last time:

* σ-algebras
* $E(Y \mid \mathcal{F})$ $\quad E(Y \mid X)$

  $Var(Y \mid \mathcal{F})$ $\quad Var(Y \mid X)$

---

def Stochastic process

→ In discrete time: sequence of random variables: $X_0, X_1, X_2, \ldots$

→ in continuous time: $\boxed{(X_t)_{t \in [0;+\infty)}}$ – indexed set of random variables.

филтрация

def: $\boxed{\text{Filtration}}$ ← models the evolution of our knowledge in time

in discrete time: sequence of σ-algebras $\mathcal{F}_0 \subseteq \mathcal{F}_1 \subseteq \mathcal{F}_2 \subseteq \mathcal{F}_3 \subseteq \ldots$

in continuous time: $(\mathcal{F}_t)_{t \in [0;\infty)}$ $\quad \mathcal{F}_s \subseteq \mathcal{F}_t$ for $s \leq t$

адаптиран

def the process (X_t) is adapted to filtration $(\mathcal{F}_t)$ if for $\forall t$ the random variable X_t is measurable wrt σ-algebra $\mathcal{F}_t$.

измерима

$\boxed{\text{at each moment } t \text{ I have enough info to calculate } X_t}$

def $(\mathcal{F}_t)$ is called „natural filtration" for (X_t) if $\forall t$ $\quad \mathcal{F}_t = \sigma((X_s)_{s \in [0;t]})$

A time moment t I know all past values of the process and nothing more.

Ex.

$X_1, X_2, X_3, X_4, \ldots$ are independent

$P(X_t=+1)=P(X_t=-1)=\frac{1}{2}$

$$\begin{cases} S_t = X_1+X_2+\dots+X_t & t\geq 1\\ S_0=0 \end{cases}$$

$$\boxed{\mathcal{F}_t = \sigma(X_1, X_2, \ldots, X_t)}$$

$M_t = S_{t+1}$

$$N_t = \begin{cases} 0 & t=0\\ S_{t-1}, & t\geq 1 \end{cases}$$

in $\mathcal{F}_5$: $\{X_1=1\}$
$\{X_1+X_2=2\}$
$\{X_5-X_1=0\}$
$\{X_5+X_3+X_1=X_2\}$

$\{S_t \leq v\} = \{X_1+X_2+\dots+X_t \leq v\}$

TRUE a) is (S_t) adapted to $(\mathcal{F}_t)$?

FALSE b) is (M_t) adapted to $(\mathcal{F}_t)$?

TRUE c) is (N_t) adapted to $(\mathcal{F}_t)$?

$t=5$ $\quad X_1, X_2, X_3, X_4, X_5$ ← you know these values.

Do you know N_5? Do you know S_4?

$t=7$ ..... we know N_7.

$\mathcal{F}_1 \subseteq \mathcal{F}_2 \subseteq \mathcal{F}_3 \subseteq \ldots$

$\mathcal{F}_1 = \{\phi, \Omega, \{X_1=1\}, \{X_1=-1\}\}$

$\mathcal{F}_2 = \{\phi, \Omega, \{X_1=1\}, \{X_1=-1\}, \{X_2=1\}, \{X_2=-1\},$
$\{X_2=X_1\}, \{X_2\neq X_1\}, \{X_2\geq X_1\}, \{X_2\leq X_1\}, \ldots\ldots \}$ (16 events)

	$X_2=+1$	$X_2=-1$
$X_1=+1$	•	•
$X_1=-1$	•	•

the number of events in $\mathcal{F}_2$ is $2^4=16$.

def. the process (X_t) is a martingale (мартингал)

if

→ discrete time $E(X_{t+1} | X_t, X_{t-1}, X_{t-2} \ldots) = X_t$

→ continuous time $E(X_{t+\Delta} | (X_s)_{s\in[0;t]}) = X_t$ for $\forall \Delta \geq 0$

---

the process (X_t) is a martingale wrt filtration $(\mathcal{F}_t)$ if

→ discrete time $E(X_{t+1} | \mathcal{F}_t) = X_t$

→ continuous time $E(X_{t+\Delta} | \mathcal{F}_t) = X_t$ for $\forall \Delta \geq 0$

Examples.

$X_1, X_2, X_3, X_4 \ldots$ are independent

$P(X_t = +1) = P(X_t = -1) = \frac{1}{2}$

$$\begin{cases} S_t = X_1 + X_2 + \ldots + X_t & t \geq 1 \\ S_0 = 0 \end{cases}$$

$\mathcal{F}_t = \sigma(X_1, X_2 \ldots X_t) = \sigma(S_1, S_2 \ldots S_t)$

a) (X_t) is a martingale?

b) (S_t) is a martingale?

c) (Y_t) where $Y_t = S_t^2 - t$ is a martingale?

a) $E(X_{t+1} | X_t, X_{t-1}, X_{t-2} \ldots X_1) \overset{\text{indep}}{=} E(X_{t+1}) =$

$= +1 \cdot \frac{1}{2} + (-1)\frac{1}{2} = 0 \neq X_t$

no

b) $E(S_{t+1} | S_t, S_{t-1}, S_{t-2}, S_{t-3} \ldots) =$ b) - yes.

$= E(\underbrace{S_t}_{\text{is known}} + \overbrace{X_{t+1} | S_t, S_{t-1}, \ldots S_1}^{\text{is independent}}) =$

$= E(S_t | S_t \ldots S_1) + E(X_{t+1} | S_t \ldots S_1) =$

$= S_t + E(X_{t+1}) = S_t + 0 = S_t$

Intuition (S_t) - is a martingale -
- your fortune in a fair game.

$$E(S_{t+1} \mid S_t \dots S_1) = S_t$$

$$\Updownarrow$$

$$E(S_{t+1} - S_t \mid S_t \dots S_1) = 0$$

---

c) $Y_t = S_t^2 - t$ useful

$$E(Y_{t+1} - Y_t \mid Y_t, Y_{t-1}, \dots Y_1) =$$

$$= E\left(S_{t+1}^2 - (t+1) - (S_t^2 - t) \mid Y_t, Y_{t-1} \dots Y_1\right) =$$

$$= E(S_{t+1}^2 - S_t^2 - 1 \mid Y_t \dots Y_1) =$$

$$= E\left((S_t + X_{t+1})^2 - S_t^2 \mid Y_t, \dots Y_1\right) - 1 =$$

$$= E\left(X_{t+1}^2 + 2 \cdot S_t \cdot X_{t+1} \mid Y_t, \dots Y_1\right) - 1 =$$

$X_{t+1} \begin{matrix} \nearrow +1 \\ \searrow -1 \end{matrix}$ $\quad X_{t+1}^2 = 1$ $\quad = 2E(S_t \cdot \overset{\text{indep.}}{X_{t+1}} \mid Y_t \dots Y_1)$

depend. indep.

$$= 2 \underbrace{E(S_t \mid Y_t \dots Y_1)}_{0 \;\; Y_t = S_t^2 - t} \cdot \boxed{E(X_{t+1} \mid Y_t \dots Y_1)} = 0 \quad \checkmark$$

$$E(X_{t+1}) = \frac{1}{2} \cdot 1 + \frac{1}{2}(-1) = 0$$

$$E(Y_{t+1} - Y_t \mid Y_t \dots Y_1) = 0 \qquad E(Y_{t+1} \mid Y_t \dots Y_1) = Y_t$$

c) yes

S_t and $Y_t \dots Y_1$ are dependent

$Y_5 = 20$ $\quad S_5^2 - 5 = 20$ $\quad S_5^2 = 25$ $\quad \begin{matrix} \nearrow S_5 = +5 \\ \searrow S_5 = -5 \end{matrix}$

$$E(S_5 \mid Y_5 = 20) = 5 \cdot \frac{1}{2} + (-5)\frac{1}{2} = 0$$

c')

$\boxed{(Y_t) \text{ is a martingale wrt } (\mathcal{F}_t) \text{ where } \mathcal{F}_t = \sigma(X_1, X_2 \dots X_t)}$

$$E(Y_{t+1} - Y_t \mid \mathcal{F}_t) = \dots\dots = 2E(S_t \mid \mathcal{F}_t) \cdot E(X_{t+1} \mid \mathcal{F}_t)$$

$$= 2 \cdot S_t \cdot 0 = 0$$

Ex.2

$X_1, X_2, X_3 \dots$ are independent

$P(X_t = +1) = 0{,}7 \quad P(X_t = -1) = 0{,}3$

$S_t = \begin{cases} 0 & t = 0 \\ X_1 + X_2 + \dots + X_t \end{cases}$ $\quad \boxed{\mathcal{F}_t = \sigma(X_1, X_2 \dots X_t)}$

a) S_t is a martingale?

b) find a constant $\boxed{\mu > 0}$ such that $M_t = \mu^{S_t}$ is a martingale wrt $(\mathcal{F}_t)$

a)? $E(S_{t+1} \mid S_t \dots S_1) =$

$\boxed{\text{no}}$

$$= E(S_t + \underbrace{X_{t+1}}_{\text{indep.}} \mid S_t \dots S_1) = S_t + \underbrace{E(X_{t+1})}_{0{,}7 \cdot 1 + 0{,}3 \cdot (-1) =}$$

$= S_t + 0{,}4$

b) $\boxed{E(M_{t+1} \mid \mathcal{F}_t) = M_t}$

$$E(\mu^{S_t + X_{t+1}} \mid \mathcal{F}_t) = \mu^{S_t}$$

$$E(\mu^{S_t} \cdot \mu^{X_{t+1}} \mid \mathcal{F}_t) = \mu^{S_t}$$

known $\quad$ indep

$$\cancel{\mu^{S_t}} \cdot E(\mu^{X_{t+1}} \mid \mathcal{F}_t) = \cancel{\mu^{S_t}}$$

$\neq 0 \qquad \neq 0$

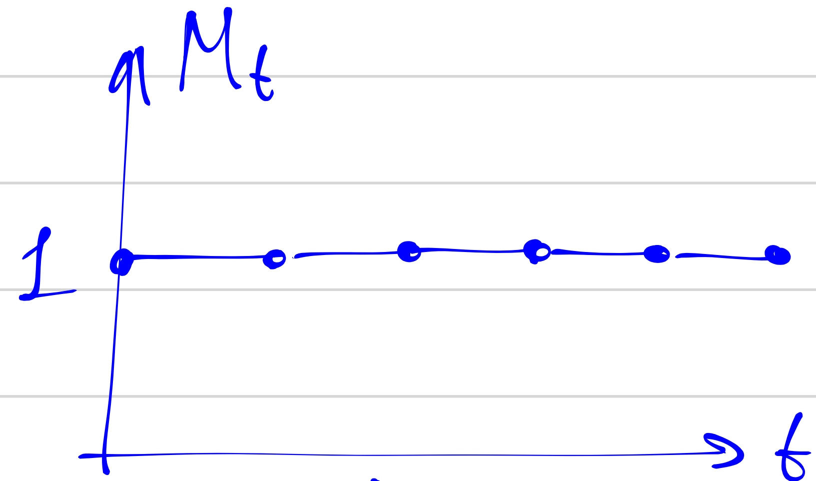
$$E(\mu^{X_{t+1}}) = 1 \qquad \boxed{0.7\mu^1 + 0.3 \cdot \mu^{-1} = 1}$$

$$\boxed{0.7\mu^{1}+0.3\cdot\mu^{-1}=1}$$

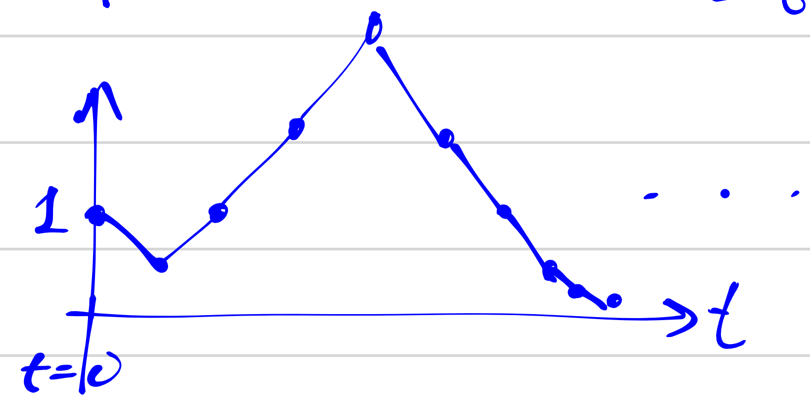
$$7\mu^2+3-10\mu=0$$

$$\mu_1=1 \qquad \mu_2=\frac{3}{7}$$

$$(M_t)=1^{S_t}=1$$



$$(M_t)=\left(\frac{3}{7}\right)^{S_t}$$



Stopping time

Int. def. Stopping time – any random moment of time that you can use to stop the game.

„You can stop at the stopping time"

not a st. time	T_1 – one day before the lowest price of dollar during current year
stopping time	T_2 – one day after the dollar reaches 50 zł/\$.

def T is a stopping time wrt filtration $(\mathcal{F}_t)$

if ① $T\in\{0,1,2,3,4,\dots,\infty\}$ (discr)

$T\in[0;+\infty]$ (cont)

② at each moment t

$$\boxed{\{T\le t\}\in\mathcal{F}_t}$$

„the moment T arrived"

Ex

$X_1, X_2, \dots$ are indep $P(X_t=+1)=P(X_t=-1)=\frac{1}{2}$

$$S_t = \begin{cases} 0 & t=0 \\ X_1+\dots+X_t \end{cases}$$

$$\mathcal{F}_t = \sigma(X_1, X_2, X_3 \dots X_t) \Rightarrow \boxed{\mathcal{F}_t = \sigma(S_1, S_2 \dots S_t)}$$

$$T_1 = \underset{t}{\operatorname{argmin}} \{S_t = 100\}$$

$$T_2 = T_1 - 1$$

$$T_3 = T_1 + 1$$

$$T_4 = 2T_1$$

which T_i is a stopping time wrt $(\mathcal{F}_t)$?

T_1 - stopp time.

T_2 - is not a stopp time

T_4, T_3 - stopp time.

M_t is a martingale

$$E(M_{t+1} \mid M_t \dots M_1) = M_t$$

or

$$E(M_{t+1} - M_t \mid M_t \dots M_1) = 0$$

$$E\left(M_{t+s} - M_t \mid (M_s)_{s\in[0,t]}\right) = 0 \text{ for } \forall s \geq 0$$

---

theorem.

If: M_t is a martingale, time is discrete

Then: $E(M_{t+s} \mid M_t, M_{t-1} \dots M_1) = M_t$ for $s \in \{1, 2, 3, \dots\}$

Proof. (by induction)

$$E(M_{t+2} \mid M_t, \dots M_1) \ ?$$

$\mathcal{F} = \sigma(M_t, \dots M_1)$

$\mathcal{H} = \sigma(M_{t+1} \dots M_1)$

$\mathcal{F} \subseteq \mathcal{H}$

$$E(M_{t+2} \mid \mathcal{F}) = E(E(M_{t+2} \mid \mathcal{H}) \mid \mathcal{F}) = E(M_{t+1} \mid \mathcal{F}) = M_t$$

....

Intuition. Doob's theorem.

If (1) → (M_t) is a martingale wrt $(\mathcal{F}_t)$

(2) → τ is a stopping time wrt $(\mathcal{F}_t)$

„I don't look into the future when I stop"

„I play a fair game"

→ (3) „I have limited resources"

then $E(M_\tau) = M_0$ / M_0 is const /

More precisely. ‼ (0) M_0 is constant.

If: ‼ (1) (M_t) is a martingale wrt $(\mathcal{F}_t)$

‼ (2) τ is a stopping time wrt $(\mathcal{F}_t)$

Doob's stopping theorem

(3) at least one condion of 3a or 3b is satisfied:

3a: ‼ $P(\tau < +\infty) = 1$ $\exists m \in \mathbb{R}$ such that

$$|M_{t \wedge \tau}| \leq m$$

3b: $E(\tau) < +\infty$ $\exists m \in \mathbb{R}$ such that

$$|M_{(t+1)\wedge\tau} - M_{t \wedge \tau}| \leq m$$

then: $E(M_\tau) = M_0$

$a \wedge b = \min(a, b)$ $5 \wedge 7 = 5$

$a \vee b = \max(a, b)$ $5 \vee 7 = 7$

$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ ⟷ $A \cap (B \cup C) =$
$= (A \cap B) \cup (A \cap C)$

$\min(a, \max(b, c)) = \max(\min(a, b), \min(a, c))$

Example 1

$X_1, X_2 \ldots$ indep $P(X_t = +1) = P(X_t = -1) = \frac{1}{2}$

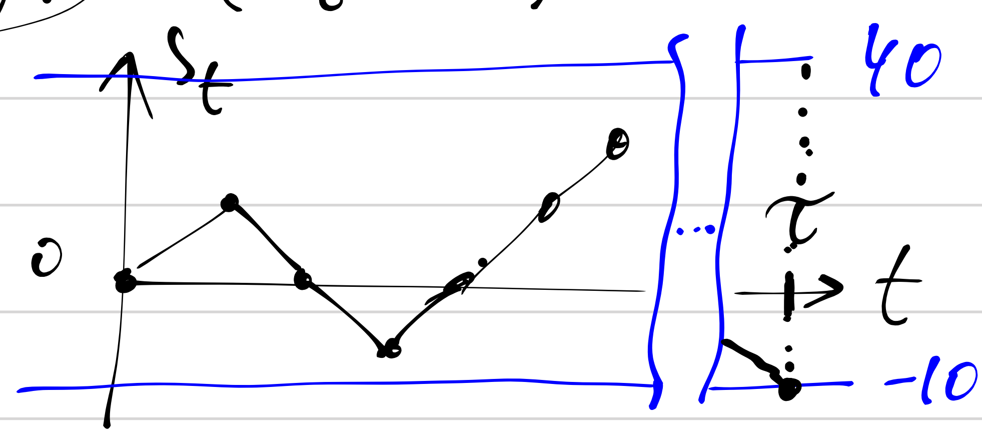
$$S_t = \begin{cases} 0 & t = 0 \\ X_1 + X_2 + X_3 + \ldots + X_t \end{cases}$$

$$\tau = \min_t \{t \mid S_t = -10, S_t = 40\}$$

$$\mathcal{F}_t = \sigma(X_1 \ldots X_t) = \sigma(S_1 \ldots S_t)$$

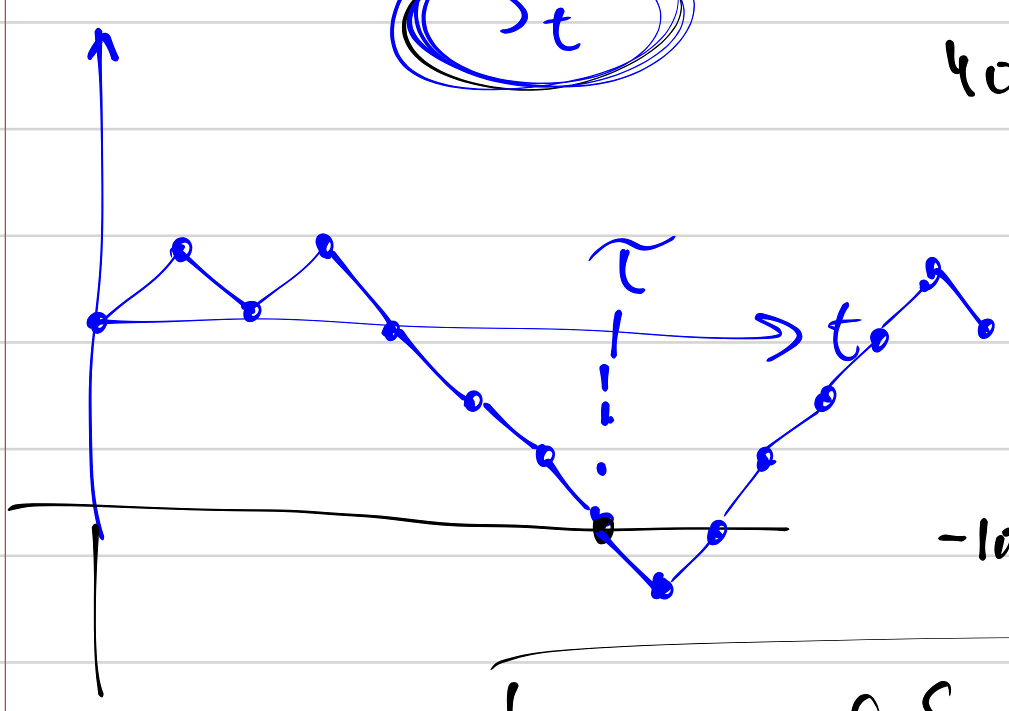
a) $P(S_\tau = -10)$? $P(S_\tau = 40)$?

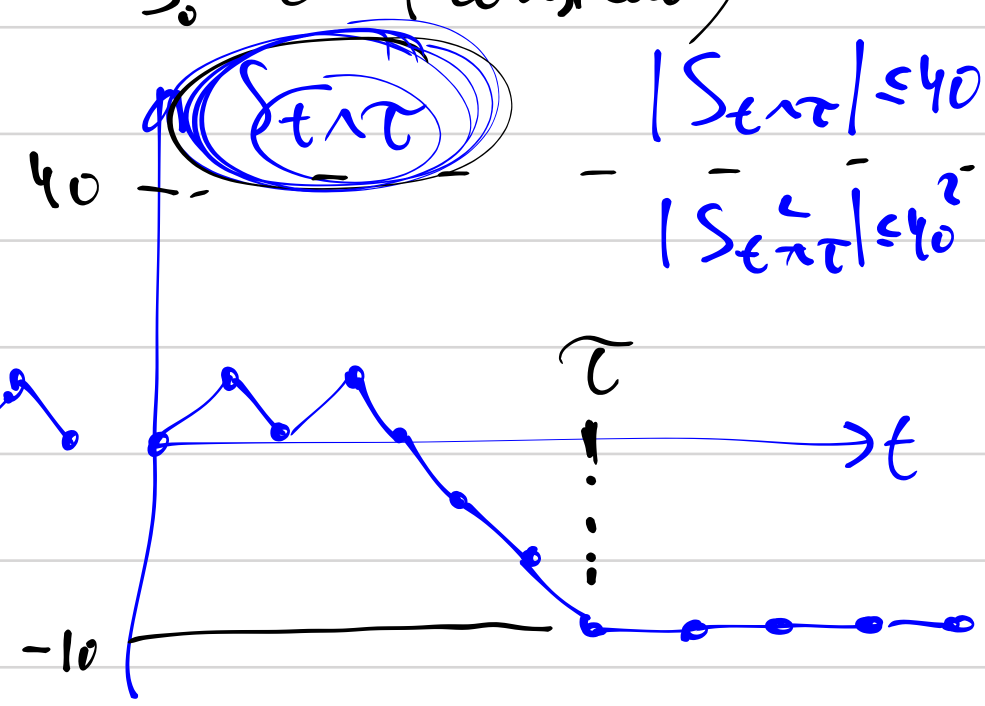
b) $E(\tau)$?



τ - stopp time !

S_t - martingale $\quad S_0 = 0$ (constant)





$|S_{t\wedge\tau}| \le 40$

$|S_{t\wedge\tau}^2| \le 40^2$

"frozen process" $\boxed{S_{t\wedge\tau} = \begin{cases} S_t & t < \tau \\ S_\tau & t \geq \tau \end{cases}}$

→ S_t is not bounded by a constant

→ $|S_{t\wedge\tau}| \le 40$

(0)(1)(2)(3a) ⟹ Doob's theorem

$$\boxed{E(S_\tau) = S_0 = 0}$$

$$E(S_\tau) = \boxed{p \cdot 40 + (1-p) \cdot (-10)} = 0$$

$p\,40 + 10p = 10$

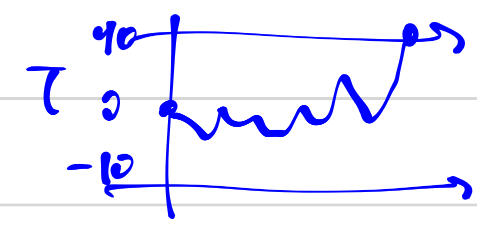
$$\boxed{p = \frac{10}{50}}$$

$P(S_\tau = 40) = \frac{1}{5}$ $\quad$ $P(S_\tau = -10) = \frac{4}{5}$

b) $E(\tau)$? $\quad$ $Y_t = S_t^2 - t$ ← martingale

(Y_t) – mart. $\quad$ τ – stopping time

$Y_0 = S_0^2 - 0 = 0$ (const)



the deltas of a „frozen process" are bounded

(3b) $$|Y_{(t+1)\wedge\tau} - Y_{t\wedge\tau}| =$$

$$= |\underbrace{S^2_{(t+1)\wedge\tau}}_{\le 40^2} - (t+1)\wedge\tau - \underbrace{S^2_{t\wedge\tau}}_{\le 40^2} + t\wedge\tau| \le$$

$|S^2_{t+1\wedge\tau} - S^2_{t\wedge\tau} + \Delta|$

$|t\wedge\tau - (t+1)\wedge\tau| \le 1$

$|\Delta| \le 1$

$|a - b + \Delta| \le$
$\le |a| + |b| + |\Delta|$

$\le 40^2 + 40^2 + 1$

$\le 40^2 + 1$

(0), (1), (2), (3b) $\Rightarrow$ $E(Y_\tau) = Y_0 = 0$

$$E(Y_\tau) = E(S_\tau^2 - \tau) = 0$$

$$E(S_\tau^2) = E(\tau)$$

$$\frac{1}{5}\cdot 40^2 + \frac{4}{5}\cdot(-10)^2 = E(\tau)$$

$$E(\tau) = \frac{40^2}{5} + \frac{4\cdot 10^2}{5} = 8\cdot 40 + 4\cdot 20 =$$
$$= 320 + 80 = 400$$

$\boxed{\text{assumption (3) is important}}$

$\boxed{\text{example}}$ „I can wait very long time"

S_t is the same

$T = \min\{t \mid S_t = 1\}$ - stopping time.

$$E(S_T) = E(1) = 1 \neq S_0 = 0.$$

3a fails: $|S_{t\wedge T}|$ not bounded.

- 10000
- 100000000...

3b fails (not obvious) $E(T) = +\infty$

$\boxed{\text{ABRACADABRA}}$ 1 keypress per second

A monkey randomly types letters (A, B, C, ... Z)

[26 letters]

$$P(X_t = „A") = P(X_t = „B") = \dots = P(X_t = „Z") = \frac{1}{26}$$

$$\tau = \min\{t \mid X_{t-10} = „A", X_{t-9} = „B" \ X_{t-8} = „R" \dots$$
$$\dots X_t = „A"\}$$

~~$E(X_t)$~~

$E(\tau)$? vs 1 year, 10 years?

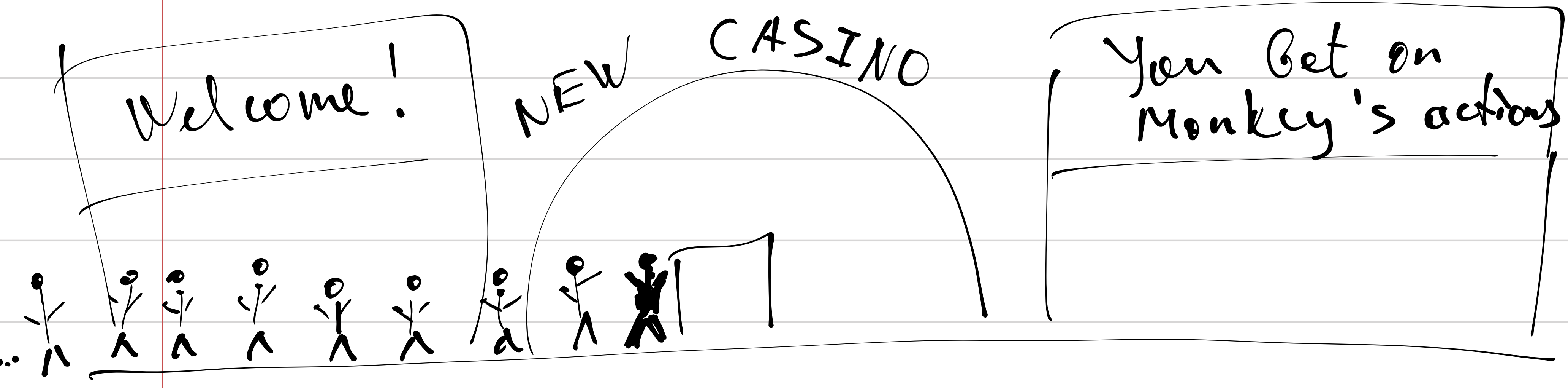
→ Invent a martingale (out of the blue) | of type $\boxed{M_t = X_t - t}$

→ Doob's theorem

→ $E(\tau)$

$$M_\tau = X_\tau - \tau$$

$$E(\tau) = E(X_\tau) \qquad E(M_\tau) = M_0$$



Rules

① you bet on the next letter → right → +bet x26
→ wrong → 0

2\$ on „R"
→ „X" → 0
→ „R" → 2×26 = 52

② at every moment time security guy will allow only 1 person with 1\$ to enter the casino.

X_t = the total seem of money of all the players inside casino.

$$E(X_{t+1} - X_t / \mathcal{F}_t) = 1$$

$$M_t = X_t - t$$

③ to calculate X_τ easily:
Every client should bet all the money on A, on B, on R, on A .....

at τ	Monkey	X A B Y Z A B X M .... ABRACADABRA
0	Client 1	A_0
0		
⋮	Client 2	A_{26} B_{26^2} R_0
⋮		
0	Client 3	A_0
0	Client 4	A_0
26^{4}	client $\tau-10$	ABRACADABRA
26^{4}	Client $\tau-3$	ABRA
0		
0	client $\tau-1$	A_0
26	Client τ	A_{26}

$$X_\tau = 26^{11} + 26^{4} + 26 = \text{const}$$

$$\boxed{M_t = X_t - t}$$

$$E(M_\tau) = E(X_\tau - \tau) = E(26^{11} + 26^4 + 26 - \tau) = $$

M_t - is martingale

X_t is not a martingale

$$= M_0 = X_0 - 0 = 0$$

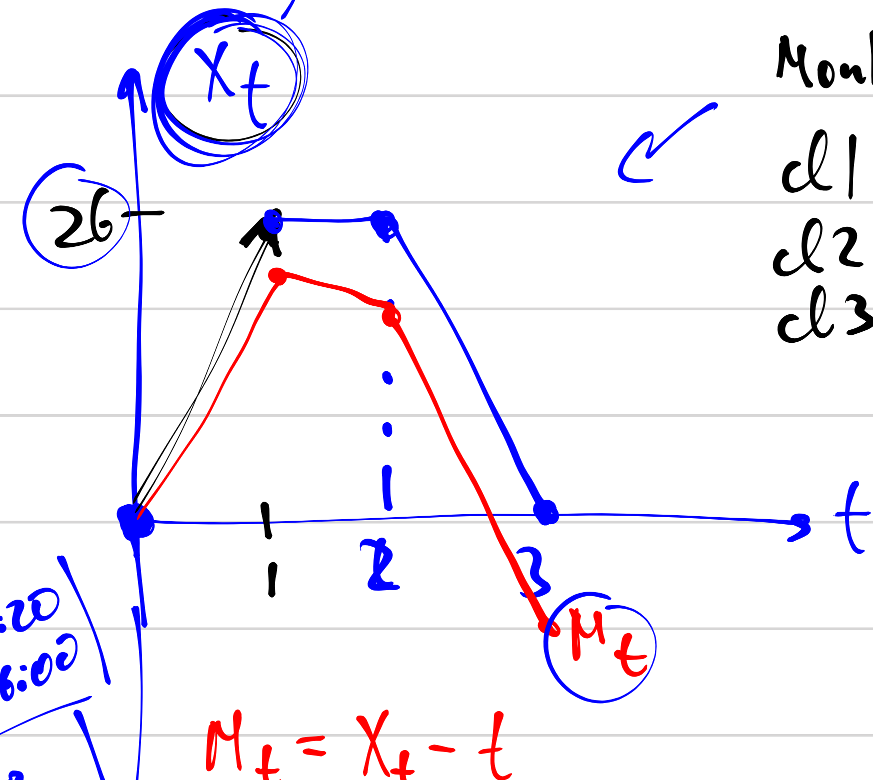
$$E(\tau) = 26^{11} + 26^{4} + 26 \quad (\text{seconds}) = 116\,385\,860 \text{ years}$$

$$E(X_{t+1} - X_t \mid \mathcal{F}_t) = 1$$

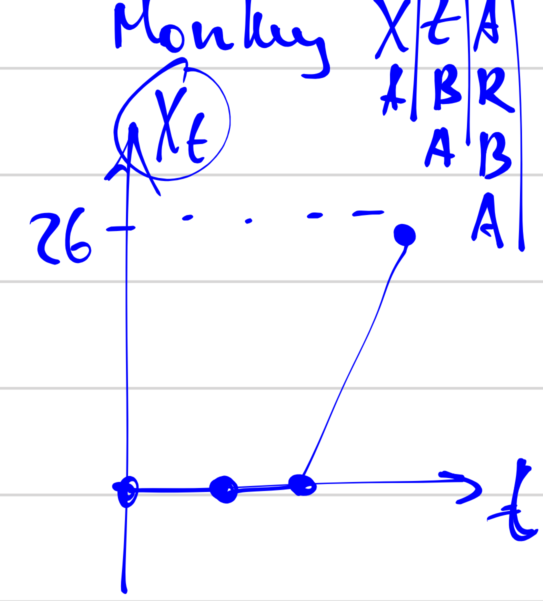
1 year $= 365 \times 24 \times 60 \times 60$

	t=1	t=2	t=3
Monkey	A	A	L
cl 1	A	B	R
cl 2		A	B
cl 3			A

	t=1	t=2	t=3
Monkey	X	Z	A
	A	B	R
		A	B
			A



$M_t = X_t - t$



Moscow
13:00 - 14:20
14:40 - 16:00
16:20 →
4.5 h

$$X_\tau = 26^{11} + 26^{4} + 26 \leftarrow \text{a constant}$$

$$M_\tau = 26^{11} + 26^{4} + 26 - \tau \leftarrow \text{a random variable}$$

Q + A

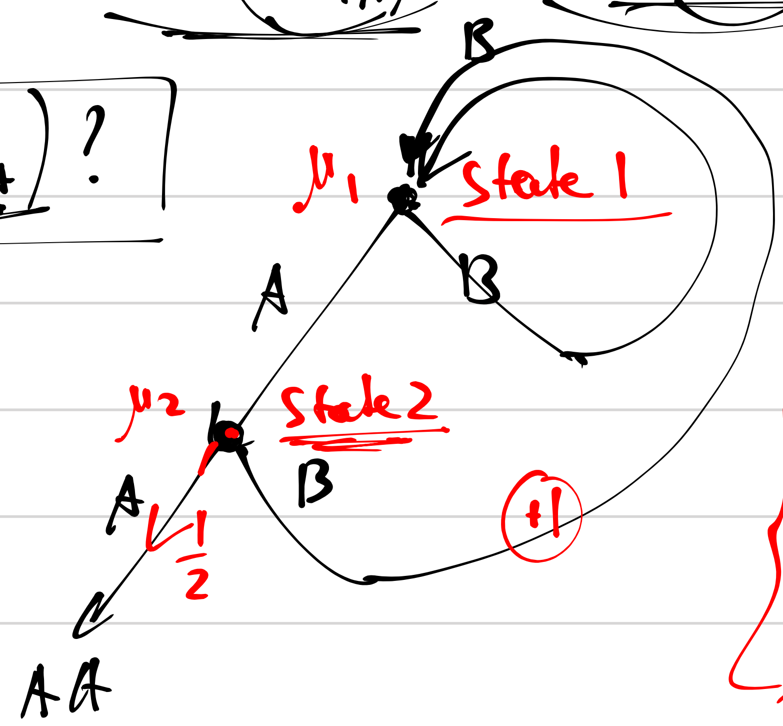
Q1

A1. $E(\tau_{AA}) > E(\tau_{AB})$!

A | B

$\frac{1}{2}$ $\frac{1}{2}$

$\boxed{E(\tau_{AA})\ ?}$



$$\begin{cases} \mu_2 = \frac{1}{2}\cdot 1 + \frac{1}{2}(1+\mu_1) \\ \mu_1 = \frac{1}{2}\cdot(1+\mu_2) + \frac{1}{2}(1+\mu_1) \end{cases}$$

$$\begin{cases} 2\mu_2 = 2+\mu_1 \\ 2\mu_1 = 2+\mu_2+\mu_1 \end{cases}$$

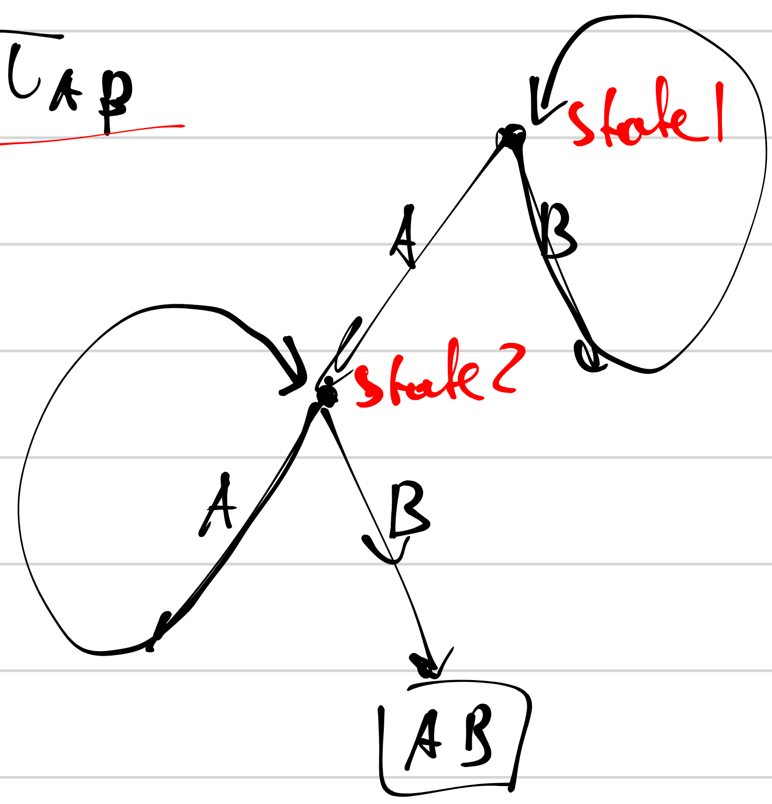
$$\begin{cases} 2\mu_2 = 2+\mu_1 \\ \mu_1 = 2+\mu_2 \end{cases}$$

$$2\cdot(\mu_1-2) = 2+\mu_1$$

$$2\mu_1 - 4 = 2+\mu_1$$

$$\boxed{\mu_1 = 6 = 2^2+2}$$

τ_{AB}



$$\begin{cases} \mu_1 = \frac{1}{2}(1+\mu_2) + \frac{1}{2}(1+\mu_1) \\ \mu_2 = \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot(1+\mu_2) \end{cases}$$

$\boxed{AAB}$

$$\begin{cases} 2\mu_1 = 2+\mu_2+\mu_1 \\ 2\mu_2 = 2+\mu_2 \end{cases}$$

$$\mu_2 = 2$$

$$\mu_1 = 4 = 2^2$$

$E(\tau^2)$?

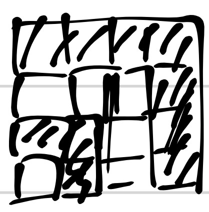
$$M_t = Y_t \boxed{-t^2} \quad \text{- martingale}$$

$$M_\tau = Y_\tau - \tau^2$$

$$E(Y_\tau) = E(\tau^2)$$

client n 1 : 1\$
client n 2 : 3\$
client n 3 : 5\$
client n 4 : 7\$

1
$1+3=4=2^2$
$1+3+5=3^2$
. . . .



$$1+3+5+7=4^2$$

Monkey XMLABZWR ...ABRACADABRA τ

$\xrightarrow{+1}$ cl 1 A_0

$\xrightarrow{+3}$ cl 2 A_0

$\xrightarrow{+5}$ cl 3 A_0

$\xrightarrow{+7}$ cl 4 $= 2\cdot 4 - 1$ $\quad A_{7\times 26}$ $B_{7\cdot 26^2}$

$(2\tau-21)26^{11}$ $\xrightarrow{2\tau-21}$ cl $\tau-10$

$(2\tau-7)\times 26^4$ $\xrightarrow{2\tau-7}$ cl $\tau-3$

$(2\tau-1)\times 26$ $\xrightarrow{2\tau-1}$ cl τ $\quad A$

$$E(M_\tau) = 0$$

$$M_t = Y_t - t^2 \qquad E(Y_\tau - \tau^2) = 0$$

total wealth of all players.

$$E\left((2\tau-21)\cdot 26^{11} + (2\tau-7)\cdot 26^4 + (2\tau-1)\cdot 26\right) = $$
$$= E(\tau^2)$$

$$\mathrm{Var}(\tau) = E(\tau^2) - (E(\tau))^2$$

stopping time

$z \sim \text{Geom}(p)$

$z=1 \quad z=2 \quad z=3 \quad \ldots\ldots$

$p \quad (1-p)\cdot p \quad (1-p)^2\cdot p$

$$E(z) = 1\cdot p + 2\cdot(1-p)\cdot p + 3\cdot(1-p)^2\cdot p + \ldots\ldots$$

$X_t \nearrow F\ (1-p)$, $X_t \searrow S\ (p)$

$z = \min\{t \mid X_t = \text{"}S\text{"}\}$

casino

$\tau = z$

$M_t = Y_t - t$

$F \to 0$

$S \to \boxed{x\left(\frac{1}{p}\right)}$

$$E(M_\tau - \tau) = 0$$

$$E\left(\frac{1}{p} - \tau\right) = 0$$

$$E(\tau) = \frac{1}{p}$$

!!

Дьяченко, Ульянов „Мера и интеграл"

глава 1 – глава 4

утверждение $E(\,)$

схема док-в утверждений про E

DCT, MCT

1. $I_A = \begin{cases} 1 \\ 0 \end{cases}$

2. X конечное число знач.

3. $X \geq 0$

4. $X = X_+ - X_-$

5. $X = X_R + i \cdot X_I$

DCT, MCT

$E(X + Y + Z) \neq$

$X \sim X'$

$Y \sim Y'$

$Z \sim Z'$

$\neq E(X' + Y' + Z')$

Контрпримеры в теории вер-стей

Стоянов

№ 6.1