<u>Упр.</u> $X \sim N(0, 8)$

$$\mathbb{E}(X^3) = \mathbb{E}(X \cdot X^2) = 8 \cdot \mathbb{E}(2X) = 16\,\mathbb{E}(X) = 0$$

$$\mathbb{E}(X^4) = \mathbb{E}(X \cdot X^3) = 8\,\mathbb{E}(3X^2) = 24\,\mathbb{E}(X \cdot X) = 24 \cdot 8\,\mathbb{E}(1) =$$

$$= 192$$

$$\mathbb{E}(X^{2026}) = \mathbb{E}(X \cdot X^{2025}) = \sigma \cdot \mathbb{E}(2025 \cdot X^{2024}) =$$

$$= \sigma \cdot 2025\,\mathbb{E}(X^{2024}) = \ldots = \sigma^{1013} \cdot 2025 \cdot 2023 \cdot 2021 \cdot \ldots \cdot 1 =$$

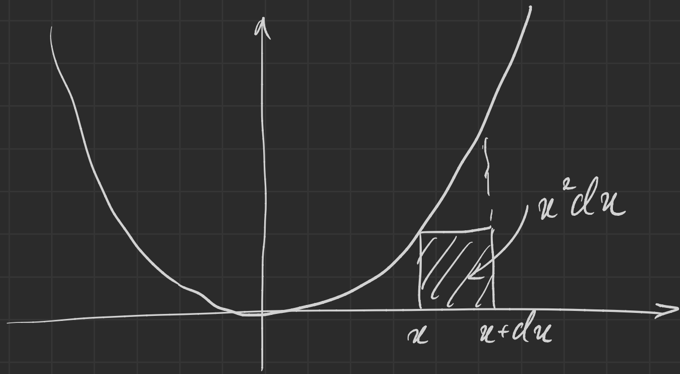
$$= \sigma^{1013}\, 2025!!$$

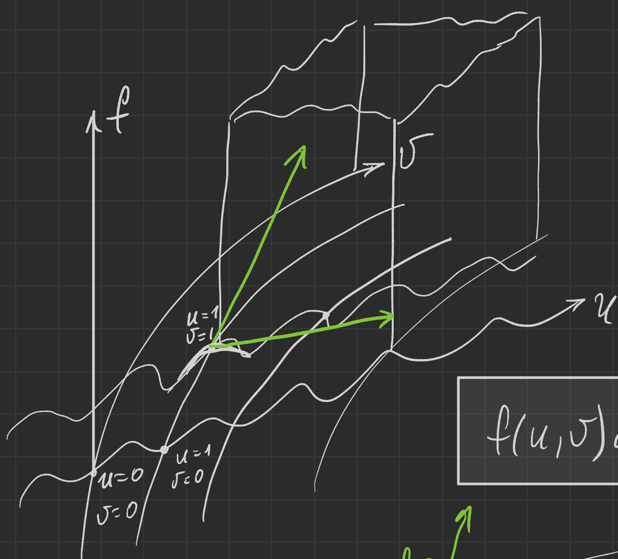
<u>лекция</u>

<u>Крэш курс про дифференциальные формы</u>

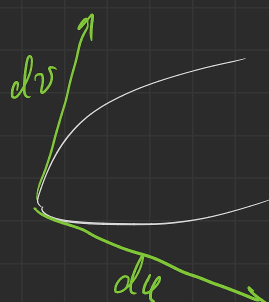
- Одна переменная

$f(x)\,dx$ $\qquad$ $x^2\,dx = \{x = 3t^2\} = 9t^4 \cdot 6t\,dt$



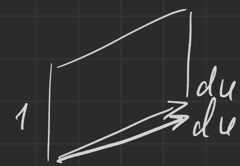


$$\boxed{f(u,v)\,du\wedge dv}$$



Свойства:

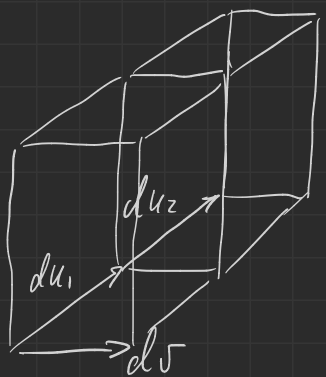
1. $1\,du \wedge du = 0$



2. билинейность

$$d(u_1+u_2)\wedge dv =$$

$$du_1\wedge dv + du_2\wedge dv$$



грани складываются

Упр. $d(a+b)\wedge d(a+b) = 0$

$$\underbrace{da\wedge da}_{0} + da\wedge db + db\wedge da + \underbrace{db\wedge db}_{0} = 0$$

3. $da\wedge db = -db\wedge da$ (следствие из 1 и 2)

4. $dg(u_1, u_2, u_3, \ldots, u_n) = g'_{u_1} du_1 + g'_{u_2} du_2 + \ldots + g'_{u_n} du_n$

Упр.

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \overset{A}{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$A = \begin{pmatrix} 5 & 2 \\ 1 & 7 \end{pmatrix}$$

$$dy_1 \wedge dy_2 \overset{?}{=} dx_1 \wedge dx_2$$

$$5x_1 + 2x_2 = y_1$$

$$x_1 + 7x_2 = y_2$$

$$d(5x_1 + 2x_2) \wedge d(x_1 + 7x_2) = \underbrace{5dx_1 \wedge dx_1}_{0} + 2dx_2 \wedge dx_1 +$$

$$+ 35 dx_1 \wedge dx_2 + \underbrace{14 dx_2 \wedge dx_2}_{0} =$$

$$= 33 dx_1 \wedge dx_2$$

$$\det A = 35 - 2 = 33$$

Теорема. Если $y = Ax$ и $A[n \times n]$, то

$$dy_1 \wedge dy_2 \wedge \ldots \wedge dy_n = \det A \, dx_1 \wedge dx_2 \wedge \ldots \wedge dx_n$$

$$\det(u_1, u_2, \dots, u_n) + \det(\tilde{u}_1, u_2, \dots, u_n) =$$

$$= \det(u_1 + \tilde{u}_1, u_2, u_3, \dots u_n)$$

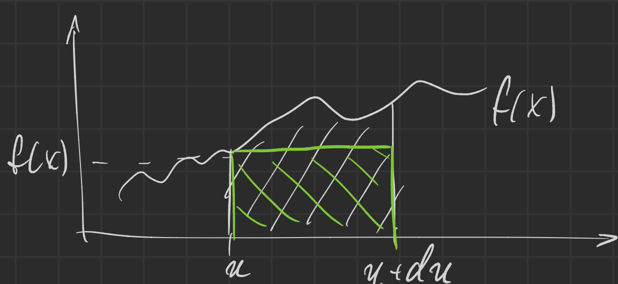
Prob. element

<u>Опр.</u> Элемент вероятности $(X_1, X_2, \dots, X_n)$

$f(x_1, x_2, \dots x_n)$ – совм. плотность

$$pe(x_1, x_2, \dots x_n) = f(x_1, x_2, \dots x_n) \cdot dx_1 \wedge dx_2 \wedge \dots \wedge dx_n$$

$$\lim_{\substack{dx_1 \to 0 \\ dx_2 \to 0}} \frac{\mathbb{P}(X_1 \in [x_1, x_1 + dx_1], X_2 \in [x_2, x_2 + dx_2])}{pe(x_1, x_2)} = 1$$



$$pe(x) = f(x)\,dx$$

<u>Упр.</u> $X_1 \sim U[0,1]$ независ.

$X_2 \sim U[0,1]$

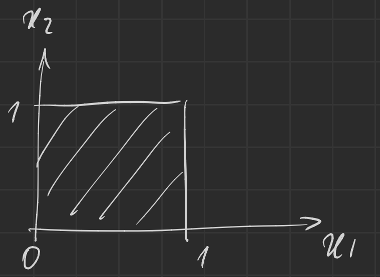
a) $f(x_1, x_2) = ?$ $\quad pe_X(x_1, x_2) = ?$

б) $y_1 = X_1 + X_2$ $\qquad pe_y(y_1, y_2) = ?$

$$y_2 = X_1 + X_2 + X_2^3$$

в) $f_y(y_1, y_2) = ?$

а) $f(x_1, x_2) = \begin{cases} 1, & 0 \le x_1 \le 1,\ 0 \le x_2 \le 1 \\ 0, & \text{иначе} \end{cases}$, $pe_x(x_1, x_2) = 1\, dx_1 \wedge dx_2$



б) $X_1 = y_1 - X_2$

$$y_2 = y_1 - X_2 + X_2 + X_2^3 = y_1 + X_2^3$$

$$X_2 = \sqrt[3]{y_2 - y_1}$$

$$X_1 = y_1 - \sqrt[3]{y_2 - y_1}$$

Утв.

$$pe_y(y_1, y_2) = pe_x(x_1, x_2) = 1 \cdot d\left(y_1 - \sqrt[3]{y_2 - y_1}\right) \wedge d\left(\sqrt[3]{y_2 - y_1}\right) =$$

$$= dy_1 \wedge d\left(\sqrt[3]{y_2 - y_1}\right) = dy_1 \wedge \left(\frac{1}{3}(y_2 - y_1)^{-\frac{2}{3}}\right) dy_2 =$$

$$f(y_1, y_2) \longrightarrow \ = \frac{1}{3}(y_2 - y_1)^{-\frac{2}{3}}\, dy_1 \wedge dy_2$$

$d\left((y_2-y_1)^{\frac{1}{3}}\right) =$

$$dF = F'_x dx + F'_y dy$$

$= \frac{1}{3}(y_2-y_1)^{-\frac{2}{3}} dy_2 - \frac{1}{3}(y_2-y_1)^{-\frac{2}{3}} dy_1$

$$f_y(y_1,y_2) = \begin{cases} \frac{1}{3}(y_2-y_1)^{-\frac{2}{3}}, & \sqrt[3]{y_2-y_1} \in [0,1],\ y_1 - \sqrt[3]{y_2-y_1} \in [0,1] \\ & y_2-y_1 \in [0,1] \\ 0, & \text{иначе} \end{cases}$$

$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -x_1 \\ +x_2 \end{pmatrix}$ иногда меняем знак

<u>Упр</u>. Преобразование Бокса-Мюллера.

$X_i \sim \mathrm{Unif}[0,1]$ незав.

$\begin{pmatrix} R \\ \Theta \end{pmatrix}$ $R = (-2\ln X_1)^{1/2}$

$\Theta = 2\pi X_2$

а) $p_e(r,t)$

б) $Y_1 = R\cos\Theta$

$Y_2 = R\sin\Theta$

$p_e(y_1,y_2) = ?$

в) Как распределены Y_1, Y_2? и где это использ?

$pe(x_1, x_2) = f(x_1, x_2)\, dx_1 \wedge dx_2 = 1\, dx_1 \wedge dx_2 =$

$\circled{=}$

$R^2 = -2\ln X_1$

$\ln X_1 = -\frac{R^2}{2}$

$X_1 = e^{-\frac{R^2}{2}}$ $\quad x_1 = e^{-\frac{r^2}{2}}$

$x_2 = \frac{t}{2\pi}$

$\circled{=}\; d\left(e^{-\frac{r^2}{2}}\right) \wedge d\left(\frac{t}{2\pi}\right) = -r e^{-\frac{r^2}{2}} dr \wedge \frac{1}{2\pi} dt =$

$= -\frac{r}{2\pi} e^{-\frac{r^2}{2}} dr \wedge dt$

$f_{R,\Theta}(r,t) = \frac{r}{2\pi} e^{-\frac{r^2}{2}}$ $\quad t \in [0, 2\pi]$

$r \in [0, +\infty)$

$pe(r,t) = -\frac{r}{2\pi} \exp\left(-\frac{r^2}{2}\right) dr \wedge dt$

б) $R = \sqrt{y_1^2 + y_2^2}$

$\Theta = \arccos\left(\frac{y_1}{R}\right)$ $\quad t = \arccos\left(\frac{y_1}{r}\right)$

$$dt = \frac{-1}{r\sqrt{1-\frac{y_1^2}{r^2}}}\, dy_1 + \ldots\, dr =$$

$$= dr \wedge dy_1 \cdot \frac{(-1)\cdot\frac{1}{r}}{\sqrt{1-\frac{y^2}{r^2}}}$$

$$dr = \frac{y_1\, dy_1 + y_2\, dy_2}{\sqrt{y_1^2+y_2^2}}$$

$$= \frac{y_2}{\sqrt{y_1^2+y_2^2}} \cdot \frac{(-1)}{\sqrt{y_1^2+y_2^2}}\, dy_2 \wedge dy_1 = \underbrace{\frac{1}{\sqrt{y_1^2+y_2^2}}\, dy_1 \wedge dy_2}_{dr \wedge dt}$$

$$pe(y_1, y_2) = -\frac{\sqrt{y_1^2+y_2^2}}{2\pi} \exp\left(\frac{-y_1^2-y_2^2}{2}\right) \cdot dr \wedge d\theta =$$

$$= \frac{-\exp\left(\frac{-y_1^2-y_2^2}{2}\right)}{2\pi}\, dy_1 \wedge dy_2$$

$$f(y_1, y_2) = \frac{\exp\left(\frac{-y_1^2-y_2^2}{2}\right)}{2\pi}, \mathbb{R}$$

$$f(y_1) = \frac{\exp\left(-\frac{y_1^2}{2}\right)}{\sqrt{2\pi}}, \quad f(y_2) = \frac{\exp\left(-\frac{y_2^2}{2}\right)}{\sqrt{2\pi}},$$

т.к. независимые.

$x_1, x_2, x_3 \sim$ незав. $N(0,1)$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

а) $f(x_1) = ?$ $\quad$ $pe(x_1, x_2) = ?$

$f(x_1, x_2) = ?$

б) $$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 & 6 \\ 4 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$pe(y_1, y_2)$? $\quad$ $f(y_1, y_2) = ?$

в) $Y = a + BX$ $\quad$ $pe_Y(y) = ?$ $\quad$ $f(y_1, \ldots, y_n) = ?$

$$Y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \quad X = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

a) $f(x_1) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_1^2}{2}\right)$

$$f(x_1, x_2) = f(x_1) \cdot f(x_2) = \frac{1}{2\pi} \exp\left(\frac{-x_1^2 - x_2^2}{2}\right)$$

$$p_e(x_1, x_2) = f(x_1, x_2)\, dx_1 \wedge dx_2 = \frac{1}{2\pi} \exp\left(\frac{-x_1^2 - x_2^2}{2}\right) dx_1 \wedge dx_2$$

Если у СВ (X_1, X_2) есть совм. плотность

① <=> ② <=> ③

1) $f(x_1, x_2) = f(x_1) \cdot f(x_2)$ нужна ф-я плотности.

2) $F(x_1, x_2) = F_1(x_1) \cdot F_2(x_2)$

$\quad P(X_1 \le x_1, X_2 \le x_2)$

3) $P(x_1 \in A_1, x_2 \in A_2) = P(X_1 \in A_1) \cdot P(X_2 \in A_2)$

подходят для любых СВ

$A_1 \subseteq \mathbb{R} \quad A_1 = [5, 18]$

$A_2 \subseteq \mathbb{R} \quad x_1 \in [5, 18]$ СВ b)

$$\underbrace{\begin{pmatrix} 3 & 6 \\ 4 & 7 \end{pmatrix}}_{= A}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$y = b + Ax$$

$$\underline{x = A^{-1}(y-b)}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = A^{-1}\left[\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right]$$

$$dx_1 \wedge dx_2 = \frac{1}{\det(A^{-1})} dy_1 \wedge dy_2 = -\frac{1}{3} dy_1 \wedge dy_2$$

$$p e_y(y_1, y_2) = p e_x(x_1, x_2) = \frac{1}{2\pi} \exp$$

$$x_1^2 + x_2^2 = \underline{x^T x} = (A^{-1}(y-b))^T \cdot A^{-1}(y-b) =$$

$$= (y-b)^T \cdot A^{-T} A^{-1}(y-b)$$

$$\boxed{A^{-T} = (A^{-1})^T = (A^T)^{-1}}$$

$$A^{-T} \cdot A^{-1} = \frac{1}{9}\Big($$

$$A^{-1} = \begin{pmatrix} 7 & -6 \\ -4 & 3 \end{pmatrix} \cdot \frac{1}{-3} \qquad A^{-T} = \begin{pmatrix} 7 & -4 \\ -6 & 3 \end{pmatrix} \cdot \frac{1}{-3}$$

$$p e_y(y_1, y_2) = \frac{1}{2\pi} \exp\left[-\frac{1}{2}\left(\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right)^T \cdot \frac{1}{9}\begin{pmatrix} 65 & -54 \\ -54 & 45 \end{pmatrix} \cdot \left(\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right)\right]\left(-\frac{1}{3}\right) dy_1 \wedge dy_2$$

$$p_\xi(x_1,x_2,x_3,\dots,x_n)=\left(\frac{1}{2\pi}\right)^{\frac{n}{2}}\exp\left(-\frac{x_1^2+x_2^2+\dots+x_n^2}{2}\right)dx_1\wedge dx_2\wedge\dots\wedge dx_n$$

$$x_1^2+x_2^2+\dots+x_n^2=x^Tx=\underline{(y-b)^TA^{-T}A^{-1}(y-b)}$$

$$dx_1\wedge dx_2\wedge\dots\wedge dx_n=\det(A^{-1})\,dy_1\wedge dy_2\wedge\dots\wedge dy_n$$

порядок важен

$$p_\xi(y_1,y_2,\dots,y_n)=\underbrace{\left(\frac{1}{2\pi}\right)^{\frac{n}{2}}\exp\left(-\frac{(y-b)^TA^{-T-1}(y-b)}{2}\right)\det(A^{-1})}_{f(y_1,\dots,y_n)}\cdot dy_1\wedge dy_2\wedge\dots\wedge dy_n$$

2) $$\begin{pmatrix}y_1\\ y_2\end{pmatrix}=\begin{pmatrix}2\\ 3\end{pmatrix}+\begin{pmatrix}3 & 6\\ 4 & 7\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$$

$EY=?$

$$Var(Y)_{ij}=?=\begin{pmatrix}Cov(Y_1,Y_1) & Cov(Y_1,Y_2) & \dots\\ Cov(Y_2,Y_1) & Cov(Y_2,Y_2) & \dots\\ \dots & \dots & \dots\end{pmatrix}=Cov(Y_i,Y_j)$$

$\mathbb{E}(X_1)=0 \qquad X_1\sim N(0,1)$

$Var\,X_1=1 \qquad Cov(X_1,X_2)=0$

$$\mathbb{E}X=\begin{pmatrix}0\\ 0\end{pmatrix}$$

$$\Downarrow\ \mathbb{E}Y=\begin{pmatrix}2\\ 3\end{pmatrix}+\underbrace{A\,\mathbb{E}X}_{=0}=\begin{pmatrix}2\\ 3\end{pmatrix} \qquad EY_1=2,\ EY_2=3$$

$$Cov(Y_1, Y_1) = Cov(2+3X_1+6X_2;\ 2+3X_1+6X_2) =$$

$$= Cov(3X_1, 3X_1) + 2Cov(3X_1, 6X_2) + Cov(6X_2, 6X_2) =$$

$$= 9\underbrace{Cov(X_1, X_1)}_{Var X_1 = 1} + 36\underbrace{Cov(X_1, X_2)}_{0} + 36\underbrace{Cov(X_2, X_2)}_{1} = 45$$

$$Cov(Y_1, Y_2) = Cov(2+3X_1+6X_2,\ 3+4X_1+7X_2) =$$

$$= Cov(3X_1, 4X_1) + Cov(6X_2, 7X_2) = 12 + 42 = 54$$

$$Cov(Y_2, Y_2) = 16 + 49 = 65$$

$$Var Y = \begin{pmatrix} 45 & 54 \\ 54 & 65 \end{pmatrix} \qquad Var X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Y – случ. вектор

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \quad EY = \begin{pmatrix} Ey_1 \\ Ey_2 \\ Ey_3 \\ Ey_4 \end{pmatrix} \quad Var Y = \begin{pmatrix} Cov(y_1, y_1) & \cdots \\ \cdots & \cdots \\ Cov(y_4, y_1) & \cdots \end{pmatrix}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad X_i \sim \text{независ } N(0, 1)$$

$$EX = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$y = b + AX$$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} + \boxed{} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\operatorname{var} X = \begin{pmatrix} 1 & 0 & \cdots & \cdots \\ 0 & 1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

$$E(y) = b$$

$$y_i = b_i + row_i(A) X$$

$$y_j = b_j + row_j(A) X$$

$$\operatorname{Cov}(y_i, y_j) = \operatorname{Cov}(b_i + row_i A x ; b_j + row_j A x) =$$

$$= \operatorname{cov}(row_i A x ; row_j A x) = (row_i A , row_j A) = row_i A \cdot (row_j A)^T$$

$$\operatorname{Cov}(row_i A x, row_j A x) = \operatorname{Cov}(5x_1 + 6x_2 + 7x_3 ; 8x_1 + 9x_2 + 10x_3)$$

$$= 5 \cdot 8 + 6 \cdot 9 + 7 \cdot 10$$

$$\operatorname{var} y = A \cdot A^T$$

Опр. $X_1 \sim N(0,1)$

$$E X_1 = 0 \quad Var X_1 = 1 \quad f(x_1) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x_1^2}{2}\right)$$

Опр.

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix} \sim N(0, I)$$

$$\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & \dots & \\ 0 & 1 & & \\ \dots & \dots & \dots & \\ 0 & 0 & & 1 \end{pmatrix}$$

вектор
ожиданий

$$Var X = I$$

$$f(x) = f(x_1, x_2, \dots, x_n) = \left(\frac{1}{2\pi}\right)^{\frac{n}{2}} \exp\left(-\frac{x_1^2 + x_2^2 + \dots + x_n^2}{2}\right)$$

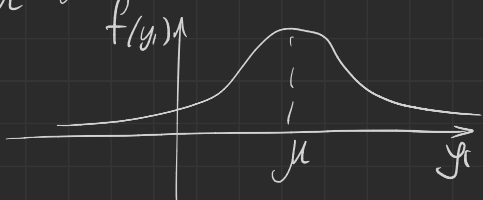
Опр. $Y_1 \sim N(\mu, \sigma^2)$ $\qquad Y_1 = \mu + \sigma X_1$

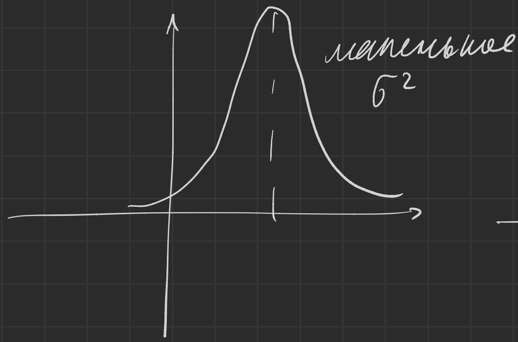
$$E Y_1 = \mu \quad Var Y_1 = \sigma^2$$

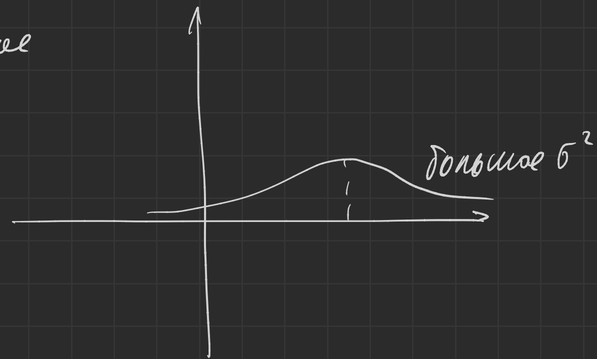
$$Var(Y_1) = Var(\mu + \sigma X_1) = Var(\sigma X_1) =$$

$$= Cov(\sigma X_1, \sigma X_1) = \sigma^2 Cov(X_1, X_1) = \sigma^2$$

$$f(y_1) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} \exp\left(-\frac{1}{2} \frac{(y_1 - \mu)^2}{\sigma^2}\right)$$







Var - разброс

$X \sim N(0, I)$ $\quad$ $Y = \mu + AX$

вектор (под X) $\quad$ вектор (под μ) $\quad$ матр. (под A)

$$EY = \mu \quad Var(Y) = A \cdot A^T = S$$

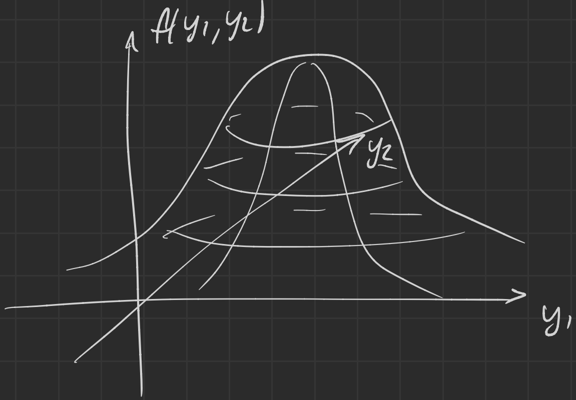
<u>Опр.</u> $Y \sim N(\mu, S)$

$$f(y) = \left(\frac{1}{2\pi}\right)^{\frac{n}{2}} \frac{1}{\det(A)} \exp\left(-\frac{1}{2}(y-\mu)^T \underbrace{A^{-T} \cdot A^{-1}}_{= S^{-1}} (y-\mu)\right)$$

$$\frac{1}{\det(A)} = \frac{1}{\sqrt{\det(S)}}$$

$$f(y) = \left(\frac{1}{2\pi}\right)^{\frac{n}{2}} \left(\frac{1}{\det(S)}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2}(y-\mu)^T S^{-1} (y-\mu)\right)$$

$$f(y_1) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\frac{(y-\mu)^2}{\sigma^2}\right)$$



Ув. теор. Иссерлиса.

Если $Y \sim N(\mu, S)$ то

$S_{1,2} = Cov(Y_1, Y_2) =$
$= E(Y_1 Y_2) - \underbrace{EY_1}_{\mu_1} \cdot \underbrace{EY_2}_{\mu_2}$

1. $E(y_1) = \mu_1$

2. $E(Y_1 \cdot Y_2) = \mu_1 \mu_2 + S_{1,2}$

3. $E(Y_1 \cdot Y_2 \cdot Y_3) = \mu_1 \cdot \mu_2 \cdot \mu_3 + S_{1,2} \cdot \mu_3 + S_{1,3} \cdot \mu_2 + S_{2,3} \cdot \mu_1$

$\cancel{\mu_1 \mu_2 \mu_3}$

4. $E(Y_1 \cdot Y_2 \cdot Y_3 \cdot Y_4) = \mu_1 \mu_2 \mu_3 \mu_4 + S_{1,2} \cdot \mu_3 \mu_4 + S_{2,3} \cdot \mu_1 \cdot \mu_4 + S_{1,4} \cdot \mu_2, \mu_3 + S_{3,4} \cdot \mu_1 \mu_2 + S_{1,4} \cdot \mu_2 \mu_3 + S_{2,4} \cdot \mu_1 \cdot \mu_3 + S_{1,3} \cdot S_{2,4} + S_{1,2} \cdot S_{3,4} + S_{1,4} \cdot S_{2,3}$

Упр. $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}; \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}\right)$

$$E(y_1 \cdot y_2 \cdot y_3) = 0 \cdot 2 \cdot 3 + 0 + 1 \cdot 0 = 0$$

$$E(y_1^2 \cdot y_3) = 0 + 3 + 0 = 3$$

EY_1

$$\begin{pmatrix} y_1 \\ y_1 \\ y_3 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \begin{array}{c} \begin{matrix} y_1 & y_1 & y_3 \end{matrix} \\ \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \end{array} \begin{matrix} y_1 \\ y_1 \\ y_3 \end{matrix}\right)$$

EY_1

EY_3