Привет :)

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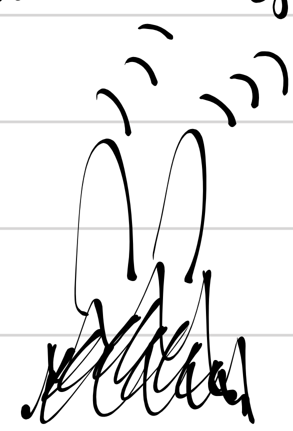
№10. $$\int_0^1 \frac{x^b - x^a}{\ln x}\,dx \overset{?}{=} \int_0^1 \frac{x^b}{\ln x}\,dx - \int_0^1 \frac{x^a}{\ln x}\,dx$$

Метод Александра Македонского:
Разделяй и властвуй!

$$s(a) = \int_0^1 \frac{x^a}{\ln x}\, dx$$

Где я это видел?

Метод пристального взгляда „Увидеть зайца"



Это заяц.

$$(x^a)'_x \stackrel{?}{=} a x^{a-1} \qquad (x^a)'_a \stackrel{?}{=}$$

$$x^a = e^{a\cdot \ln x} \qquad (x^a)'_a = (e^{a \ln x})'_a = \ln x \cdot x^a$$

$$\int (x^a)\, da = \frac{x^a}{\ln x} + C$$

$$\int_1^a x^u\, du = \frac{x^a}{\ln x} - \boxed{\frac{x}{\ln x}}$$

а на это мы забьём!

$$s(a) = \int_0^1 \frac{x^a}{\ln x}\, dx = \int_0^1 \left[\int_1^a x^u\, du + \frac{x}{\ln x}\right] dx$$

Исх-ный вопрос: $s(b) - s(a) \stackrel{?}{=} \varphi(b) - \varphi(a)$

$$\boxed{\varphi(a) = \int_0^1 \int_1^a x^u\, du\, dx}$$

$$q(a) = \int_0^1 \int_1^a x^u \, du \, dx = \int_1^a \boxed{\int_0^1 x^u \, dx} \, du =$$

$$= \int_1^a \frac{x^{u+1}}{u+1} \Big|_{x=0}^{x=1} du =$$

$$= \int_1^a \frac{1}{u+1} - \frac{0}{u+1} \, du = \int_1^a \frac{1}{u+1} \, du = \ln(u+1) \Big|_{u=1}^{u=a}$$

$$q(a) = \ln(a+1) - \ln 2$$

$$Q = q(b) - q(a)$$

д/з

$$s(p) = \boxed{\int_0^{\pi/2} \ln(p^2 - \sin^2 x) \, dx} \quad |p| \geq 1$$

мало

Feynman integration trick

!

$$\int_0^{\pi/2} \ln(1 - \sin^2 x) \, dx$$

исходный вопрос

$\to p / p^2$

Сам

$\frac{\partial}{\partial p}$

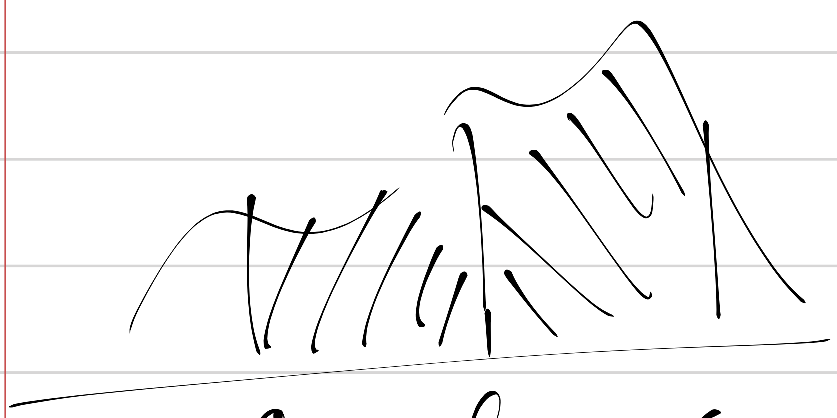
полезно

$$s(p) = \boxed{\int_0^{\pi/2} \ln(p - \sin x) \, dx} + \int_0^{\pi/2} \ln(p + \sin x) \, dx$$

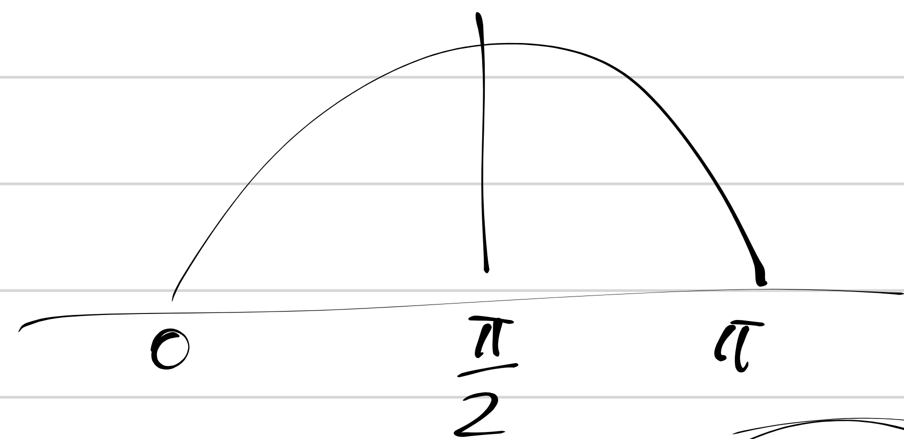
$$\boxed{a(p)} \qquad \boxed{a'(p)} = \int_0^{\pi/2} \frac{1}{p - \sin x} \, dx$$

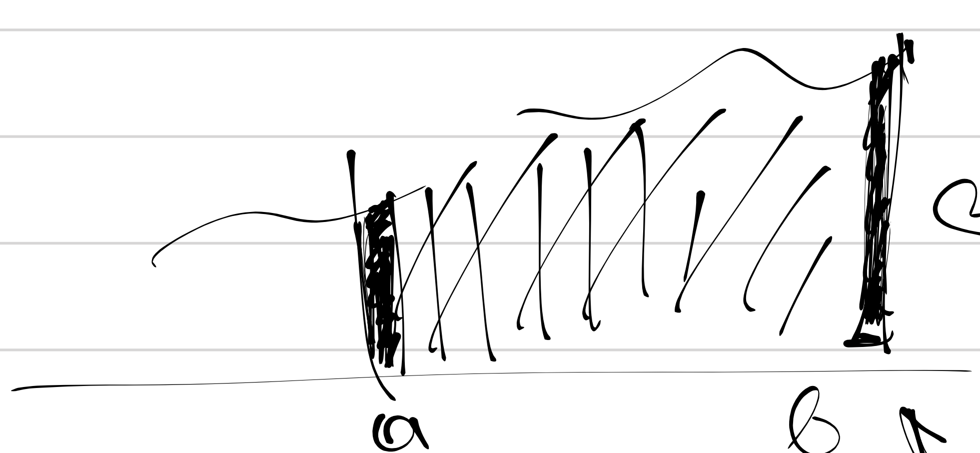
$$a'(p) = \int_0^{\pi/2} \frac{1}{p-\sin x}\,dx$$ → свобода !!

Кушаем пирог по кусочкам



$$\int_a^b f\,dx + \int_b^c f\,dx = \int_a^c f\,dx$$

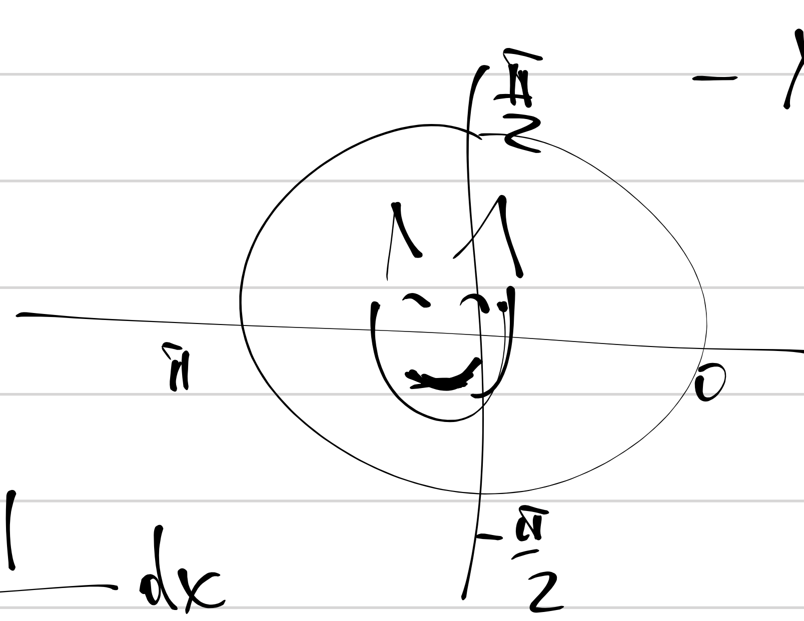




$$\int_a^b f(x)\,dx$$))

$$\int_a^b f(a+b-x)\,dx$$

$$a'(p) = \int_0^{\pi/2} \frac{1}{p-\sin x}\,dx = \int_0^{\pi/2} \frac{1}{p-\sin(0+\frac{\pi}{2}-x)}\,dx =$$



— Хорошо, меняем sin на cos?

→ ↕ „да".

$$= \int_0^{\pi/2} \frac{1}{p-\cos x}\,dx$$

$$a'(p) = \int_0^{\pi/2} \frac{1}{p - \cos x}\,dx = \int_0^{\pi/2} \frac{1}{p - 2\cos^2\frac{x}{2} + 1}\,dx =$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$$

$$\frac{1}{\cos^2 u} = \begin{cases} (\operatorname{tg} u)'_u \\ \operatorname{tg}^2 u + 1 = \frac{\sin^2 + \cos^2}{\cos^2} = \frac{1}{\cos^2 u} \end{cases}$$

$$= \int_0^{\pi/2} \frac{1}{p+1 - 2\cos^2\frac{x}{2}}\,dx = \left(\begin{array}{l} \frac{x}{2} = u \\ dx = 2du \\ x \in [0; \frac{\pi}{2}] \\ u \in [0; \frac{\pi}{4}] \end{array} \right) =$$

$$= \int_0^{\pi/4} \frac{2\,du}{(p+1) - 2\cos^2 u} = \int_0^{\pi/4} \frac{2 \cdot \boxed{\frac{1}{\cos^2 u}\,du}}{(p+1)\cdot \frac{1}{\cos^2 u} - 2}$$

$t = \tan u$

$u \in [0; \frac{\pi}{4}]$

$t \in [0; 1]$

$$= \int_0^1 \frac{2 \cdot dt}{(p+1)(t^2+1) - 2}$$

$\boxed{\dots}$

$$d\tan u = \frac{1}{\cos^2 u}\,du \qquad \frac{1}{\cos^2 u} = \tan^2 u + 1$$

$$\tan u = t$$

$$dt = \frac{1}{\cos^2 u}\,du$$

$$\frac{\sin^2}{\cos^2} + 1 = \frac{\sin^2 + \cos^2}{\cos^2} = \frac{1}{\cos^2} \qquad t^2 + 1 = \frac{1}{\cos^2 u}$$

$$\frac{1}{t^2-7} = \frac{1}{2\sqrt{7}}\left(\frac{1}{t-\sqrt{7}} + \frac{-1}{t+\sqrt{7}}\right)$$

$$\frac{1}{t^2-7} = \frac{1}{2\sqrt{7}}\left(\frac{t+\sqrt{7}-t+\sqrt{7}}{t^2-7}\right)$$

$$\int \frac{1}{t^2+7}\,dt = \int \frac{1/7}{\left(\frac{t}{\sqrt{7}}\right)^2+1}\,dt = \underset{?}{k} \cdot \operatorname{arctg}\frac{t}{\sqrt{7}} + C$$

№6.

$$s(p) = \int_0^{\pi} \frac{\ln(1+p\sin x)}{\sin x}\,dx \longrightarrow s(0)=0$$

$$\int_0^{\pi} \frac{\ln(1+\sin x)}{\sin x}\,dx \quad (p)$$

$$s'(p) = \int_0^{\pi} \frac{1}{\sin x}\cdot\frac{1}{1+p\sin x}\cdot \sin x\,dx$$

$$= \int_0^{\pi} \frac{1}{1+p\sin x}\,dx =$$

аналог только что !!

$$s'(p) = \ldots = \ldots = \ldots = \ldots = h(p)$$

$$s(p) = H(p) + C \quad \text{находим из } s(0)=0$$

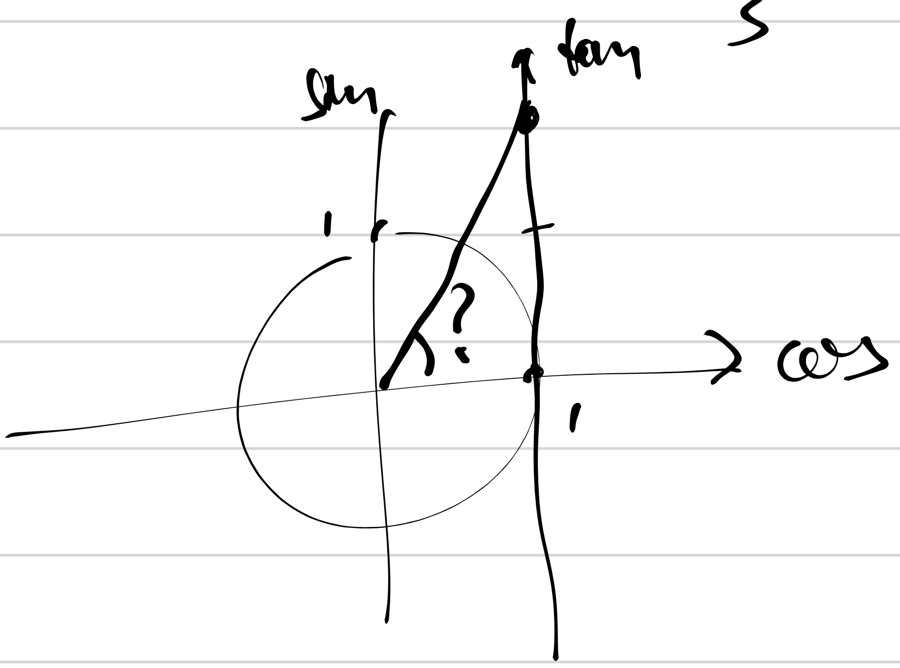
№2

а можно ли?

Коберт разрешил!

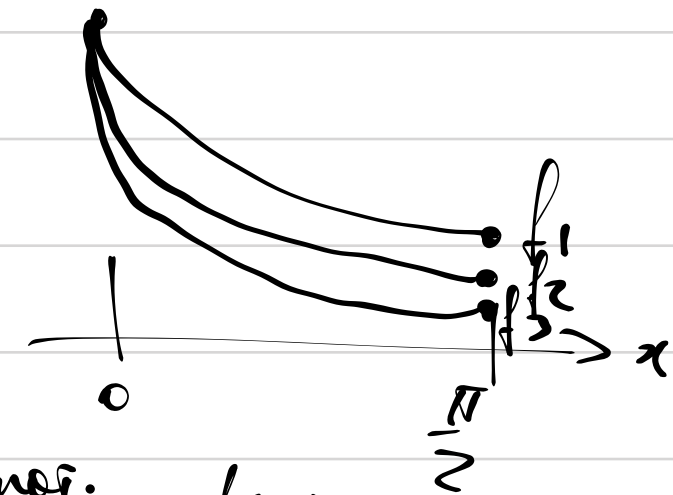
$$\lim_{y\to 0} \int_y^{\sqrt3+y} \frac{dx}{1+x^2+y^2} = \int_0^{\sqrt3} \frac{dx}{1+x^2} = \operatorname{arctg} x \Big|_{x=0}^{x=\sqrt3} =$$

$$= \operatorname{arctan}\sqrt3 - \operatorname{arctan} 0 = \frac{\pi}{3} - 0 = \frac{\pi}{3}$$



№ 3*

$$\lim_{n\to\infty} \int_0^{\pi/2} \underset{\underset{0}{\vee|}}{\overset{\overset{?}{\vee|}}{\boxed{e^{-n\sin x}}}}\, dx$$



$$f_n(x) = e^{-n\sin x} \xrightarrow{\text{пот.}} f(x)$$

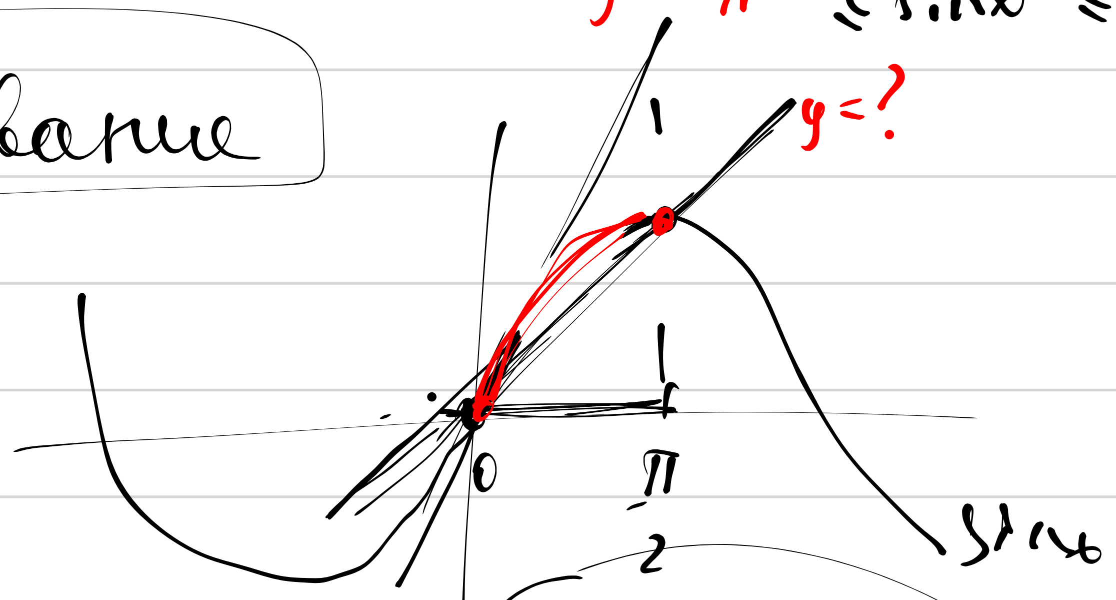
$x \in [0; \frac{\pi}{2}]$

$\sin x \in [0; 1]$

$$\lim_{n\to\infty} f_n(x) = \begin{cases} 1, & \text{если } x = 0 \\ 0, & \text{если } x \in (0; \frac{\pi}{2}] \end{cases} = f(x)$$

$y = x \cdot \frac{2}{\pi} \leq \sin x \leq x$

Минорирование



$$0 \leq \int_0^{\pi/2} e^{-n\sin x}\, dx \leq \int_0^{\pi/2} e^{-n\cdot x\cdot\frac{2}{\pi}}\, dx =$$

$$= \left.\frac{e^{-n\cdot x\frac{2}{\pi}}}{-n\frac{2}{\pi}}\right|_{x=0}^{x=\frac{\pi}{2}} = \frac{e^{-n}}{-n\frac{2}{\pi}} - \frac{1}{-n\frac{2}{\pi}} \xrightarrow[n\to\infty]{} 0$$