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total: 10

1	2	3	4
2	3	3	2

Problem 1

$$y_i = \beta_1 + \beta_2 X_i + u_i$$

$$y_i^* = \beta_1^* + \beta_2^* X_i^* + v_i$$

$$y_i^* = \frac{y_i - a}{c} \quad X_i^* = \frac{X_i - b}{d}$$

We know that OLS ests are:

$$\hat\beta_1 = \bar y - \hat\beta_2 \bar X \; ; \; \hat\beta_2 = \frac{\sum(X_i - \bar X)(y_i - \bar y)}{\sum(X_i - \bar X)^2}.$$

From OLS and *:

$$\hat\beta_2^* = \frac{\sum(X_i^* - \bar X^*)(y_i^* - \bar y^*)}{\sum(X_i^* - \bar X^*)^2}$$ +

Lets note that $\sum X_i^* = \frac{\sum X_i - nb}{d} \Rightarrow \bar X^* = \frac{\bar X - b}{d}$

and $\sum y^* = \frac{\sum y_i - na}{c} \Rightarrow \bar y^* = \frac{\bar y - a}{c}$ Then:

$$\hat\beta_2^* = \frac{\sum\left(\frac{X_i - b - \bar X + b}{d}\right)\left(\frac{y_i - a - \bar y + a}{c}\right)}{\sum\left(\frac{X_i - b - \bar X + b}{d}\right)^2} = \frac{\frac{1}{cd}\sum(X_i - \bar X)(y_i - \bar y)}{\frac{1}{d^2}\sum(X_i - \bar X)^2} =$$

$= \frac{d}{c}\hat\beta_2$ +1. Then: $\hat\beta_1^* = \bar y^* - \hat\beta_2^* \bar X^* = \frac{\bar y - a}{c} - \frac{d}{c}\hat\beta_2 \cdot \frac{\bar X - b}{d} = \frac{\bar y - a - \hat\beta_2 \bar X + \hat\beta_2 b}{c} = \frac{\hat\beta_1 + \hat\beta_2 b - a}{c}$

$\hat\beta_1$

+1

1)

Problem 2.

$y_i = \beta_1 + \beta_2 X_i + u_i \Rightarrow$

$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$. From the 1st and the last columns:

$$\begin{cases} 4 = \hat{\beta}_1 + \hat{\beta}_2 \cdot 3 \\ 11 = \hat{\beta}_1 + \cancel{10}\,\hat{\beta}_2 \end{cases} \Rightarrow 2\hat{\beta}_2 = 4 - 11 \Rightarrow \hat{\beta}_2 = -3{,}5 \Rightarrow$$

$\Rightarrow \hat{\beta}_1 = \cancel{18}\ 14{,}5$ +

+

So our model is:

$\hat{y}_i = \cancel{\beta}\ 14{,}5 - 3{,}5 X_i$

Restore the missing values: + $\approx 1{,}86$

$8 = 14{,}5 - 3{,}5 X_i \Rightarrow X_i = \frac{6{,}5}{3{,}5} = \frac{13}{7}$ (2nd column X_i)

$7 = 14{,}5 - 3{,}5 X_i \Rightarrow X_i = \frac{7{,}5}{3{,}5} = \frac{75}{35} = \frac{15}{7}$ (3st column X_i)

+ $\approx 2{,}14$

~~B~~ From OLS we know:

$\hat{\beta}_2 = \frac{\sum (X_i - \bar{X})(y_i - \bar{y})}{\sum (X_i - \bar{X})^2}$ and $\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{X}$

1st proof: $\bar{y} = \bar{\hat{y}}$ in OLS cause $(\bar{\hat{u}} = 0)$

but $\frac{29}{4} \neq \frac{30}{4} \Rightarrow \hat{\beta}_1$ and $\hat{\beta}_2$ not OLS estimators

$\bar{X} = \cancel{\frac{3+4+1}{3}}\ \frac{3 + \frac{13}{7} + \frac{15}{7} + 1}{4} = \frac{4+4}{4} = 2$

$\bar{y} = \frac{4+8+7+11}{4} = 7{,}5$

The second proof

So $\hat{\beta}_1$ should be (if it is OLS):

+

$\hat{\beta}_1 = 7{,}5 \underbrace{+3{,}5}_{\hat{\beta}_2} \cdot 2 = 14{,}5$ (1) (how it used to be)

$\hat{\beta}_2 = \frac{(-1 \cdot 3{,}5) + (-\frac{1}{7}) \cdot (\frac{1}{2}) + (-\frac{1}{2}) \cdot (\frac{1}{7}) + (-1) \cdot (3{,}5)}{(-1)^2 + (\frac{1}{7})^2 + (\frac{1}{7})^2 + (1)^2} = \frac{-\frac{1}{7} - 7}{\frac{100}{49}} =$

$= -\frac{56}{7} / \frac{100}{49} = -\frac{56 \cdot 49}{100 \cdot 7} = -\frac{98}{25} = -3{,}92$ not OLS

$\Rightarrow \hat{\beta}_1$ and $\hat{\beta}_2$ not OLS

3,5 ✗

Problem 3 (1, 3, 4):

1. So, ~~the~~ coef of Tenure is equal to ≈ 2. It means that ~~the~~ one additional year of work exp ~~gives $\approx +2$ thousands~~ increases the wage on 2 thousand ruble. This coef is significant cause zero is not in conf interval. Intercept which ~~is~~ equals to 38 means that a person with no work experience ~~will~~ will have a wage about 38 thousand rubles. This ~~coef~~ is signif, too (same reasons). (+1)

3. $R^2 = \frac{ESS}{TSS} = \frac{71331,4126}{128255,824} \approx 0,586$ (+1)

4. R^2 value shows how good our model describes our data. ~~It~~ It's a part of variance of our target that we've explained (that's why $R^2=1$ is very good is $R^2=0$ is bad). Our R^2 is not very good, but not as bad

(+1)

Problem 4.

$$y_i = \beta X_i + u_i \quad \text{iid } u_i \sim N(0, \sigma_u^2)$$

Our model will be: $\hat{y}_i = \hat{\beta} X_i$

we will minimise sum of squares of residuals:

$$OLS = \frac{1}{n}\sum(\hat{y}_i - y_i)^2 = \frac{1}{n}\sum(\hat{\beta}X_i - y_i)^2 \to \min_{\hat{\beta}}$$

$$OLS'_{\beta} = 0 \Leftrightarrow \sum 2(\hat{\beta}X_i - y_i)\cdot X_i =$$

const in this

$$= 2\left(\sum \hat{\beta} X_i^2 - \sum y_i X_i\right) = 0 \Leftrightarrow$$

$$\Leftrightarrow \hat{\beta}\cdot\sum X_i^2 = \sum y_i X_i \Leftrightarrow \hat{\beta} = \frac{\sum y_i X_i}{\sum X_i^2}$$

$$OLS''_{\beta} : 2\sum X_i^2 \geq 0 \Rightarrow$$ we found a minimum (what we wanted)

$$\left(\sum\left(2(\hat{\beta}X_i - y_i)X_i\right)\right)' = 2\sum\hat{\beta}X_i^2 - \underbrace{2\sum y_i X_i}_{\text{not depends on } \hat{\beta}}$$