Привет 1

→ трюк Фейнмана $I(p) = \qquad \to I'(p)$

→ умение увидеть экспоненту $I'(p) = \ldots\ldots \to I(p)$

$$\int \ldots \frac{1}{x} \ldots\ldots dx = \int \ldots\ldots \int_0^\infty \exp(-yx)\,dy \ldots\ldots dx$$

→ корзиночный метод

$$\int_0^\infty \frac{\sin px}{x}\,dx = \ldots = \frac{\pi}{2}$$

$$\int_0^\infty \frac{\cos(px)}{1+x^2}\,dx = \ldots = \frac{\pi}{2}\cdot\exp(-p)$$

$p \geq 0$

....

похоже

№10*

$$\int_0^\infty \left(\frac{\sin px}{x}\right)^2 dx = I(p) \to I'?$$

$$= I'(p) \to I?$$

$$I(p) = \int_0^\infty \left(\frac{\sin px}{x}\right)^2 dx \qquad \left(\frac{\sin px}{x}\right)^2 \leq \frac{1}{x^2}$$

$$I'(p) \overset{?}{=} \int_0^\infty \frac{2\sin px}{x}\cdot\cos px\,dx =$$

$$= \int_0^\infty \frac{\sin(2px)}{x}\,dx = \int_0^\infty \frac{\sin(2px)}{2px}\cdot d(2px) = \frac{\pi}{2} = I'(p)$$

$$I(p) = \frac{\pi}{2}\cdot p + C \qquad C? \qquad p=0 \quad \int_0^\infty \left(\frac{\sin 0}{x}\right)^2 dx = 0$$

$$I(0) = 0$$

ответ: $I(p) = \frac{\pi}{2} p$

№9* $\int_0^\infty \frac{dx}{(x^2+p)^n}$. $p>0$ $n\in\mathbb{N}$

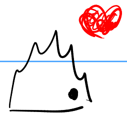
→ т. Фейнмана

→ рабоче-крестьянский

$$I_n(p)=\int_0^\infty \frac{dx}{(x^2+p)^n}$$

$$I'(p)=\int_0^\infty (-n)\cdot\frac{1}{(x^2+p)^{n+1}}\,dx$$

$$I'(p)=(-n)\cdot\int_0^\infty\frac{1}{(x^2+p)^{n+1}}\,dx$$



$(p>0)$

$$I_1(p)=\int_0^\infty\frac{dx}{x^2+p}=$$

$$=\left\{x=\sqrt{p}\cdot u\right\}=\int_0^\infty\frac{d(\sqrt{p}u)}{pu^2+p}=\frac{\sqrt{p}}{p}\int_0^\infty\frac{du}{u^2+1}=\frac{\sqrt{p}}{p}\cdot\frac{\pi}{2}$$

$$I_n'(p)=(-n)\cdot I_{n+1}(p)$$

$$\boxed{I_{n+1}=\frac{1}{-n}\cdot I_n'}$$

$$I_1=p^{-\frac{1}{2}}\cdot\frac{\pi}{2}$$

D ↓

$$I_2=\frac{1}{-1}\cdot\left(-\frac{1}{2}\right)\cdot p^{-\frac{3}{2}}\cdot\frac{\pi}{2}$$

D ↓

$$I_3=\frac{1}{1}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{2}\cdot p^{-5/2}\cdot\frac{\pi}{2}$$

(n-1) раз дифф

D ↓

$$I_n=\frac{\pi}{2}\cdot\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{7}{2}\cdot\ldots\cdot\frac{(2n-3)}{2}\cdot p^{-(2p-1)/2}\cdot\frac{1}{(n-1)!}$$

Р.к.

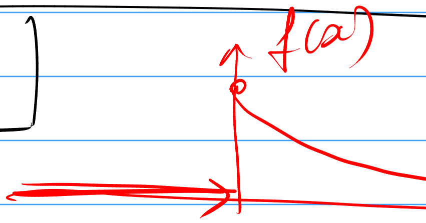
$$I_n=\int_0^\infty\frac{du}{(u^2+1)^n}=\int_0^\infty\frac{u^2+1-u^2}{(u^2+1)^n}\,du=\underbrace{\int_0^\infty\frac{1}{(u^2+1)^{n-1}}\,du}_{I_{n-1}}-\int_0^\infty\frac{u^2}{(u^2+1)^n}\,du$$

$$I_n = I_{n-1} - \left[\int_0^\infty \overset{\text{Diff}}{u} \cdot \overset{\text{Int}}{\frac{u}{(u^2+1)^n}}\, du\right]$$

$$I_n = I_{n-1} - \left[\underbrace{u \cdot \frac{1}{(u^2+1)^{n-1}} \cdot \frac{1}{2} \cdot \left(\frac{1}{1-n}\right)\Bigg|_0^\infty} - \int_0^\infty 1 \cdot \frac{1}{(u^2+1)^{n-1}} \cdot \frac{1}{2} \cdot \frac{1}{1-n}\, du\right]$$

$$I_n = I_{n-1} + \frac{1}{2} \cdot \frac{1}{1-n} I_{n-1} \quad \ldots$$

кнопка про №7 и №8



ф. плотн

т. вер.

$X \sim Expon.(\lambda)$

$$E(X^2) = ? \quad \int_0^\infty x^2 \cdot \lambda \cdot \exp(-\lambda x)\, dx \qquad f(x) = \begin{cases} \lambda \cdot \exp(-\lambda x), & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$E(X^{2022}) = ? \quad \int_0^\infty x^{2022} \lambda \cdot \exp(-\lambda x)\, dx$$

простые интегралы (!)

$$M(u) = E(\exp(uX)) = \int_0^\infty \exp(ux) \cdot \lambda \cdot \exp(-\lambda x)\, dx = \quad \boxed{u < \lambda}$$

$$= \lambda \int_0^\infty \exp((u-\lambda)x)\, dx = \frac{\lambda}{u-\lambda} \exp((u-\lambda)x)\Big|_{x=0}^{x=\infty} =$$

$$= \frac{\lambda}{u-\lambda} \cdot (0 - 1) = \frac{\lambda}{\lambda - u}$$

$$M'(u) = E(X \cdot \exp(uX))$$

$$M''(u) = E(X^2 \cdot \exp(uX)) \Rightarrow M''(0)$$

$$M^{(2022)}(0) = E(X^{2022} \exp(0X))$$

$$\int_0^\infty x^2 \lambda e^{-\lambda x}\, dx \qquad \int_0^\infty x^{2022} \lambda \cdot e^{-\lambda x}\, dx$$

Надо-то:

$$E(X^2) = ? \quad \frac{2}{\lambda^2}$$

$$E(X^{2022}) = ? \quad \frac{2022!}{\lambda^{2022}}$$

$$\boxed{\frac{1}{1-q} = 1 + q + q^2 + q^3 + q^4 + \ldots}$$

Мой первый ряд Тейлора (9 класс)

$$\boxed{M(u)} = \frac{\lambda}{\lambda-u} = \frac{1}{1-\frac{u}{\lambda}} = 1 + \frac{u}{\lambda} + \left(\frac{u}{\lambda}\right)^2 + \left(\frac{u}{\lambda}\right)^3 + \ldots$$

$\lambda > 0$

$$M'(0) = \frac{1}{\lambda} \qquad M''(0) = \frac{2}{\lambda^2} \qquad M^{(2022)}(0) = \frac{2022!}{\lambda^{2022}}$$

№8 $Z \sim N(0; \sigma^2)$

$E(Z^4)$? $= \int_{-\infty}^{\infty} z^4 \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{1}{2\sigma^2}(z-0)^2\right) dz$

$E(Z^{2022})$?

$p = +\frac{1}{2\sigma^2}$

№8

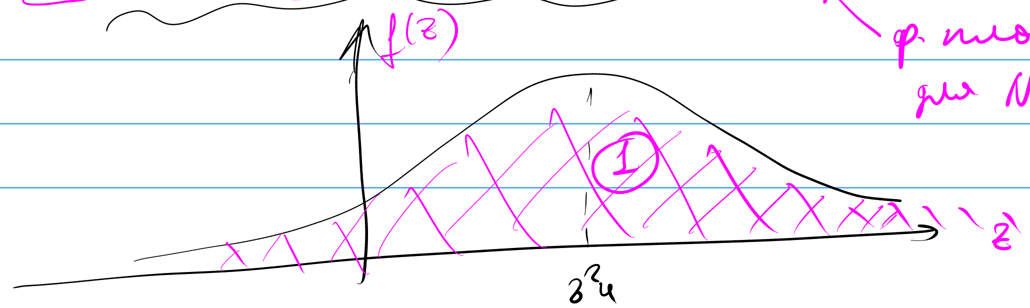
$$M(u) = \boxed{E(\exp(uZ))} = \int_{-\infty}^{\infty} \exp(uz) \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{1}{2\sigma^2} z^2\right) dz =$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}\left(z^2 - 2uz\sigma^2\right)\right) dz =$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}\left(z^2 - 2z\cdot\sigma^2 u + \sigma^4 u^2\right) + \frac{\sigma^4 u^2}{2\sigma^2}\right) dz$$

$$= \exp\left(\frac{\sigma^2 u^2}{2}\right) \cdot \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(-\frac{1}{2\sigma^2}\left(z - \sigma^2 u\right)^2\right) dz}_{1} = \exp\left(\frac{\sigma^2 u^2}{2}\right)$$

ф. плотн для $N(\sigma^2 u; \sigma^2)$



$$E(\exp(uZ)) = \exp\left(\frac{u^2\sigma^2}{2}\right) = M(u) \quad \text{при} \quad Z \sim N(0;\sigma^2)$$

$$E(Z^4) \stackrel{?}{=} M''''(0) \qquad M''''(u) = E(Z\cdot Z\cdot Z\cdot Z \exp(uZ))$$

$$E(Z^{2022}) \stackrel{?}{=}$$

$$u^4 \cdot \sigma^4 \cdot \frac{1}{8} \qquad M^{(4)}(0) = E(Z^4)$$

$$M^{(2022)}(0) = E(Z^{2022})$$

$$M(u) = \exp\left(\frac{u^2\sigma^2}{2}\right) = \boxed{1 + \frac{u^2\sigma^2}{2}} + \left(\frac{u^2\sigma^2}{2}\right)^2 \cdot \frac{1}{2!} + \left(\frac{u^2\sigma^2}{2}\right)^3 \frac{1}{3!} + \ldots$$

$$\exp(t) = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots \qquad E(Z^{357}) = 0$$

$$E(Z^4) = M^{(4)}(0) = 4! \cdot \sigma^4 \cdot \frac{1}{8}$$

$$E(Z^{2022}) = \int_{-\infty}^{\infty} z^{2022} \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2\sigma^2}\cdot z^2}\, dz$$

$$= M^{(2022)}(0) = \frac{1}{1011!} \cdot 2022! \cdot \frac{\sigma^{2022}}{2^{1011}} = 2021!! \cdot \sigma^{2022}$$

$$\frac{2022 \cdot 2021 \cdot 2020 \cdot \ldots \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 2 \cdot \ldots \cdot 2 \cdot 2} \cdot \frac{1}{1011 \cdot 1010 \cdots 1}$$

$$2021 \cdot 2019 \cdot 2017 \ldots 1013 \cdot 1011 \cdot \ldots \cdot 1$$

№6

$$\int_0^\infty \frac{x \sin(px)}{1+x^2}\, dx = A(p)$$

$$A(p) = -\int_0^\infty \frac{\cos(px)}{1+x^2}\, dx$$

№3

$$B(p) = \int_0^\infty \frac{\sin(px)}{x} \cdot e^{-x}\, dx$$

$$B'(p) = \int_0^\infty \cos(px) \cdot e^{-x}\, dx$$