Привет!!

Красота

№7 $\int_0^\infty \frac{\sin(px)}{x}\,dx \qquad p>0$

$px=t$

$x\in[0,\infty) \quad t\in[0,+\infty)$

$x=\frac{t}{p} \qquad dx=\frac{dt}{p}$

$$\int_0^\infty \frac{\sin px}{x}\,dx = \int_0^\infty \frac{\sin t}{t/p}\cdot\frac{dt}{p} = \int_0^\infty \frac{\sin t}{t}\,dt$$

две идеи

Трюк Фейнмана:

→ вставить внутрь параметр y, получить $I(y)$, так, чтобы $I'(y)$ был попроще!

$$I(y)=\int_0^\infty \frac{\sin t}{t}\cdot \boxed{\exp(-ty)}\,dt$$

$$I'(y)=\int_0^\infty \frac{\sin t}{\cancel{t}}\cdot(-\cancel{t})\cdot\exp(-ty)\,dt = \int_0^\infty \sin t\cdot(-\exp(-ty))\cdot dt$$

→ два раза по частям.

я везде вижу экспоненту

$$\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$$

Почему $\frac{1}{t}$ — это экспонента?

$\frac{1}{t}=\exp^{\ln\frac{1}{t}}$

$$\frac{1}{t}=\int_0^\infty \exp(-ty)\,dy = \frac{\exp(-ty)}{-t}\Bigg|_{y=0}^{y=\infty} = 0-\left(-\frac{1}{t}\right) = \frac{1}{t}$$

$$I(t)=\int_0^\infty \sin t\int_0^\infty \exp(-ty)\,dy\,dt$$

$\boxed{\text{Ясно cos ч}}$

$$\begin{cases} \exp(it) = \cos t + i\sin t \\ \exp(-it) = \cos t - i\sin t \end{cases}$$

$$\sin t = \frac{\exp(it) - \exp(-it)}{2i}$$

$$\int_0^\infty \frac{\sin t}{t}\,dt = \int_0^\infty \frac{\exp(it) - \exp(-it)}{2i} \cdot \int_0^\infty \exp(-ty)\,dy \cdot dt =$$

$$= \;\ldots\ldots$$

№8 $\displaystyle\int_0^\infty \frac{\sin(x^2)}{x}\,dx$

$$\int_0^\infty \frac{\sin(x^{2022})}{x}\,dx = \int_0^\infty \frac{\sin(x^{2022})}{x^{2022}} \cdot x^{2021}\,dx =$$

$$= \frac{1}{2022} \cdot \int_0^\infty \frac{\sin u}{u}\,du = \frac{\pi/2}{2022}$$

$u = x^{2022}$ $\quad x \in [0;\infty)$ $\quad u \in [0;\infty)$

$du = 2022 \cdot x^{2021}\,dx$

$$\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx = \left(\text{по частям к } \int_0^\infty \frac{\sin x}{x}\,dx\right)$$

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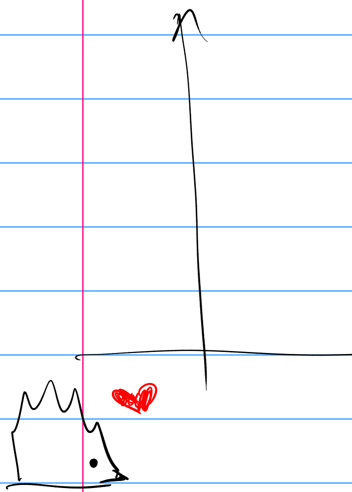
Меня послали на Землю, чтобы я рассказал:



$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ — вект. скоростей !

(1) $R \cdot \underline{X} \sim \underline{X}$, если R — вращение/отражения

(2) комп-та вектора X незав-мы.

$f(x_1, x_2)$?

(1) $f(x_1, x_2)$ зависит только от длины $|x|$ или от $(x_1^2 + x_2^2)$

$$f(x_1, x_2) = h(x_1^2 + x_2^2)$$

(2) $\rightarrow f_1(x_1) \cdot f_2(x_2) = f_1(x_1) \cdot f_1(x_2) = h_1(x_1^2) h_1(x_2^2)$

$$h(x_1^2 + x_2^2) = h_1(x_1^2) \cdot h_1(x_2^2)$$

$x_1^2 = a \quad x_2^2 = b$

$$h(a+b) = h_1(a) \cdot h_1(b)$$

уже здесь пахнет экспонентой

$$h(a+0) = h_1(a) \cdot h_1(0)$$

$$h'(a+b) = h_1(a) \cdot h_1'(b) \qquad b=0.$$

$$h'(a) = h_1(a) \cdot h_1'(0)$$

$$\boxed{k h(a) = h'(a)}$$

$$h'(a) = k \cdot h(a) \qquad \frac{dh}{da} = k \cdot h$$

$$\frac{dh}{h} = k \cdot da$$

$$\ln|h| = ka + C$$

$$h = \exp(ka) \cdot c$$

$$f(x_1, x_2) = \exp(k(x_1^2 + x_2^2)) \cdot c$$

$$\boxed{\begin{matrix} 1 \\ 2 \end{matrix}} \iff \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}; \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\right)$$

незав A и B $\quad P(A \cap B) = P(A) \cdot P(B)$

$$P(X_1 \le x_1, X_2 \le x_2) = P(X_1 \le x_1) \cdot P(X_2 \le x_2)$$

$$F(x_1, x_2) = F_1(x_1) \cdot F_2(x_2)$$

(по x_1 и x_2)

$$\downarrow$$

$$f(x_1, x_2) = f_1(x_1) \cdot f_2(x_2)$$

№11

$$f(x_1, x_2) = \exp(k(x_1^2 + x_2^2)) \cdot c$$

$$Var(X_1) = D(X_1) = \sigma^2 \quad \Rightarrow k, c\,?$$

$\boxed{k<0}$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp(k(x_1^2 + x_2^2)) \cdot c \cdot dx_1 dx_2 = 1$$

$$E(X_1^2) = \sigma^2$$

$$\boxed{\int_{-\infty}^{\infty} \sqrt{c} \cdot x_1^2 \cdot \exp(kx_1^2)\, dx_1 = \sigma^2}$$

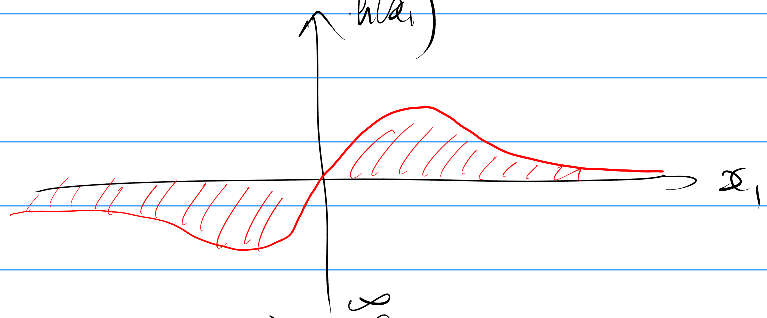
$$f(x_1,x_2)=c\cdot\exp\left(k(x_1^2+x_2^2)\right) \qquad \boxed{k<0}$$

$$\to f_1(x_1)\cdot f_2(x_2)$$

$$f_1(x_1)=\sqrt{c}\cdot\exp(kx_1^2) \qquad f_2(x_2)=\sqrt{c}\cdot\exp(kx_2^2)$$

$$E(X_1)=\int_{-\infty}^{\infty} x_1\cdot f_1(x_1)\,dx_1=\int_{-\infty}^{\infty}\underbrace{x_1\cdot\exp(kx_1^2)\cdot\sqrt{c}}_{h(x_1)}\,dx_1=0$$

по симметрии



$$\boxed{\sigma^2}=Var(X_1)=E(X_1^2)=\int_{-\infty}^{\infty} x_1^2\cdot\sqrt{c}\cdot\exp(kx_1^2)\,dx_1=$$

$$=\int_{-\infty}^{\infty}\underbrace{\boxed{x_1}}_{\text{произв} \to 1}\cdot\underbrace{\boxed{\sqrt{c}\cdot\exp(kx_1^2)\cdot x_1}}_{\text{интегрир} \to \sqrt{c}\exp(kx_1^2)/2k}\cdot dx_1=$$

$\boxed{k<0}$

$$=\underbrace{x_1\cdot\sqrt{c}\cdot\exp(kx_1^2)/2k\Big|_{x_1=-\infty}^{x_1=+\infty}}_{=0}-\int_{-\infty}^{\infty}\sqrt{c}\cdot\exp(kx_1^2)/2k\,dx_1$$

$$-\frac{1}{2k}\cdot\underbrace{\boxed{\int_{-\infty}^{\infty}\sqrt{c}\cdot\exp(k\cdot x_1^2)\,dx_1}}_{=1}=$$

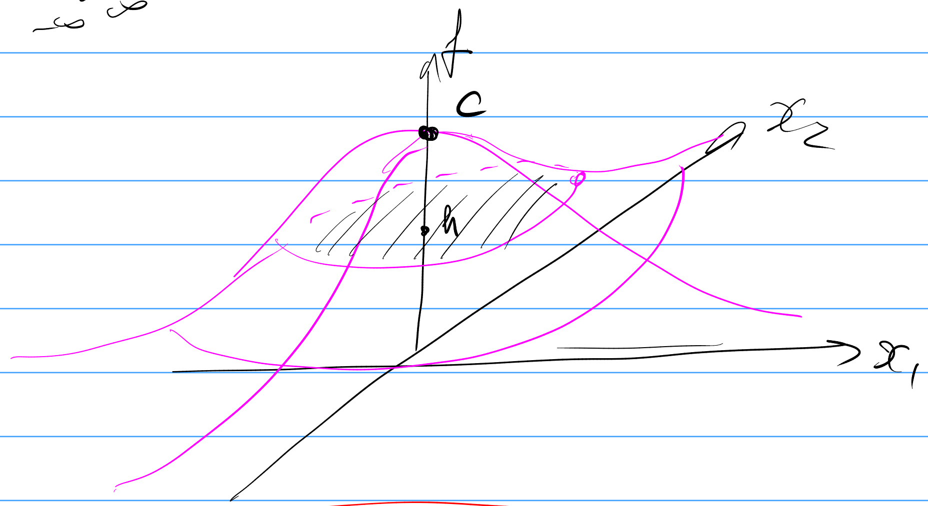
$$\sigma^2=-\frac{1}{2k}$$

$$k=-\frac{1}{2\sigma^2}$$

$$f(x_1,x_2)=c\cdot\exp\left(-\frac{1}{2\sigma^2}(x_1^2+x_2^2)\right)$$

$\boxed{\text{11 класс!}}$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c\cdot \exp\left(-\frac{1}{2\sigma^2}(x_1^2+x_2^2)\right) dx_1\, dx_2 = 1 \qquad c=?$$



$$\boxed{\int_0^c \pi R^2(h)\cdot dh = 1}$$

$$-\frac{1}{2\sigma^2}(x_1^2+x_2^2) = \ln h - \ln c$$

$$c\exp\left(-\frac{1}{2\sigma^2}(x_1^2+x_2^2)\right) = h$$

$$\exp\left(-\frac{1}{2\sigma^2}(x_1^2+x_2^2)\right) = \frac{h}{c}$$

$$x_1^2+x_2^2 = -2\sigma^2(\ln h - \ln c) = R^2(h)$$

$$\int_0^c -2\sigma^2(\ln h - \ln c)\pi\, dh = 1$$

$$\int_0^c \ln\frac{h}{c}\, dh = -\frac{1}{2\sigma^2\pi}$$

$$\frac{h}{c} = t$$

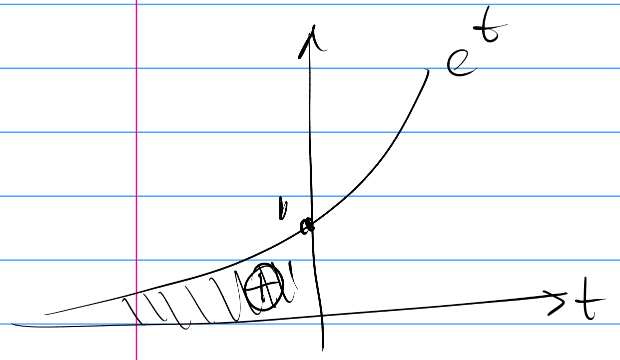
$$t\in[0;1]$$

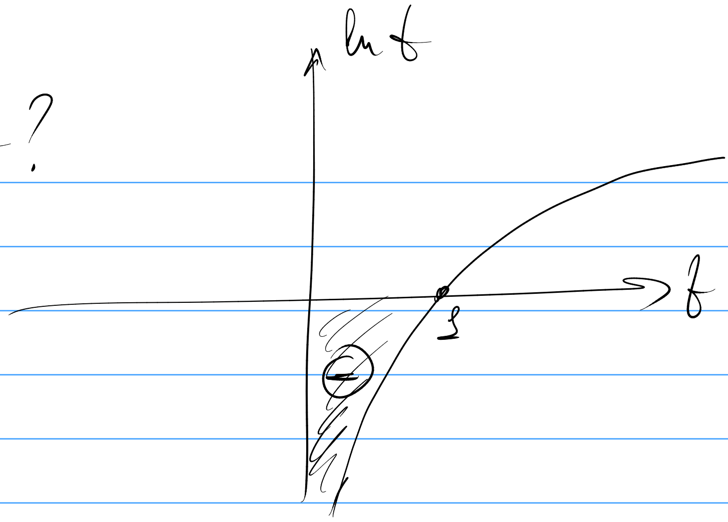
$$dh = c\cdot dt$$

$$\int_0^1 \ln t\cdot c\cdot dt = -\frac{1}{2\sigma^2\pi}$$

$$\int_0^1 \ln t\cdot dt = -\frac{1}{2c\sigma^2\pi}$$

$$-\frac{1}{\pi 2c\sigma^2} = \int_0^1 \ln t \, dt = ?$$

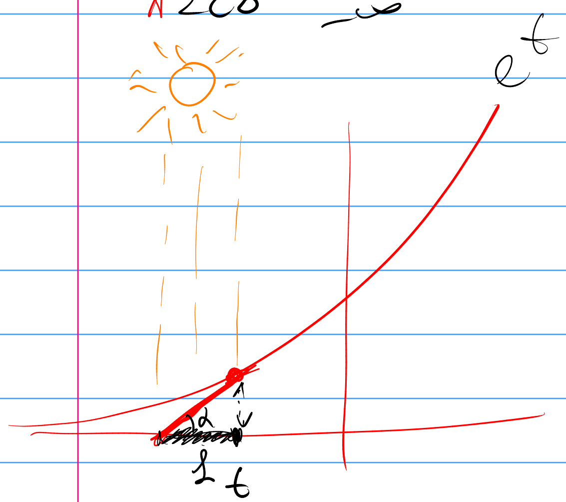




$$c = \frac{1}{2\sigma^2}$$

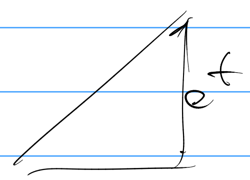
$$\frac{1}{\pi 2c\sigma^2} = \int_{-\infty}^{0} e^t \, dt = 1$$

Принцип Маликона
Маликона Мкоуаканяна.



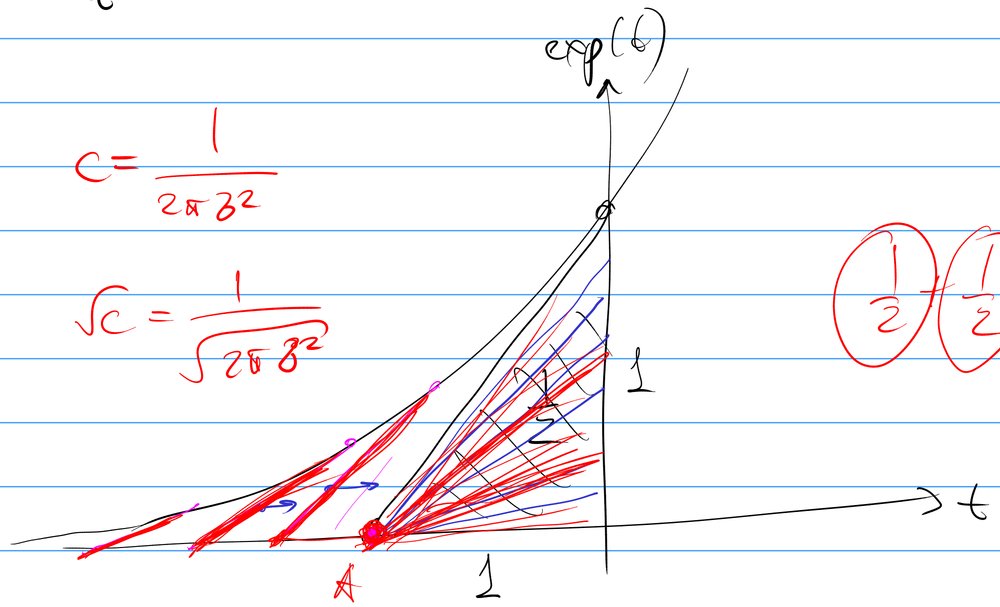
$$tg\,\alpha = (e^t)' = e^t$$

тень косат. $= 1$



$$c = \frac{1}{2\pi\sigma^2}$$

$$\sqrt{c} = \frac{1}{\sqrt{2\pi\sigma^2}}$$



$$\frac{1}{2} + \frac{1}{2} = 1$$

№9

$$\int_0^\infty \frac{\cos(\alpha x)}{\beta^2+x^2}\,dx$$

1. сведем число пар-в.

$$\ldots \to \int_0^\infty \frac{\cos(\alpha x)}{1+x^2}\,dx = F'(\alpha)$$

$$F''(\alpha) = \int_0^\infty \frac{-\sin(\alpha x)\cdot x}{(1+x^2)}\,dx$$

$$F(\alpha) = \int_0^\infty \frac{\sin(\alpha x)}{x(1+x^2)}\,dx$$

$$F - F'' = \int_0^\infty \frac{\sin \alpha x}{(1+x^2)x} + \frac{\sin(\alpha x)\cdot x}{1+x^2}\,dx =$$

$$= \int_0^\infty \frac{\sin(\alpha x)}{(1+x^2)}\cdot \underbrace{\left(\frac{1}{x}+x\right)}_{\frac{1+x^2}{x}}\,dx = \boxed{\int_0^\infty \frac{\sin(\alpha x)}{x}\,dx = \frac{\pi}{2}}$$

$$F - F'' = \frac{\pi}{2}$$

$F?$

$$F' = g(F) \qquad F'' = g'\cdot F' = g'\cdot g$$

$$g - g'F' = \frac{\pi}{2}$$

$$g - g'g = \frac{\pi}{2}$$

угадали

$$\boxed{g(t) = \frac{\pi}{2}}$$

$$g(t) = \frac{\pi}{2} + h(t)$$

$$\frac{\pi}{2} + h(t) - h'\cdot\left(\frac{\pi}{2} + h(t)\right) = \frac{\pi}{2}$$

$$h = \frac{dh}{dt}\cdot\left(\frac{\pi}{2}+h\right)$$

$$\int dt = \int \frac{dh}{h}\left(\frac{\pi}{2}+h\right)$$

...