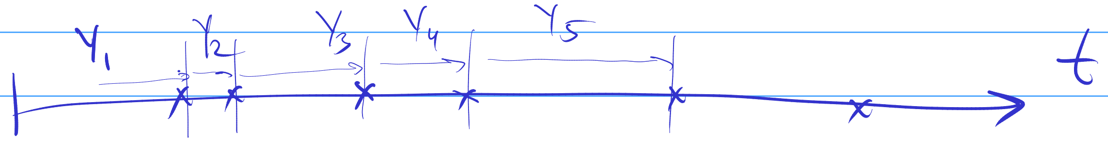
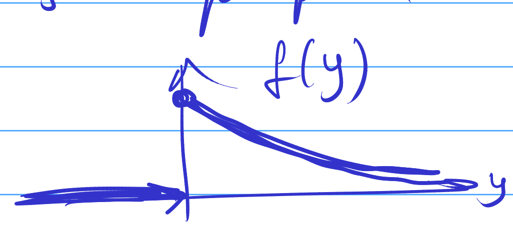
Привет !!

Мотивационный момент!



Y_i ~ незав-мы и экспонец-но распр-ны с пар-м λ.

$$f_{Y_i}(y)=\begin{cases}\lambda \cdot e^{-\lambda y}, & y\geq 0\\ 0, & \text{иначе}\end{cases}$$



а) $Y_1+Y_2+\dots+Y_n = S_n$ $\quad f_{S_n}(s)$?

б) $p+q=n$ $\quad \underbrace{Y_1+Y_2+\dots+Y_p}_{\text{для себя}}+\underbrace{Y_{p+1}+\dots+Y_{p+q}}_{\text{на продажу}}=S_n$

$$R_{p,q}=\frac{S_p}{S_{p+q}}$$ — доля времени, потр-ная на жарку блинов для себя.

$R_{1,2}$

Найдите ф. плотности $R_{p,q}$?

$\begin{pmatrix}x\\y\end{pmatrix}\longrightarrow\begin{pmatrix}\varphi\\ \tau\end{pmatrix}$ $\qquad \left|\det J_{\binom{x}{y}=f\binom{\tau}{\varphi}}\right|$

$$\iint\limits_{\sim} \dots\, dx\,dy=\iint\limits_{\sim} \dots \cdot \boxed{} \cdot d\tau\, d\varphi$$

диф. форма $dx \wedge dy$

опр. $\det\begin{pmatrix} \downarrow & \downarrow \\ v & v \\ | & | \end{pmatrix} = 0$

$$dx \wedge dx = 0$$
$$dx \wedge dy = -dy \wedge dx$$

$$-\det\begin{pmatrix} | & | \\ a & b \\ | & | \end{pmatrix} = \det\begin{pmatrix} | & | \\ b & a \\ | & | \end{pmatrix}$$

к дифф.

$$\int_0^\infty \int_0^\infty e^{-x^2-y^2}\, dx\, dy = \text{(с точн. знака)}$$

$$= \int_0^\infty \int_0^\infty e^{-x^2-y^2}\, dx \wedge dy = \int_0^\infty \int_0^{\frac{\pi}{2}} e^{-r^2\cos^2\varphi - r^2\sin^2\varphi} \cdot$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$$\cdot \left(dr \cdot \cos\varphi - r\sin\varphi\, d\varphi\right) \wedge \left(dr \cdot \sin\varphi + r\cos\varphi\, d\varphi\right) =$$

$$= \int_0^\infty \int^{\pi/2} e^{-r^2} \left(dr\cos\varphi \wedge r\cos\varphi\, d\varphi - r\sin\varphi\, d\varphi \wedge dr\sin\varphi\right) =$$

$$= \int_0^\infty \int_0^{\pi/2} e^{-r^2} \left(\cos^2\varphi \cdot r\, dr \wedge d\varphi + \sin^2\varphi \cdot r \cdot dr \wedge d\varphi\right) =$$

$$= \int_0^\infty \int_0^{\pi/2} e^{-r^2} r\, dr \wedge d\varphi$$

(!) $dx \wedge dy \wedge dz$ $\qquad$ $dx \wedge dy$

5	1	?
6	3	?
2	8	?

Задача.

Y_1, Y_2, Y_3

офе по продолжу.

$$f(y_1, y_2, y_3)\, dy_1 \wedge dy_2 \wedge dy_3 =$$

$$= \lambda e^{-\lambda y_1} \cdot \lambda e^{-\lambda y_2} \cdot \lambda e^{-\lambda y_3}\, dy_1 \wedge dy_2 \wedge dy_3$$

$$\underbrace{Y_1+Y_2+Y_3}_{\|} \qquad \frac{Y_1}{Y_1+Y_2} \qquad \frac{Y_1+Y_2}{Y_1+Y_2+Y_3}$$

$$S_3, \quad R_{1,12}, \quad R_{12,123}$$

$$Y_1 \overset{?}{=} S_3 \cdot R_{1,12} \cdot R_{12,123}$$

$$Y_2 = S_3 \cdot R_{12,123} \cdot (1 - R_{1,12})$$

$$Y_3 = S_3 \cdot (\overbrace{1 - R_{12,123}}^{Y_1+Y_2})$$

$$dy_1 \wedge dy_2 = d(s_3 \cdot r_{1,12} \cdot r_{12,123}) \wedge d(S_3 \cdot r_{12,123} - S_3 \cdot r_{12,123} \cdot r_{1,12}) =$$

$$= d(S_3\, r_{1,12} \cdot r_{12,123}) \wedge d(S_3 \cdot r_{12,123}) =$$

$da \wedge da = 0$

$$= \left(dr_{1,12} \cdot S_3\, r_{12,123} + r_{1,12} \cdot d(S_3 \cdot r_{12,123})\right) \wedge d(S_3\, r_{12,123}) =$$

$$= S_3\, r_{12,123} \cdot dr_{1,12} \wedge d(S_3\, r_{12,123})$$

$$dy_1 \wedge dy_2 \wedge dy_3 = S_3\, r_{12,123} \cdot dr_{1,12} \wedge d(S_3\, r_{12,123}) \wedge d(S_3 - S_3\, r_{12,123}) =$$

$$= S_3\, r_{12,123} \cdot dr_{1,12} \wedge (\cancel{r_{12,123}\, dS_3} + S_3\, dr_{12,123}) \wedge dS_3 =$$

$dy_1 \wedge dy_2 \wedge dy_3 = s_3^2 \cdot \tau_{12,123} \cdot d\tau_{1,12} \wedge d\tau_{12,123} \cdot ds_3$

τ_{12}^{1}

τ_{123}^{12}

$f(s_3, \tau_{1,12}, \tau_{12,123}) =$

$= \boxed{\lambda^3 \cdot e^{-\lambda s_3} \cdot s_3^2 \, \tau_{12,123}}$ ← !

$\lambda = 1$

$= e^{-s_3} \cdot s_3^2 \cdot \tau_{12,123} \cdot \boxed{1}$

$\kappa\ s_3$ $\quad R_{12,123}$ $\quad R_{1,12}$ — $\frac{1}{B(1,1)} \cdot \tau^0 \cdot (1-\tau)^0$

$f_{s_3} = \frac{1}{\Gamma(3)} \cdot e^{-s_3} \cdot s_3^2$

$\Gamma(3) = \int_0^\infty e^{-s} s^2 ds$

$\Gamma(x) = \int_0^\infty e^{-s} s^{x-1} ds$

$\Gamma(5)$?

$Y_1 + Y_2 \dots Y_5 = S_5$

$f(t) = e^{-t} \cdot t^4 \cdot \frac{1}{\Gamma(5)}$

$R \in [0;1]$ — ф-ция вр-ни пор-нная на пар-ии ρи себе.

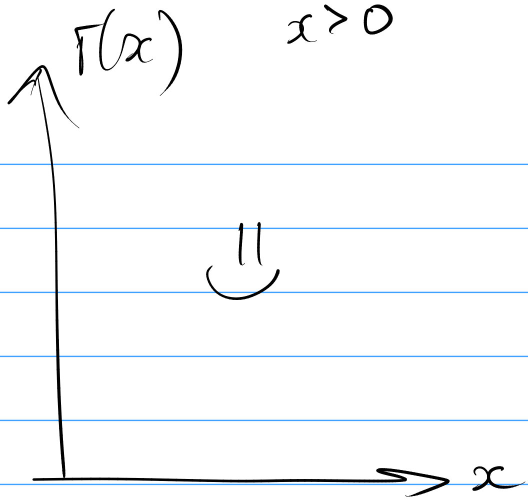
$p \quad q$

$\boxed{f(\tau) = \frac{1}{B(p,q)} \cdot \tau^{p-1} \cdot (1-\tau)^{q-1}}$

$R_{12,123}$ $\quad f = \frac{1}{B(2,1)} \cdot \tau^{2-1} \cdot (1-\tau)^0$

$$\boxed{\Gamma(x) = (x-1)\cdot\Gamma(x-1)}$$

$x > 0$



сложные инт → сводим к $B(5,2)$

$\Gamma(3)$

$$\int_0^1 x^3\sqrt{1-x^3}\,dx$$

$\text{сложн}^{?}\cdot(1-\text{сложн})^{?}$

$x^3 = t \quad x = t^{1/3}$

$$\int_0^1 t\cdot(1-t)^{\frac12}\cdot d\left(t^{\frac13}\right) = \int_0^1 t\cdot(1-t)^{\frac12}\cdot\frac13 t^{-\frac23}\cdot dt =$$

$$= \boxed{\frac13\int_0^1 t^{\frac13}(1-t)^{\frac12}\,dt} = \frac13 B\left(\frac43, \frac32\right)$$

$$B(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}\,dx$$

$$I = \int_0^1 \ln\Gamma(x)\,dx = \left(\text{см. лекцию } \Gamma(x)\cdot\Gamma(1-x) = \frac{\pi}{\sin(\pi x)}\right)$$

$$2I = \int_0^1 \ln\Gamma(x)\,dx + \int_0^1 \ln\Gamma(x)\,dx =$$

$$= \int_0^1 \ln\Gamma(x)\,dx + \int_0^1 \ln\Gamma(1-x)\,dx =$$

$$= \int_0^1 \ln\big(\Gamma(x)\cdot\Gamma(1-x)\big)\,dx =$$

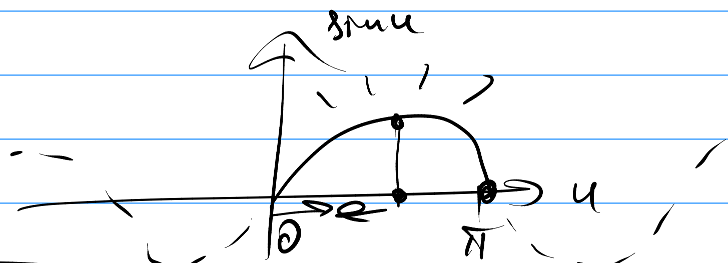
$$= \int_0^1 \ln\frac{\pi}{\sin\pi x}\,dx =$$

$$= \underbrace{\boxed{\int_0^1 \ln\pi\,dx}}_{\ln\pi} - \int_0^1 \ln\sin\pi x\,dx =$$

$\pi x = u$

$dx = \frac{du}{\pi}$

$$= \ln\pi - \int_0^{\pi} \ln\sin u \cdot \frac{1}{\pi}\,du =$$



$$= \ln\pi - \frac{2}{\pi}\underbrace{\boxed{\int_0^{\pi/2} \ln\sin u\,du}}_{J}$$

$$2J = \int_0^{\pi/2} \ln\sin u\,du + \int_0^{\pi/2} \ln\sin\left(\frac{\pi}{2}-u\right)du =$$

$$= \int \ln \sin u \, du + \int \ln \cos u \, du =$$

$$= \int_0^{\pi/2} \ln \frac{\sin 2u}{2} \, du = \frac{2}{2}\int_0^{\pi/2} \ln \sin 2u \, du - \ln 2 \cdot \frac{\pi}{2} =$$

$$= \frac{1}{2} \int_0^{\pi/2} \ln \sin 2u \, d(2u) - \ln 2 \cdot \frac{\pi}{2} = \frac{1}{2} J - \ln 2 \frac{\pi}{2}$$

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