

You have 40 minutes to complete the test. Please explain each step of your derivations and state all the assumptions employed. Note that different problems can give you different points. Maximum for the test is 10 points.

Problem 1

Some practitioners of econometrics consider regressions with transformed variables. For example, if the original model specification is:

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

the revised specification is:

$$Y_i^* = \beta_1^* + \beta_2^* X_i^* + v_i$$

where:

$$Y_i^* = \frac{Y_i - a}{c} \quad \text{and} \quad X_i^* = \frac{X_i - b}{d}.$$

Knowing that a, b, c, d are some constants and $c \neq 0, d \neq 0$, express the OLS estimators $\hat{\beta}_1^*$, $\hat{\beta}_2^*$ in terms of the OLS estimators $\hat{\beta}_1, \hat{\beta}_2$ [2 points].

Problem 2

A novice econometrician estimated a classical linear model

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

using 4 observations and obtained the following results:

Y_i	3	7	9	10
X_i	3	XXX	XXX	1
\hat{Y}_i	4	8	7	11

Help this econometrician find the estimates of the regression coefficients $\hat{\beta}_1$ and $\hat{\beta}_2$ [1 point] and restore the missing values in table [1 point]. Can you confirm that these estimations were made using the OLS method? [1 point]

Problem 3

The output below gives the result of regressing *WAGE*, individual monthly wage measured in thousand rubles, on *TENURE*, total years of work experience a person has.

. regress WAGE TENURE

Source	SS	df	MS	Number of obs	=	100
Model	71331.4826	1	71331.4826	F(1, 98)	=	122.80
Residual	56924.3411	98	580.860624	Prob > F	=	0.0000
				R-squared	=	XXX
				Adj R-squared	=	XXX
Total	128255.824	99	1295.51337	Root MSE	=	24.101

WAGE	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
TENURE	2.168977	.1957268	XXX	XXX	1.780564	2.557391
_cons	38.34598	4.65056	XXX	XXX	29.11709	47.57486

Unfortunately, some things are missing. Looking at this output, do the following tasks:

1. Give an interpretation of the coefficients estimations [1 point]
2. Tell whether coefficients are statistically significant or not (if necessary, you may assume that these hypotheses are tested at a 5% significance level, $t_{crit} \approx 2$) [1 point]
3. Find the R^2 value [0.5 points]
4. Explain in your own words what the R^2 value shows [0.5 points]

Problem 4

We have a linear model

$$Y_i = \beta X_i + u_i$$

where β is a fixed parameter, u_i is a disturbance term that is independently and identically distributed with expected value 0 and population variance σ_u^2 and $i = 1, \dots, n$ is the observation index.

Derive the OLS $\hat{\beta}$ estimator. Answers without a solution will not be accepted. You need to provide a full solution. [2 points]