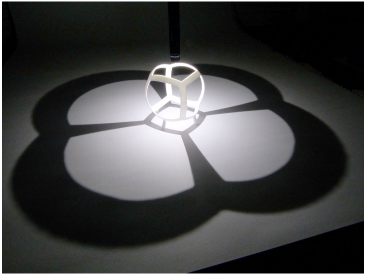
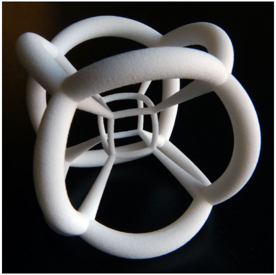
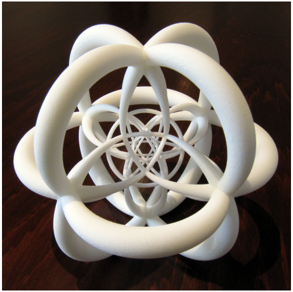




Henry Segerman
Oklahoma State University

Sculpture in 4-dimensions



What is 4–dimensional space?

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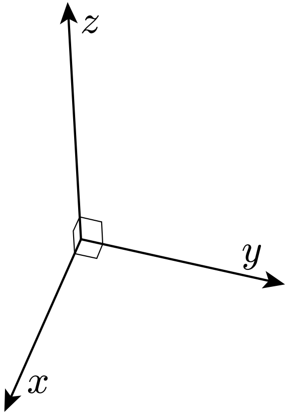
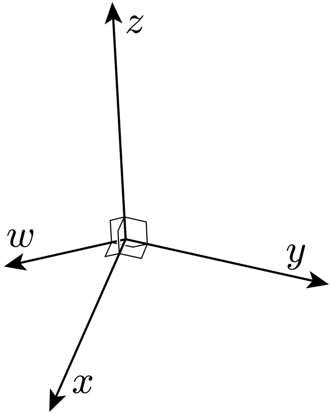
We describe a point in 3-dimensional space using three numbers, say (x, y, z).



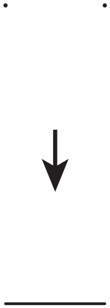
What is 4–dimensional space?

We describe a point in 3-dimensional space using three numbers, say (x, y, z).

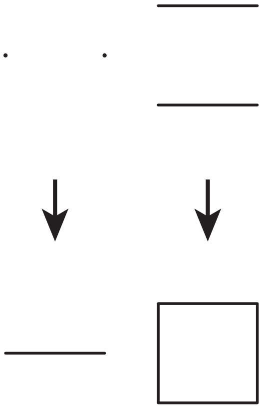
A point in 4-dimensional space is given by four numbers, say (w, x, y, z).



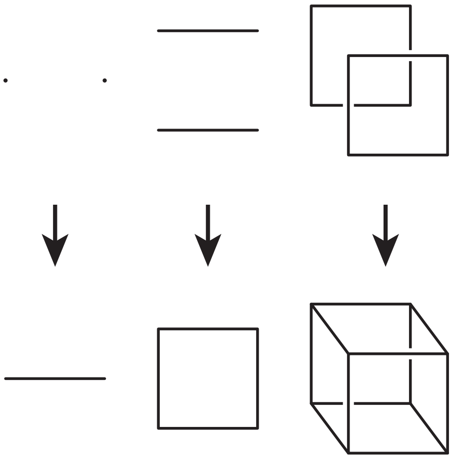
Example: how to make a hypercube



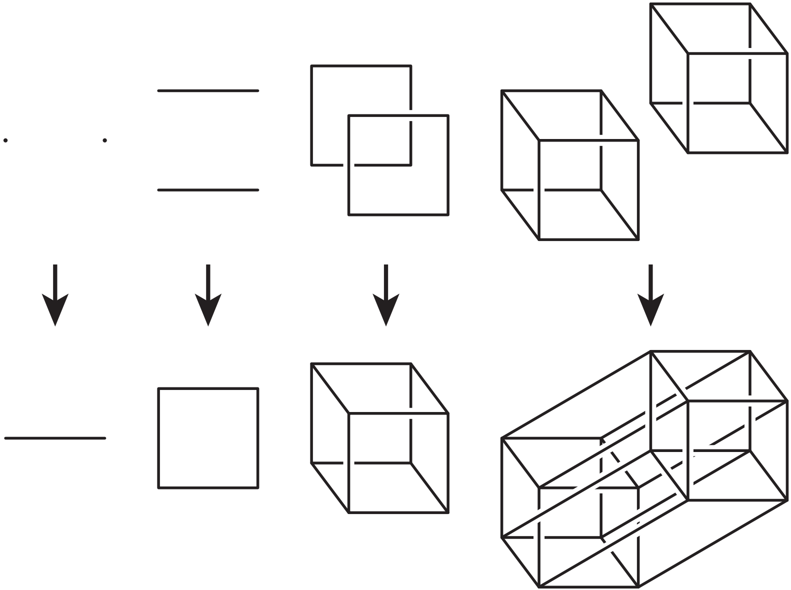
Example: how to make a hypercube



Example: how to make a hypercube



Example: how to make a hypercube



How can we see 4-dimensional things?

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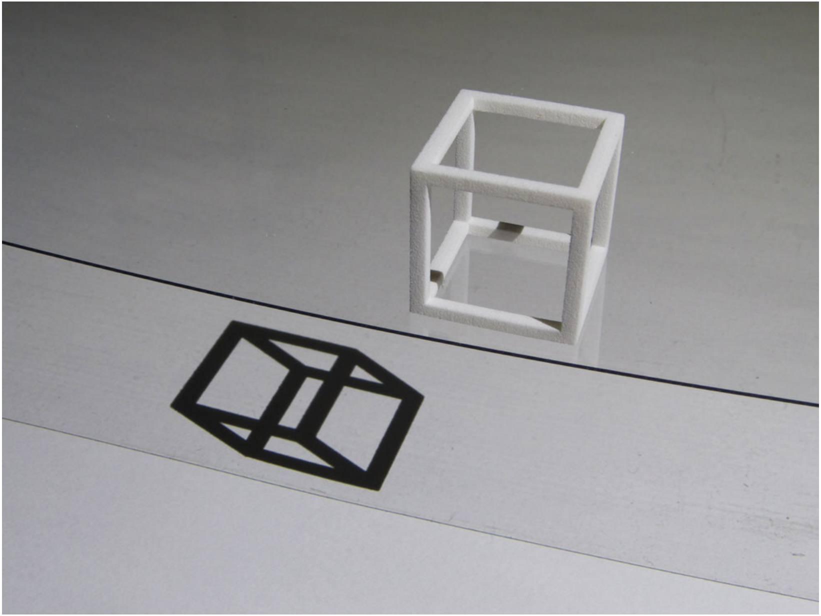
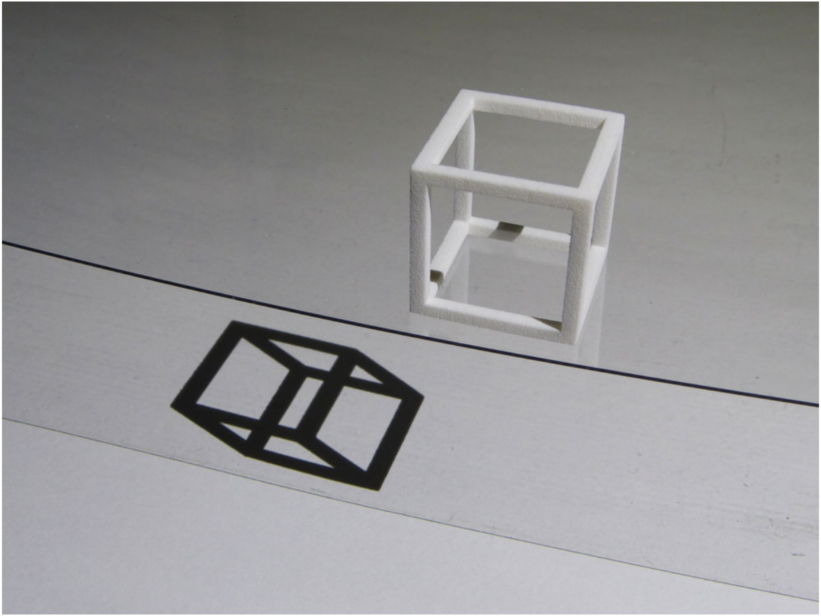
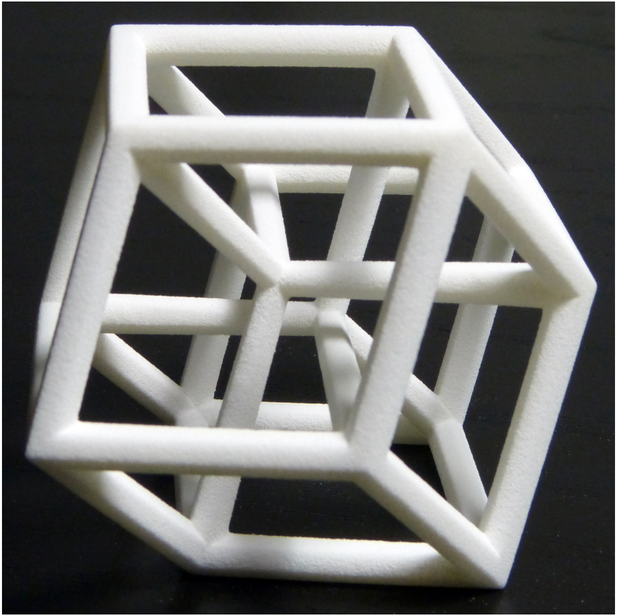


Orthogonal projection of a cube

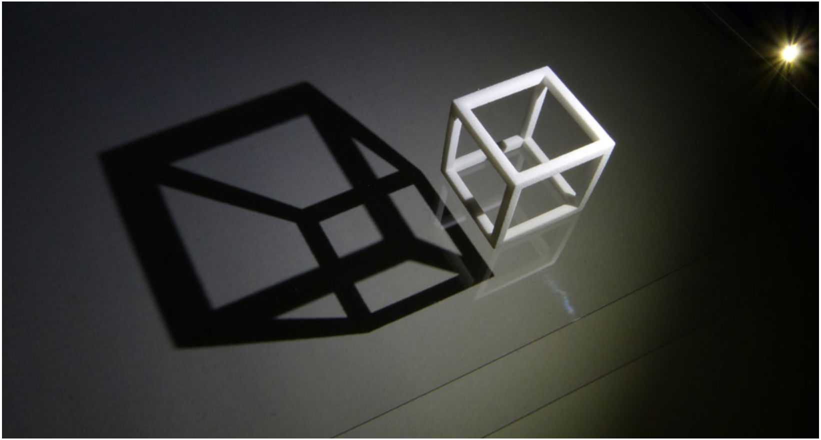


Orthogonal projection of a hypercube

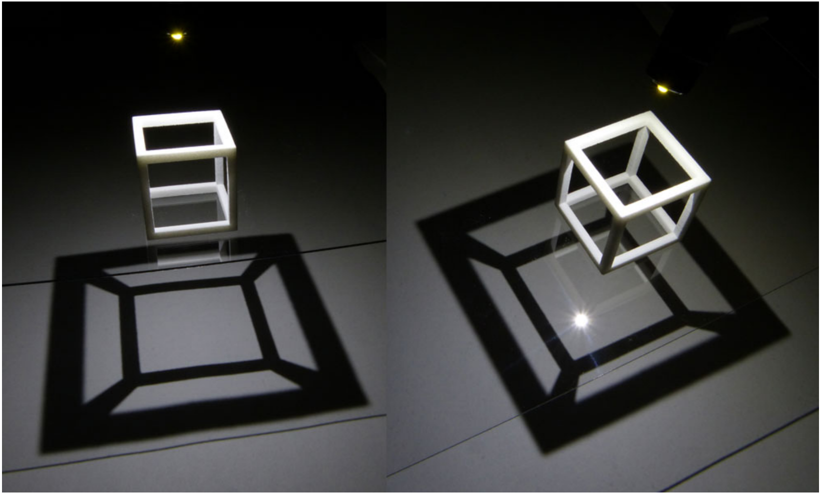


Hypercube B by Bathsheba Grossman.

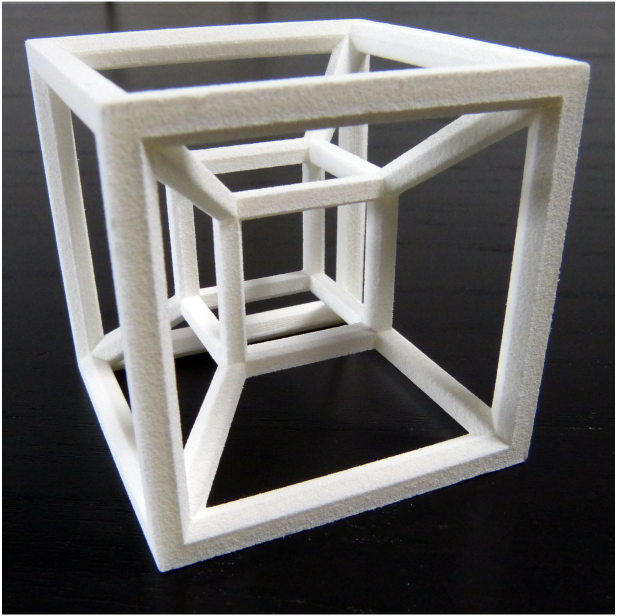
Perspective projection of a cube



Perspective projection of a cube



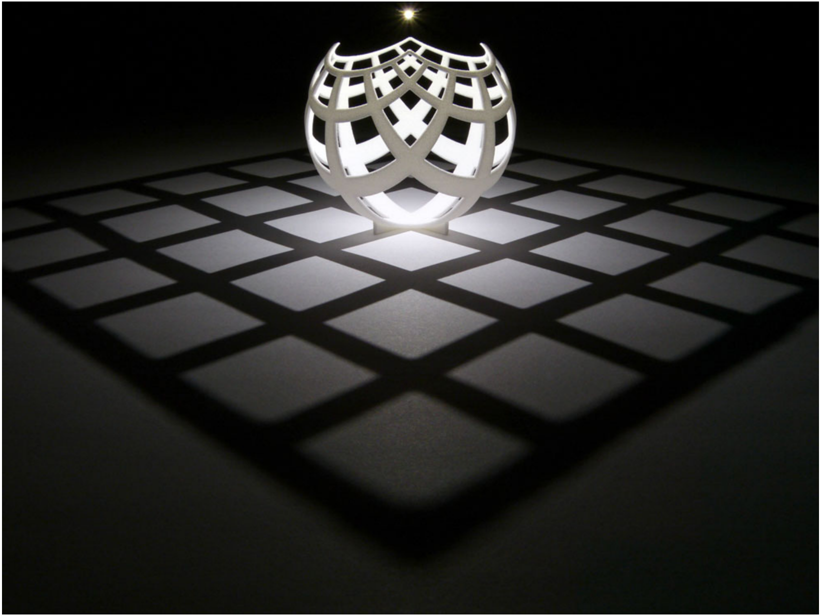
Perspective projection of a hypercube



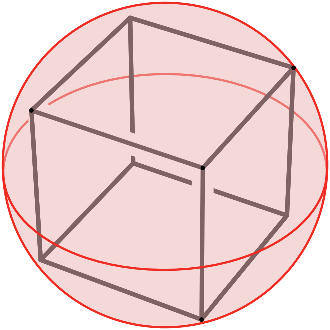
Hypercube A by Bathsheba Grossman.

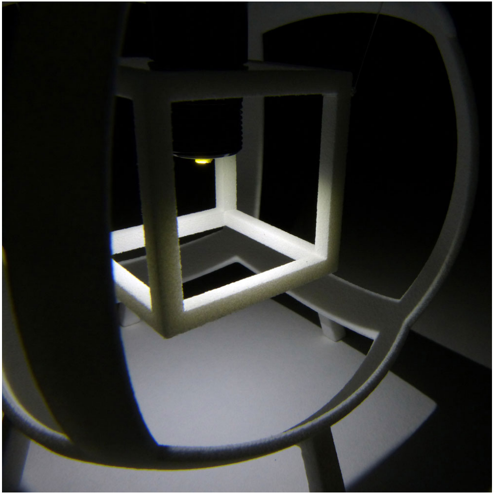
Stereographic projection

Stereographic projection



First radially project the cube to the sphere...





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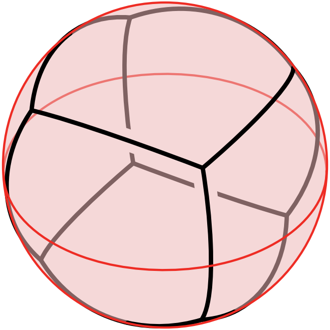
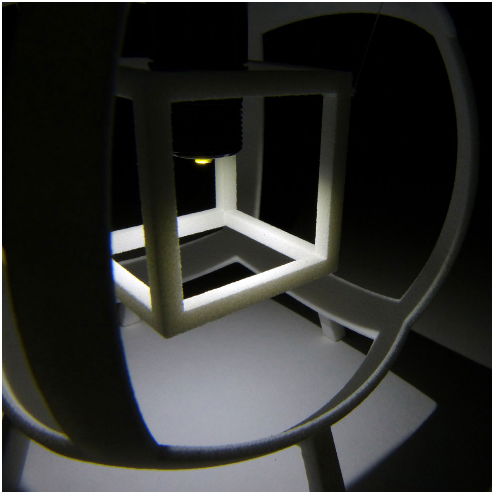
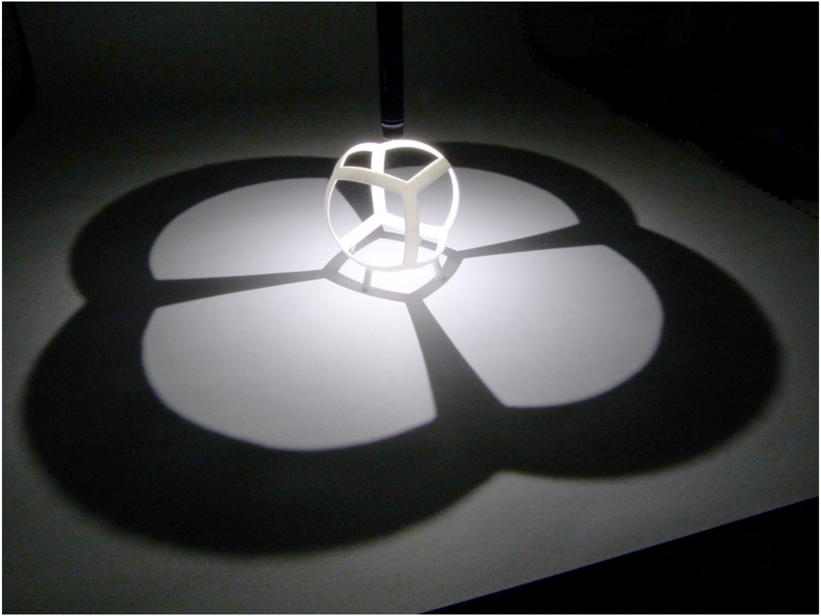




Then stereographically project to the plane



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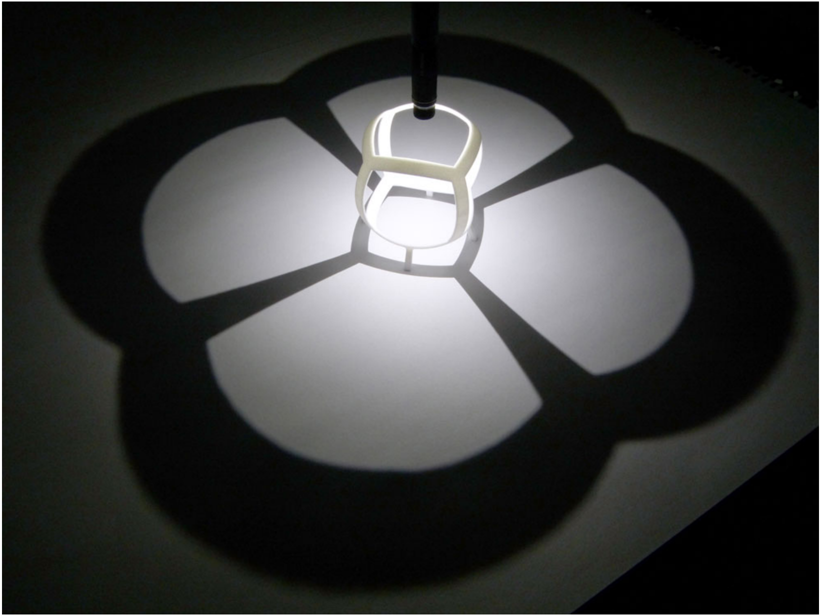
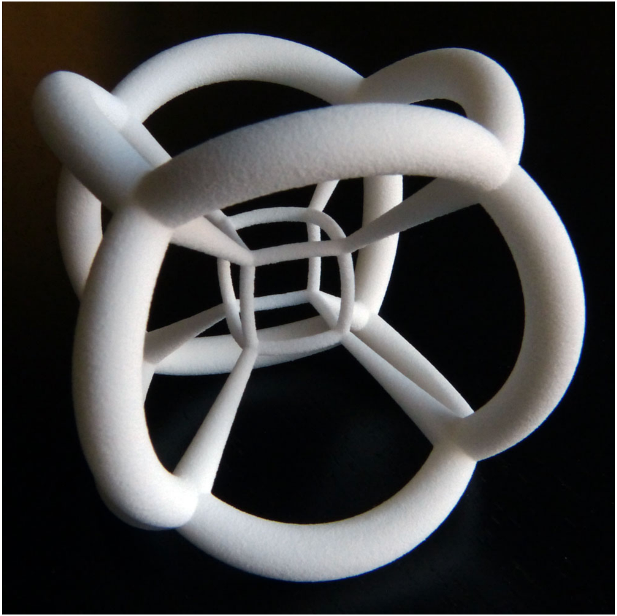


Do the same thing one dimension up to see a hypercube



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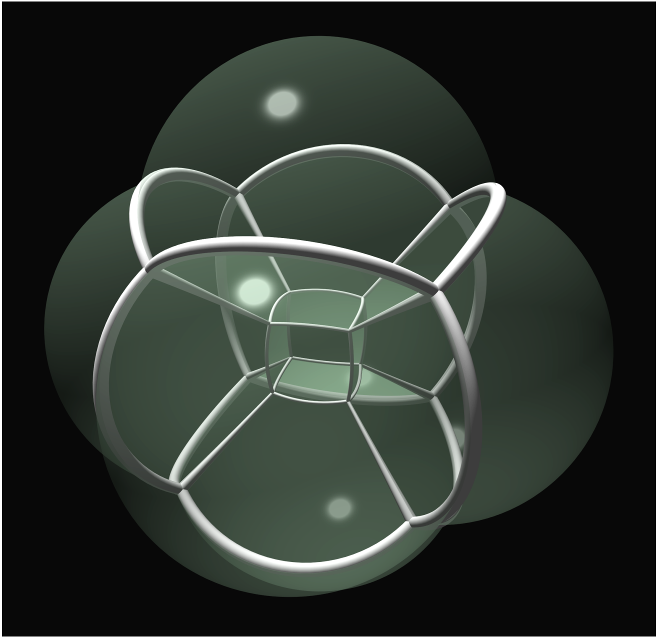
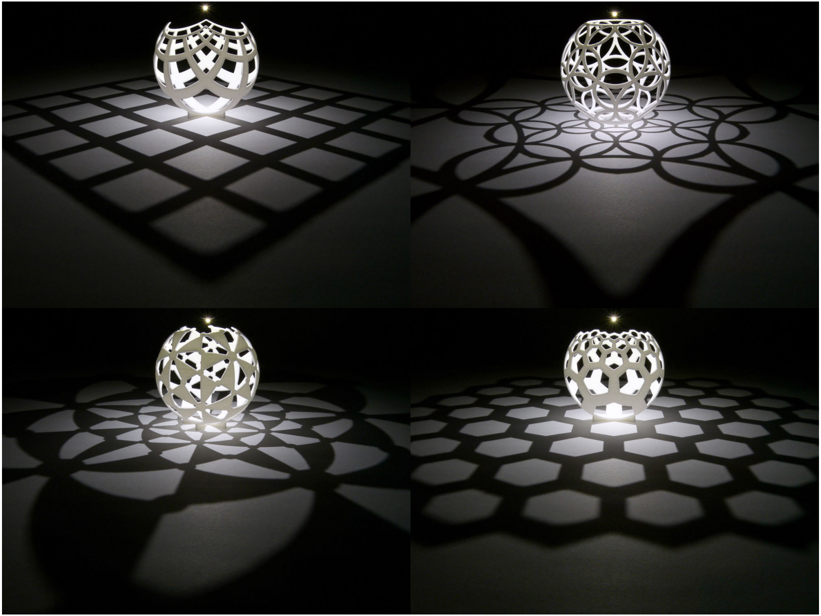


More amazing properties of stereographic projection



Visualising the sphere in 4-dimensional space

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A sphere is the set of points at a fixed distance from a center point.

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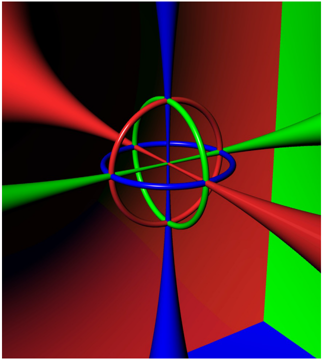
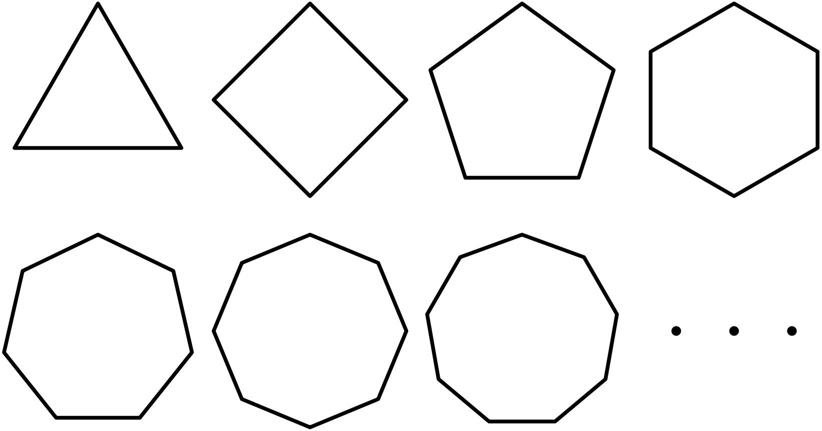
- The sphere in 4-dimensional space is "the same as" 3-dimensional space, plus a point.



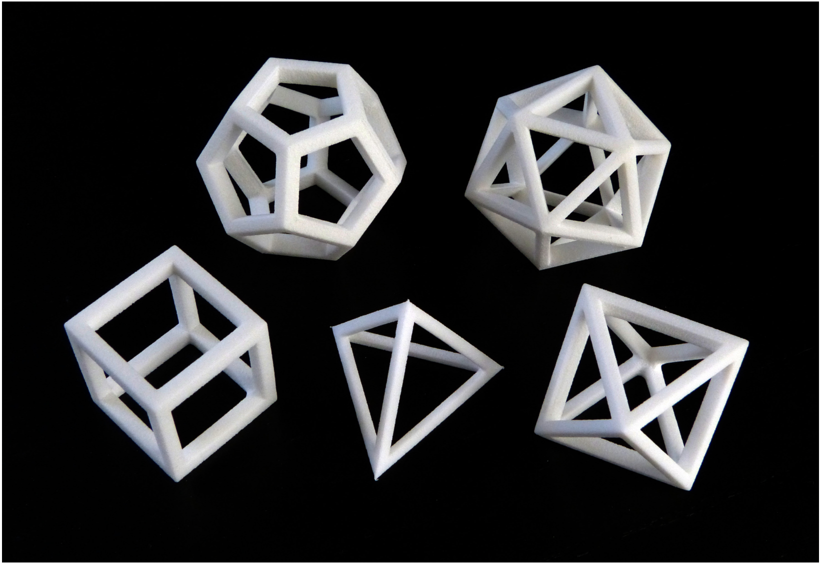
Regular Polytopes

In 2-dimensions: Regular polygons

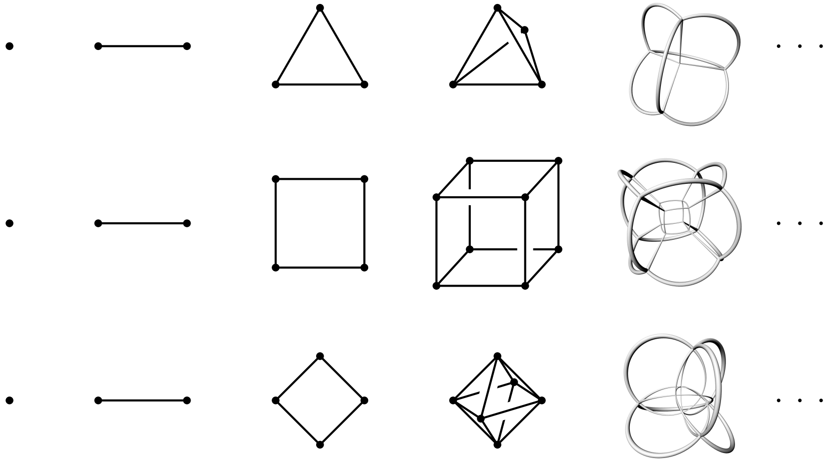


Regular Polytopes

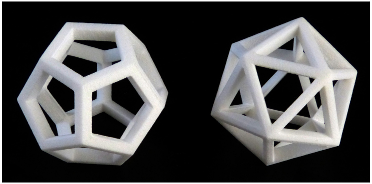
In 3-dimensions: Regular polyhedra



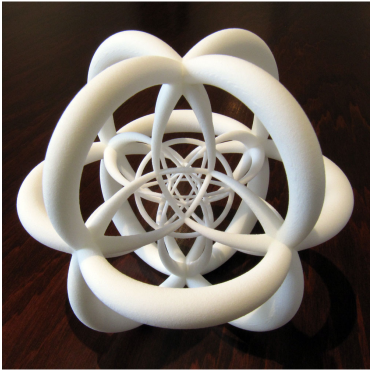
Three families of regular polytopes



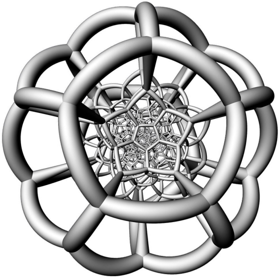
The only exceptions!



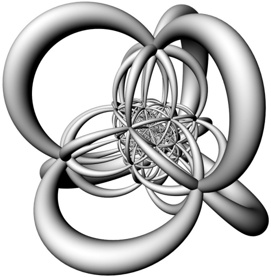
Dodecahedron Icosahedron



24-cell

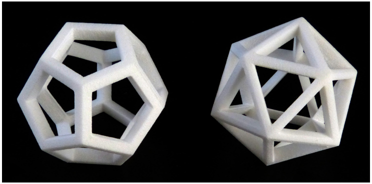


120-cell

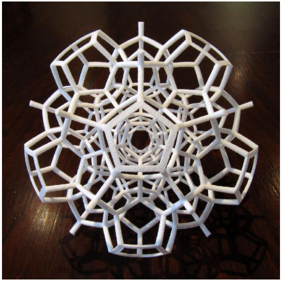


600-cell

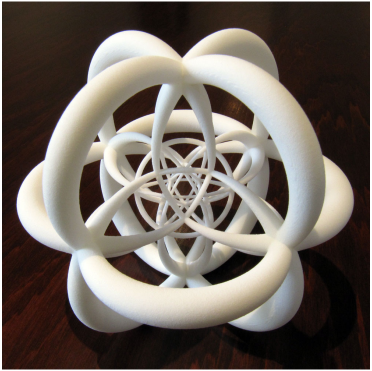
The only exceptions!



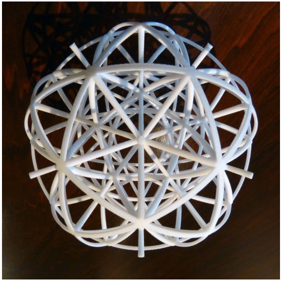
Dodecahedron Icosahedron



Half of a 120-cell



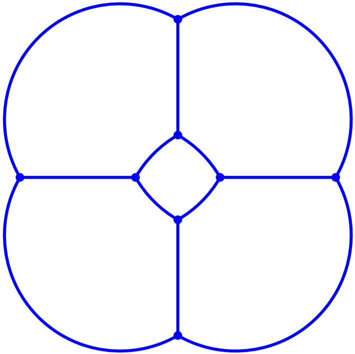
24-cell



Half of a 600-cell

Duality for 3-dimensional polyhedra

vertices	$\leftrightarrow$	faces
edges	$\leftrightarrow$	edges
faces	$\leftrightarrow$	vertices



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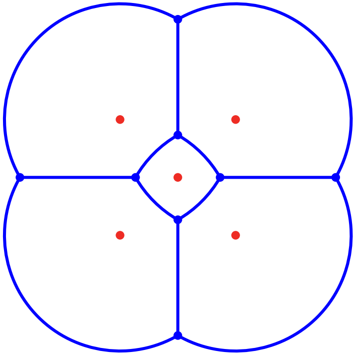

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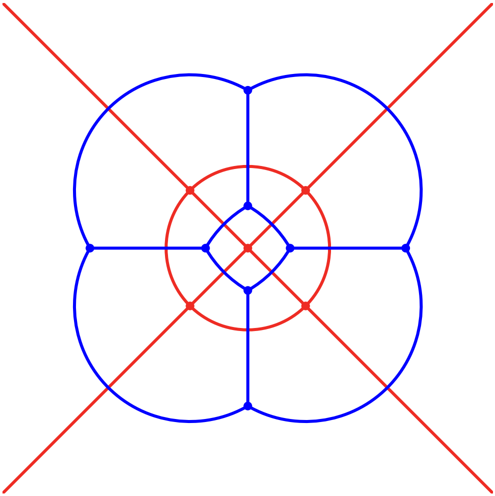

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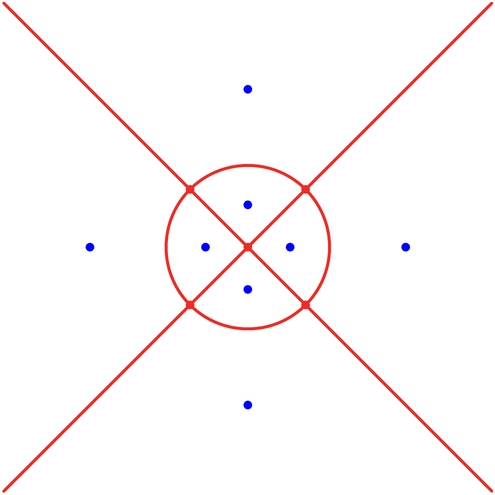

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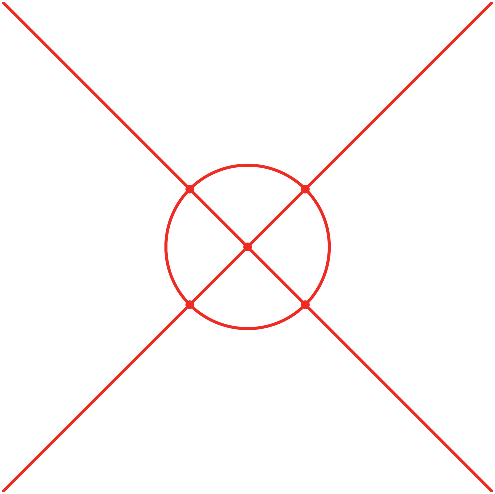
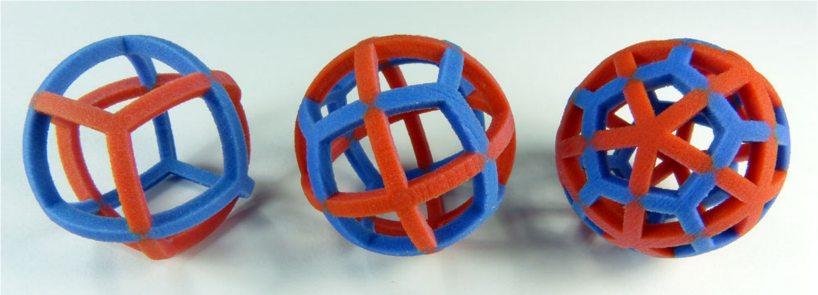
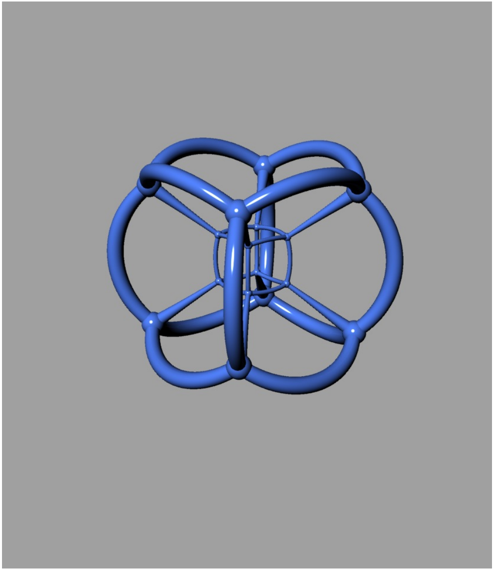


Duality for 3-dimensional polyhedra



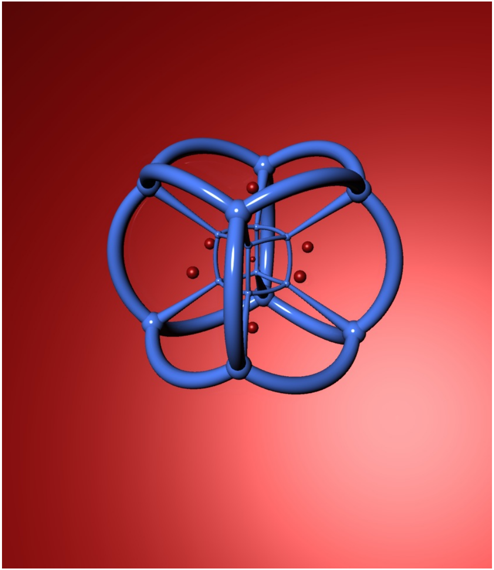
Duality for 4-dimensional polytopes

vertices	$\leftrightarrow$	cells
edges	$\leftrightarrow$	faces
faces	$\leftrightarrow$	edges
cells	$\leftrightarrow$	vertices



Duality for 4-dimensional polytopes

vertices	$\leftrightarrow$	cells
edges	$\leftrightarrow$	faces
faces	$\leftrightarrow$	edges
cells	$\leftrightarrow$	vertices



Duality for 4-dimensional polytopes

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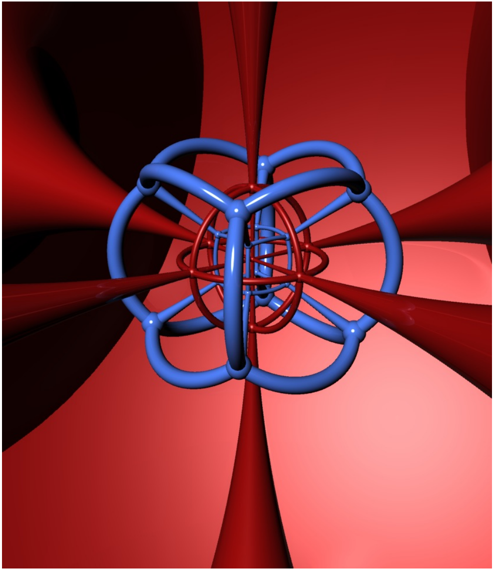

Duality for 4-dimensional polytopes

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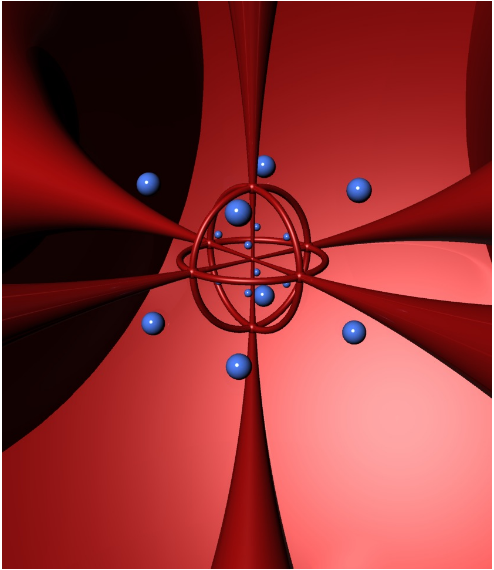

Duality for 4-dimensional polytopes

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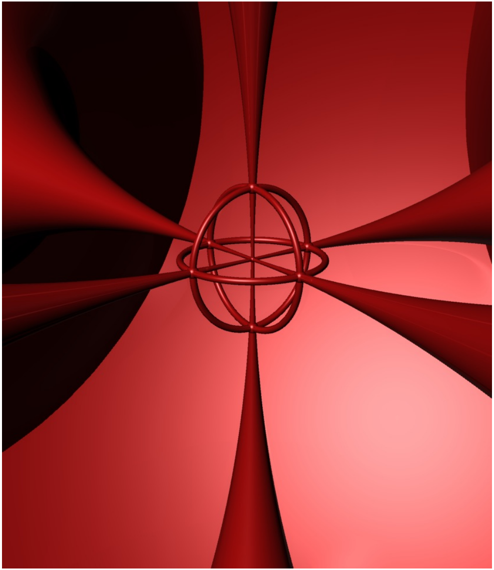
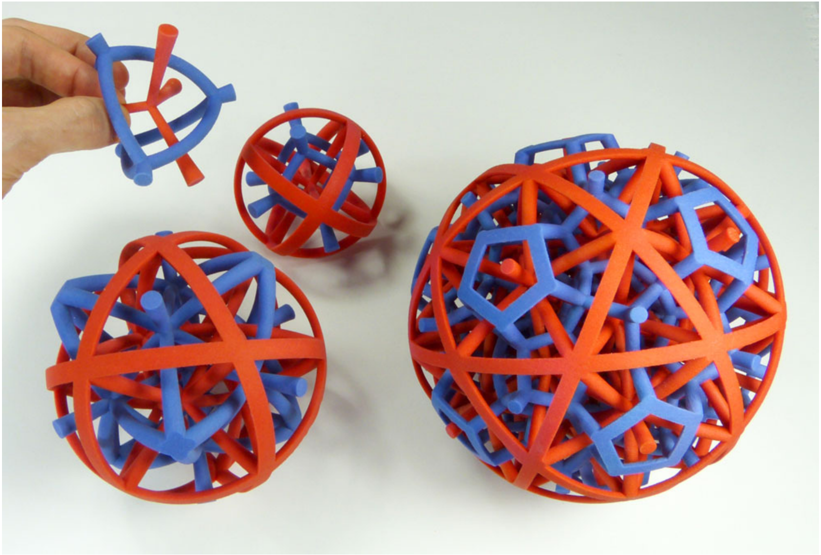
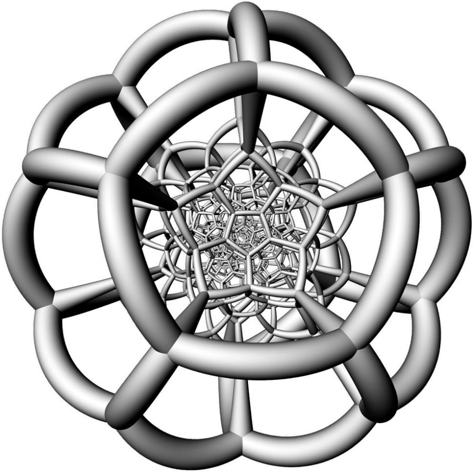


Duality for 4-dimensional polytopes



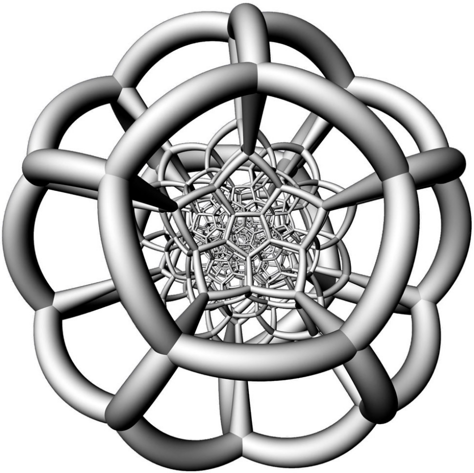
Puzzling the 120-cell



Puzzling the 120-cell

The 120–cell has

- 120 dodecahedral cells,
- 720 pentagonal faces,
- 1200 edges, and
- 600 vertices.



Spherical layers in the 120–cell

One way to understand the 120–cell is to look at the layers of dodecahedra around the central dodecahedron.

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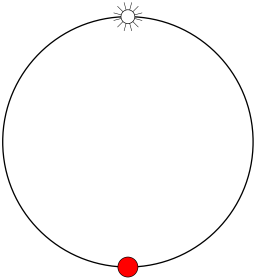
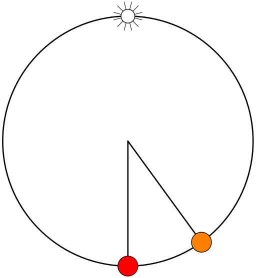
- 1 central dodecahedron



Spherical layers in the 120–cell

One way to understand the 120–cell is to look at the layers of dodecahedra around the central dodecahedron.

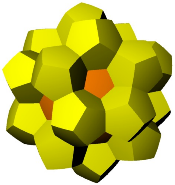
- 1 central dodecahedron
- 12 dodecahedra at angle $\pi/5$

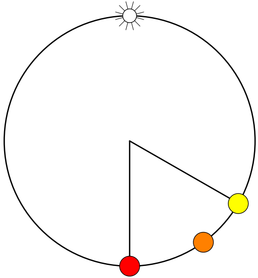


Spherical layers in the 120–cell

One way to understand the 120–cell is to look at the layers of dodecahedra around the central dodecahedron.

- 1 central dodecahedron
- 12 dodecahedra at angle $\pi/5$
- 20 dodecahedra at angle $\pi/3$





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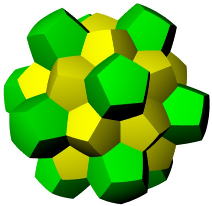
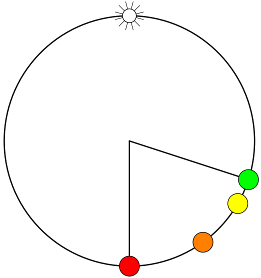
- 1 central dodecahedron
- 12 dodecahedra at angle $\pi/5$
- 20 dodecahedra at angle $\pi/3$
- 12 dodecahedra at angle $2\pi/5$

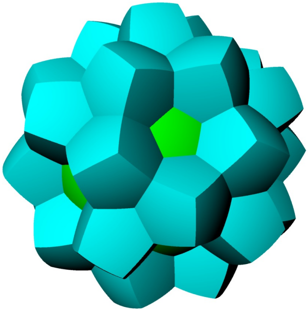


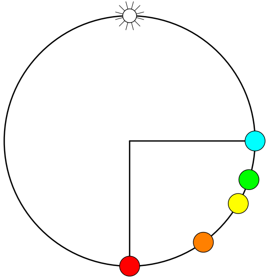


Spherical layers in the 120–cell

One way to understand the 120–cell is to look at the layers of dodecahedra around the central dodecahedron.

- 1 central dodecahedron
- 12 dodecahedra at angle $\pi/5$
- 20 dodecahedra at angle $\pi/3$
- 12 dodecahedra at angle $2\pi/5$
- 30 dodecahedra at angle $\pi/2$





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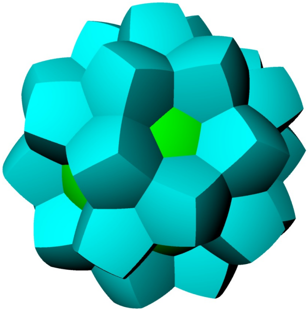
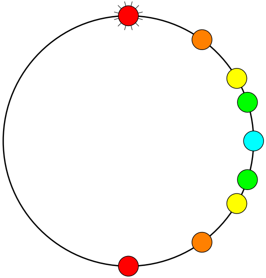
- 30 dodecahedra at angle $\pi/2$

The pattern is mirrored in the last four layers.

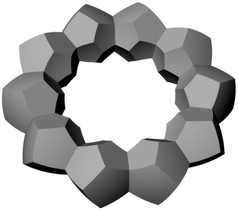
$1+12+20+12+30+12+20+12+1 = 120$





Rings of dodecahedra in the 120–cell

A second way to understand the 120–cell is by making it up out of rings of 10 dodecahedra.



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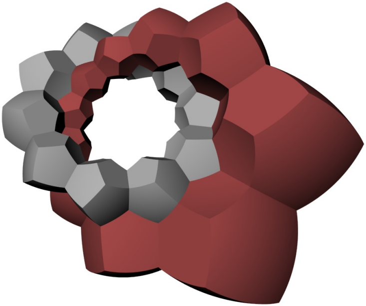

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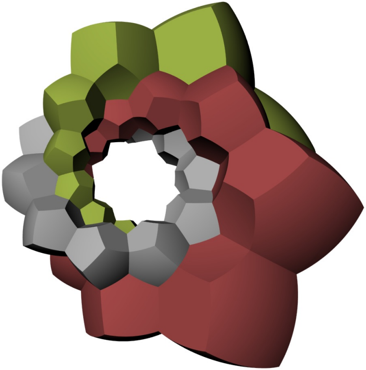

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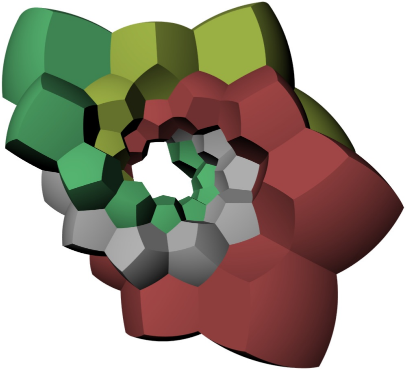

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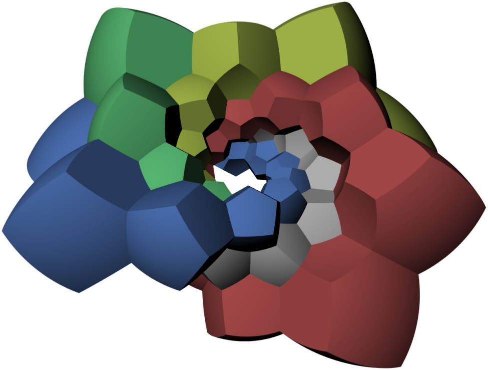
The rings wrap around each other.

Each ring is surrounded by five others.



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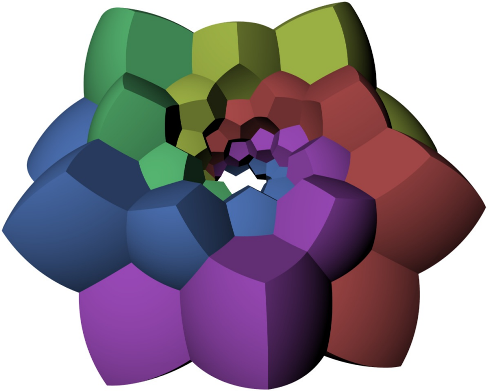

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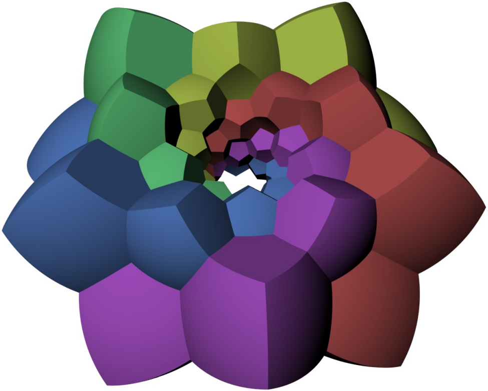

These six rings make up half of the 120–cell. The other half consists of five more rings that wrap around these, and one more ring "dual" to the original grey one.

$$1 + 5 + 5 + 1 = 12 = 120/10$$

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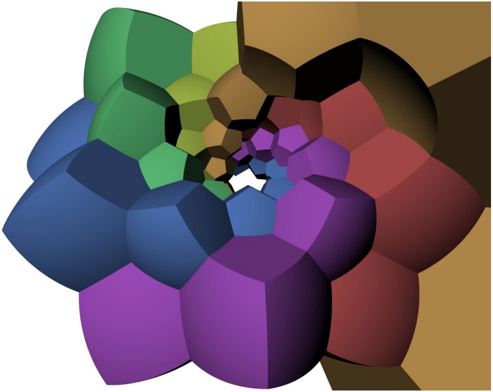

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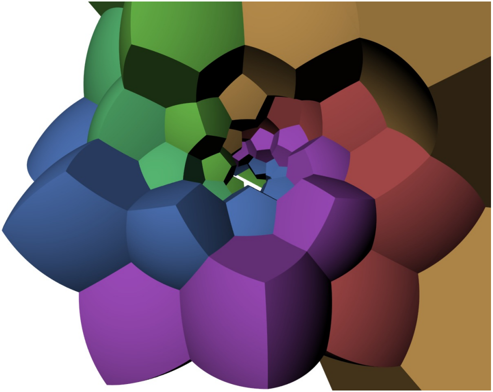

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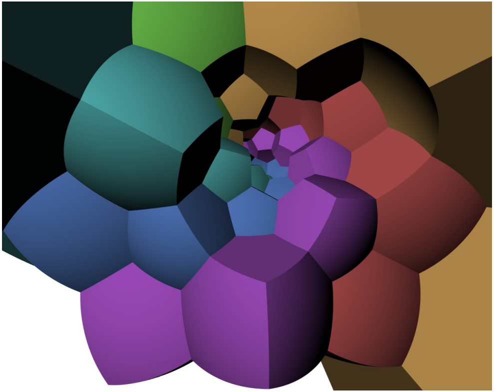

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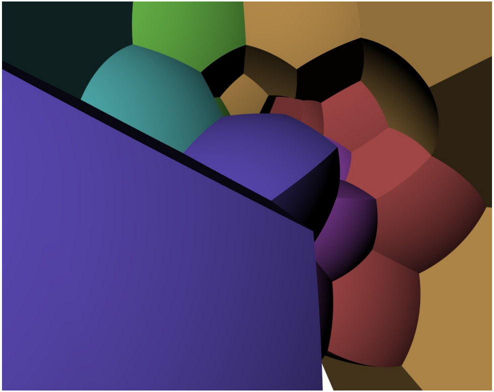

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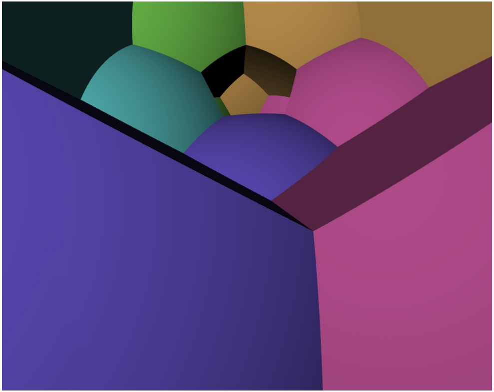

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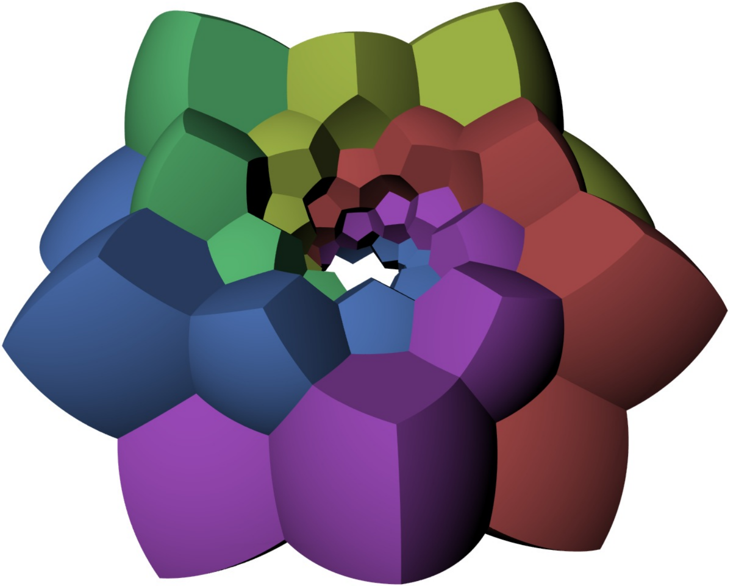

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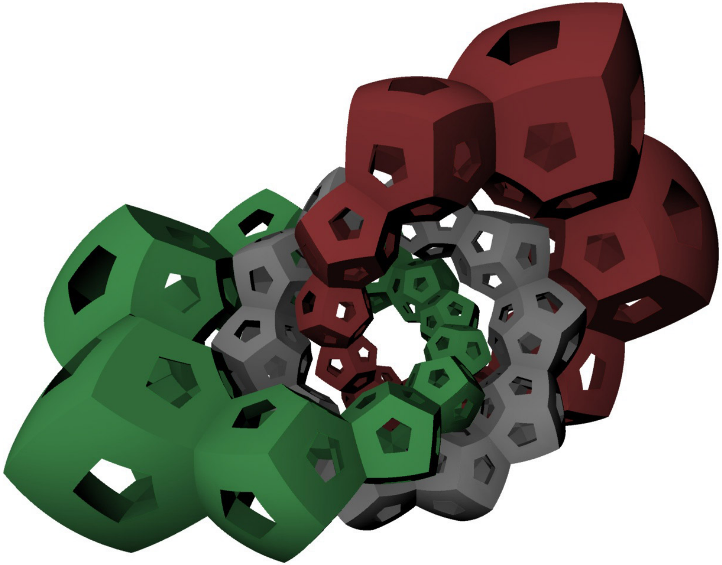
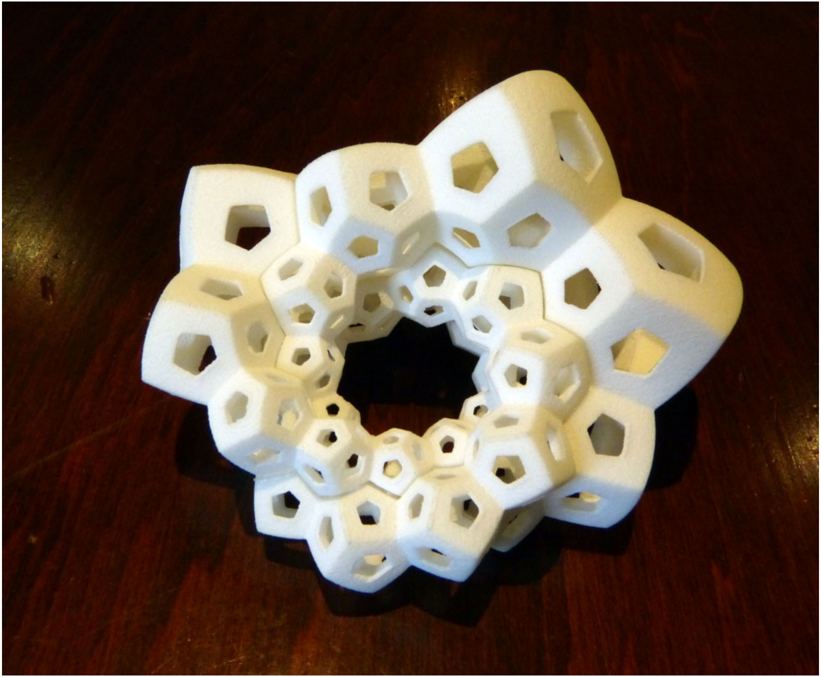
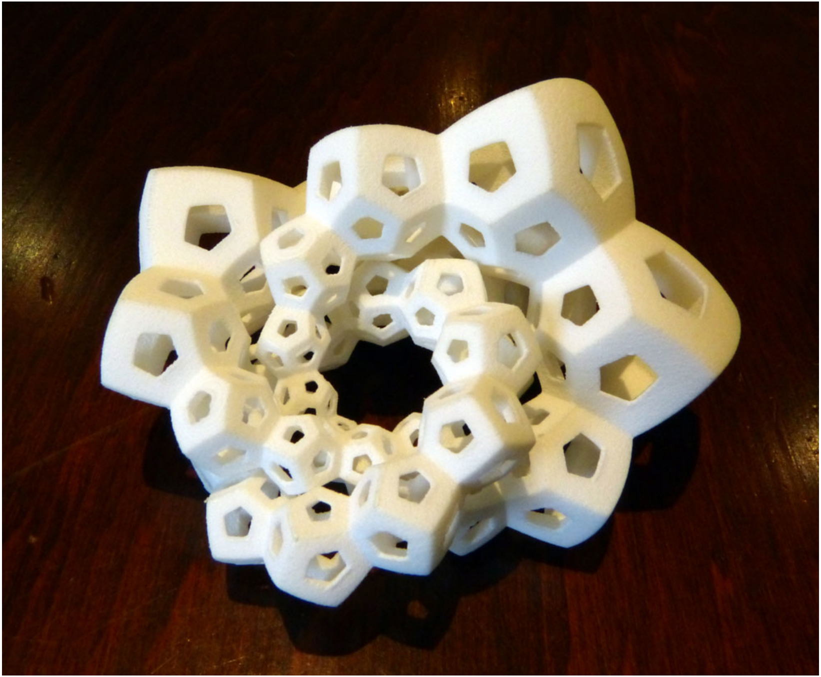
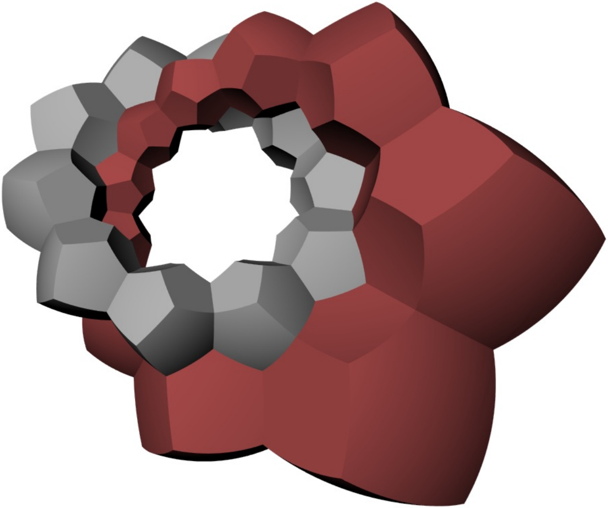


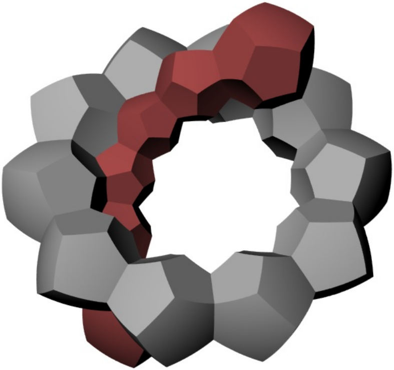




To print all five we use a trick...



To print all five we use a trick... don't print the whole ring. We call part of a ring a rib.



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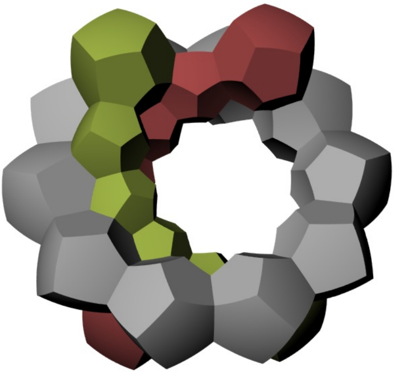

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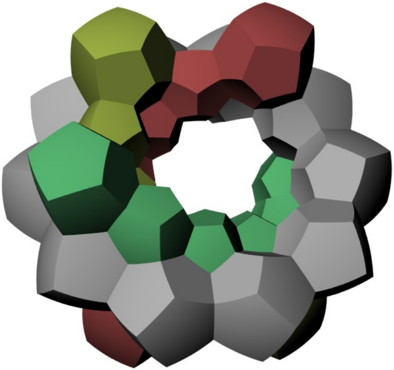

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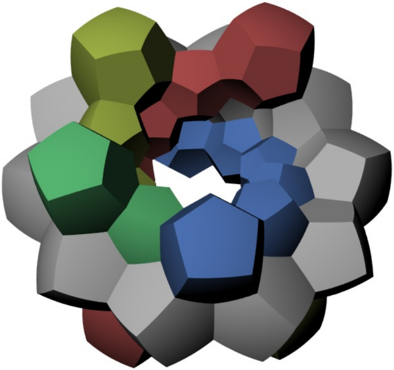

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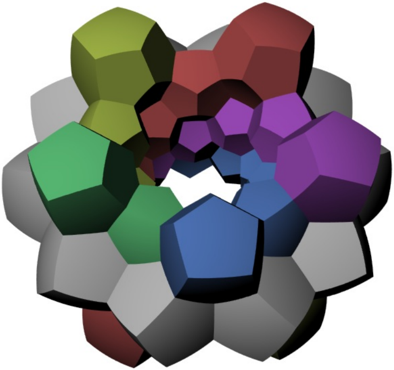

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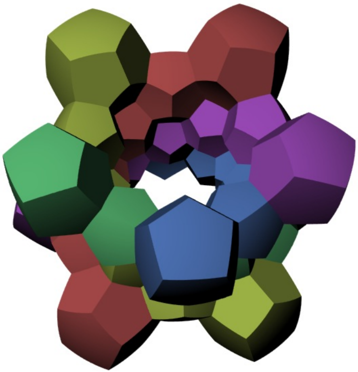

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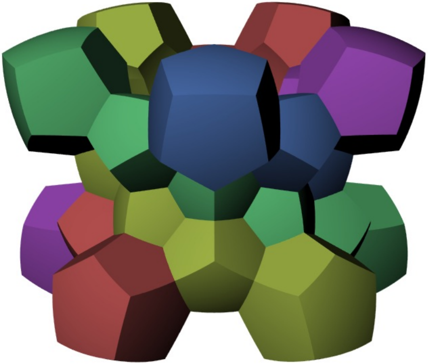
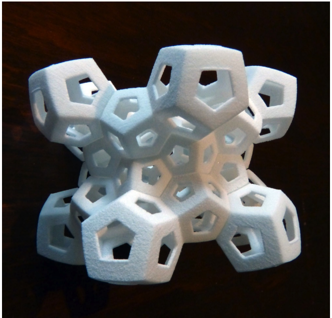
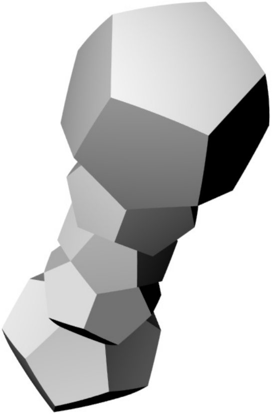


Dc30 Ring puzzle






Another decomposition, with even shorter ribs.



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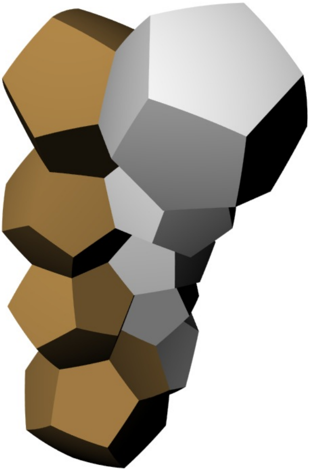

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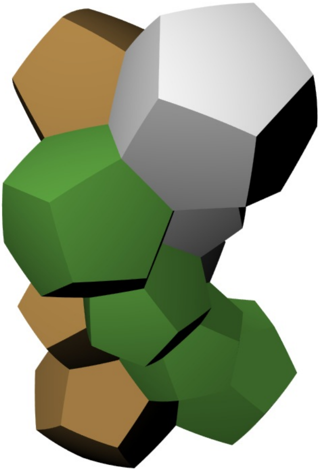

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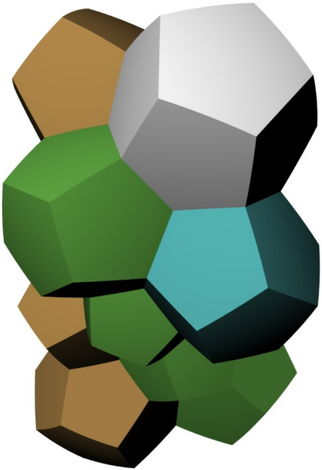

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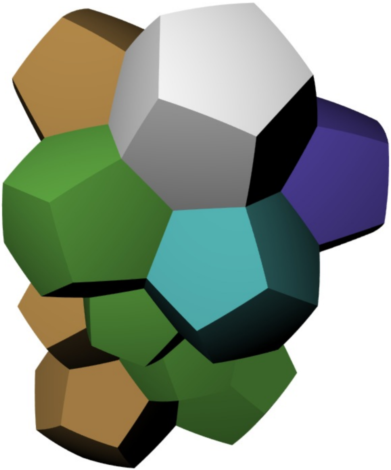

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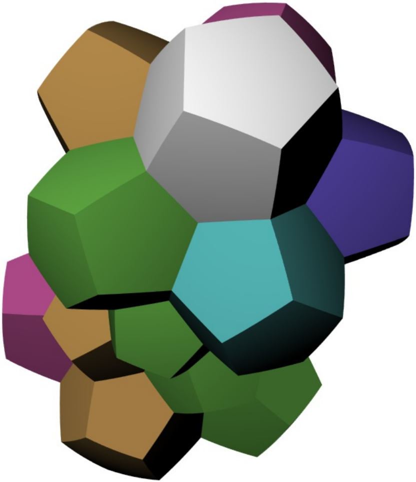

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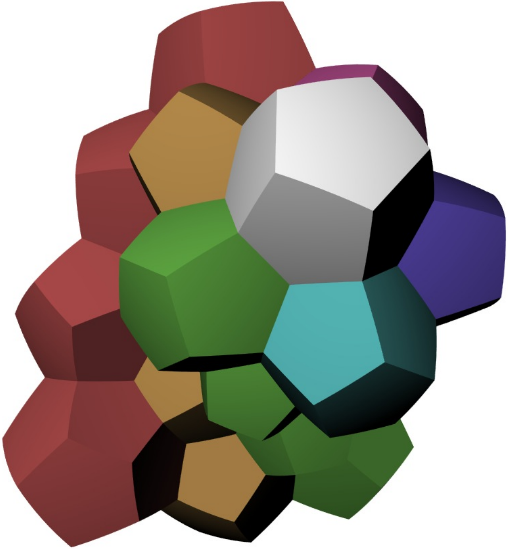

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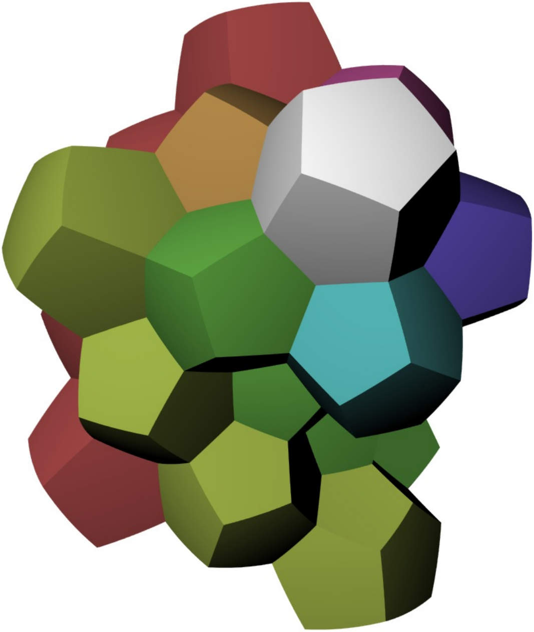

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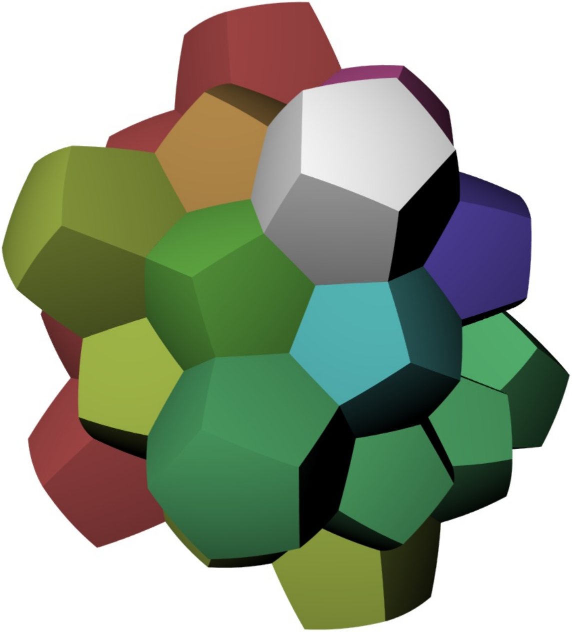

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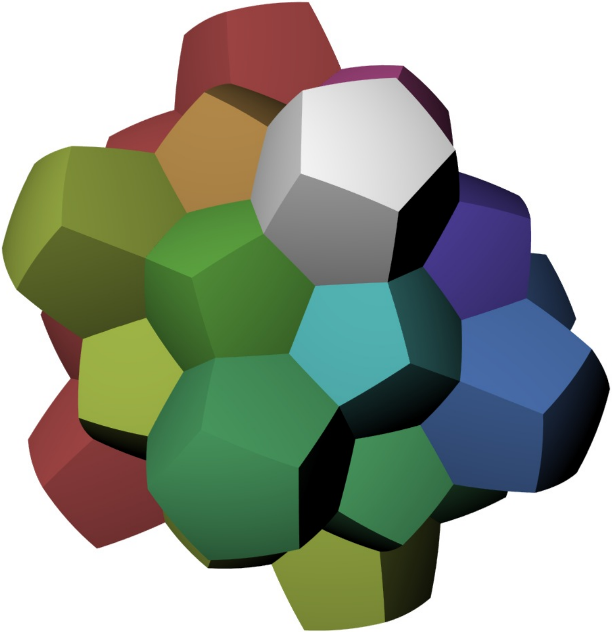

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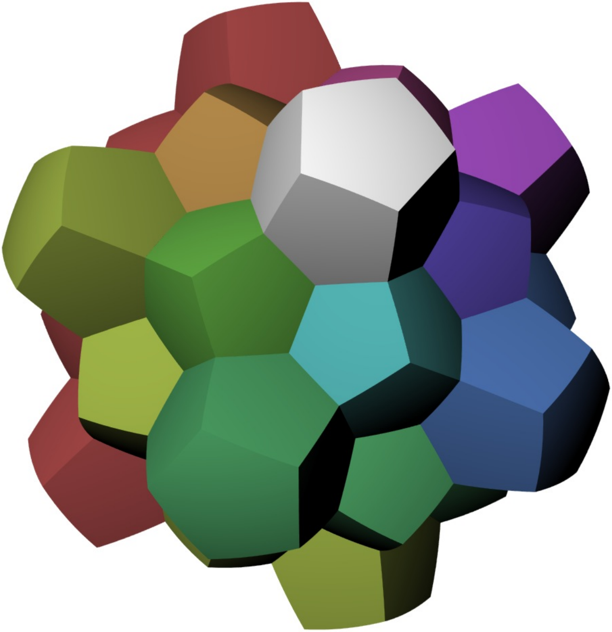
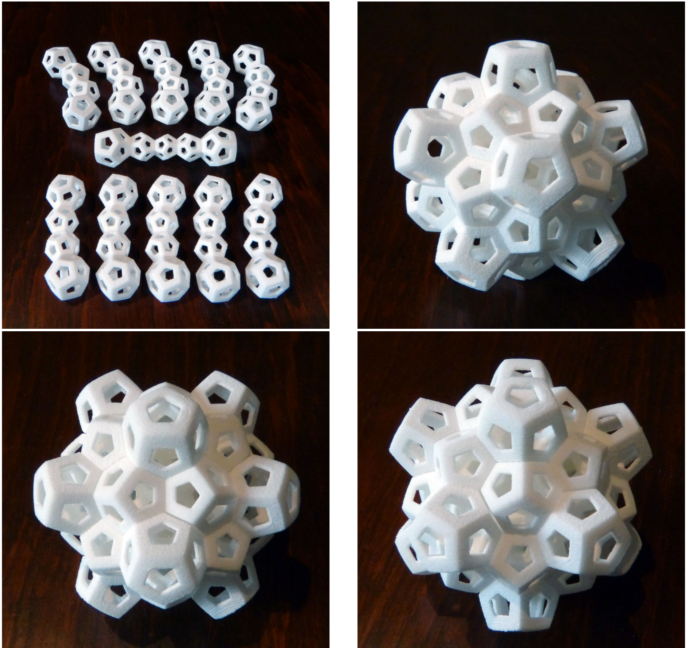
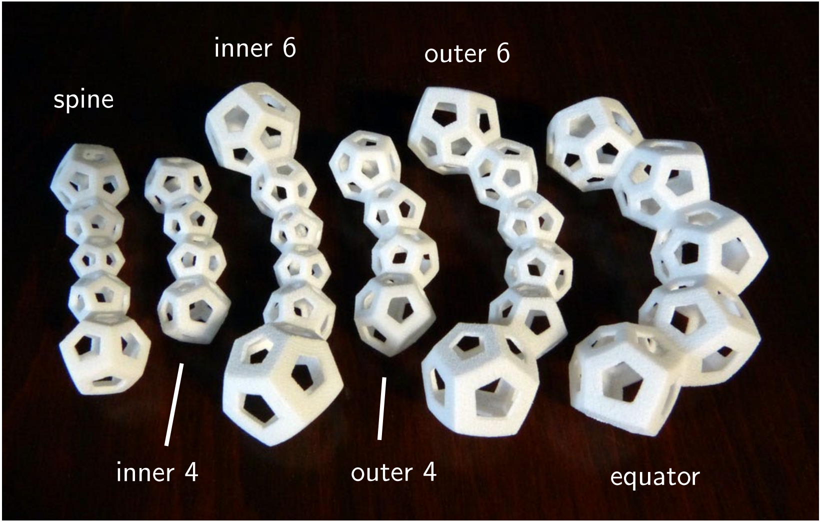


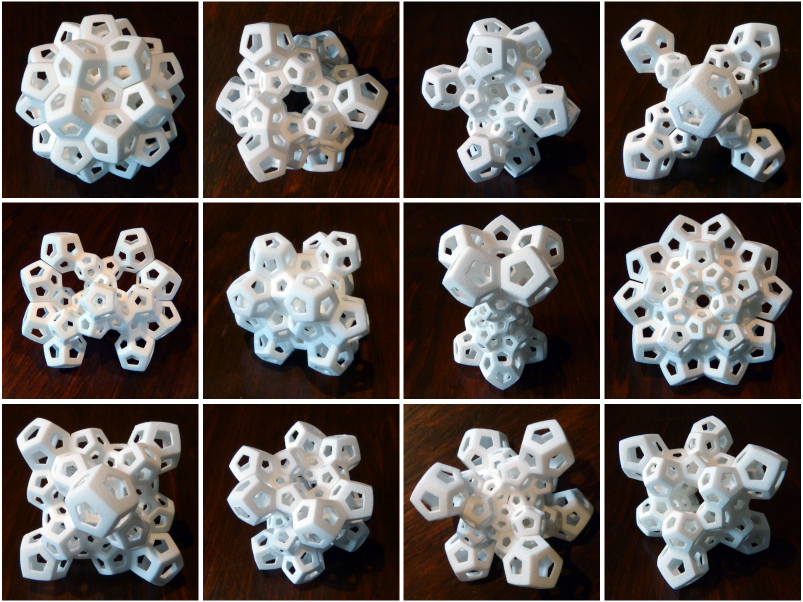
Dc45 Meteor puzzle



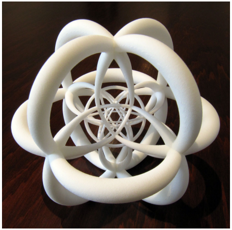
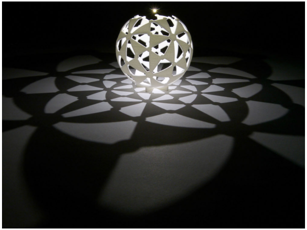
Six kinds of ribs



These make many puzzles, which we collectively call Quintessence.



Thanks!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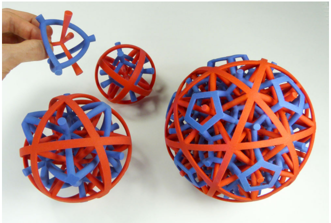

http://homepages.warwick.ac.uk/~masgar/