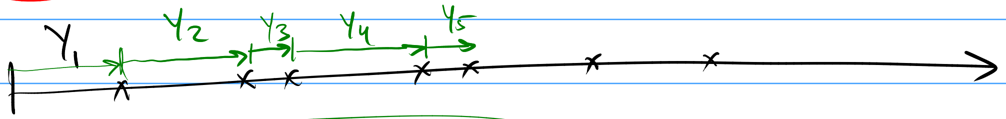
Привет :)

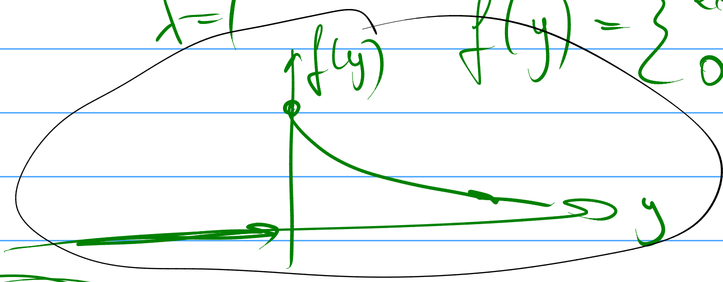
$$B(p,q) = ? \quad \int_0^1 \tau^{p-1}\cdot(1-\tau)^{q-1}\,d\tau \qquad p>0 \quad q>0$$

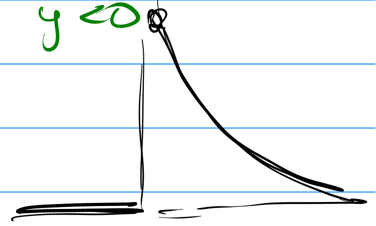
$$\Gamma(x) = ? \quad \int_0^\infty e^{-t}\cdot t^{x-1}\,dt \qquad x>0$$



Y_i ~ незав и один. распр $\qquad Y_i \sim \exp(\lambda)$

$\lambda=1 \qquad f(y) = \begin{cases} \exp(-y) & y\geq 0 \\ 0 & y<0 \end{cases}$





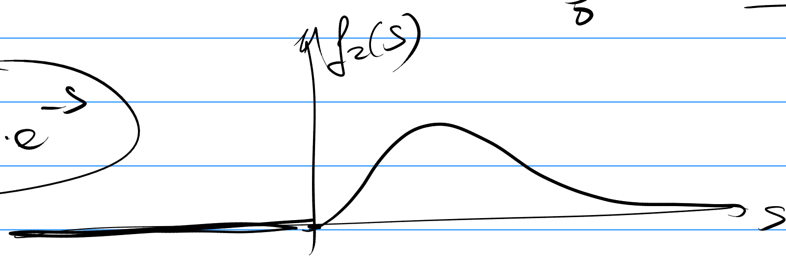
Вопрос 1 $\qquad S_n = Y_1+Y_2+\ldots+Y_n$

$f_{S_n}(s)$? $\qquad \to \Gamma(n)$

$$f_2(s) = \int_0^s f(x)\cdot f(s-x)\,dx =$$

$$= \int_0^s e^{-x}\cdot e^{-(s-x)}\,dx = e^{-s}\cdot\int_0^s e^0\,dx = se^{-s}$$

$$f_2(s) = s\cdot e^{-s}$$



$$f_3(s) = \int_0^s f_2(x)\cdot f(s-x)\,dx = \int_0^s xe^{-x}\cdot e^{-(s-x)}\,dx =$$

$$\int_0^1 x \sqrt[3]{1-x^3}\,dx \longrightarrow \Gamma(n)=\int_0^\infty e^{-t}\cdot t^{n-1}\,dt$$

$$\longrightarrow B(p,q)=\int_0^1 \tau^{p-1}\cdot(1-\tau)^{q-1}\,d\tau$$

$x^3=u$

$x=(u)^{1/3}$

$$\int_0^1 x^3\sqrt{1-x^3}\,dx=\int_0^1 u\cdot\sqrt{1-u}\cdot d\left(u^{1/3}\right)=$$

$$=\int_0^1 u\cdot\frac{1}{3}u^{-\frac{2}{3}}\cdot(1-u)^{\frac{1}{2}}\,du=\frac{1}{3}\int_0^1 u^{\frac{1}{3}}\cdot(1-u)^{\frac{1}{2}}\,du=$$

$$=\frac{1}{3}\cdot B\left(\frac{4}{3},\frac{3}{2}\right)$$

$$\int_0^\infty x^p e^{-qx^2}\,dx = \Rightarrow \Gamma(n)$$

$t=qx^2 \qquad x=t^{\frac{1}{2}}\cdot q^{-\frac{1}{2}}$

$$=\int_0^\infty \left(t^{\frac{1}{2}}q^{-\frac{1}{2}}\right)^p\cdot e^{-t}\,d\left(t^{\frac{1}{2}}q^{-\frac{1}{2}}\right)=$$

$$=\int_0^\infty t^{\frac{p}{2}}q^{-\frac{p}{2}}e^{-t}\,t^{-\frac{1}{2}}\cdot\frac{1}{2}\cdot q^{-\frac{1}{2}}\,dt = q^{-\frac{p+1}{2}}\int_0^\infty t^{\frac{p-1}{2}}e^{-t}\,dt$$

$$=q^{-\frac{p+1}{2}}\cdot\Gamma\left(\frac{p-1}{2}+1\right)$$

Свойства:

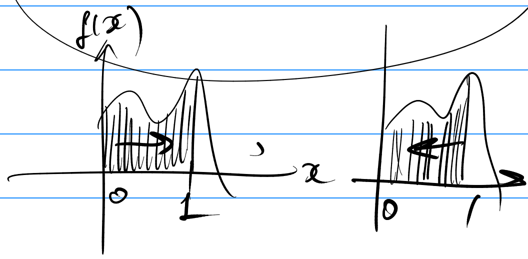
$$\Gamma(x)\cdot\Gamma(1-x)=\frac{\pi}{\sin(\pi x)}$$

См лекцию

$$\Gamma(x)=(x-1)\cdot\Gamma(x-1)$$

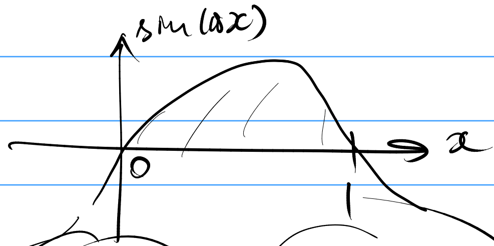
....

$$\int_0^1 f(x)\,dx=\int_0^1 f(1-x)\,dx$$



$$\|^{**}\qquad \int_0^1 \ln\Gamma(x)\,dx = I$$

$$\ln\Gamma(x)+\ln\Gamma(1-x)=\ln\pi-\underbrace{\ln\sin(\pi x)}$$



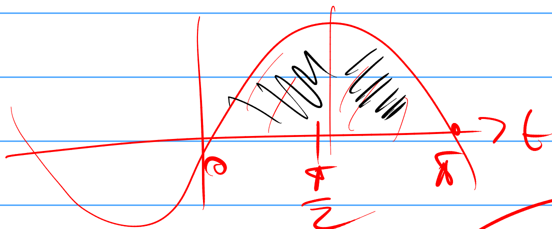
I

$$\int_0^1 \ln\Gamma(x)\,dx+\int_0^1 \ln\Gamma(1-x)\,dx=\underbrace{\int_0^1 \ln\pi\,dx}_{\ln\pi}-\int_0^1 \ln\sin(\pi x)\,dx=$$

$\frac{d\pi x}{\pi}$

$$2I=\ln\pi-\int_0^\pi \ln\sin t\,dt\cdot\frac{1}{\pi}=$$

$$=\ln\pi-\frac{1}{\pi}\left(2\int_0^{\frac{\pi}{2}} \ln\sin t\,dt\right)=$$



$$2I=\ln\pi-\frac{2J}{\pi}$$

J

$$2I=\ln\pi-\frac{1}{\pi}\left(\int_0^{\frac{\pi}{2}} \ln\sin t\,dt+\int_0^{\frac{\pi}{2}} \ln\underbrace{\sin\left(\frac{\pi}{2}-t\right)}_{\cos t}dt\right)=\ln\pi-\frac{2J}{\pi}$$

$$2J=\int_0^{\frac{\pi}{2}} \ln(\sin t\cdot\cos t)\,dt=\int_0^{\pi/2} \ln\left(\frac{\sin 2t}{2}\right)dt=$$

$$= \frac{1}{2}\int_0^{\pi/2} \ln(\sin 2t) - \ln 2 \; d(2t) = \frac{1}{2}\int_0^{\pi/2} \ln \sin u - \ln 2 \, du =$$

$$= \frac{1}{2} J - \frac{1}{2}\ln 2 \cdot \frac{\pi}{2}$$

$$\begin{cases} 2I = \ln \pi - \frac{2J}{\pi} \\ 2J = \frac{1}{2}J - \frac{\pi \ln 2}{4} \end{cases}$$

$$\frac{3}{2}J = -\frac{\pi \ln 2}{4}$$

$$J = -\frac{\pi \ln 2}{6}$$

$$I = \frac{\ln \pi}{2} - \frac{J}{\pi} =$$

$$= \frac{\ln \pi}{2} + \frac{\ln 2}{6}$$

Св-во $B(p,q) =$? сноска ?

$$= \int_0^\infty \frac{y^{p-1}}{(1+y)^{p+q}} \, dy$$

сноска !!

$$B(p,q) = \int_0^1 z^{p-1}(1-z)^{q-1} dz$$

$$z = \frac{y}{1+y}$$

$$P(y=1|x) = \frac{\exp(\beta_0 + \beta_1 \cdot x)}{1 + \exp(\beta_0 + \beta_1 \cdot x)}$$

5 $\int_0^\infty \frac{x^p}{1+x^q} dx = \circledast$ B

$x^q = u \qquad x = u^{\frac{1}{q}}$

$\boxed{\text{идея}}$

$(\text{что})^{?} \cdot (1 - \text{что})^{?}$

$$\underbrace{\frac{x}{1+x}}_{(1-\tau)} + \underbrace{\frac{1}{1+x}}_{\tau} = 1$$

$$(1-\tau)^{\alpha} \cdot \tau^{1-\alpha} = \left(\frac{x}{1+x}\right)^{\alpha} \cdot \left(\frac{1}{1+x}\right)^{1-\alpha} = \frac{x^{\alpha}}{1+x}$$

$$= \int_0^{\infty} \frac{\left(u^{\frac{1}{q}}\right)^{p}}{1+u} \cdot d\left(u^{\frac{1}{q}}\right) =$$

$$= \int_0^{\infty} \frac{u^{p/q} \cdot \frac{1}{q} \cdot u^{\frac{1}{q}-1} du}{1+u}$$

$$\frac{u^{?}}{1+u} du$$

$$= e^{-s}\cdot\int_0^s x\,dx = e^{-s}\cdot\frac{s^2}{2}$$

$$f_n(s) = \frac{e^{-s}\cdot s^{n-1}}{(n-1)!} = \frac{1}{\Gamma(n)}\cdot e^{-s}\cdot s^{n-1}$$

норм. конст для плотн

Задачка 2

$\underbrace{Y_1, \dots, Y_p}_{p \text{ пенсионеров}}$ $\underbrace{Y_{p+1}, \dots, Y_{p+q}}_{q\text{- студентов}}$

$$R = \frac{Y_1 + \dots + Y_p}{Y_1 + \dots + Y_p + Y_{p+1} + \dots + Y_{p+q}}$$

$Y_i \sim$ незав. эксп $\lambda = 1$

$f_R(r)$? =

$R \in [0;1]$

$$R = \frac{W_1}{W_1 + W_2}$$

$$f_1(w_1) = \frac{1}{\Gamma(p)}\cdot e^{-w_1}\cdot w_1^{p-1}$$

$$f_2(w_2) = \frac{1}{\Gamma(q)}\cdot e^{-w_2}\cdot w_2^{q-1}$$

$$\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} \underset{g}{\overset{h}{\rightleftarrows}} \begin{pmatrix} S = W_1 + W_2 \\ R = \frac{W_1}{W_1 + W_2} \end{pmatrix}$$

$f_{R,S}$?

$$f(w_1, w_2)\cdot dw_1 \wedge dw_2 \quad = \frac{1}{\Gamma(p)\Gamma(q)}\cdot e^{-w_1}\cdot e^{-w_2}\cdot w_1^{p-1}\cdot w_2^{q-1}\, dw_1 \wedge dw_2 =$$

$$W_1 = R\cdot S$$

$$W_2 = S - W_1 = S - RS = (1-R)\cdot S$$

$$= \frac{1}{\Gamma(p)\Gamma(q)} \left(e^{-\tau s} \cdot e^{-(1-\tau)s}\right) \cdot (\tau s)^{p-1} \cdot ((1-\tau)s)^{q-1} \cdot$$

$da \wedge da = 0$

$da \wedge db = -db \wedge da$

$$\cdot\, d(\tau s) \wedge d(s - \tau s) =$$

$$= \frac{1}{\Gamma(p)\Gamma(q)} \cdot e^{-s} \cdot s^{p-1+q-1} \cdot \tau^{p-1} \cdot (1-\tau)^{q-1} \cdot d(\tau s) \wedge ds$$

$$= \frac{1}{\Gamma(p)\Gamma(q)} \cdot e^{-s} \cdot s^{p+q-2} \cdot \tau^{p-1} (1-\tau)^{q-1} \cdot (d\tau \cdot s + \tau \cdot ds) \wedge ds =$$

$$= \frac{1}{\Gamma(p)\Gamma(q)} \cdot e^{-s} \cdot s^{p+q-2} \cdot \tau^{p-1} (1-\tau)^{q-1} \cdot s \cdot d\tau \wedge ds$$

$$= \underbrace{\frac{1}{\Gamma(p)\Gamma(q)} \boxed{e^{-s} \cdot s^{p+q-1}} \cdot \boxed{\tau^{p-1} \cdot (1-\tau)^{q-1}}}_{f(\tau, s)} \, d\tau \wedge ds$$

R и S незав.

$f_S(s)$ с точностью до констант

$f_R(\tau)$ с точн. до конст.

$S \sim Y_1 + \dots + Y_{p+q}$

$$= \underbrace{\boxed{\frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} \cdot \tau^{p-1} (1-\tau)^{q-1}}}_{f_R(\tau)} \cdot \underbrace{\boxed{\frac{1}{\Gamma(p+q)} \cdot e^{-s} \cdot s^{p+q-1}}}_{f_S(s)}$$

$$B(p, q) = \int_0^1 \tau^{p-1} (1-\tau)^{q-1} d\tau = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$