You have 40 minutes to complete the test. Please explain each step of your derivations and state all the assumptions employed. Note that different problems can give you different points. Maximum for the test is 10 points.

## **Problem 1**

You are given the following data on 2,000 respondents:

- hourly earnings (Y);
- educational attainment (years) (EDUC);
- total expenditure in year (TE);
- value of the respondent's house (H);
- mother's and father's educational attainment (MEDUC and FEDUC);
- Weight (W);
- Sex (S);
- whether the main job was in the government sector or the private (G).

As a policy analyst, you are asked to investigate whether there is a difference in estimated impact of education on earnings for different genders.

Write one equation you can estimate to test this hypothesis. Explain why you chose these variables, type of model (nonlinear or linear). [1.5 points]

Tell whether a Chow test can be used to test this hypothesis. If not, explain why. If yes, show how you would perform this test. [1 point]

### **Problem 2**

An econometrician estimated two models with 100 observations (s.e. in brackets):

$$Log\hat{Y} = -0.5 + 3.1 Log X_1 + 0.4 X_2$$

$$(-0.2) \quad (0.3) \qquad (0.1)$$

Assuming the disturbance term has a standard normal distribution, calculate the 95 per cent confidence intervals for coefficients estimates for  $Log X_1$  and  $X_2$ . Write an interpretation of one of the intervals. [1 point]

$$\hat{Y} = -0.5 + 2X_1 + 3.1X_2 + 3D + 2.1X_1 * D$$
 where D is dummy variable  $(-0.2)$   $(0.5)$   $(2.4)$   $(1.3)$   $(1.0)$ 

Give interpretation of the coefficients estimates for **both** models. [2 points]

#### **Problem 3**

An econometrician gained some data from a university and estimated a model based on 300 observations:

$$GPA_i = \beta_0 + \beta_1 * CLASS_i + \varepsilon_i$$
,

where GPA is the average grade of a student, CLASS is the percentage of attended classes,  $\varepsilon$  is a disturbance term.

The econometrician thinks that talent also must influence the average grade a student gets, however, this factor is impossible to observe and measure. Thus, the researcher supposes that there is an endogeneity problem since the effect of talent goes to the disturbance term and talent might correlate with the percentage of attended classes.

What will happen with the coefficients estimates in case of the endogeneity problem? [0.5 points]

The researcher discovered that the university could provide him with the data on the distance from a student's house to the university. Explain why this distance factor can be used as an instrumental variable. [1 point]

# **Problem 4**

An econometrician estimated the following model with 100 observations:

$$E = \beta_0 + \beta_1 X + \beta_2 Z + u$$

Suspecting that the regression was subject to heteroskedasticity where  $\sigma_u = f(Z)$ , the researcher prepared the following data:

RSS from a regression based on the 50 observations with the smallest values of Z	200
RSS from a regression based on the 50 observations with the largest values of Z	250
R <sup>2</sup> from auxiliary regression (squared errors from basic regression – dependent variable	0,3
and $X, X^2, Z, Z^2, X*Z$ - independent variables)	

Perform the Goldfeld–Quandt test and White test and state your conclusions. [2 points]

How do you deal with heteroskedasticity? Which methods do you know? Write at least two methods [1 point]

# Probabilities for the standard normal distribution

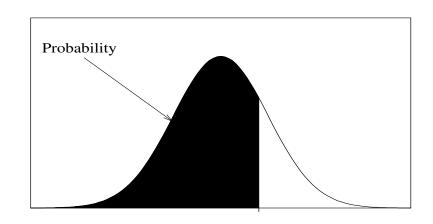


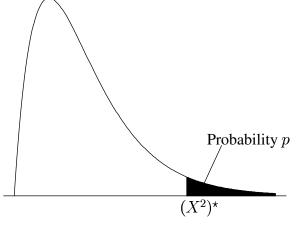
Table entry for z is the probability lying to the left of z

Z

0.1 0. 0.2 0. 0.3 0. 0.4 0. 0.5 0. 0.6 0. 0.7 0.	.00 0.5000 0.5398 0.5793 0.6179 0.6554 0.6915	.01 0.5040 0.5438 0.5832 0.6217 0.6591 0.6950	.02 0.5080 0.5478 0.5871 0.6255 0.6628	.03 0.5120 0.5517 0.5910 0.6293	.04 0.5160 0.5557 0.5948	.05 0.5199 0.5596	.06 0.5239 0.5636	.07 0.5279	.08 0.5319	.09 0.5359
0.1 0. 0.2 0. 0.3 0. 0.4 0. 0.5 0. 0.6 0. 0.7 0.	0.5398 0.5793 0.6179 0.6554 0.6915	0.5438 0.5832 0.6217 0.6591	0.5478 0.5871 0.6255	0.5517 0.5910	0.5557				0.5519	0.5559
0.2 0. 0.3 0. 0.4 0. 0.5 0. 0.6 0. 0.7 0.	0.5793 0.6179 0.6554 0.6915	0.5832 0.6217 0.6591	0.5871 0.6255	0.5910		0.5590		0.5675	0.5714	0.5753
0.3 0. 0.4 0. 0.5 0. 0.6 0. 0.7 0.	0.6179 0.6554 0.6915	0.6217 0.6591	0.6255		0.3948	0.5987		0.5675	0.5714	
0.4 0. 0.5 0. 0.6 0. 0.7 0.	0.6554	0.6591		0.0293	0.6221		0.6026 0.6406	0.6064	0.6103	0.6141 0.6517
0.5 0. 0.6 0. 0.7 0.	.6915		U.DDZA		0.6331	0.6368		0.6443	0.6480	
0.6 0. 0.7 0.		1109111		0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.7 0.	1.7237		0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
	7500	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
$\sim$	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
	.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
	.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
	.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
	.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
	.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
	.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
	.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
	.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0 0.	.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
	.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2 0.	.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3 0.	.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4 0.	.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5 0.	.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6 0.	.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7 0.	.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8 0.	.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9 0.	.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0 0.	.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1 0.	.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2 0.	.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3 0.	.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4 0.	.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

# Probabilities for the $\chi^2$ -distribution

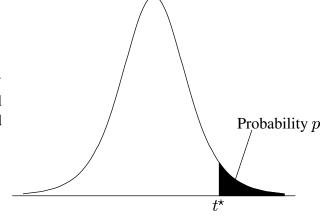
Table entry for p is the point  $(X^2)^*$  with probability p lying above it



	Tail probability $p$											
df	.25	.2	.15	.1	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.52	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.69
80	88.13	90.41	93.11	96.58	101.88	106.63	108.07	112.33	116.32	120.10	124.84	128.26
100	109.14	111.67	114.66	118.50	124.34	129.56	131.14	135.81	140.17	144.29	149.45	153.17

### Probabilities for the t-distribution

Table entry for p and C is the point  $t^*$  with probability p lying above it and probability C lying between  $-t^*$  and  $t^*$ 



	Tail probability p												
df	.25	.2	.15	.1	.05	.025	.02	.01	.005	.0025	.001	.0005	
1	1.000	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	127.321	318.309	636.619	
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599	
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924	
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610	
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869	
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959	
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408	
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041	
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781	
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587	
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437	
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318	
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221	
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140	
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073	
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015	
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965	
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922	
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883	
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850	
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819	
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792	
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768	
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745	
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725	
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707	
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690	
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674	
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659	
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646	
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551	
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496	
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460	
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416	
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390	
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300	
$\infty$	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291	
	50%	60%	70 %	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%	
						Cont	fidence le	evel C					

F(m,n)-distribution critical values for 5% significance level

m\n	1	2	3	4	5	10	20	30	40	50	60	70	80	90	100
1	161	18,5	10,1	7,71	6,61	4,96	4,35	4,17	4,08	4,03	4,00	3,98	3,96	3,95	3,94
2	199	19,0	9,55	6,94	5,79	4,10	3,49	3,32	3,23	3,18	3,15	3,13	3,11	3,10	3,09
3	216	19,2	9,28	6,59	5,41	3,71	3,10	2,92	2,84	2,79	2,76	2,74	2,72	2,71	2,70
4	225	19,2	9,12	6,39	5,19	3,48	2,87	2,69	2,61	2,56	2,53	2,50	2,49	2,47	2,46
5	230	19,3	9,01	6,26	5,05	3,33	2,71	2,53	2,45	2,40	2,37	2,35	2,33	2,32	2,31
10	242	19,4	8,79	5,96	4,74	2,98	2,35	2,16	2,08	2,03	1,99	1,97	1,95	1,94	1,93
20	248	19,4	8,66	5,80	4,56	2,77	2,12	1,93	1,84	1,78	1,75	1,72	1,70	1,69	1,68
30	250	19,5	8,62	5,75	4,50	2,70	2,04	1,84	1,74	1,69	1,65	1,62	1,60	1,59	1,57
40	251	19,5	8,59	5,72	4,46	2,66	1,99	1,79	1,69	1,63	1,59	1,57	1,54	1,53	1,52
50	252	19,5	8,58	5,70	4,44	2,64	1,97	1,76	1,66	1,60	1,56	1,53	1,51	1,49	1,48
60	252	19,5	8,57	5,69	4,43	2,62	1,95	1,74	1,64	1,58	1,53	1,50	1,48	1,46	1,45
70	252	19,5	8,57	5,68	4,42	2,61	1,93	1,72	1,62	1,56	1,52	1,49	1,46	1,44	1,43
80	253	19,5	8,56	5,67	4,41	2,60	1,92	1,71	1,61	1,54	1,50	1,47	1,45	1,43	1,41
90	253	19,5	8,56	5,67	4,41	2,59	1,91	1,70	1,60	1,53	1,49	1,46	1,44	1,42	1,40
100	253	19,5	8,55	5,66	4,41	2,59	1,91	1,70	1,59	1,52	1,48	1,45	1,43	1,41	1,39

# F(m,n)-distribution critical values for 95% significance level

m\n	1	2	3	4	5	10	20	30	40	50	60	70	80	90	100
1	0,01	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
2	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05
3	0,10	0,10	0,11	0,11	0,11	0,11	0,12	0,12	0,12	0,12	0,12	0,12	0,12	0,12	0,12
4	0,13	0,14	0,15	0,16	0,16	0,17	0,17	0,17	0,17	0,18	0,18	0,18	0,18	0,18	0,18
5	0,15	0,17	0,18	0,19	0,20	0,21	0,22	0,22	0,22	0,23	0,23	0,23	0,23	0,23	0,23
10	0,20	0,24	0,27	0,29	0,30	0,34	0,36	0,37	0,38	0,38	0,38	0,38	0,38	0,39	0,39
20	0,23	0,29	0,32	0,35	0,37	0,43	0,47	0,49	0,50	0,51	0,51	0,52	0,52	0,52	0,52
30	0,24	0,30	0,34	0,37	0,39	0,46	0,52	0,54	0,56	0,57	0,57	0,58	0,58	0,59	0,59
40	0,24	0,31	0,35	0,38	0,41	0,48	0,54	0,57	0,59	0,60	0,61	0,62	0,62	0,63	0,63
50	0,25	0,31	0,36	0,39	0,42	0,49	0,56	0,59	0,61	0,63	0,63	0,64	0,65	0,65	0,66
60	0,25	0,32	0,36	0,40	0,42	0,50	0,57	0,61	0,63	0,64	0,65	0,66	0,67	0,67	0,68
70	0,25	0,32	0,37	0,40	0,43	0,51	0,58	0,62	0,64	0,65	0,66	0,67	0,68	0,69	0,69
80	0,25	0,32	0,37	0,40	0,43	0,51	0,59	0,62	0,65	0,66	0,67	0,68	0,69	0,70	0,70
90	0,25	0,32	0,37	0,40	0,43	0,52	0,59	0,63	0,65	0,67	0,68	0,69	0,70	0,71	0,71
100	0,25	0,32	0,37	0,41	0,43	0,52	0,60	0,64	0,66	0,68	0,69	0,70	0,71	0,71	0,72