

## Problem 1.

$$\beta_1 = 2\beta_4$$

$$\beta_2 = \beta_3$$

Metrics<sub>i</sub> =  $\beta_0 + \beta_1 \text{ Maths}_i + \beta_2 \text{ Linear}_i + \beta_3 \text{ Prob}_i + \beta_4 \text{ Stat}_i + \epsilon_i$

Restricted: Metrics<sub>i</sub> =  $\beta_0 + 2\beta_4 \text{ Maths}_i + \beta_2 \text{ Linear Algebra}_i + \beta_2 \text{ Prob Theory}_i + 2\beta_4 \text{ Statistics}_i + \epsilon_i =$

$$= \beta_0 + 2\beta_4 (\text{Maths}_i + \text{Stat}_i) + \beta_2 (\text{Linear}_i + \text{Prob}_i) + \epsilon_i$$

$$H_0: \beta_1 - 2\beta_4 = 0 \text{ and } \beta_2 - \beta_3 = 0$$

$H_1$ : either  $\beta_1 - 2\beta_4$  or  $\beta_2 - \beta_3 \neq 0$ , or both  $\neq 0$

$$F(2, N-5) = \frac{(RSS_R - RSS_{UR})/2}{RSS_U / (N-5)}$$

$\uparrow$  num of restrictions       $\uparrow$  num of coefs in unrestricted model

these two are equivalent (where SPRING, SUMMER, AUTUMN are the seasons for the season)  
 $\beta_1 \cdot \text{INCOME}$

## Problem 2.

COFFEE =  $\beta_0 + \beta_1 \cdot \text{SEASON} \cdot \text{INCOME} \leftarrow (RSS_2)$   
 $\text{COFFEE} = \beta_0 + \beta_1 \cdot \text{INCOME} + \beta_2 \cdot \text{SPRING} \cdot \text{INCOME} + \beta_3 \cdot \text{SUMMER} \cdot \text{INCOME} + \beta_4 \cdot \text{AUTUMN} \cdot \text{INCOME}$   
 $\beta_0$  - expenditures on COFFEE if INCOME = 0 (implausible if INCOME = 0 is not included in the sample)

$\beta_1$  - added how much more will be spent on COFFEE if INCOME increases for 1, ~~for~~, ~~correcting for~~  
~~SEASON (example: if SEASON = 2,  $\beta_1 = 4$ )~~

~~$\beta_1 \cdot \text{SEASON}$~~  Consequently, COFFEE increases by  $\beta_1$  if SEASON = 1 (winter), by  $\beta_1 \cdot 2$  in spring,  $\beta_1 \cdot 3$  in summer and by  $\beta_1 \cdot 4$  in autumn for each 1 point increase in INCOME.

Same linear relationship for all seasons:  $\text{COFFEE} = \beta_0 + \beta_1 \cdot \text{INCOME}$  (RSS<sub>1</sub>)

$$H_0: \text{SEASON} = 1$$

$$H_1: \text{SEASON} \neq 1$$

without dummies      with dummies

$$F(3, N-5) = \frac{(RSS_1 - RSS_2)/3}{RSS_2 / (N-5)}$$

$\uparrow$  num of coefs for the dummies       $\uparrow$  num of coefs in the model with dummies



### Problem 3.

Both mistakes are misspecifying the model: the first model's mistake - omitting a crucial variable, the second one's - adding an unnecessary one.

The consequences ~~for~~ of the 1<sup>st</sup> mistake - coefficients will be biased and standard errors are invalid (as well as  $t$ ,  $F$ -tests). The consequences ~~for~~ of the 2<sup>nd</sup> mistake - coefs are not biased but inefficient ( $\sigma^2$  is larger because of corr between the regressors), st. errors are valid, as well as tests. All in all, the 1<sup>st</sup> mistake is more dangerous, as the relationship predicted is wrong (coefs are biased), while the 2<sup>nd</sup> mistake gives that the true relationship, but with ~~with pred~~ gives bigger st. errors.

### Problem 4.

"Teacher Quality and Learning Outcomes in Kindergarten"

Research question is to determine the effect of the quality of teachers (several observable and unobservable characteristics were used) on the ~~level of education~~, performance in school.

Data: 24 000 students from studying in ~~specific~~ certain schools with certain classes structure in Ecuador