Привет! Семинар 3. :)

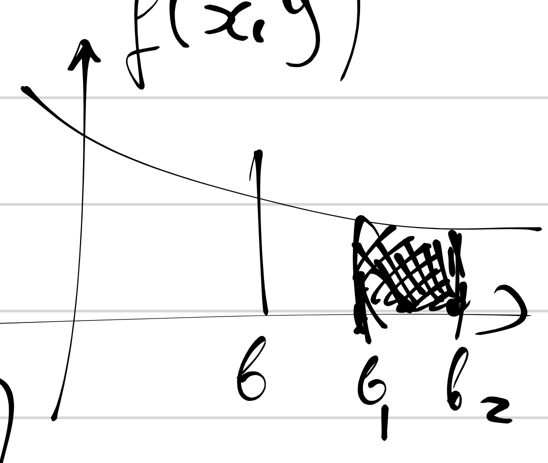
Равном сходимость интеграла !!

Крит Коши.

$$\int_0^\infty f(x,y)\,dx$$ равномерно сх-ся на Y, $y \in Y$

⊕ необх и достаточный.

хвост(b,y) $\displaystyle\int_b^\infty f(x,y)\,dx \xrightarrow[b\to\infty]{\text{на } Y} 0$



1. GG: выбирает b.
2. Cauchy: выбир $b_1, b_2 > b$
   BG: выбирает $y \in Y$

$$\left|\int_{b_1}^{b_2} f(x,y)\,dx\right| \cdot \quad \begin{matrix} GG \\ < \varepsilon \end{matrix} \qquad \begin{matrix} C+BG \\ > \varepsilon \end{matrix}$$

$$\forall \varepsilon \ \exists b \ \forall b_1, b_2 > b \ \forall y \in Y \ \left|\int_{b_1}^{b_2} f\,dx\right| < \varepsilon$$

№1 - №4 $\displaystyle\int_0^{+\infty} \frac{dx}{x^p+1}$

№1 $p \in (1; +\infty)$

№4 $p \in [p_0; +\infty)$, $p_0 > 1$

1. GG: b
2. C+BG: b_1, b_2, p

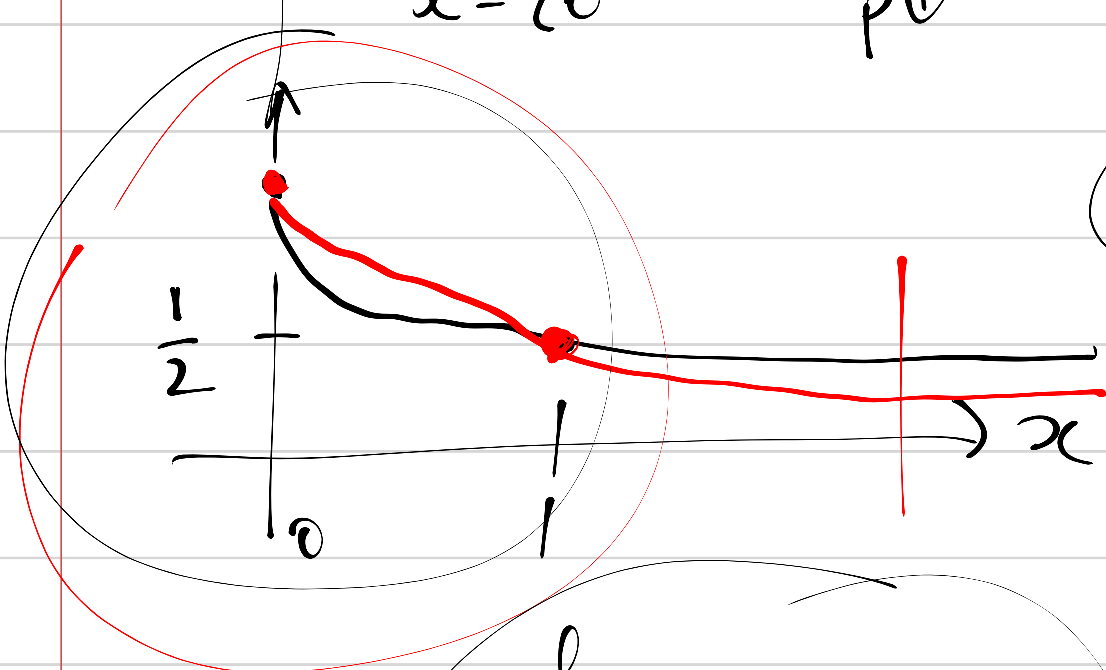
$$\int_{b_1}^{b_2} \frac{1}{x^p+1}\,dx$$

№1 $p \in (1;\infty)$

$$\int_{b_1}^{b_2} \frac{1}{x^p+1}\,dx$$

$x \in (0,1)$ $p\uparrow$ знам $\downarrow$ дробь $\uparrow$

$x = 20$ $p\downarrow$ знам $\downarrow$ дробь $\uparrow$



$$F(b,y) = \int_b^\infty f(x,y)\,dx$$

$$F(b,y) \overset{?}{\underset{b\to\infty}{\rightrightarrows}} 0$$

считаем $x>1$ sup $\displaystyle\int_{b_1}^{b_2} \frac{1}{x^p+1}\,dx =$

$b_2 \to \infty$

$b_1 \to b$

$p \to 1$

$$= \int_b^\infty \frac{1}{x+1}\,dx = \ln|x+1| \Big|_{x=b}^{x=+\infty} > \varepsilon$$

№4 $p \in (p_0; +\infty)$ $p_0 > 1$ $p_0 = 1.01$

$\boxed{x>1}$

$$\int_{b_1}^{b_2} \frac{1}{x^p+1}\,dx$$

Критер. Вейерштрасса. $f_{ob}(x)$

$\int_0^\infty f_{ob}(x,y)\,dx$ сх. равн

если $|f(x,y)| \le f_{ob}(x,y)$ то и

$\int_0^\infty f(x,y)\,dx$ с.р.в.

$$\int_{b_1}^{b_2} \frac{1}{x^p+1}\,dx$$

$$\frac{1}{x^p+1} \le \frac{1}{x^{p_0}+1} < \frac{1}{x^{p_0}}$$

$\boxed{[!\ p_0 = 1.01,\ \text{не меняем хор } p_0]}$

$$\int_{b_1}^{b_2} \frac{1}{x^{p_0}}\,dx = \frac{x^{-p_0+1}}{-p_0+1}\Bigg|_{x=b_1}^{x=b_2} = \frac{b_2^{\overbrace{-p_0+1}^{<0}} - b_1^{\overbrace{-p_0+1}^{<0}}}{-p_0+1}$$

„∃b"

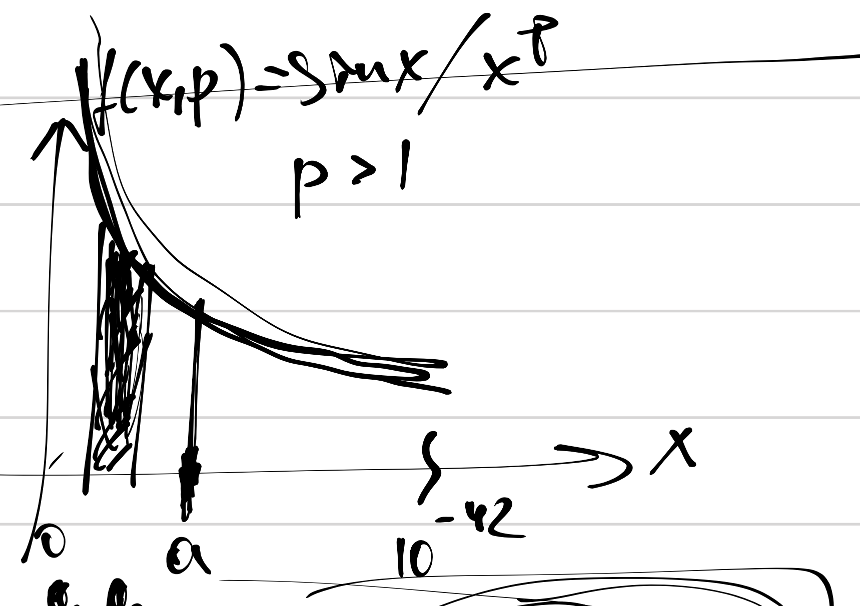
1. БГ: b. $\qquad b_2 > b > 1$
2. Cauchy: b_1, b_2 $\qquad b_2^{-p_0+1} < b^{-p_0+1} < \varepsilon$

   БГ: p

№2 – №5 $\qquad \int_0^1 \frac{\sin x}{x^p}\,dx$

№2: $p \in (0;2)$

№5: $p \in (0, p_0]$ $\qquad p_0 < 2$



! идея! График поможет

$$\frac{\sin x}{x^p} \approx x^{1-p}$$

1. БГ: a
2. Cauchy: a_1 a_2 $\qquad \sup \int_{a_1}^{a_2} \frac{\sin x}{x^p}\,dx \qquad \int_0^a \frac{\sin x}{x^2}\,dx$

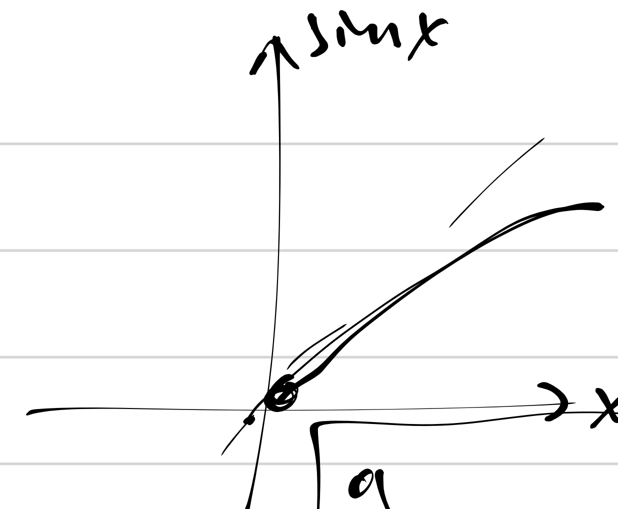
   БГ: p

$a_1 \to 0$

$a_2 \to a$

$p \to 2$

при малых x



$$\boxed{0.911 \cdot x \le \sin x \le x}$$

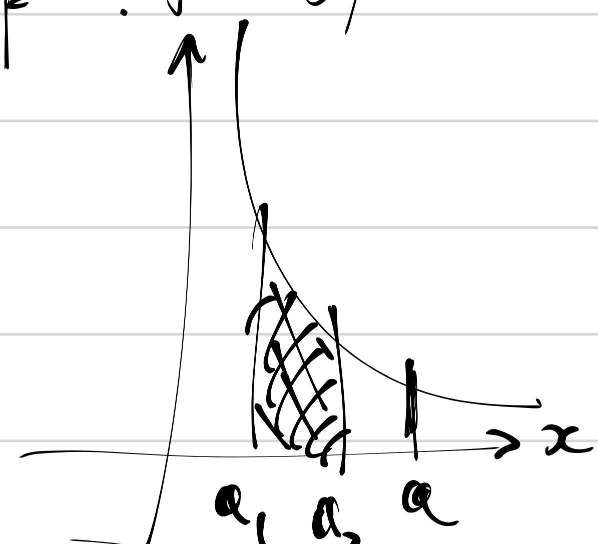
$$\int_0^a \frac{\sin x}{x^2}\,dx \ge \int_0^a \frac{0.911 x}{x^2}\,dx = \boxed{\int_0^a 0.911 \cdot \frac{1}{x}\,dx}$$

$$= 0.911 \ln x \Big|_{x=0}^{x=a} = +\infty > \varepsilon$$

№5

1. 66 : a
2. Cauchy + B6: a_1, a_2, p . $f(x,y)$

$$\int_{a_1}^{a_2} \frac{\sin x}{x^p}\,dx$$



$a_2 \to a$

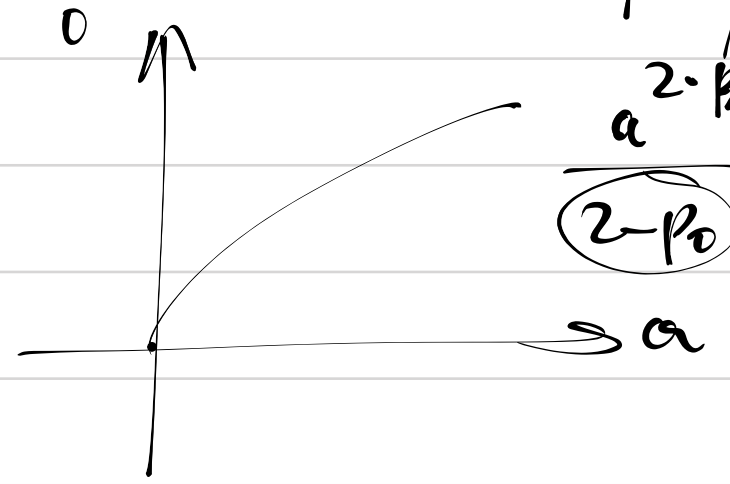
$a_1 \to 0$

$p \to p_0$ $\quad p_0 < 2$

$$\sup \boxed{} = \int_0^a \frac{\sin x}{x^{p_0}}\,dx \le$$

$$\le \int_0^a \frac{x}{x^{p_0}}\,dx = \int_0^a x^{1-p_0}\,dx = \frac{x^{2-p_0}}{2-p_0}\Big|_{x=0}^{x=a} =$$

$$= \frac{a^{2-p_0}}{2-p_0}$$



$$\frac{a^{2-p_0}}{2-p_0}$$

$$\boxed{a = \left((2-p_0)\,\varepsilon\right)^{\frac{1}{2-p_0}}}$$

$$\frac{a^{2-p_0}}{2-p_0} = \varepsilon$$

Дирихле | Абеля

$$\int_0^{+\infty} f(x,y)\cdot g(x,y)\,dx$$

Дирихле $\overset{?}{\longleftrightarrow}$ **Абель**

$A_Д$. $\int_0^{\infty} f\,dx$ огр.

$\exists M$ т.что $\forall b, \forall y$

$$\left|\int_0^b f\,dx\right| < M.$$

$Б_Д$ $g(x,y)$ мон по x для $\forall y$.

$$g(x,y) \underset{x\to\infty}{\rightrightarrows} 0$$

$A_А$. $\int_0^{\infty} f\,dx$ сх-ся равн-но. !!

$Б_А$ $g(x,y)$ мон по x для $\forall y$

$g(x,y)$ огранич.

$\exists M$. что $\forall y, x$

$$|g(x,y)| < M$$

№7

$$H(p) = \int_0^{\infty} \frac{\sin x}{x}\cdot e^{-px}\,dx$$

$p \in [0; +\infty)$

$H'(p)$

⋮

$f = \frac{\sin x}{x}$ $g = e^{-px}$

$$\boxed{\int_0^{\infty} \frac{\sin x}{x}\,dx = \frac{\pi}{2}}$$

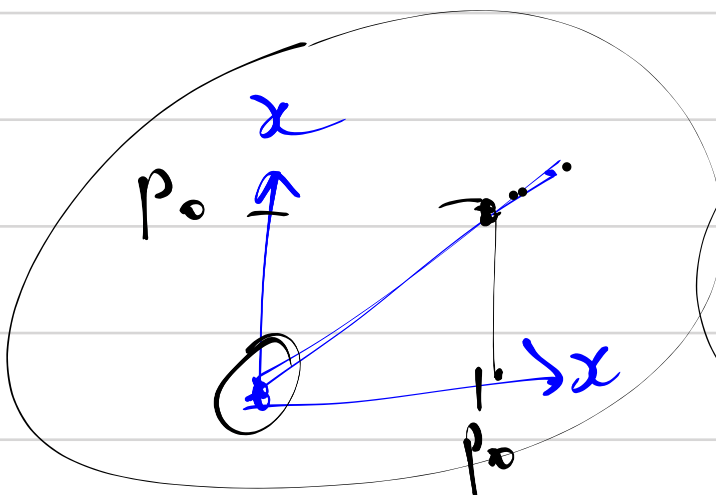
нет p!

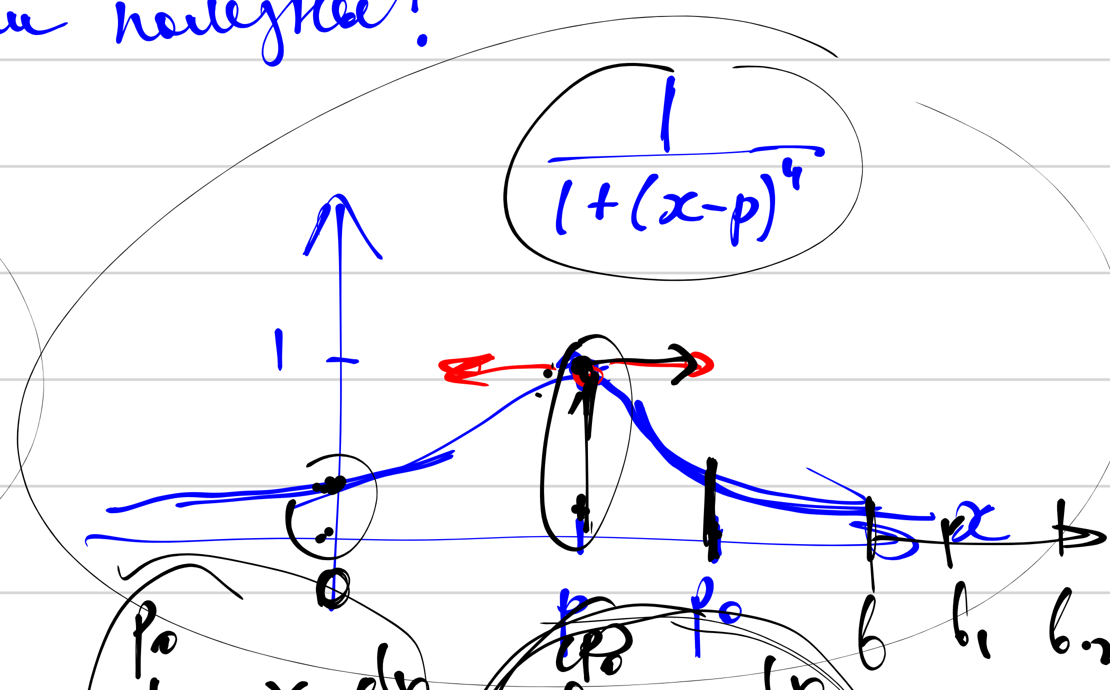
!!

№10* $\displaystyle\int_0^{\infty} \frac{x\,dx}{1+(x-p)^4}$ $\qquad \boxed{p \in (-\infty; p_0]}$

$p_0 > 0.$

! иногда графики полезны!





$$\int_0^{\infty} \frac{x\,dx}{1+(x-p)^4} = \int_0^{p_0} \frac{x\,dx}{1+(x-p)^4} + \int_{p_0}^{\infty} \frac{x\,dx}{1+(x-p)^4}.$$

$$\int_{p_0}^{\infty} \frac{x}{1+(x-p)^4}\,dx \le$$

сам. худший $p \to p_0$

$$\int_{p_0}^{\infty} \frac{x}{1+(x-p_0)^4}\,dx \le \int_{p_0}^{\infty} \frac{x}{(x-p_0)^4}\,dx =$$

$dx = du$

$x - p_0 = u$

$$= \int_0^{\infty} \frac{u+p_0}{u^4}\,du =$$

Вейерштр.

$$= \int_0^{\infty} \frac{1}{u^3}\,du + \int_0^{\infty} \frac{p_0}{u^4}\,du$$