$\boxed{\text{Листочек 5}}$

$\boxed{\text{идеи:}}$

$$\int_0^\infty \text{sin } p, x \ldots\ldots dx$$

→ трюк Фейнмана: $\boxed{\begin{matrix} I(p) & \text{– сложный} \\ I'(p) & \text{– легче} \end{matrix}}$ $= \int \ldots dx$

$I'(p)$ – сложный $= \int \ldots dx$

$I(p)$ – легче

→ Везде вижу экспоненту

$$\int \ldots \frac{1}{x} \ldots\ldots dx$$

$$\int_0^\infty \exp(-yx)\, dy = \frac{\exp(-yx)}{-x}\Bigg|_{y=0}^{y=\infty} =$$

$$= \frac{1}{x}$$

→ коржики с пирожками

$$\int_0^\infty \frac{\sin x}{x}\, dx = \ldots\ldots = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\cos bx}{1+x^2} = \ldots\ldots = \frac{\pi}{2} \cdot \exp(-b)$$

. . .

→ по частям

...

(10^*)

$$\int_0^\infty \left(\frac{\sin px}{x}\right)^2 dx$$

похоже на
$$\frac{\pi}{2} = \int_0^\infty \frac{\sin x}{x}\,dx$$

$I(p)$
$I'(p)$?

$I'(p)$
$I(p)$?

интуиция

$$\left(\sin(px)\right)'_p = x\cdot\cos(px)$$

$$\left(\frac{\sin(px)}{x}\right)^2 \le \frac{1}{x^2}$$

$$I(p) = \int_0^\infty \left(\frac{\sin px}{x}\right)^2 dx$$

$$I'(p) \overset{?}{=} \int_0^\infty 2\left(\frac{\sin px}{x}\right)^1 \cdot \left(\frac{1}{x}\cdot x\cdot \cos(px)\right) dx =$$

$$= \int_0^\infty \frac{2\sin(px)\cdot\cos(px)}{x}\,dx = \int_0^\infty \frac{\sin(2px)}{x}\,dx =$$

$$= \int_0^\infty \frac{\sin(2px)}{\cancel{2p}x}\cdot d(\cancel{2p}x) = \int_0^\infty \frac{\sin u}{u}\,du = \frac{\pi}{2} = I'(p)$$

$$\boxed{I(p) = \frac{\pi}{2}p + C}$$

$$I(p) = \int_0^\infty \left(\frac{\sin(px)}{x}\right)^2 dx$$

легко найти при $p=0$

$$I(0) = \int_0^\infty 0\,dx = 0$$

$C = 0$

$$I(p) = \frac{\pi}{2}\cdot p$$

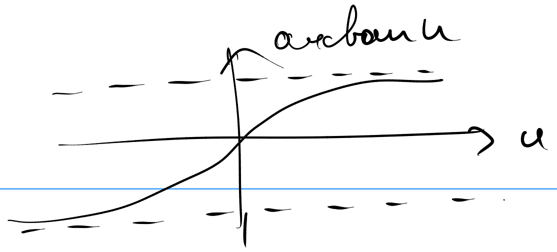
$(N9^*)$

$$\int_0^\infty \frac{dx}{(x^2+p)^n} \qquad p>0 \qquad n\in\mathbb{N}$$

$n=1$

$$\int_0^\infty \frac{dx}{x^2+p} = \qquad x = \sqrt{p}\cdot u$$

$$= \int_0^\infty \frac{\sqrt{p}\,du}{p\cdot u^2+p} = \frac{\sqrt{p}}{p}\cdot\int_0^\infty \frac{du}{u^2+1} = p^{-\frac{1}{2}}\cdot \operatorname{arctan}\Big|_0^\infty =$$



$$= p^{-\frac{1}{2}} \cdot \frac{\pi}{2}$$

$$I_n = \int_0^\infty \frac{1}{(x^2+p)^n}\,dx = \int_0^\infty (x^2+p)^{-n}\,dx$$

$I(p) \to I'(p)$!!

$I'(p) \to I(p)$!!

$$I'_n = \int_0^\infty (-n)\cdot(x^2+p)^{-n-1}\,dx = (-n)\cdot I_{n+1}$$

$$I'_n = (-n)\cdot I_{n+1}$$

$$\boxed{I_{n+1} = \left(-\frac{1}{n}\right) I'_n}$$

$$I_1 = p^{-\frac{1}{2}}\frac{\pi}{2}$$

$$I_2 = \frac{\pi}{2}\left(-\frac{1}{2}\right)p^{-\frac{3}{2}}\cdot\left(-\frac{1}{1}\right)$$

$$I_3 = \frac{\pi}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)p^{-\frac{5}{2}}\cdot\left(-\frac{1}{1}\right)\left(-\frac{1}{2}\right)$$

...

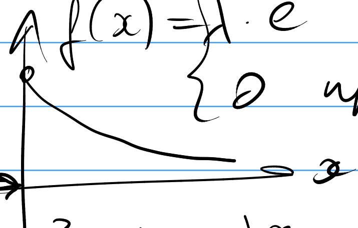
$$I_n = \frac{\pi}{2}\cdot p^{-n+\frac{1}{2}}\cdot\frac{(2n-1)!!}{2^{n-1}}\cdot\frac{1}{(n-1)!}$$

$9!! = 9\cdot7\cdot5\cdot3\cdot1$ double factorial

Упр. → №7, №8

$X \sim Exp(\lambda)$

$$f(x) = \begin{cases}\lambda\cdot e^{-\lambda x} & \text{при } x \geq 0\\ 0 & \text{при } x < 0\end{cases}$$



Надо найти:

$$E(X^2)\ ? = \int_0^\infty x^2\cdot\lambda\cdot e^{-\lambda x}\,dx$$

$$E(X^{2022})\ ? = \int_0^\infty x^{2022}\cdot\lambda\cdot e^{-\lambda x}\,dx$$

$\boxed{\text{Трюк Фейнмана.}}$

Возьмём более простую

$$M(u) = E(\exp(uX)) = \int_0^\infty \exp(ux)\cdot\lambda\cdot\exp(-\lambda x)\,dx$$

$u < \lambda$

ф. производящая моменты

$$= \lambda \int_0^\infty \exp((u-\lambda)x)\,dx =$$

$$= \lambda \left.\frac{\exp((u-\lambda)x)}{u-\lambda}\right|_{x=0}^{\infty} = \frac{\lambda}{\lambda-u}$$

нашёл

$$M(u) = E(\exp(uX)) = \frac{\lambda}{\lambda-u} \quad (u<\lambda) \quad \lambda>0$$

$$M'(u) = E(X\cdot\exp(uX))$$

$$M''(u) = E(X\cdot X\cdot\exp(uX))$$

$$E(X^2) = M''(0)$$

$$E(X^{2022}) = M^{(2022)}(0)$$

$$M(u) = \frac{\lambda}{\lambda-u} = \frac{1}{1-\frac{u}{\lambda}} = 1 + \left(\frac{u}{\lambda}\right) + \left(\frac{u}{\lambda}\right)^2 + \left(\frac{u}{\lambda}\right)^3 + \ldots$$

выпадут в $M''(0)$ выпадут в $M''(0)$

9 класс Первый ряд Тейлора.

$$\frac{1}{1-q} = 1 + q + q^2 + q^3 + q^4 + \ldots \quad \text{при } |q|<1$$

$$M''(0) = \frac{2}{\lambda^2} \qquad M^{(2022)}(0) = \frac{2022!}{\lambda^{2022}}$$

$$Var(X) = E(X^2) - (E(X))^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$$

(7) $\int_0^\infty x^2\cdot e^{-px}\,dx$

№6

$$X \sim N(0; \sigma^2)$$

$$E(X^4)\ ?$$

$$\nearrow \quad E(X^{2022})\ ?$$

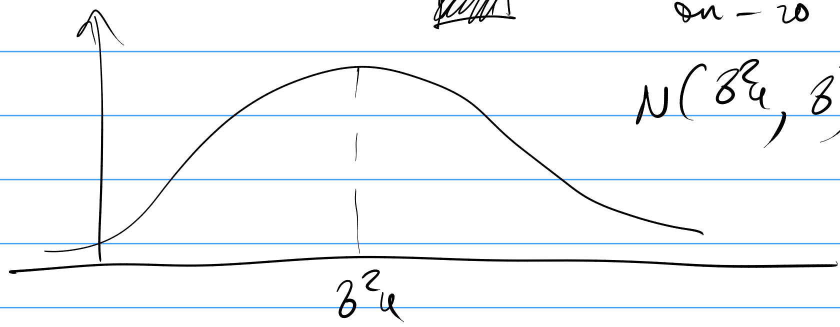
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$$

эти сложные, а мы найдем более простой !!

$$M(u) = E(\exp(uX)) = \int_{-\infty}^{\infty} \exp(ux) \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{1}{2\sigma^2}x^2\right)dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}\left(x^2 - 2\sigma^2 ux + (\sigma^2 u)^2 - (\sigma^2 u)^2\right)\right) \cdot dx = \exp\left(\frac{\sigma^4 u^2}{2\sigma^2}\right) \cdot \boxed{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{1}{2\sigma^2}(x-\sigma^2 u)^2\right)dx} =$$

метод пристального взгляда от — го охотника



$$M(u) = E(\exp(uX)) = \exp\left(\frac{\sigma^2 u^2}{2}\right)$$

$$E(X^4) = M''''(0)$$

$$E(X^{2022}) = M^{(2022)}(0)$$

$$\exp(t) = 1 + t + \frac{t^2}{2!} + \ldots$$

$$= 1 + \frac{\sigma^2}{2} \cdot u^2 + \left(\frac{\sigma^2}{2}\right)^2 \cdot \frac{1}{2!} \cdot u^4 + \left(\frac{\sigma^2}{2}\right)^3 \cdot \frac{1}{3!} \cdot u^6 + \ldots$$

$$M(u) = 1 + \left(\frac{\sigma^2 u^2}{2}\right) + \left(\frac{\sigma^2 u^2}{2}\right)^2 \cdot \frac{1}{2!} + \left(\frac{\sigma^2 u^2}{2}\right)^3 \frac{1}{3!} + \ldots$$

$$1 + ?u^2 + ?u^4 + ?u^6 + \ldots$$

$$E(X^4) = M''''(0) = \left(\frac{\sigma^2}{2}\right)^2 \cdot \frac{1}{2!} \cdot 4!$$

$$E(X^{2022}) = M^{(2022)}(0) = \left(\frac{\sigma^2}{2}\right)^{1011}\cdot\frac{1}{1011!}\cdot 2022! =$$

$$= \sigma^{2022}\cdot\frac{2022\cdot 2021\cdot 2020\cdot 2019\cdot\ldots\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1011\cdot 2\cdot 1010\cdot 2\cdot 1009\cdot\ldots\cdot 2\cdot 2\cdot 2\cdot 1} =$$

$$= \sigma^{2022}\cdot 2021!!$$

№8

$$\int_0^{\infty} x^4\cdot e^{-px^2}\,dx$$

$$\underline{E(X^4)} = \int_{-\infty}^{\infty} x^4\cdot\frac{1}{\sqrt{2\pi\sigma^2}}\cdot e^{-\frac{1}{2\sigma^2}x^2}\,dx \qquad p=\frac{1}{2\sigma^2}$$

Упр 6

$$\int_0^{\infty}\frac{x\sin(px)}{1+x^2}\,dx \leftarrow I'(p)$$

$$\nwarrow I(p)$$

$$\boxed{J(p)=\int_0^{\infty}\frac{\cos(px)}{1+x^2}\,dx} \qquad J'(p)=\int_0^{\infty}\frac{-\sin(px)\cdot x}{1+x^2}\,dx$$

$\boxed{\text{в порядке}}$ знаем ответ