Variant A

1. Let S be the $n \times n$ «shipbuilding timber» matrix, i.e. the square matrix with all elements equal to 1 and I be the $n \times n$ identity matrix. Let $A = aI + bS$ where a and b are scalar parameters.

   (a) (2%) Express the matrix A^2 as a linear combination of matrices I and S

$$S^2 = (aI + bS)^2 = a^2I^2 + b^2S^2 + abIS + abSI = a^2I + b^2nS + 2abS = a^2I + (b^2n + 2ab)S$$

   (b) (6%) Find the inverse of A if it is known that it exists and can be represented as a linear combination of I and S

Допустим обратная к A матрица имеет вид $A^{-1} = cI + dS$.

$$(aI + bS)(cI + dS) = acI + (ad + dc + bdn)S$$

Чтобы этот результа равнялся I нам нужно чтобы:

$$\begin{cases} ac = 1 \\ ad + db + bdn = 0 \end{cases}$$

Выражаем c и d через a и b:

$$\begin{cases} c = 1/a \\ d = -b/(a^2 + b) \end{cases}$$

Итого,

$$A^{-1} = a^{-1}I - \frac{b}{a^2 + b}S$$

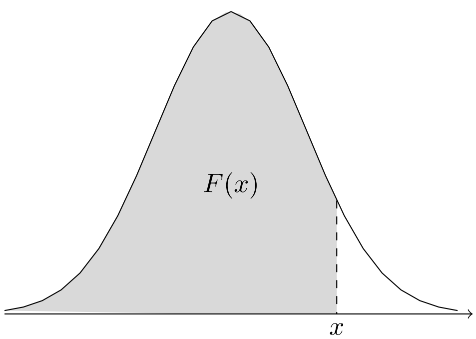
   (c) (2%) Using your result in previous part or otherwise find the inverse of

$$\begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

Здесь $A = -2I + S$. Значит

$$A^{-1} = -0.5I - \frac{1}{4+1}S = \begin{pmatrix} -0.7 & -0.2 & -0.2 & -0.2 \\ -0.2 & -0.7 & -0.2 & -0.2 \\ -0.2 & -0.2 & -0.7 & -0.2 \\ -0.2 & -0.2 & -0.2 & -0.7 \end{pmatrix}$$

Удачи!



x	$F(x)$	x	$F(x)$	x	$F(x)$	x	$F(x)$
0.050	0.520	0.750	0.773	1.450	0.926	2.150	0.984
0.100	0.540	0.800	0.788	1.500	0.933	2.200	0.986
0.150	0.560	0.850	0.802	1.550	0.939	2.250	0.988
0.200	0.579	0.900	0.816	1.600	0.945	2.300	0.989
0.250	0.599	0.950	0.829	1.650	0.951	2.350	0.991
0.300	0.618	1.000	0.841	1.700	0.955	2.400	0.992
0.350	0.637	1.050	0.853	1.750	0.960	2.450	0.993
0.400	0.655	1.100	0.864	1.800	0.964	2.500	0.994
0.450	0.674	1.150	0.875	1.850	0.968	2.550	0.995
0.500	0.691	1.200	0.885	1.900	0.971	2.600	0.995
0.550	0.709	1.250	0.894	1.950	0.974	2.650	0.996
0.600	0.726	1.300	0.903	2.000	0.977	2.700	0.997
0.650	0.742	1.350	0.911	2.050	0.980	2.750	0.997
0.700	0.758	1.400	0.919	2.100	0.982	2.800	0.997

Рис. 1: Таблица значений функции распределения для стандартной нормальной величины