You have 40 minutes to complete the test. Please explain each step of your derivations and state all the assumptions employed. Note that different problems can give you different points. Maximum for the test is 10 points.

For ***problems 1 and 2*** *use the following numbers:*

N (number of observations)	$\widehat{\beta_0}$	$\widehat{s.e.\beta_0}$	$\widehat{\beta_1}$	$\widehat{s.e.\beta_1}$	$\widehat{\beta_2}$	$\widehat{s.e.\beta_2}$	R^2
10 * (number of letters in your name in English + number of letters in your surname in English)	number of letters in your name in English + 0.1	the order number[1] of your name's first letter in the alphabet / 10	number of letters in your surname in English + 0.2	the order number of your surname's first letter in the alphabet / 10	number of letters in your surname in English – number of letters in your name in English + 0.3	(the order number of your name's first letter in the alphabet + the order number of your surname's first letter in the alphabet) / 10	(your group number (e.g., 1, 2, 3) + 4) / 10

Problem 1

An econometrician estimated this model with N observations:

$$\hat{Y} = \underset{(\widehat{s.e.\beta_0})}{\widehat{\beta_0}} + \underset{(\widehat{s.e.\beta_1})}{\widehat{\beta_1}X_1} + \underset{(\widehat{s.e.\beta_2})}{\widehat{\beta_2}X_2}$$

Assuming the disturbance term has a standard normal distribution, calculate the 95 per cent confidence interval for β_1 and β_2. [1 point]

What can you conclude from this calculation? [1 point]

Problem 2

An econometrician estimated the model with N observations (persons). In this model: EARNINGS – hourly earnings of person ($), S – Number of years of study. Along with the coefficient estimates, the researcher also got the R^2 value.

$$\widehat{EARNINGS} = \underset{(\widehat{s.e.\beta_0})}{\widehat{\beta_0}} + \underset{(\widehat{s.e.\beta_1})}{\widehat{\beta_1}\,S}, \qquad R^2$$

Assuming the disturbance term has a standard normal distribution, perform an F test on the goodness of fit of the equation writing down the null and alternative hypotheses. What can you conclude from this calculation? [1 point]

Give an interpretation of the coefficients estimates. [1 point]

[1] See the ordered alphabet after the problem set

Problem 3

An econometrician estimated two linear models based on the same 90 observations (persons). In this model: GRADE – the grade a student got for his econometric test (points, out of 10), H – hours of studying before the test.

Model 1: $\widehat{GRADE} = \underset{(0,2)}{2.3} + \underset{(0,4)}{3,25}\, H,$ $\quad R^2 = 0.43$

Model 2: $\widehat{GRADE} = \underset{(0,3)}{4,64}\, H,$ $\quad R^2 = 0.52$

Which model would you use? Explain. [2 points]

Problem 4

A researcher investigating the determinants of the demand for public transport in a certain city has the following data for 100 residents for the previous calendar year: expenditure on public transport, E, measured in dollars; number of days worked, W; and number of days not worked, NW *(by definition, NW is equal to 365 – W).* He attempts to fit the following model:

$$E = \beta_0 + \beta_1 W + \beta_2 NW + u$$

Explain why it is impossible to fit this equation. Give intuitive explanations. [1 point]

The researcher estimated model using the OLS method. What can we say about the OLS estimator of coefficients if u is a disturbance term that is independently and identically distributed with expected value a ≠ 0. [1 point]

Problem 5

Prove that the OLS estimator of coefficients in a multiple regression is unbiased if the Gauss–Markov conditions are satisfied (Use the matrix notation). [1 point]

Derive the variance of the coefficients (Use the matrix notation). [1 point]

Number Substitution Cypher

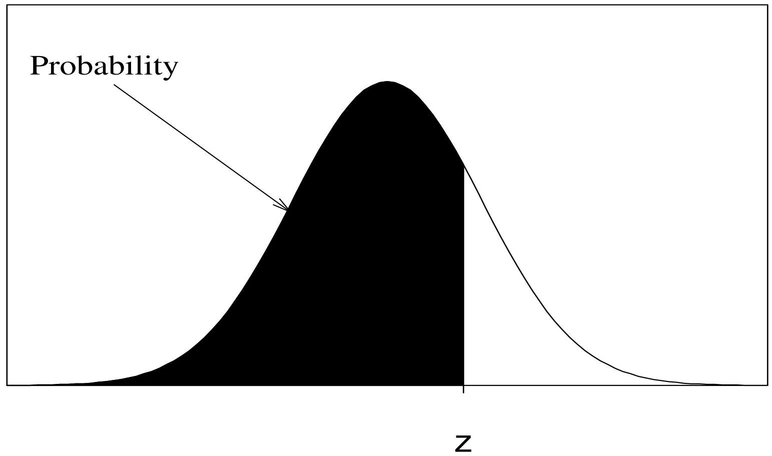
A	B	C	D	E	F	G	H	I	J
1	2	3	4	5	6	7	8	9	10

K	L	M	N	O	P	Q	R	S	T
11	12	13	14	15	16	17	18	19	20

U	V	W	X	Y	Z
21	22	23	24	25	26

Probabilities for the standard normal distribution

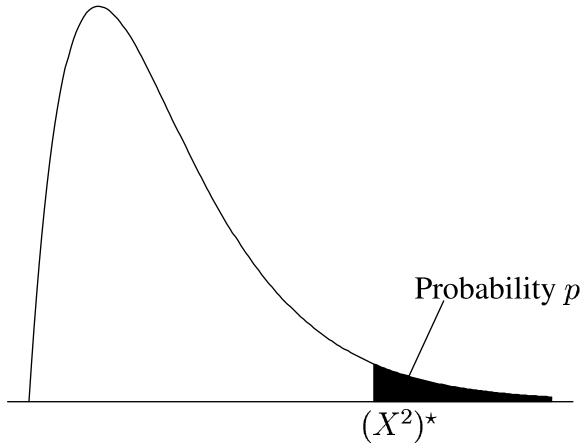
Table entry for z is the probability lying to the left of z



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Probabilities for the χ^2-distribution

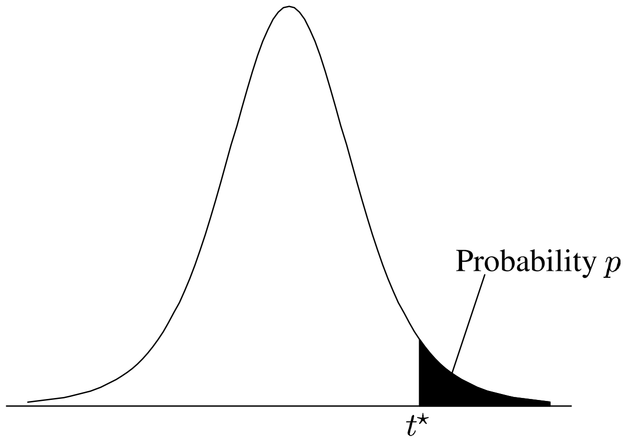
Table entry for p is the point $(X^2)^\star$ with probability p lying above it



	Tail probability p											
df	.25	.2	.15	.1	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.52	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.69
80	88.13	90.41	93.11	96.58	101.88	106.63	108.07	112.33	116.32	120.10	124.84	128.26
100	109.14	111.67	114.66	118.50	124.34	129.56	131.14	135.81	140.17	144.29	149.45	153.17

Probabilities for the t-distribution

Table entry for p and C is the point $t^\star$ with probability p lying above it and probability C lying between $-t^\star$ and $t^\star$



	Tail probability p											
df	.25	.2	.15	.1	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291
	50%	60%	70 %	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

F(m,n)-distribution critical values for 5% significance level

m\n	1	2	3	4	5	10	20	30	40	50	60	70	80	90	100
1	161	18,5	10,1	7,71	6,61	4,96	4,35	4,17	4,08	4,03	4,00	3,98	3,96	3,95	3,94
2	199	19,0	9,55	6,94	5,79	4,10	3,49	3,32	3,23	3,18	3,15	3,13	3,11	3,10	3,09
3	216	19,2	9,28	6,59	5,41	3,71	3,10	2,92	2,84	2,79	2,76	2,74	2,72	2,71	2,70
4	225	19,2	9,12	6,39	5,19	3,48	2,87	2,69	2,61	2,56	2,53	2,50	2,49	2,47	2,46
5	230	19,3	9,01	6,26	5,05	3,33	2,71	2,53	2,45	2,40	2,37	2,35	2,33	2,32	2,31
10	242	19,4	8,79	5,96	4,74	2,98	2,35	2,16	2,08	2,03	1,99	1,97	1,95	1,94	1,93
20	248	19,4	8,66	5,80	4,56	2,77	2,12	1,93	1,84	1,78	1,75	1,72	1,70	1,69	1,68
30	250	19,5	8,62	5,75	4,50	2,70	2,04	1,84	1,74	1,69	1,65	1,62	1,60	1,59	1,57
40	251	19,5	8,59	5,72	4,46	2,66	1,99	1,79	1,69	1,63	1,59	1,57	1,54	1,53	1,52
50	252	19,5	8,58	5,70	4,44	2,64	1,97	1,76	1,66	1,60	1,56	1,53	1,51	1,49	1,48
60	252	19,5	8,57	5,69	4,43	2,62	1,95	1,74	1,64	1,58	1,53	1,50	1,48	1,46	1,45
70	252	19,5	8,57	5,68	4,42	2,61	1,93	1,72	1,62	1,56	1,52	1,49	1,46	1,44	1,43
80	253	19,5	8,56	5,67	4,41	2,60	1,92	1,71	1,61	1,54	1,50	1,47	1,45	1,43	1,41
90	253	19,5	8,56	5,67	4,41	2,59	1,91	1,70	1,60	1,53	1,49	1,46	1,44	1,42	1,40
100	253	19,5	8,55	5,66	4,41	2,59	1,91	1,70	1,59	1,52	1,48	1,45	1,43	1,41	1,39

F(m,n)-distribution critical values for 95% significance level

m\n	1	2	3	4	5	10	20	30	40	50	60	70	80	90	100
1	0,01	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
2	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05	0,05
3	0,10	0,10	0,11	0,11	0,11	0,11	0,12	0,12	0,12	0,12	0,12	0,12	0,12	0,12	0,12
4	0,13	0,14	0,15	0,16	0,16	0,17	0,17	0,17	0,17	0,18	0,18	0,18	0,18	0,18	0,18
5	0,15	0,17	0,18	0,19	0,20	0,21	0,22	0,22	0,22	0,23	0,23	0,23	0,23	0,23	0,23
10	0,20	0,24	0,27	0,29	0,30	0,34	0,36	0,37	0,38	0,38	0,38	0,38	0,38	0,39	0,39
20	0,23	0,29	0,32	0,35	0,37	0,43	0,47	0,49	0,50	0,51	0,51	0,52	0,52	0,52	0,52
30	0,24	0,30	0,34	0,37	0,39	0,46	0,52	0,54	0,56	0,57	0,57	0,58	0,58	0,59	0,59
40	0,24	0,31	0,35	0,38	0,41	0,48	0,54	0,57	0,59	0,60	0,61	0,62	0,62	0,63	0,63
50	0,25	0,31	0,36	0,39	0,42	0,49	0,56	0,59	0,61	0,63	0,63	0,64	0,65	0,65	0,66
60	0,25	0,32	0,36	0,40	0,42	0,50	0,57	0,61	0,63	0,64	0,65	0,66	0,67	0,67	0,68
70	0,25	0,32	0,37	0,40	0,43	0,51	0,58	0,62	0,64	0,65	0,66	0,67	0,68	0,69	0,69
80	0,25	0,32	0,37	0,40	0,43	0,51	0,59	0,62	0,65	0,66	0,67	0,68	0,69	0,70	0,70
90	0,25	0,32	0,37	0,40	0,43	0,52	0,59	0,63	0,65	0,67	0,68	0,69	0,70	0,71	0,71
100	0,25	0,32	0,37	0,41	0,43	0,52	0,60	0,64	0,66	0,68	0,69	0,70	0,71	0,71	0,72