probability $\mathbb{P}(X \geq 1)$ for $n = 50$ using $B = 10000$ simulations.

   (b) Estimate $\mathbb{E}(X)$ and $\mathbb{V}\text{ar}(X)$ for $n = 50$ using $B = 10000$ simulations.

   (c) Plot the estimated value of $\mathbb{E}(X)$ as a function of n for n from 1 to 200 using $B = 10000$ simulations.