130 | 4,1 | 1,3 | 9,2 | 1,9 | 5,3 | 0,9 | 0,7 | Stolyanov Mark 193 (BEC)

Deadline: ~~19:~~ 20: ~~26~~ 30

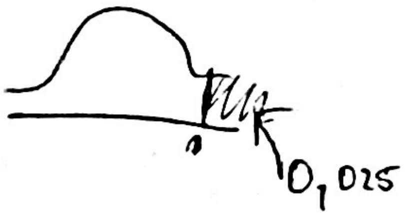
P1.

$$\hat{y} = 4{,}1 + 9{,}2X_1 + 5{,}3X_2$$
$$\quad (1{,}3) \quad (1{,}9) \quad (0{,}9)$$

$df = n - k = 130 - 3 = 127$

$u_i \sim N(0, \sigma^2)$

two-sided int $\Rightarrow$ $|t_{crit}| \approx 1{,}98$



β_1:

$$\underbrace{9{,}2}_{\hat\beta_1} - 1{,}98 \cdot 1{,}9 < \beta_1 < 9{,}2 + 1{,}98 \cdot 1{,}9 \quad (se(\hat\beta_1)\, t_c)$$

$$5{,}438 < \beta_1 < 12{,}962 \Rightarrow$$

$\Rightarrow \hat\beta_1$ is significant (zero is not in interval)

β_2:

$$5{,}3 - 0{,}9 \cdot 1{,}98 < \beta_2 < 5{,}3 + 0{,}9 \cdot 1{,}98$$

$$3{,}518 < \beta_2 < 7{,}082 \Rightarrow$$

$\Rightarrow \hat\beta_2$ is significant, too (zero is not in conf interval of level 95%)

P2.

$$\widehat{Earnings} = \underset{\hat\beta_0}{4{,}1} + \underset{\hat\beta_1}{9{,}2}\,S \qquad R^2 = 0{,}7$$
$$\quad (1{,}3) \quad (1{,}9)$$

F-test:

$H_0: \beta_1 = 0$

$H_1: \beta_1 \neq 0$

$$F(k-1, n-k) = F(1, 128) = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} = \frac{0{,}7/1}{0{,}3/128} = \approx 298{,}67$$

$F(1,128)_{crit} \approx 3{,}94$.

$298{,}67 \gg 3{,}94 \Rightarrow$ coefficients is highly significant at level 5%

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(null hypothesis is rejectet at level 5%)
Interpretation:
One additional year of studying causes increase in Earnings by 9,2 $ (slope)
People with no studying have earnings 4,1 $ (intercept)

P3 - $N = 90$

I would choose model 2 cause it has ~~mo~~ bigger R^2, it means that this model better desribe variance of grades. Also, s.e. (H) in the second model is less, it is better (less variance and random)

~~4,64 / 0,3~~ Also, I don't believe that people with no practicing of studying econometrics can get more than a zero for the test (best specification is without intercept) personally

P.4. $N = 100$

$$E = \beta_0 + \beta_1 W + \beta_2 \underset{\substack{\uparrow \\ 365 - W}}{NW} + u$$

It is impossible, cause there is a ~~strong~~ exact multicollinearity (NW is linear function of W), so $\hat{\beta}_1 = \frac{0}{0}$ then (It's impossible to define what effect was done, for example, for decreasing

2

Of E ~~ist~~ decrease in W or increase NW: and what exact effects ~~β_i~~ W and NW have on E (~~not~~ the same meaning cannot explain variation in E: are they attributable to variations in W or variations in NW, or both)

$$\hat{\beta}_2 = \frac{\cancel{\Sigma(E_i-\bar{E}}\text{ something}}{\Sigma(W_i-\bar{W})^2\Sigma(NW_i-\overline{NW})^2-(\Sigma(W_i-\bar{W})(NW_i-\overline{NW}))^2}$$

$NW_i - \overline{NW} = -(W_i - \bar{W})$

$$= \frac{\text{something}}{(\Sigma(W_i-\bar{W})^2)^2-(\Sigma(W_i-\bar{W})(W_i-\bar{W}))^2} = \frac{\text{something}}{0}$$

if researcher do nothing with multicollinearity then ~~he~~ couldn't fit the model.

It he will fight multicollinearity, for example, by dropping NW from regression, then if $u \sim N(0, \sigma^2)$ let $\hat{\beta}_0 = k$, $\hat{\beta}_1 = c$.

If u now $\sim N(a, \sigma^2)$, then $\hat{\beta}_1 = k+a$, $\hat{\beta}_1$ remain the same: cause intuitively: now: $y = \beta_0 + \beta_1 W + a + \varepsilon$ ← new random component

was: $E = \beta_0 + \beta_1 W + u \Rightarrow$ then $E = \beta_0 + \beta_1 W + a + \varepsilon =$

$y^* = y + a$ $= \underbrace{(\beta_0 + a)}_{\beta_0^* \text{ now}} + \beta_1 W + \varepsilon$. or now $\hat{\beta}_0^* = \bar{y}^* - \hat{\beta}_1 \bar{X} = \bar{y} + a - \hat{\beta}_1 \bar{X} = \beta_0 + a$

PS.

$$\hat{\beta} = (X^T X)^{-1} X^T y \qquad \cancel{\beta_2 = \Sigma(X-\bar{X})(\beta_0+\beta_1}$$

$$E(\hat{\beta}) = E(\underbrace{(X^T X)^{-1} X^T}_{\text{non-stochastic}} y) = (X^T X)^{-1} X^T E(X\beta + u) =$$

$Eu = 0$

$$= (X^T X)^{-1} X^T X \beta = \beta \quad \text{q.e.d}$$

$Var(y) = Var(X\beta + u)$
$= Var(u) = \sigma_u^2$

$$Var(\hat{\beta}) = Var((X^T X)^{-1} X^T y) = (X^T X)^{-1} X^T Var(y) ((X^T X)^{-1} X^T)^T$$

$$= \sigma_u^2 (X^T X)^{-1} X^T X (X^T X)^{-1} = \sigma_u^2 (X^T X)^{-1}$$

P.S. to PS. $$\hat{\beta}_1^* = \frac{\Sigma(X_i - \bar{X})(y_i + a - \bar{y} - a)}{\Sigma(X_i - \bar{X})^2} = \hat{\beta}_1$$

$\hat{\beta}_1$ remain the same.