Сем 4

Именная простота

(7) Инт Дирихле $\int_0^\infty \frac{\sin px}{x} dx = \int_0^\infty \frac{\sin u}{u} \cdot du$

$px = u$

$x = \frac{u}{p}$

$dx = \frac{du}{p}$

$\frac{dx}{x} = \frac{du}{u}$

~~способ 1~~

Трюк Ребнмана
добавим внутрь параметр, так, чтобы $I(y)$ $I'(y)$ была простой!

$$I(y) = \int_0^\infty \frac{\sin u}{u} \cdot \exp(-uy) \cdot du$$

$$\exp(☺) = e^{☺}$$

$$I'(y) = \int_0^\infty \sin u \cdot (-1) \cdot \exp(-uy)\, du =$$

! два раза по частям ! $I'(y)$

потом восстановим $I(y)$.

$$= \sin(u) \cdot \frac{-1 \cdot \exp(-uy)}{-y} \Bigg|_{u=0}^{u=+\infty} - \int_0^\infty \cos u \cdot \frac{-1 \cdot \exp(-uy)}{-y} du$$

$$= \boxed{-1} \int_0^\infty \cos u \, \frac{\exp(-uy)}{y} du = \cos u \, \frac{\exp(-uy)}{-y^2} \Bigg|_{u=0}^{u=+\infty} - \int_0^\infty (-\sin u) \cdot \frac{\exp(-uy)}{-y^2} du$$

$\boxed{-1}$ $\boxed{-1}$ $\boxed{-1}$

$$I'(y) = 1 \cdot \frac{1}{y^2} - \frac{1}{y^2} \cdot I'(y)$$

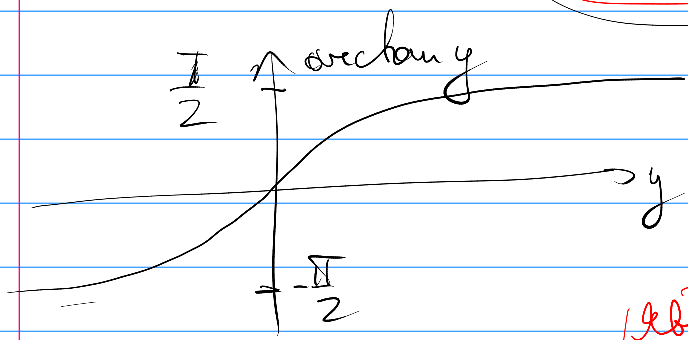
$$I'(y) = \frac{1/y^2}{1 + 1/y^2} = \frac{1}{y^2+1}$$

косяк в знаке !!

$$I'(y) = \frac{1}{1+y^2} \Rightarrow \boxed{I(y) = \arctan(y) + C}$$

$$I(y) = \int_0^\infty \frac{\sin u}{u} \cdot \exp(-uy)\,du$$

$y \to +\infty$ $\quad \boxed{I(y) \xrightarrow[y\to\infty]{} 0}$



$$\boxed{I(y) = \arctan y - \frac{\pi}{2}}$$

$$I(0) = -\frac{\pi}{2}$$

Явно косяк !! в знаке где-то ошиблись.

$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}$$

№8

$$\int_0^\infty \frac{\sin(x^{2022})}{x}\,dx = \int_0^\infty \frac{\sin(x^{2022})}{x^{2022}}\cdot x^{2021}\,dx =$$

$=\frac{1}{x}$

$$= \left\{ \begin{aligned} u &= x^{2022} \\ du &= 2022x^{2021}\,dx \end{aligned} \right\} = \int_0^\infty \frac{\sin u}{u}\cdot\frac{1}{2022}\,du = \frac{1}{2022}\cdot\frac{\pi}{2}$$

№9

$$\int_0^\infty \frac{\cos ax}{b^2+x^2}\,dx = \int_0^\infty \frac{\cos ax}{1+\left(\frac{x}{b}\right)^2}\cdot\frac{1}{b^2}\,dx = \int_0^\infty \frac{\cos(abu)}{1+u^2}\,\frac{b}{b^2}\cdot du =$$

$$= \left\{ \begin{aligned} u &= \frac{x}{b} \\ dx &= du\cdot b \end{aligned} \right\}$$

перемасштабирование

$ab = y$

$$= \frac{1}{b}\int_0^\infty \frac{\cos(abu)}{1+u^2}\,du = \boxed{\frac{1}{b}\int_0^\infty \frac{\cos(yu)}{1+u^2}\,du}$$

Трюк Фейнмана

$$F'(y) = \int_0^\infty \frac{\cos(yu)}{1+u^2}\,du$$

0

$$F(y) = \int_0^\infty \frac{\sin(yu)/u}{1+u^2}\,du$$

$$F''(y) = \int_0^\infty \frac{u(-\sin(yu))}{1+u^2}\,du$$

$F'(0) = \frac{\pi}{2}$

$F(0) = 0$

$F+F''$

$$F - F'' = \int_0^\infty \frac{\sin(yu)}{1+u^2}\left(\frac{1}{u}+u\right)du =$$

$$= \int_0^\infty \frac{\sin(yu)}{1+u^2}\left(\frac{1+u^2}{u}\right)du = \int_0^\infty \frac{\sin(yu)}{u}\,du = \frac{\pi}{2}$$

$+$ $F - F'' = \frac{\pi}{2}$ $F'(y)$?

гот. решения. лин. дифф.

$$F - F'' = \frac{\pi}{2}$$

1) $F = \frac{\pi}{2}$ Ч. Р.

2) $F(y) = \frac{\pi}{2} + \underbrace{H(y)}$

поправка к Ч. Р.-ю.

$$\frac{\pi}{2} + H(y) - H''(y) = \frac{\pi}{2}$$

$$H(y) = H''(y)$$ → замену.

устно!

$$H(y) = \exp(y) \qquad H(y) = \exp(-y)$$

$$H' = \exp(y) \qquad H' = \exp(-y)\cdot(-1)$$

$$H'' = \exp(y) = H \qquad H'' = \exp(-y)$$

$$H(y) = C_1\cdot\exp(y) + C_2\cdot\exp(-y)$$

$$F(y) = \frac{\pi}{2} + C_1\cdot\exp(y) + C_2\cdot\exp(-y)$$

$$F'(y) = 0 + C_1\cdot\exp(y) - C_2\cdot\exp(-y)$$

$$\boxed{F'(0) = \frac{\pi}{2} \qquad F(0) = 0}$$

$F'(y) \;\ldots\; F(y)$

$$\begin{cases} \frac{\pi}{2} + C_1 + C_2 = 0 \\ C_1 - C_2 = \frac{\pi}{2} \end{cases} \Rightarrow C_1,\ C_2 \ldots$$

$$\boxed{F'(y) = \ldots\ldots}$$

№11 $\int\limits_{-\infty}^{\infty} e^{-(ax^2+2bx)}dx =$

идея! Снижаем число пар-ров.

$$= \int\limits_{-\infty}^{\infty} e^{-a\left(x^2 - 2\frac{b}{a^2}x\right)} dx = \int\limits_{-\infty}^{\infty} e^{-a\left(x^2-2\tau x\right)} dx =$$

$\frac{b}{a^2} = \tau$

$$= \int\limits_{-\infty}^{\infty} e^{-a\left(x^2-2\tau x+\tau^2-\tau^2\right)} dx =$$

$$= \int\limits_{-\infty}^{\infty} e^{-a(x-\tau)^2+a\tau^2} dx = e^{a\tau^2}\int\limits_{-\infty}^{\infty} e^{-a(x-\tau)^2} dx =$$

$x-\tau = t \qquad x\in(-\infty;\infty)$

$dx = dt \qquad t\in(-\infty;+\infty)$

$$= e^{a\tau^2}\int\limits_{-\infty}^{\infty} e^{-at^2} dt = e^{a\tau^2}\int\limits_{-\infty}^{\infty} e^{-u^2}\cdot\frac{du}{\sqrt{a}} =$$

$\sqrt{a}t = u$

$du = \sqrt{a}\cdot dt$

$\tau^2 = \frac{b^2}{a^4}$

$$= \frac{e^{a\tau^2}}{\sqrt{a}} \boxed{\int\limits_{-\infty}^{\infty} e^{-u^2} du}$$

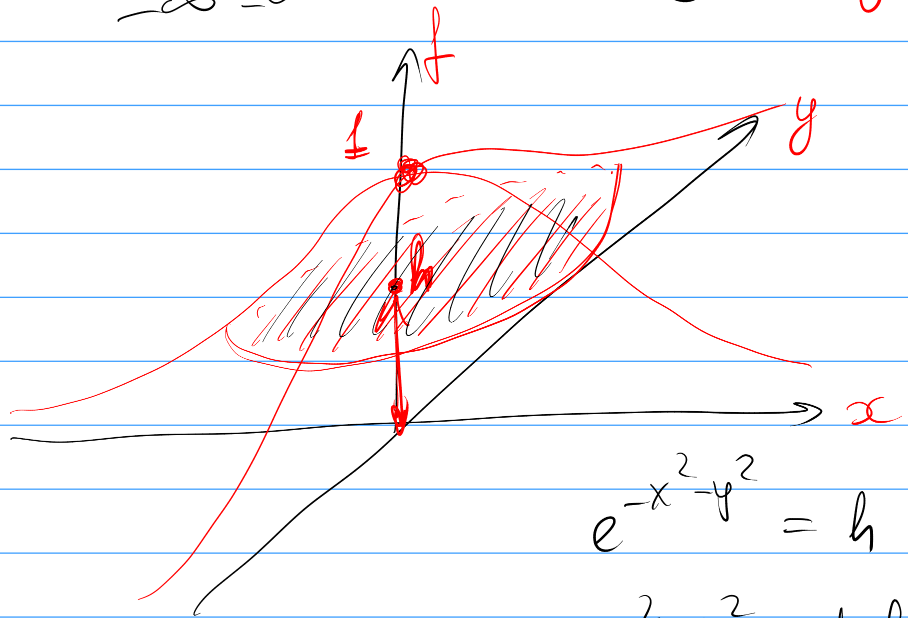
$$I = \int_{-\infty}^{\infty} e^{-u^2}\,du$$

→ трюк Рейнмана

→ полярные координаты

→ объемы.

$$I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dx\,dy = \int_0^1 \pi R^2(h)\,dh;$$



$$e^{-x^2-y^2} = h$$

$$-x^2-y^2 = \ln h$$

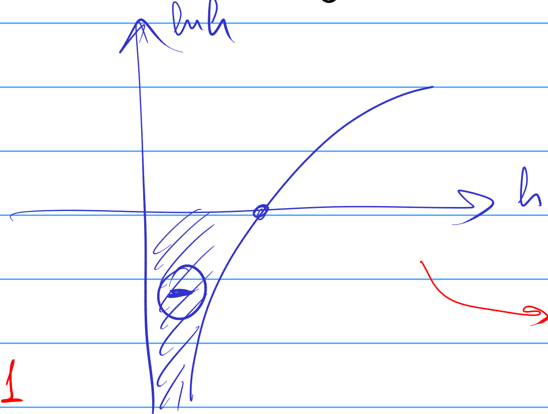
$$R^2 = x^2+y^2 = -\ln h$$

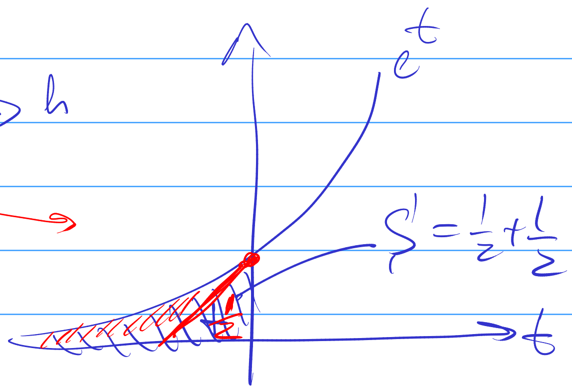
$$\pi R^2 = -\pi \ln h$$

$$I^2 = \int_0^1 -\pi \underbrace{\ln h}_{\wedge 0}\,dh = -\pi\int_0^1 1\cdot\ln h\,dh$$

→ по частям

$$\int_0^1 \ln h\,dh$$



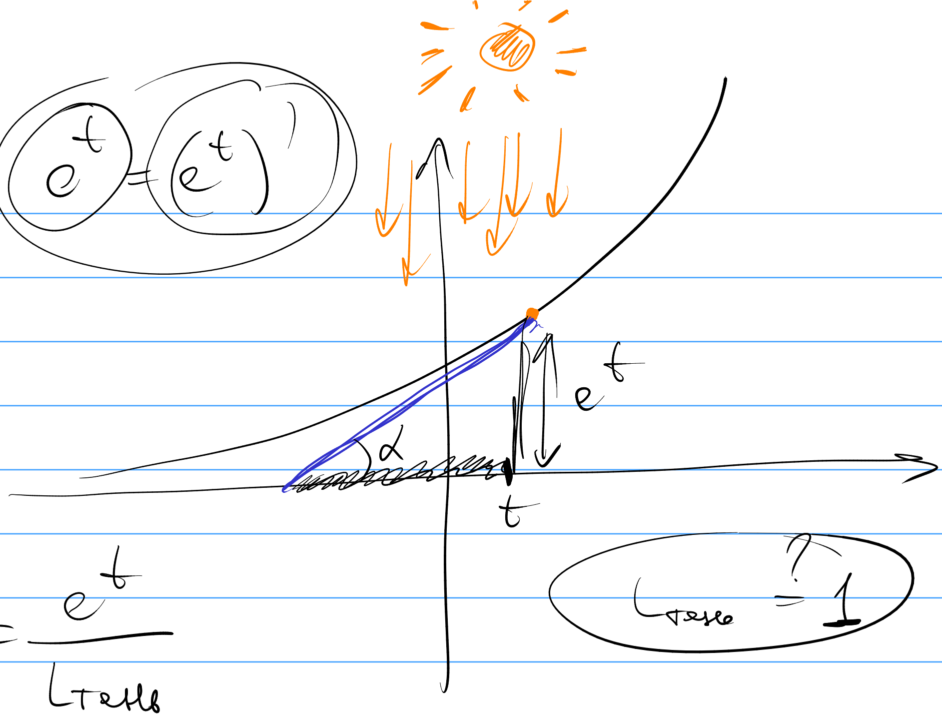


$$I^2 = \pi\int_{-\infty}^{0} e^t\,dt = \pi\cdot 1$$

$$I = \sqrt{\pi}$$

?

$e^t = (e^t)'$



$L_{\text{тени}} \overset{?}{=} 1$

$\operatorname{tg}\alpha = e^t = \dfrac{e^t}{L_{\text{тень}}}$

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